Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 34 (1981)

Artikel: Modelling of the cracked elastic state of reinforced concrete

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DOI: https://doi.org/10.5169/seals-26898

Modelling of the Cracked Elastic State of Reinforced Concrete

Modélisation de l'état fissuré élastique du béton armé

Modellierung von Stahlbeton im Zustand II.

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SUMMARY
Based on observations of tests and analytical experiences, simple assumptions for a realistic description of the cracked elastic state of reinforced concrete are proposed.

RÉSUMÉ
A partir des observations expérimentelles et des expériences analytiques on propose des dispositions simples pour décrire le comportement réel du béton armé dans l'état fissuré élastique.

ZUSAMMENFASSUNG
Ausgehend von Versuchsbeobachtungen und rechnerischen Erfahrungen werden einfache Einsatze zur wirklichkeitsnation Erfassung des elastisch gerissenen Zustandes von Stahlbeton vorgeschlagen.

1. INTRODUCTION

Constitutive models of cracked reinforced concrete are numerously known. A survey of usual assumptions is found by Eibl and Ivânyi [6], Gerstle [8] , Argyris et al [1]. A special field of modeling is the phase of cracked elastic state with the following characteristics:

- progressive cracking, possibly perament change of crack orientation
- linear-elastic steel behavior
- significant effect of tension stiffening on the deformation behavior
- increasing non-linear behavior of concrete in compression.

In particular, the progressive cracking causes large changes of stiffnesses which requires an incremental load increase for numerical analysis to keep the stability of the solution. Additional difficulties are to note down at iteration processes relative to permanent changes of crack orientations and the appearance of secondary cracks. Often the appropriate realistic physical basis to judge these questions is missing, too. The consideration of bond between reinforcing bars and concrete for an analytical model needs special reflections because realistic physical assumptions, such as Ngo and Scordelis [14], go numerically to great expense. Just the same remark is valid for aspects of shear transfer through the crack due to aggregate interlock and dowel action.

In the following, some simplifying statements of the modeling of reinforced concrete constructions in the cracked elastic state are discussed, basing on the results of many tests on reinforced slabs of realistically dimensions and analytical experience with finite-element methods.

2. EXPERIMENTAL RESULTS

Tests on various two-way reinforced concrete slabs and panels are connected with numerous experimental problems. Most of the previous results are faulty or useless as the profound analysis of Lenschow [13] has shown because the test rig restrains the deformation of the test slabs. The tests of Lenschow and the later ones of Clark [5], too, are mainly used for formulating yield criteria; the small scale of test specimens does not allow conclusions for the cracked elastic state. For answering the questions of

- primary and secondary crack formation
- crack orientation
- crack width

and solving the problem of section designing and analytical modeling, Kordina and Ivânyi have directed investigations in Braunschweig on uni- and biaxial bending tests on slabs ($M_2 = 0$ respectively $M_2/M_1 = -1$). The test specimens have a thickness of d = 14 cm and are two-way reinforced.

In the following only a few results concerning the cracking process and crack orientation are discussed.

The test specimen and the testing conditions for $M_2 = 0$ - developped by Lenschow - are shown in fig. 1. Torsional moments ($M_2/M_1 = - 1{,}0$) are brought up in a similar way.

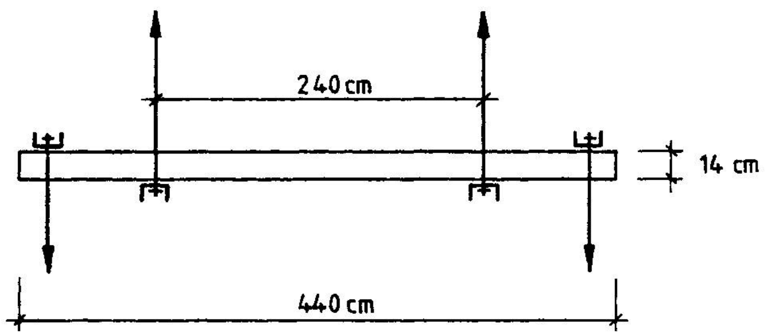

Fig. 1 Test slab

For various orientation of reinforcing steel to the principal moments with isotropic and orthotropic arrangement of reinforcement, strains of concrete and steel and crackwidth in several hundreds of places are measured. The load is brought up incrementally till yield state of reinforcement. The crack orientations are analysed by a stereometric method basing on Stroeven [17], for every load increment all together and also for every single one. Therefore, in Angle intervals of 10^o a grid plan is revolved round one point and the number of cuts along every line is summed up. The main crack orientation is to be found at the minimum number of cuts (fig. 2). This method is the only reliable one, the experience of the past has shown that many not serios estimations yield to faults.

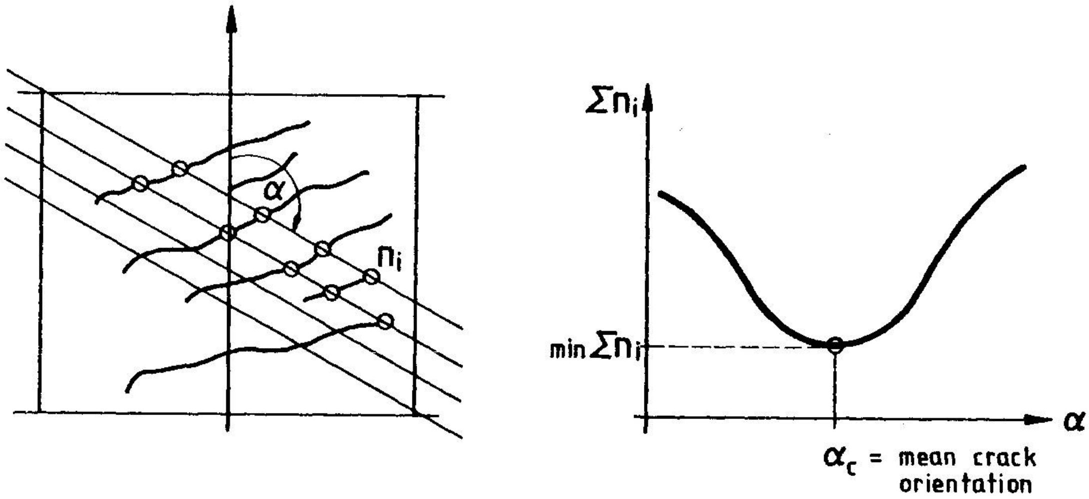

Fig. 2 Finding of crack orientation

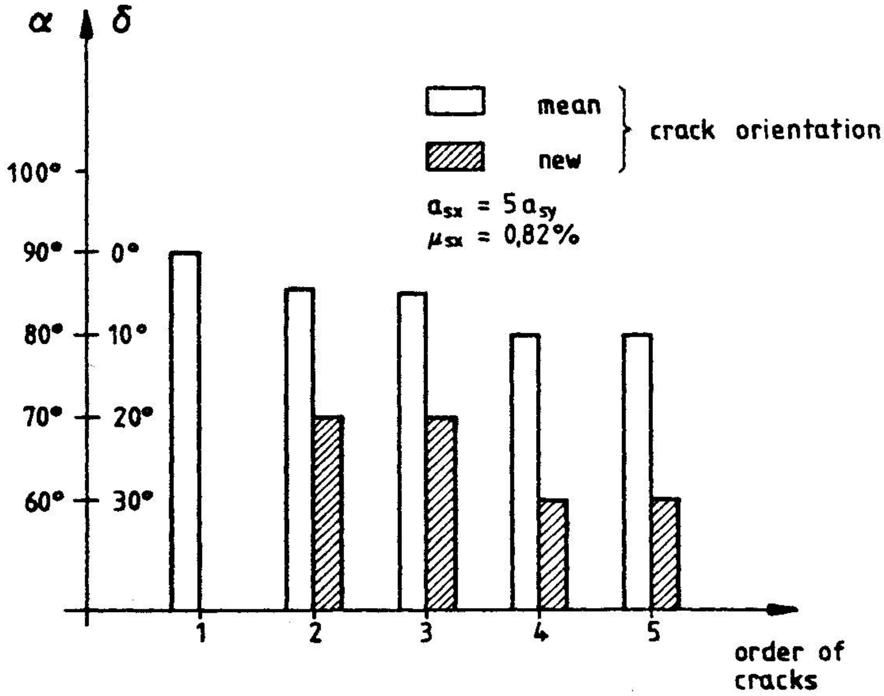

Fig. 3 Analyse of crack orientation (Test slab No.7, notations s. Fig. 6)

Fig. 3 shows the change of crack orientation of an anisotropic reinforced and uniaxial stressed slab (M_2 = 0) for every increment alone and the main crack orientation, too. The result shows that the secondary crack orientation causes only small changes concerning the main crack orientation, although the influence is most clear on that demonstrated marked anisotropy.

3. DISCUSSION OF TEST RESULTS

Baumann [2] takes the view that shear transfer along the primary cracks causes the development of secondary cracks with the result of a main crack orientation corresponding to H = O in the elastic cracked condition (fig. 4). This assumption is not realistic because shear forces along the crack can make only further cracks under very high reinforcing ratio.

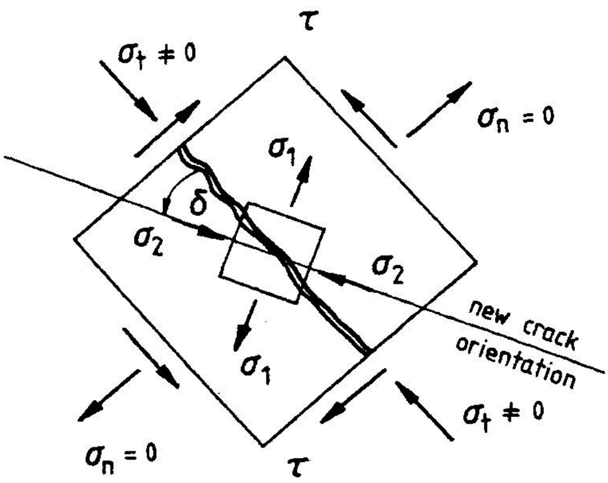

Fig. 4 Formation of secondary cracks according to Baumann [2]

To clarify the most important parameters concerning the secondary cracking, Fastabend [7] made investigations on a plain-stress model to define the state between two primary cracks. The reinforcing bars (truss elements) have been connected with nodal points of membrane elements by the aid of bond links (beam elements). Therefore, a linearizising bond law of Noakowski [15]was used. The edge forces have been developped on the equilibrium and compatibility conditions based on Baumann with the assumption of various shear stiffness for uniaxial tension. The model is shown in fig.5. Some analytical results are comparable with experiments. Table 1 shows the covering of both of them. The colums "main crack orientation" indicates that there is no fundamental change on the primary crack orientation caused by the development of secondary cracks. Analytical solutions for secondary crack orientations,

Table 1

Spec. No	$\frac{a_{sx}}{a_{sy}}$	α	cal δ	meas δ	mean δ	μ_{sx} [%]
7	5	30^o	19^o	20^o	5^o	0,82
4	5	45^o	11^o	10^o	0^o	
9	1	30^o	11^o	10^o	5^o	

(Notations see Fig. 6)

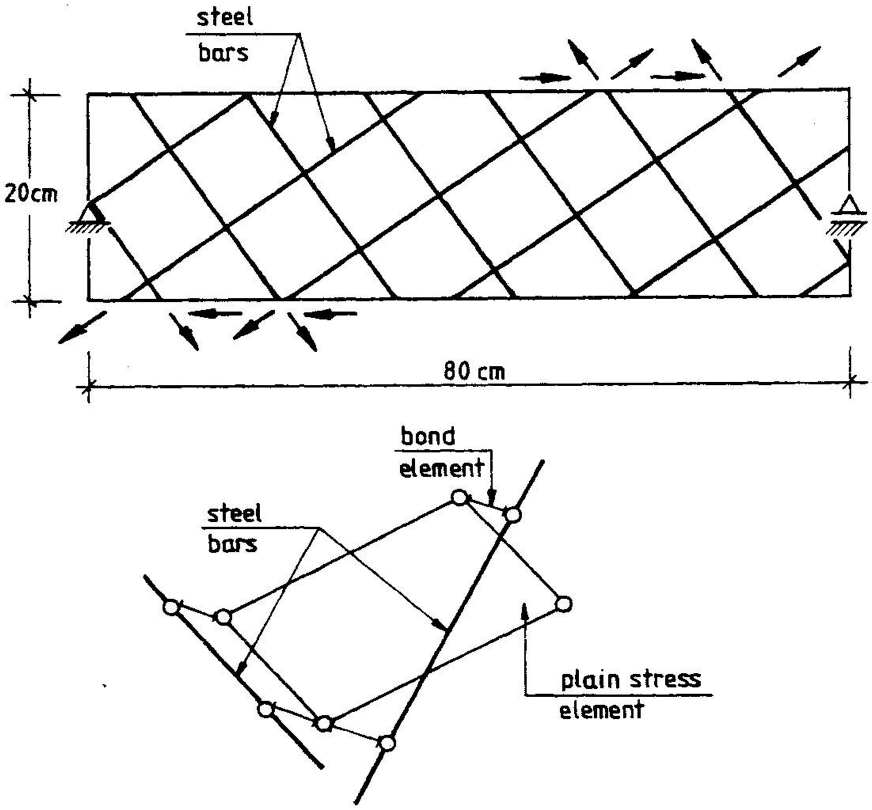

Fig. 5 Finite Element Model for analytical study of crack orientation [7]

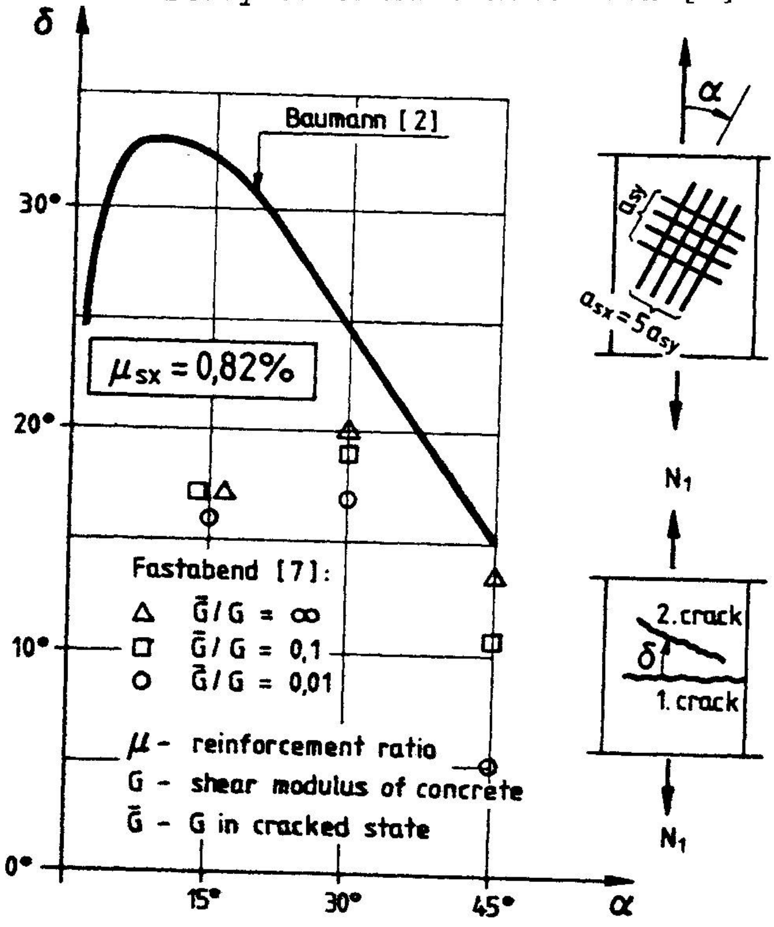

Fig. 6 Comparisation of the results according to [2] and [7]

according to [2] and [7], depend on the orientation of reinforcing steel to the principal moments for uniaxial bending are compared in fig. 6. The secondary crack orientation based on Baumanns solutions, as can be seen, indicate a fundamentally different development.

Similar results confirmed by Gilbert and Warner [9] show that secondary cracks with new orientations do not arise even throught assumption of any shear stiffness. Only a true modeling of bonding yields a physically correct behavior of the cracked elastic state (see 4.3).

4. PROPOSAL MODELING OF CRACKED ELASTIC STATE

4.1 Primary crack conditions

The tensile strength of concrete varies in dependence on special facts such as concrete quality (Rüsch [16]) and stress state (Kupfer et al [12]) statistic parameters as there are size effects (Bažant [3]) and last not least on the strain gradient (Iványi [10]). Most of these influence are not taken into consideration for numerical analysis because the detailed strength dates are missing. Even the check on a real experiment does not allow statements that the tensile strengthes found out on test specimens yield a realistic crack condition.

Another problem of numerical analysis is the fact that cracking in equally stressed fields begins nearly at the same time which may cause an instability of numerical analysis. In reality, these phenomena do not arise because the tensile strength of concrete is a stochastic variable.

Knowing these problems the following recommendation is made in order to describe the primary cracking on FEM analysis with "smeared" cracks.

- Proceeding from an ingeniously mean value of concrete tensile strength, there are to generate, e.g. according to the Gaussian distribution, individual tensile strengthes of elements or layers by the aid of a random generator in a very simple way.

Consequence of the different tensile strengths of each element is the development of cracks in the calculation only by degrees which does not mean instability for the analytical model.

Usually that order prevents that elements or layers appear cracked and uncracked by turns within the iteration cycles.

- The element tensile strengthes shall be valid irrespective of the stress state. Knowing the scatter of test results, too many specifications of dependences are superfluous.

- The primary crack orientation is to be chosen perpendicularly to the largest main strain of the element. Variability of primary crack orientations during iterations cycles are to be prevented.

- At stress states $\sigma_1/\sigma_2>0$ an orthogonal double cracked condition is to be supposed at the beginning of $\sigma_2>0,6\div0,8\cdot\sigma_1$ because this behavior is corresponding to the experiments (s. Lenschow's isotatic tests).

4.2 Secundary crack conditions

Following the results in part 2 of this paper it can be said that the development of new cracks in different direktions does not influence the behavior of the specimen fundamentally (Fig. 3). Numerically, this behavior is to be realized only in an insufficient way if smeared cracks are supposed because in an analytically double cracked state an unrealistic element behavior occurs.That does mean that at the numerical analysis the possibility of progressive cracking should be limited to prevent a physically wrong behavior. In the following some different ways are shown:

- a simple method to prevent "numerical" secondary cracks is to pull up the tensile strength of a primary cracked element, for example about 30 p.c.
- Another solution is to allow secondary cracks only above a divergence of 30° from primary crack orientation.

In both cases, a double cracked condition appears only if stronger emcompassments are caused as a result of the cracks occured or changing of the loading conditions. If such behavior is to be excluded at all (s. Jofried and McNeice [11]), it is also justified to renounce on further control concerning the tensile stress of primary cracked elements.

4.3 Consideration of bond

Usually, the calculation of complete constructions on micro-idealizing of bond (Ngo and Scordalis [14])must be resigned.

A simple possibility is to consider tension stiffening

- to get a falling branch of the stress-strain-line of concrete intension

or

- to simulate the dependence of steel stress on average steel strains (fig. 7).

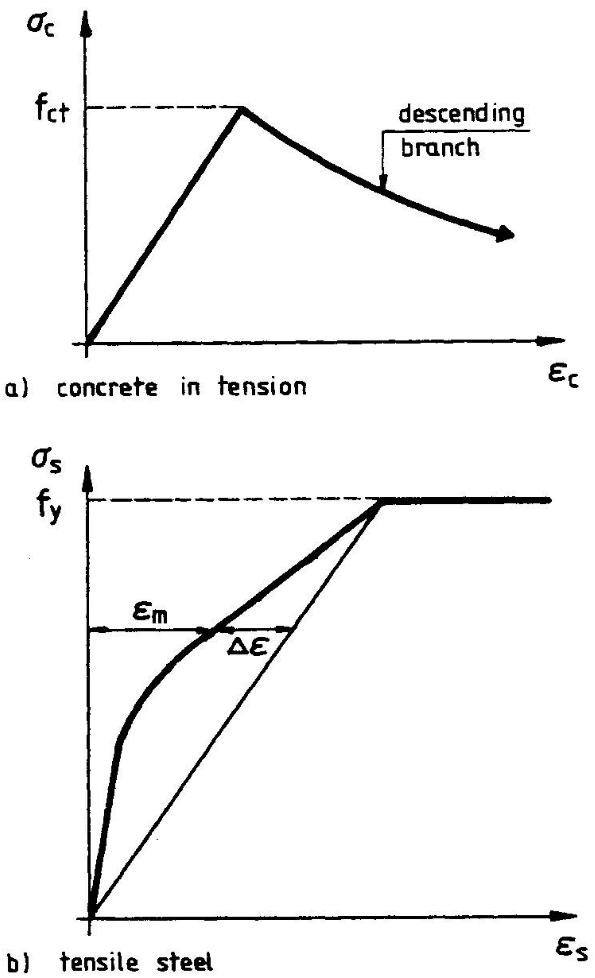

Fig. 7 Assumptions for tensile stiffening of reinforced concrete

Physically, the consideration of bond at the reinforcing level is to be prefered: In multi-axial stress states the only way for describing the realistic concrete stress states is to do it that way. Gilbert and Warner [9] report moreover about the expectation of numerical advantages resulting from the restriction of the number of iterations on the steel layer concerning this method. In that way, stress transfer perpendicular to the crack orientation on the concrete elementis impossible.Nevertheless for the correct calculation of the stresses in a cracked concrete element, it is necessary to add the strain differences of the steel layers $\Delta\varepsilon=\varepsilon_s-\varepsilon_m$ (fig. 7) as strain producer in direction of the reinforcing bars.

4.4 Shear stiffness in cracked state

The physical connection between crack width and crack shear stiffness is complicated and not exact definiable. Furthermore , numerous analytical results show no particular influence on the mechanical behavior of the varied shear stiffnesses with in far limits. Nevertheless, numerical reasons require a minimum of shear stiffness to avoid accidental instabilities of analysis.

With regard to these discussions very sofisticated assumptions concerning the shear stiffness are not recommendable. Basically in the analysis of once or double cracked elements $\beta=\bar{G}/G=0,2\div0,4$ has to be set.

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