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Biaxially Loaded Slender Reinforced Conrete Columns

Colonnes en béton armé élancées et chargées d'une façon biaxiale

Schlanke Stahlbetonstützen unter doppelter Biegung

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SUMMARY
The paper describes an analytical method for the calculation of the "exact" ultimate load of a reinforced concrete column under conditions of biaxial bending. Nonlinear material properties, both for concrete and steel, are assumed. The analysis aims at monitoring the load versus deflection response of the column up to collapse. At each stage of loading the equilibrium deflected shape is calculated, using the second order Newton Raphson method. The moment thrust curvature relations required in the process are computed using Gauss quadrature, resulting in very rapid calculations. The question of accuracy of the method is discussed. The method is shown to give good agreement with test results from two previously reported papers.

RÉSUMÉ
Cette publication décrit une méthode analytique de calcul "exact" de la charge ultime d'une colonne en béton armé soumise à la flexion biaxiale. Propriétés non-linéaires du béton et de l'acier sont pris en considération. L'analyse vise à simuler le comportement de la colonne jusqu'à la rupture. Après chaque étage du chargement, la flexion de la colonne est calculée par la méthode Newton-Raphson du second ordre. Les relations entre flexion composée et courbure ont été programmées par la méthode des quadratures de Gauss, ce qui permet un calcul très rapide. La précision de la méthode est discutée. Les résultats numériques correspondent aux données expérimentales de deux recherches précédentes.

ZUSAMMENFASSUNG
Der Beitrag beschreibt eine analytische Methode zur Berechnung von Stahlbetonstützen unter doppelter Biegung, wobei nichtlineares Verhalten von Beton und Stahl berücksichtigt werden. Mithilfe der Newton-Raphson-Methode zweiter Ordnung wird die Gleichgewichtslage unter jedem Belastungsschnitt berechnet. Die Methode zeigt ausgezeichnete Übereinstimmung mit Versuchsergebnisse.

INTRODUCTION

The problem considered in this paper is that of a reinforced concrete beam-column with biaxial end restraints which may be dissimilar at the two ends of the column. It is assumed that the cross section is rectangular. Longitudinal reinforcement is represented as notionally concentrated point areas. Complete flexibility in the choice of material stress-strain characteristics is allowed. Warping and torsional effects are, however, not considered. Proportional or non-proportional loading paths can be specified.

A solution to the problem can be based on the notion that the ultimate load of a column corresponds to that level of loading for which the column stiffness in terms of its capacity to sustain additional loads is reduced to zero. The column stiffness is not calculated explicitly. Instead, the column deflected shape in equilibrium with the applied loading is calculated for increasing levels of applied loading, starting with a sufficiently low level. As expected, the rate of change of deflections with respect to the changes in loading increases with increasing levels of the applied loading. Eventually, for a load level higher than the ultimate load of the column, no equilibrium deflected shape is obtainable. Thus, the calculations are aimed at determining the ultimate load as the highest load for which an equilibrium deflected shape can be obtained.

When considering material and geometric nonlinearities, the determination of the deflected shape of the column in equilibrium with the external loading requires the solution of two separate subproblems.

First, an algorithm has to be selected to relate the curvatures in the column to the internal stress resultants. This forms the moment thrust curvature relationship for the cross section and its material properties. The inclusion of arbitrary nonlinear material stress strain characteristics renders the moment thrust curvature relationships unobtainable in the form of simple formulae. Inevitably, a numerical procedure has to be adopted. One commonly used algorithm, adopted by several investigators [1,5,14], with minor variations, is based on the subdivision of the cross section into a grid of sufficiently small areas, followed by the calculation of strains and stresses at the centroids of these areas, and finally the direct summation of forces and moments so obtained. It has been found that such an algorithm employs about two-thirds to three-quarters of the total time required for a complete analysis of a given column. It is clear that any technique aimed at reducing the computation time required for the moment thrust curvature relationship will lead to savings in the overall computational effort. Description of one such rapid algorithm forms a main part of this paper.

The second subproblem relates to the determination of the deflected shape satisfying the condition that the internal stress resultants at any point have to be in equilibrium with the external loading. Here, two types of approach can be adopted. The first is an integration type approach, and resembles the well known Newmark's method [8] for elastic beam columns. The second is based on finite differences, and is aimed at arriving at the values of the deflections at discrete points along the length of the column by some sort of iteration. Other options available for example, to use a first order iteration with Aitken's or some other type of accelerator for convergence, or to use a second order Newton Raphson type iteration. The author has found that, providing that at any stage of the computation process use is made of the results obtained thus far, the Newton Raphson iteration is very rapid and it is this method which is described later in the paper.

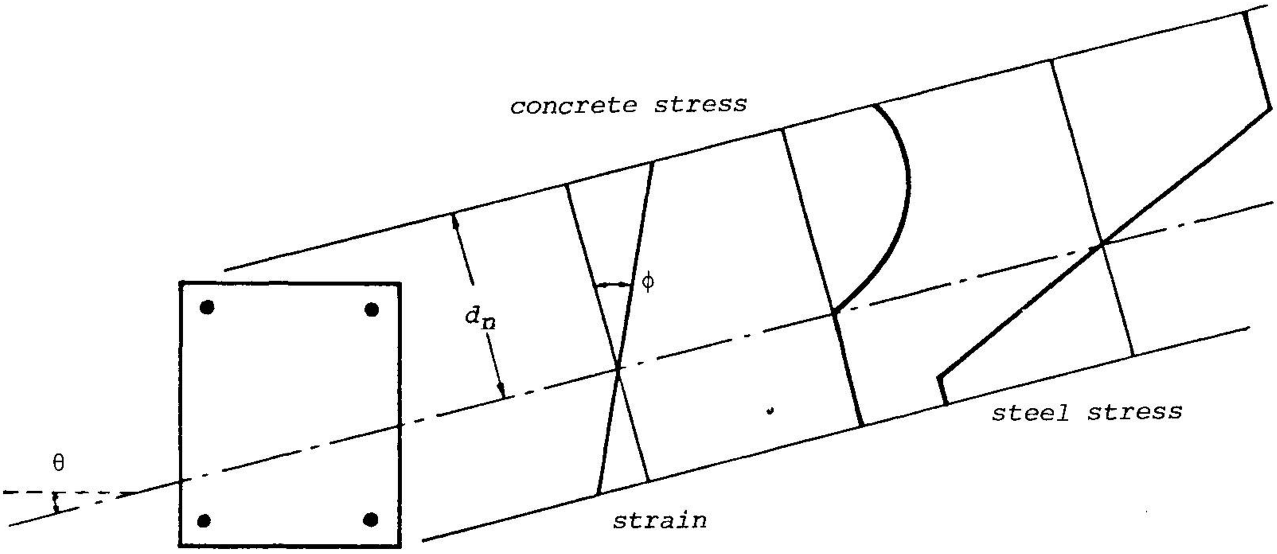


FIGURE 1 - Strain and Stress Distribution under Biaxial Curvatures

MOMENT THRUST CURVATURE RELATIONS

Where torsion and warping of the cross section are ignored, as in the present study, the biaxial moment thrust curvature relations involve six quantities: axial thrust P , biaxial moments M_x and M_y, biaxial curvatures ϕ_x and ϕ_y, and a sixth parameter required to establish the strain distribution, and hence the stress distribution, at a cross section uniquely. A convenient choice for the sixth variable becomes the distance d_n of the neutral axis from a predetermined point in the cross section, for example from the most highly stressed corner (Figure 1). Assuming that plane sections before bending remain plane after bending, the two biaxial curvatures can be compounded to yield the total curvature ϕ and the direction of the neutral axis θ:

$$\phi = (\phi_x^2 + \phi_y^2)^{\frac{1}{2}} \tag{1}$$

$$\theta = tan^{-1}(\phi_y/\phi_x) \tag{2}$$

With the strain distribution in the cross section thus established, the strain at the centroid of a small area dA can be obtained and that, in turn, can be used to calculate the stress σ acting on the elemental area from the stress strain characteristic, which may be nonlinear, for the material concerned. The three stress resultants are then given by:

$$P = \int_A \sigma dA \tag{3}$$

$$M_x = \int_A \sigma x dA \tag{4}$$

$$M_y = \int_A \sigma y dA \tag{5}$$

These equations, together, represent the moment thrust curvature relations. In general, by assigning known values to any three of the six unknowns the other three can be evaluated uniquely. In the present analysis, at each stage in the determination of the deflected shape of the column by the method to be outlined later, the curvatures ϕ_x and ϕ_y , and the applied thrust P are regarded as the independent variables. The problem is thus reduced to varying d_n in such a manner so as to yield the correct thrust P. The biaxial moments M_x and M_y are then available as by-products of the above process. The author has found that Newton's iteration technique is very efficient in this regard.

Whatever be the technique adopted for carrying out the iteration between d_n and P, the most important step involved remains the evaluation of the three integrals in Equations (3)-(5). Referring to the typical reinforced concrete column cross section shown in Figure 1, it is plain that a closed form solution is not easy, if not impossible, especially when arbitrary nonlinear material stress-strain characteristics are involved.

USE OF GAUSS QUADRATURE

According to the Gauss quadrature formulae, a definite integral between the limits -1 and +1 can be replaced by a weighted sum of the values of the integrand at certain specified points:

$$\int_{-1}^{1} f(\xi)d\xi = \sum_{i=1}^{m} H_i f(\xi_i) \tag{6}$$

where H_i are the weighting coefficients and $\xi = \xi_i$ are the specified Gauss points. The integration is exact if $f(\xi)$ is a polynomial of degree up to $(2m-1)$. Values of ξ_i and H_i are available [6,16] in tabular form for values from 2 to 16.

A double integral can be replaced by a double summation as follows:

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)d\xi d\eta = \sum_{i=1}^{m}\sum_{j=i}^{m} H_i H_j f(\xi_i,\eta_j) \tag{7}$$

The square area implied by the limits -1 and +1 in the two directions can be successfully mapped on to a quadrilateral area (Figure 2) by the following transformations using the so-called natural co-ordinates (ξ,η):

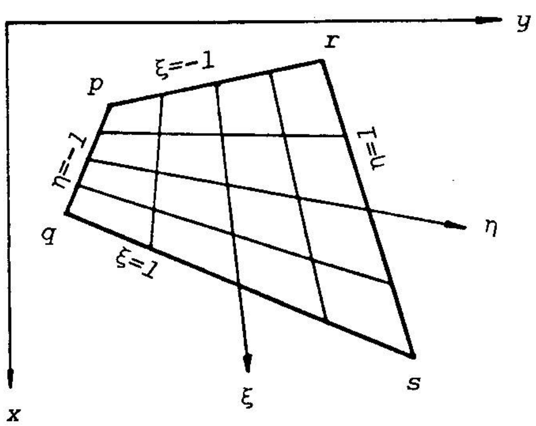


FIGURE 2 - Natural Co-ordinate System for a Quadrilateral

$$x = \frac{1}{4}\left[(1-\xi)(1-\eta)x_p + (1+\xi)(1-\eta)x_q + (1-\xi)(1+\eta)x_r + (1+\xi)(1+\eta)x_s\right] \tag{8}$$

$$y = \frac{1}{4}\left[(1-\xi)(1-\eta)y_p + (1+\xi)(1-\eta)y_q + (1-\xi)(1+\eta)y_r + (1+\xi)(1+\eta)y_s\right] \tag{9}$$

where (x_p, y_p), (x_q, y_q), (x_r, y_r), and (x_s, y_s) are the co-ordinates of the points p, q, r, and s, respectively.

The elemental area $dxdy$ in the cartesian co-ordinates is related to the area $d\xi d\eta$ in the natural co-ordinate system through the relation:

$$dxdy = |J| \; d\xi d\eta \tag{10}$$

where $|J|$ represents the determinant of the Jacobian matrix:

$$[J] = \frac{1}{4}\begin{bmatrix} -(1-\eta) & (1-\eta) & -(1+\eta) & (1+\eta) \\ -(1-\xi) & -(1+\xi) & (1-\xi) & (1+\xi) \end{bmatrix} \begin{bmatrix} x_p & y_p \\ x_q & y_q \\ x_r & y_r \\ x_s & y_s \end{bmatrix} \tag{11}$$

MOMENT THRUST CURVATURE RELATIONS USING GAUSS QUADRATURE

There are several alternative ways by which the Gauss quadrature formulae can be applied to obtain the integrals in Equations (3)-(5). Description of one such method was given in an earlier paper [11], in which a rectangular area was first subdivided into trapezoidal or triangular areas by lines parallel to the neutral axis and coinciding either with the corners of the rectangle or with any points marking a deviation in the material stress strain curve. The method had the merit of generality in its application to a variety of cross sections with material stress strain characteristics in the form of linear or curved segments. The following method, though perhaps slightly less accurate than the method given in Reference [11], is simpler to program and is considerably faster for an equal number of Gauss points chosen in the analysis. As indiated earlier, accuracy can always be improved by increasing the number of Gauss points used in a given problem, and in this connection some guidance is given later in the paper.

COMPUTATION PROCEDURE

In the context of reinforced concrete column sections, it is assumed that the integration is being carried over a rectangular area and the longitudinal reinforcing bars are considered as point area. The steel stress values obtained for these point areas are adjusted for the area of concrete replaced by the steel bars.

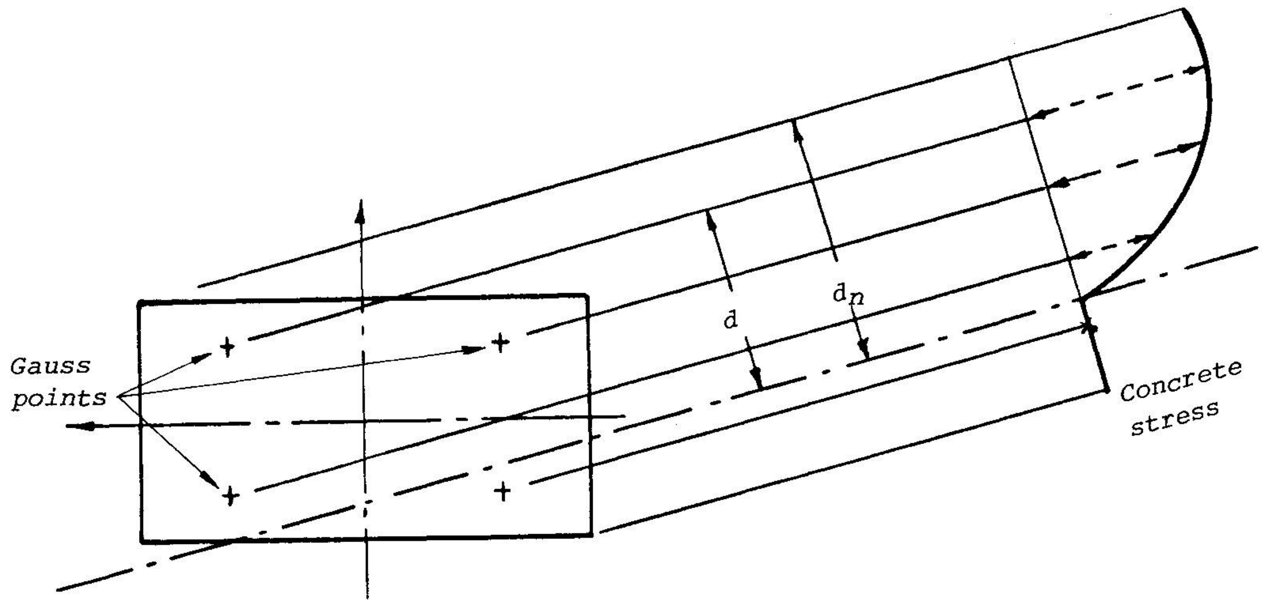


FIGURE 3 - Stress at Gauss Points

Assuming that ϕ and θ (or ϕ_x and ϕ_x) are known, the integration for a given value of d_n can be carried out as follows:

Step 1. For each Gauss point (Figure 3), repeat Steps 2-7.

Step 2. Determine the cartesian coordinates of the Gauss point from the predetermined natural coordinates using Equations (8) and (9).

Step 3. From the coordinates obtained, calculate the distance d from the neutral axis.

Step 4. Calculate the imposed strain ($\varepsilon = \phi d$) at the Gauss point.

Step 5. Using the material stress strain curve, calculate the stress at the Gauss point.

Step 6. Evaluate the Jacobian $|J|$ at the Gauss point using Equation (11).

Step 7. Obtain the summations implied by the Equations (7) and (3)-(5).

Step 8. As in Step 4, calculate the strain at the centroid of each of the reinforcing bars.

Step 9. From the material stress strain characteristics, calculate the steel stress and then the concrete stress, and hence the net stress for each reinforcing bar.

Step 10. Augment the summations obtained in Step 7 appropriately.

It is worth noting that at a given cross section, in Step 6, once the number of Gauss points is fixed, the value of the Jacobian $|J|$ and hence of the product $|J|H_i H_j$ needs to be evaluated only once for each Gauss point, thus offering additional savings in computational effort.

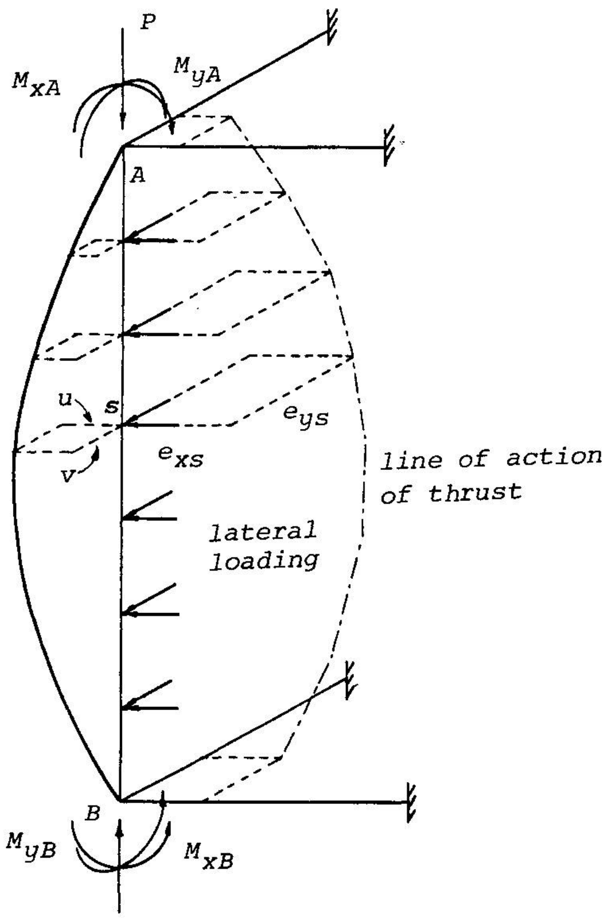


FIGURE 4 - Sign Convention for a Biaxially Loaded Restrained Column

DETERMINATION OF THE EQUILIBRIUM DEFLECTED SHAPE

Figure 4 shows the deflected shape of a beam column with generalised end conditions under generalised end loading together with some lateral loads. The deflections at the top and bottom of the column are assumed to be zero. Implicitly the following analysis is restricted to columns with ends which are not allowed to translate, but can rotate subject to the beam restraints. Thus columns suject to sway are not covered. under the initial conditions, the column initial deflected shape is assumed to be in equilibrium.

At any stage of loading, the column undergoes additional deflections, and hence end rotations. The end rotations induce moments in an opposite sense to the directions of the rotations. After allowing for the corrections to the applied end moments, the column deflected shape can be thought of as being in equilibrium with an eccentrically located thrust which has a variable line of action, as shown in Figure 4. This line of thrust would follow the notional bending moment diagrams in the two bending planes, due to the applied loading but corrected for the restraining end moments. In the absence of any lateral loads, the effective line of thrust would be a straight line passing through the effective end eccentricities at the two ends. The solution to the problem requires that the internal stress resultants induced by the biaxial curvatures in the column be balanced at each point along the length of the column by the applied thrust acting at a total biaxial eccentricity, obtained by summing the deflections and the deviations of the line of action of the thrust with respect to the column axis in both the bending planes.

BASIC RELATIONSHIPS

The biaxial curvatures at any point along the column length can be approximated by:

$$\phi_{tx} = -\partial^2 u/\partial x^2 \tag{12}$$

$$\phi_{ty} = -\partial^2 v/\partial y^2 \tag{13}$$

where, u and v are the total deflections in the two bending planes. If u_o and v_o are the initial deflections, the initial curvatures can be similarly expressed by:

$$\phi_{ox} = -\partial^2 u_o/\partial x^2 \tag{14}$$

$$\phi_{oy} = -\partial^2 v_o/\partial y^2 \tag{15}$$

The net curvatures are obtained by subtracting the initial curvatures from the total curvatures:

$$\phi_x = \phi_{tx} - \phi_{ox} \tag{16}$$

$$\phi_y = \phi_{ty} - \phi_{oy} \tag{17}$$

The internal stress resultants induced by the net curvatures can be obtained at all points in the column by knowing the values of these curvatures from the moment thrust curvature relations adopting the procedure outlined earlier. Let M_x and M_y be the bending moment stress resultants so obtained. For equilibrium:

$$u = (M_x/P) - e_x \tag{18}$$

$$v = (M_y/P) - e_y \tag{19}$$

where e_x and e_y are the biaxial eccentricities of the effective line of action of the thrust P for the particular column cross section measured with respect to the originally straight column axis.

Recognising that M_x and M_y are nonlinear functions of u and v, the solution to the problem now requires a simultaneous solution of Equations (18) and (19) at all points along the column length.

The column length is subdivided into n equal parts, each of length h . The resulting nodes are labelled 1,2,....,(n+1). The equilibrium is then satisfied at the (n-1) internal nodes only. Clearly, the larger the value of n, the more accurate the solution obtained will be.

At each station, the biaxial curvatures can be calculated by adopting the finite difference approximations:

$$\phi_{sx} = -(u_{s-1} + 2u_s - u_{s+1})/h^2 \tag{20}$$

$$\phi_{sy} = -(v_{s-1} + 2v_s - v_{s+1})/h^2 \tag{21}$$

Keeping in mind the implied moment thrust curvature relations, Equations (18) and (19) may now be stated thus:

$$u_s = U(u_{s-1}, v_{s-1}, u_s, v_s, u_{s+1}, v_{s+1}) \tag{22}$$

$$v_s = V(u_{s-1}, v_{s-1}, u_s, v_s, u_{s+1}, v_{s+1}) \tag{23}$$

or, even more generally as:

$$\{w\} = \{W(w_1, w_2, \ldots\ldots w_{2n-1}, w_{2n})\} \tag{24}$$

where, $w_{2s-1} = u_s$ and $w_{2s} = v_s$. The solution to nonlinear problems of the type expressed by Equation (24) can be obtained by the Newton Raphson method. The method states that if $\{w^k\}$ represents an approximate solution to Equation (24), a better approximation $\{w^{k+1}\}$ can be obtained by the following equation:

$$\{w^{k+1}\} = \{w^k\} - [I - K]^{-1} \{w^k - W(w^k)\} \tag{25}$$

In Equation (25), $[I]$ is an identity matrix and $[K]$ is a Jacobian matrix, the elements of which are:

$$K_{ij} = \frac{\partial W_i}{\partial w_j} = \frac{\Delta W_i}{\Delta w_j} \tag{26}$$

The elemental definition is of more relevance where the derivatives have to be calculated numerically, as in the present case. For the next iteration, $\{w^k\}$ is replaced by $\{w^{k+1}\}$, and the process is repeated until satisfactory convergence has been obtained.

An inspection of Equations (22) and (23) would reveal that, at each node s, the evaluation of the Jacobian matrix requires one basic and 6 incremental computations of the moment thrust curvature relations. However, by keeping Δu numerically the same at all stations, and noting that

$$\frac{\Delta\phi_{sx}}{\Delta u_{s-1}} = \frac{\Delta\phi_{sx}}{\Delta u_{s+1}} = -2\,\frac{\Delta\phi_{sx}}{\Delta u_s} \tag{27}$$

and similar other relationships, the entire procedure can be reduced to one basic and only two incremental computations of the moment thrust curvature relations at each node. Clearly, this simple device reduces the computation time required by more than half.

In the absence of any end restraints, the Jacobian matrix in Equation (25) will be a banded matrix. However, when end restraints are present, a change in displacements near the column ends would alter the end slope and hence the restraining moments at the ends, and this in turn would alter the computed displacements $\{W\}$ at all stations. In this case the Jacobian matrix would no longer be banded. If, for example, a difference formula of the type

$$\psi_{1x} = (4w_3 - w_5)/2h \tag{28}$$

is used to calculate the end slopes, it is clear that the derivatives with respect to the eight end deflection components involved would have to be evaluated. Once again, by noting that

$$\frac{\partial\psi_{1x}}{\partial w_5} = -\frac{1}{4}\,\frac{\partial\psi_{1x}}{\Delta w_3} = \frac{1}{2h} \tag{29}$$

and similar other equations related to the end deflections involved, it is possible to calculate elements of the Jacobian $[K]$ influenced by end restraint effects, without having to do any further moment thrust curvature calculations.

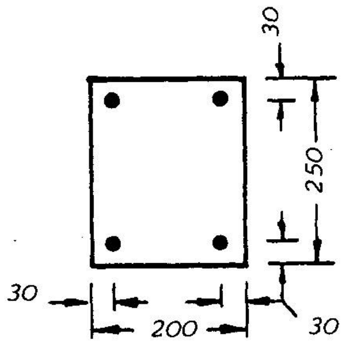
Because of the incremental nature of the computations involved, the end restraint characteristic does not have to be perfectly elastic. Nonlinear moment rotation curves for the end restraints can be handled with no penalty on the efficiency of the procedure. The procedure has been described in considerable detail in Reference [13].

STABILITY ANALYSIS

The procedure described above for the determination of the deflected shape of the column can be applied, in general, for the nonlinear analysis of any structure subjected to a given loading. Stability analysis further requires that the deflection response of the structure be monitored for increasing loads until, at some stage, equilibrium is no longer possible. Thus, if α is defined as a load fator on the initial loading $\{F_O\}$, the structure is analysed for varying loads $\{F_\alpha\}$, given by:

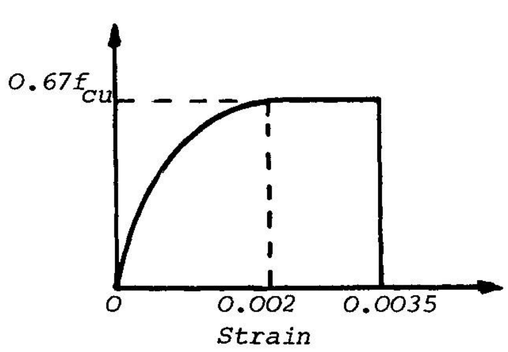
$$\{F_\alpha\} = ((\alpha-1)[G] + [I])\ \{F_O\} \qquad (30)$$

in which $[I]$ is an identity matrix and $[G]$ is a diagonal matrix, the elements of which are either 1 or 0, depending upon whether the corresponding load component varies with α or not. The highest value of α so obtained would be the load factor corresponding to the ultimate limit state of collapse. In the case of the biaxially loaded column the vector $\{F_\alpha\}$ consists of five components namely, the applied thrust and the biaxial moments at the two ends.



SECTION	STEEL	LENGTH
L	1%	5000
H	6%	2500

f_{cu} = 30 MPa, f_y = 250 MPa



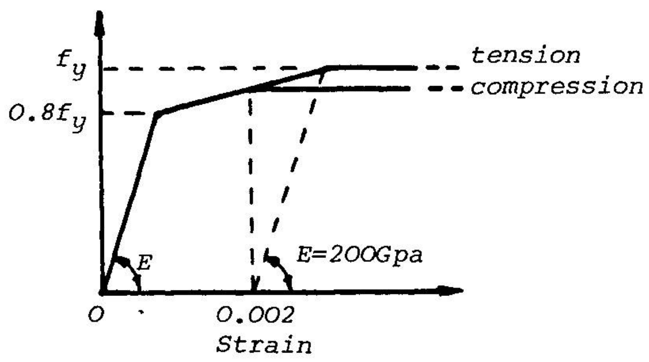


FIGURE 5 - Basic Data for Columns Referred in Table 1

ACCURACY OF THE PROPOSED METHOD

The method outlined above has been used to calculate the failure loads of columns of two cross sections shown in Figure 5. The percentage of steel in the two sections was 1% and 6% respectively. The material properties for both cases were the same. For steel a trilinear stress strain characteristic in line with the recommendations in CP110 [2] was chosen with a characteristic yield stress of 250 MPa and a Young's modulus of 200 GPa. For concrete, a parabolic rectangular characteristic in line with the CEB-FIP recommendations [3] was chosen with a characteristic strength of 30 MPa, a peak strain of 0.002, and a crushing strain of 0.0035. For the column with 1% steel, a column length of 5000 mm was chosen, whereas for the other section a stockier length of 2500 mm was adopted. The columns were assumed to be loaded with equal eccentricities of magnitudes 50 mm and 100 mm respectively, at both ends in the plane of the major axis. Both columns were assumed to have an out of plane initial deflection of magnitude 0.001 time the column length, resulting in biaxial mode of bending.

Table 1 gives the results for the two columns for different number of Gauss points, varying from 2 to 6, and for different number of column segments along the length, varying from 8 to 20. A close look at the failure loads obtained shows that for a given number of column segments, use of 4 Gauss points in each direction yields results within 0.5% of the convergence value. It appears that use of only 2 Gauss points tends to overestimate the failure load by about 2-3%. Use of 3 Gauss points, on the other hand, gives similar error on the safe side. On the question of the number of column segments along the length, it is noted that no significant improvement in accuracy is obtained by taking more than about 8 column segments. This, of course, would be true for columns in single curvature bending only. For columns in double curvature bending, a suitable number of column segments along the length would be about 16. It is worth noting that in the case of double curvature bending, having fewer segments may result in errors on the unsafe side [13].

TABLE 1 - Results with Varying Number of Gauss Points and Column Segments

Section	Number of Column Segments	Failure Load (kN) for Number of Gauss Points				
		2	3	4	5	6
[L]	8	428.6	408.2	418.3	417.7	418.3
	10	430.3	409.0	418.6	418.2	418.5
	12	429.7	409.3	419.1	418.6	418.9
	16	430.3	409.5	419.4	418.8	419.3
	20	430.6	409.5	419.5	419.0	419.4
[H]	8	688.1	696.6	683.8	686.7	686.5
	10	688.2	696.7	683.9	686.8	686.6
	12	688.2	696.8	683.9	686.8	686.6
	16	688.2	696.8	683.9	686.8	686.7
	20	688.2	696.9	684.0	686.8	686.7

COMPARISON WITH EXPERIMENTAL RESULTS

A very large number of test results are available for slender reinforced concrete columns. Very few of these, however, involved columns in biaxial bending. Two series of tests described in recent literature have been chosen to show typical agreement with experimental results [4,9].

In the first series by Pannell and Robinson, seven small scale columns with biaxial eccentricities were tested. In the second series, by Cranston and Sturrock, the columns had a narrow rectangular shape, and although the applied eccentricity was only in the major axis, small imperfections in the minor axis could easily have triggered biaxial mode of failure. For this reason, a minor axis initial lack of straightness of magnitude 0.001 times the column length has been adopted for the latter series. In both cases, for steel an elastic perfectly plastic stress strain characteristic has been used. For concrete, a stress strain curve of the type shown in Figure 5 has been adopted, except that the factor 0.67 on the cube strength has not been used. In all other respects, the data given in the original publications [4,9] has been used.

Table 2 shows the comparison between the experimental and analytical results. It will be noted that for the 12 columns analysed, the computed results are always on the conservative side. The mean value of the ratio of the theoretical and experimental failure loads is around 0.69, which indicates that the results based on the proposed method can be accepted with confidence.

TABLE 2 - Comparison with Test Results

Reference	Column Label	Failure load (kN) Test	Failure load (kN) Theory	Ratio Theory/Test
[9]	11	40	22.1	0.553
	12	40	26.1	0.653
	13	20	13.8	0.689
	14	20	12.2	0.610
	15	50	29.3	0.586
	16	30	24.0	0.800
	17	20	15.8	0.790
[4]	3	273	224.4	0.822
	4	463	301.0	0.650
	5	349	272.2	0.780
	6	321	195.2	0.608
	7	378	274.0	0.725
			Mean =	0.689
			Standard Deviation =	0.093

APPLICATIONS

Several studies in the past few years have shown that analytical techniques similar to the one described in the paper agree well with experimental results [4,7,12]. It is recognised, however, that such complex methods are unlikely to be used in the design office. The usefulness of these methods lies in their application to parametric studies of a wide range of practical columns, and then summarising the results by means of simple formulae which inevitably would be less accurate, and would also have limited range of applicability, but would be amenable for use in design office.

The method described in this paper has been used to verify the accuracy of a new design procedure [15]. Current areas of investigation include the problem of columns of varying profile along the length as used in bridge structures.

CONCLUSIONS

The exact method of analysis for biaxially restrained reinforced concrete columns with nonlinear material properties in the past has suffered from the drawback of requiring large amount of computer time so that parametric studies undertaken have had to be of limited scope. As shown in this paper, use of Gauss quadrature at the stage of computing the moment thrust curvature relations can reduce the computational effort significantly. It is shown that using only 4 Gauss points in any direction yields results within 0.5% of the convergence value for a given number of column segments. It is shown that for columns in single curvature bending, even for the biaxial case, use of 8 column segments along the length yields results within 0.1% of the convergence value. For columns in double curvature bending, it is suggested that use of 16 column segments would yield comparable level of accuracy. The method has been shown to give good agreement with two separate series of tests available in the literature, the computed results always being on the conservative side.

APPENDIX I - REFERENCES

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