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Advanced Mechanics of Reinforced Concrete in Structural Fire Analysis

Méthodes nouvelles de la mécanique du béton armé pour l'analyse des structures soumises au feu

Neue Verfahren im Stahlbeton für die Berechnung von Tragwerken im Brandfall

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SUMMARY
A numerical model for the analysis of concrete structures in a fire environment is presented. This model is based on the finite element method with subdivision of the cross section in a rectangular mesh. A particular formulation is used for the evaluation of thermal effects in a discretized structure and for the step-by-step analysis of a structure submitted to increasing temperatures. A comparison between theoretical and experimental results is made for a continuous T beam loaded and heated unsymmetrically.

RÉSUMÉ
On présente un modèle numérique pour l'étude du comportement au feu des structures en béton. Ce modèle est basé sur la méthode des éléments finis avec subdivision de la section droite en mailles rectangulaires. On propose une formulation particulière pour l'étude des effets thermiques dans une structure discrétisée et pour l'analyse incrémentielle d'une structure soumise à température croissante. On effectue une comparaison entre résultats théoriques et expérimentaux pour une poutre en T continue chargée et chauffée de manière non symétrique.

ZUSAMMENFASSUNG
Ein numerisches Modell für die Untersuchung des Brandverhaltens von Betontragwerken wird vorgelegt. Dieses Modell beruht auf der finiten Elemente Methode mit Diskretisierung des Querschnittes in rechteckige Maschen. Eine besondere Formulierung für die Untersuchung der thermischen Effekte in einem diskretisierten Tragwerk und für die inkrementelle strukturelle Analyse unter zunehmender Temperatur wird vorgeschlagen.
Ein Vergleich zwischen theoretischen und experimentellen Ergebnissen für einen durchlaufenden, unsymmetrisch erwärmten und belasteten T-Träger wird durchgeführt.

1. INTRODUCTION

The thermomechanical behaviour of reinforced concrete structures in a fire environment is a very complicated problem. The best way to verify this statement is to attend a fire test. The phenomena observed during the experiment show that an analytical treatment of this problem is rather involved.

The usual way of determining the fire resistance of structural elements is to perform fire tests. Nevertheless the information needed for a rational design for fire safety cannot be provided only by results from standard tests. As testing facilities are rather limited such tests are not sufficient to represent the behaviour of all structural elements under fire conditions, since the evaluation of structural response should also account for different conditions of restraint due to the building system. Furthemore fire tests are rather expensive.

The need for analytical predictions of thermal and structural responses has grown more and more intensively. Though much progress has been made in the field of mechanics of reinforced concrete [2], very few research has up to now been devoted to fire problems due to the lack of knowledge of the material properties at elevated temperatures and the difficulty of modeling the structural behaviour. Significant advances have recently been made in the works of Anderberg [1], Bresler [3], Kordina and Klingsch [8].

In this paper a numerical model for the evaluation of the fire resistance of reinforced concrete structures is presented. A comparison between calculated and experimental results shows the efficiency of the model.

2. MATERIAL PROPERTIES AT ELEVATED TEMPERATURES

A structural fire resistance problem may be subdivided in two distinct parts :

1. a thermal problem i.e. evaluation of the temperature distribution in the element during the development of fire ;

2. a mechanical problem, i.e. study of the mechanical behaviour of the element due to the temperature increase calculated at point 1 and evaluation of the fire endurance of the element.

To solve these problems analytically, it is necessary to collect data on thermophysical (point 1) and mechanical (point 2) properties of the materials used, i.e. steel and concrete. Furthemore, due to the high temperatures reached, the variations of temperature affect significantly the properties of these materials, and this must be taken into account in the numerical model.

Experimental research has been made in various laboratories [1] [3] [8], and also in Belgium [5], but the experimental results display appreciable differences mainly in the case of concrete. Simplified methods for the evaluation of the fire endurance only require the determination of classical mechanical properties, i.e. ultimate strength in tension and compression, yielding stress and modulus of elasticity. In a step-by-step numerical analysis, these characteristics are not sufficient, and information concerning the instantaneous stress-strain relation is necessary.

Experimental investigations (cf [5]) show that the stress-strain diagram depends not only on the temperature reached, but also on preloading. This factor has clearly a favourable influence on the strength of concrete at elevated temperatures.

Thermal creep of concrete and steel has also a non negligible influence. Creep models have been proposed for both materials, but they appear quite involved. Furthermore research people do not still agree on the form of the creep laws and on the importance of the phenomenon. Thus this question remains open.

As temperature and stress vary from one point to another, a rigorous analysis of the problem should be based on a stress (σ) versus strain (ε) relation of the following type :

$$f(\sigma, \tilde{\sigma}, \varepsilon, \theta, t) = 0 \qquad (1)$$

where $\tilde{\sigma}$: stress history
θ : temperature
t : time

Such a relation characterizing the material behaviour under varying loading and temperature conditions has not yet been determined.

The analytical models used in this study are described in reference [5].

3. TEMPERATURE DISTRIBUTION IN THE ELEMENTS

The first problem to be solved is the modeling of the environment created by a fire. Several factors influence the severity and duration of a fire, but it is not reasonable to take them into account in performing fire tests. In order to simplify comparisons between fire test results from different laboratories, it has been decided to refer to the same thermal program. This is known as the standard ISO curve adopted in the ISO 834 Recommendations [7] and defined by the following temperature versus time relation :

$$\theta - \theta_o = 345 \log_{10} (8\, t + 1) \qquad (2)$$

t = time in minutes
θ_o = initial temperature

The distribution of temperature in the elements is governed by the following non linear partial differential equation :

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial \theta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial \theta}{\partial z}\right) + Q = c\rho \frac{\partial \theta}{\partial t} \qquad (3)$$

with various boundary conditions

$\lambda_x, \lambda_y, \lambda_z$ (x, y, z, θ) : coefficients of thermal conductivity
$c\,(x, y, z, \theta)$: specific heat
$\rho\,(x, y, z, \theta)$: density
$Q(x, y, z, \theta)$: distribution of internal sources

The most difficult problem is the determination of the density of heat flow Φ transmitted to the element.This can be written as follows :

$$\Phi = h(\theta_e - \theta) + \sigma_o\, \varepsilon_{es}\, (T_e^4 - T^4) \qquad (4)$$

θ_e, T_e : temperature and absolute temperature of the environment
h : coefficient of convection

σ_o : Stefan-Boltzman constant

ε_{es} : resultant emissivity factor between the environment and the surface of the element

Due to the high temperatures reached, the radiation term becomes rapidly preponderant. The preceding parameters cannot be determined analytically, but the following values have been adopted on the basis of numerical tests :

$$h = 25\ W/m^2 .\ K \qquad \varepsilon_{es} \approx 0,5 \tag{5}$$

The temperature distribution has to be obtained by numerical methods. The finite element and finite difference methods have been used in this study [5]. In both cases, reasonable agreement has been found between theoretical and experimental results.

4. NUMERICAL PROCEDURE FOR THE THERMOMECHANICAL ANALYSIS OF REINFORCED CONCRETE STRUCTURES IN A FIRE ENVIRONMENT

4.1. General structure of the computer program

The general structure of the computer program is given in figure 1. It can be derived from the analysis of a structural element submitted to fire. Before fire external loads are applied on the element. After starting of fire the increase of temperature induces several thermal effects and a reduction of the bearing capacity. The numerical procedure must be able to analyse these phenomena and to predict failure corresponding to the fire endurance of the element.

The 1st part of the computer program is devoted to the evaluation of the structural behaviour of the beam at ambient temperature under static loads. This is an extension to beams of the method developed previously for slabs [4]. Preference has been given to the formulation proposed in a parallel study by Hand et al. [6].

The 2nd and 3rd parts correspond to the thermomechanical response of the structural element during the development of fire. This analysis is made step-by-step. For the thermal analysis a finite difference program has been incorporated in the general program.

The increase of temperature leads to mechanical transformations analysed in part 3. The variation of displacements, strains, thermal stresses is calculated. An iterative procedure appears which will be described in § 4.3. If the element does not collapse, a new increment of time is applied, corresponding to an increase of the environmental temperature and the procedure is repeated.

Failure is detected by applying the classical Ryan and Robertson criterion on the rate of displacement

$$\frac{\Delta f}{\Delta t} \leq \frac{\ell^2}{9000\ h} \tag{6}$$

ℓ = length of the span
h = effective height

4.2. Evaluation of thermal effects in a discretized structure

The procedure used for the evaluation of thermal effects in a discretized structure is based on the classical assumption that plane sections remain plane. The

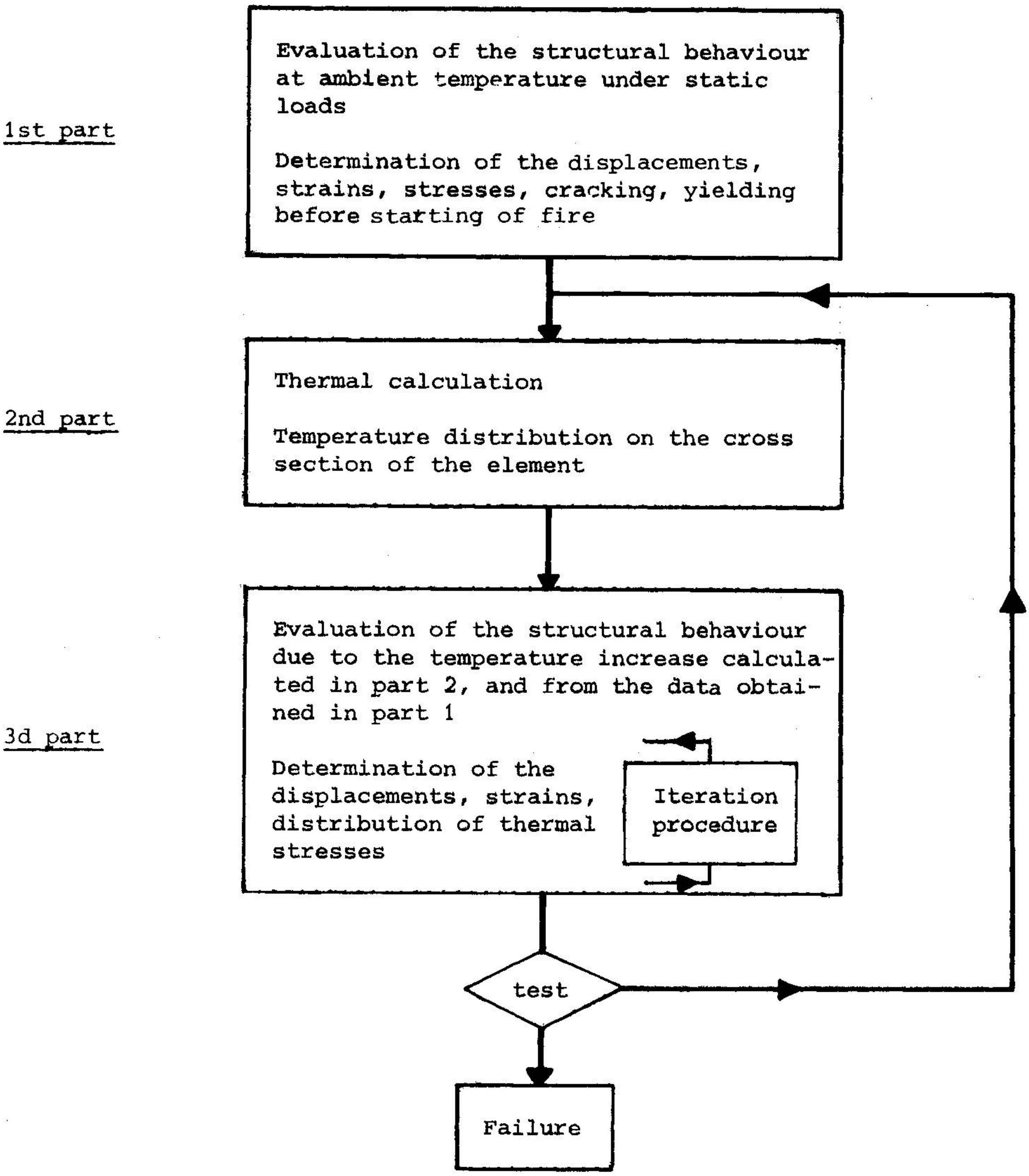

Figure 1 : General structure of the computer program

increase of displacement during a time step is given by :

$$\Delta u = \Delta u_o - z \frac{d\Delta w}{dx} \tag{7}$$

The "effective" strains, i.e. strains related stresses can be obtained by adding to the strains associated with the preceding displacement field, those corresponding to full restraint of the element :

$$\Delta\varepsilon = -\alpha\,\Delta\Theta + \frac{d\Delta u}{dx} = -\alpha\,\Delta\Theta + \Delta\varepsilon_o + z\,\Delta\chi \tag{8}$$

α : coefficient of thermal expansion
χ : curvature

The additional thermal stresses can be derived from the preceding equation :

$$\Delta\sigma = E\,\Delta\varepsilon = E(-\alpha\,\Delta\Theta + \Delta\varepsilon_o + z\,\Delta\chi) \tag{9}$$

The quantities $\Delta\varepsilon_o$ and $\Delta\chi$ must be determined so as to satisfy equilibrium equations. In a statically determinate beam these conditions reduce to :

$$\int_\Omega \Delta\sigma\, d\Omega = 0 \qquad \int_\Omega \Delta\sigma\, z\, d\Omega = 0 \tag{10}$$

By substituting (9) in (10), one gets :

$$\begin{aligned} \Delta\varepsilon_o \int_\Omega E\, d\Omega + \Delta\chi \int_\Omega E\, z\, d\Omega &= \int_\Omega E\,\alpha\,\Delta\Theta\, d\Omega \\ \Delta\varepsilon_o \int_\Omega E\, z\, d\Omega + \Delta\chi \int_\Omega E\, z^2\, d\Omega &= \int_\Omega E\,\alpha\,\Delta\Theta\, z\, d\Omega \end{aligned} \tag{11}$$

The discretization can be introduced here. The beam is divided in finite elements and the cross section is divided in subslices forming a rectangular mesh (figure 2). Let nc be the number of horizontal layers and nt the number of vertical layers. The discretization in a rectangular mesh corresponds to evaluating in a discretized way the integrals of equation (11), which leads to :

$$\begin{aligned} \int_\Omega E\, d\Omega &= \sum_{i=1}^{nc} \sum_{j=1}^{nt} E_{ij}\,\Omega_{ij} = E\Omega^* \\ \int_\Omega E\, z\, d\Omega &= \sum_{i=1}^{nc} \sum_{j=1}^{nt} E_{ij}\, S_{ij} = ES^* \\ \int_\Omega E\, z^2\, d\Omega &= \sum_{i=1}^{nc} \sum_{j=1}^{nt} E_{ij}\, I_{ij} = EI^* \\ \int_\Omega E\,\alpha\,\Delta\Theta\, d\Omega &= \sum_{i=1}^{nc} \sum_{j=1}^{nt} E_{ij}\,\alpha_{ij}\,\Delta\Theta_{ij}\,\Omega_{ij} = \Delta N_\Theta \\ \int_\Omega E\,\alpha\,\Delta\Theta\, z\, d\Omega &= \sum_{i=1}^{nc} \sum_{j=1}^{nt} E_{ij}\,\alpha_{ij}\,\Delta\Theta_{ij}\, S_{ij} = \Delta M_\Theta \end{aligned} \tag{12}$$

By using the preceding notations, equations (11) may be written as follows :

$$\begin{Bmatrix} \Delta N_\Theta \\ \Delta M_\Theta \end{Bmatrix} = \begin{bmatrix} E\Omega^* & ES^* \\ ES^* & EI^* \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_o \\ \Delta\chi \end{Bmatrix} \tag{13}$$

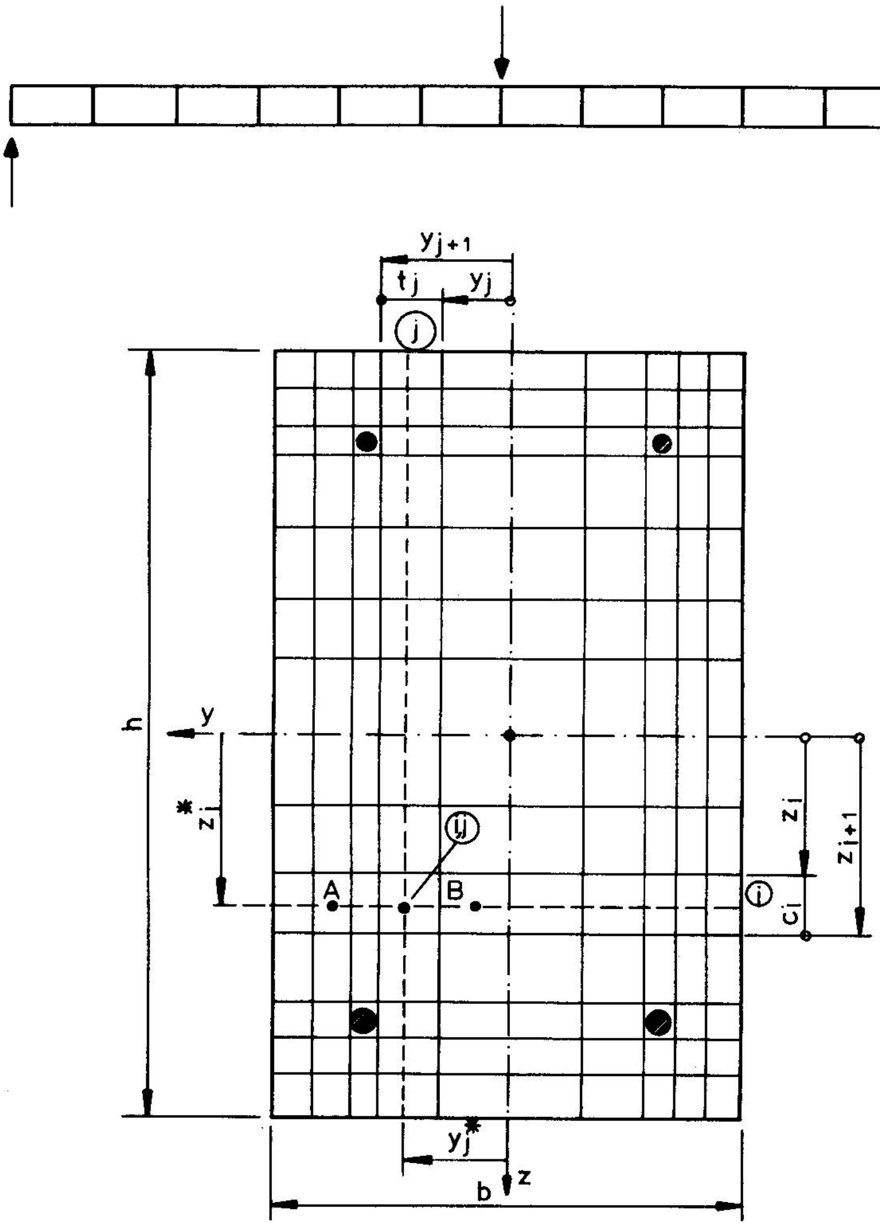

Figure 2 : Discretization of the beam and of the cross section

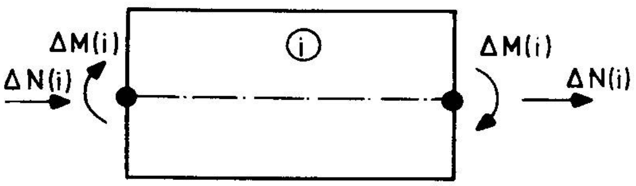

Figure 3 : Thermal equivalent nodal forces

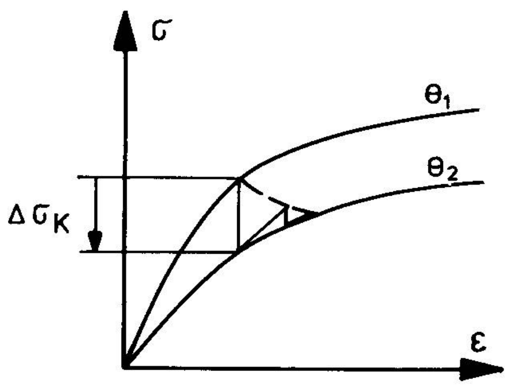

Figure 4 : Iterative procedure

which gives the thermal equivalent forces in a discretized element (figure 3). For the whole structure these forces must be combined between adjacent elements. As can be seen a coupling between membrane and flexural effects appears similar to the one described in [4] and [6].

4.3. Numerical iterative prodedure

In a classical thermoelastic problem thermal effects are calculated by applying the well-known Duhamel's principle. The structure is first restrained against any displacement ; equivalent restraint forces are computed and next applied to the element.

The problem is more involved in this case for several reasons :

- The restraint forces must be evaluated for a discretized structure as indicated in the preceding paragraph ;
- The structure is analysed step-by-step. This means that Duhamel's principle must be applied at each time or temperature increment ;
- Due to several nonlinearities (stress-strain diagram, cracking, crushing), an iterative procedure occurs leading to progressive structural modifications during each increment.

At the beginning of each time or temperature increment, the structure tangent stiffness matrix is updated to reflect any changes in material properties which have taken place. Inelastic effects within the increment are taken into account by the initial stress method, in which pseudo-loads are iteratively redistributed through the structure using the tangent stiffness computed at the beginning of each increment and modified during the iterative procedure.

At the beginning of the time or temperature increment the correction forces consist in thermal equivalent forces $\{\Delta F_\theta\}$ that can be obtained from equation (13). The temperature variations during a time step cause a subsequent change in the stress-strain law (figure 4). This temperature dependent shift gives rise to another set of correction forces.

The associated correction stress is the difference $\Delta\sigma_K$ between two stress-strain laws for the same strain and different temperatures (see figure 4). This stress difference is transformed into equivalent forces by the usual following procedure :

$$\Delta N_K = \sum_{i=1}^{nc} \sum_{j=1}^{nt} (\Delta\sigma_K)_{ij} \, \Omega_{ij} \tag{14}$$

$$\Delta M_K = \sum_{i=1}^{nc} \sum_{j=1}^{nt} (\Delta\sigma_K)_{ij} \, S_{ij}$$

and for the equivalent nodal forces

$$\{\Delta F_K\} = \int_\Omega [B]^T \begin{Bmatrix} \Delta N_K \\ \Delta M_K \end{Bmatrix} d\Omega \tag{15}$$

For the i^{th} time increment, the first approximation to the incremental displacements is calculated from

$$[K]_i^{(o)} \{\Delta u\}_i^{(o)} = \{\Delta F\}_i^{(o)} \tag{16}$$

where $[K]_i^{(o)}$ is the tangent stiffness matrix at the beginning of the i^{th} increment.

$$\{\Delta F\}_i^{(o)} = \{\Delta F_\Theta\}_i + \{\Delta F_K\}_i \tag{17}$$

These nodal forces are redistributed through the structure. But inelastic actions occur due to the nonlinearity of stress-strain diagrams, cracking and crushing of concrete, which leads to an iterative procedure. Numerical tests show that the iterations are mainly due to the formation of new cracks. The stress released in a crack is temperature dependent and is given by

$$\{\Delta\sigma\}_{cr} = R_b(\theta) \tag{18}$$

Successive iterations take the form

$$[K]_i^{(r)} \{\Delta u\}_i^{(r)} = \{\Delta F\}_i^{(r)} \tag{19}$$

where $[K]_i^{(r)}$ is the stiffness matrix updated at the beginning of the r^{th} iteration in the i^{th} increment

$\{\Delta F\}_i$ is mainly due to the formation of new cracks in the preceding iteration.

The total incremental displacements for the i^{th} time increment (after N iterations) is then given by :

$$\{\Delta u\}_i = \sum_{r=o}^{N} \{\Delta u\}_i^{(r)} \tag{20}$$

The incremental stresses are obtained by adding those corresponding to the preceding incremental displacements to the restraint stresses $- E\ \alpha\ \Delta\theta$.

5. EXAMPLE : T BEAM ON 3 SUPPORTS

To demonstrate the accuracy of the numerical results which can be obtained from the described procedure a continous T beam on 3 supports has been analysed and the theoretical results compared with test results obtained at the Technical University of Braunschweig [9].

The loading and heating system is presented in figure 5. The beam is loaded and heated unsymmetrically. The thermal program is applied according to the ISO 834 Recommendations [7]. The dimensions of the cross section and reinforcement arrangement are indicated on figure 6. The beam is subdivided in 20 finite elements. The whole beam has to be considered due to the lack of symmetry, but only one half of the cross section for the division in subslices.

Figure 7 shows the temperature increase in the tension steel near the corners. There is a good agreement between theoretical and experimental results. As can be seen the temperature increases rapidly at these points : ~ 600°C after 1 hour and more than 800°C after 2 hours.

The structural behaviour of a continuous beam or a beam with fixed ends is completely different from that of a simply supported beam. In the latter case, large deflections occur due to the steep thermal gradient on the cross section. In a hyperstatic beam, the same thermal gradient produces an increase of the bending moment on the supports, while the deflections remain rather small.

The variation of the bending moment on the central support of the T beam analysed here is indicated on figure 8. After ~ 1 hour, the bending moment tends to become constant. This corresponds to the formation of a plastic hinge on the central support. Again there is a good agreement between theoretical and experimental results.

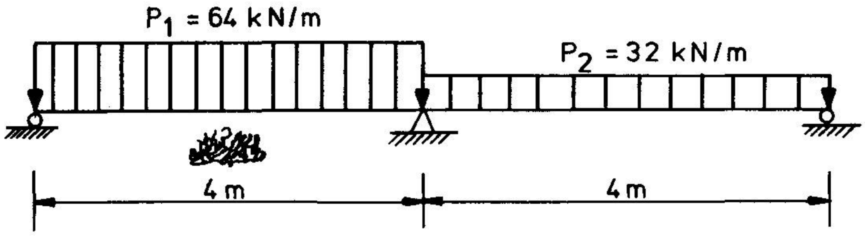

Figure 5: Fire test on continuous T beam loaded and heated unsymmetrically [9]

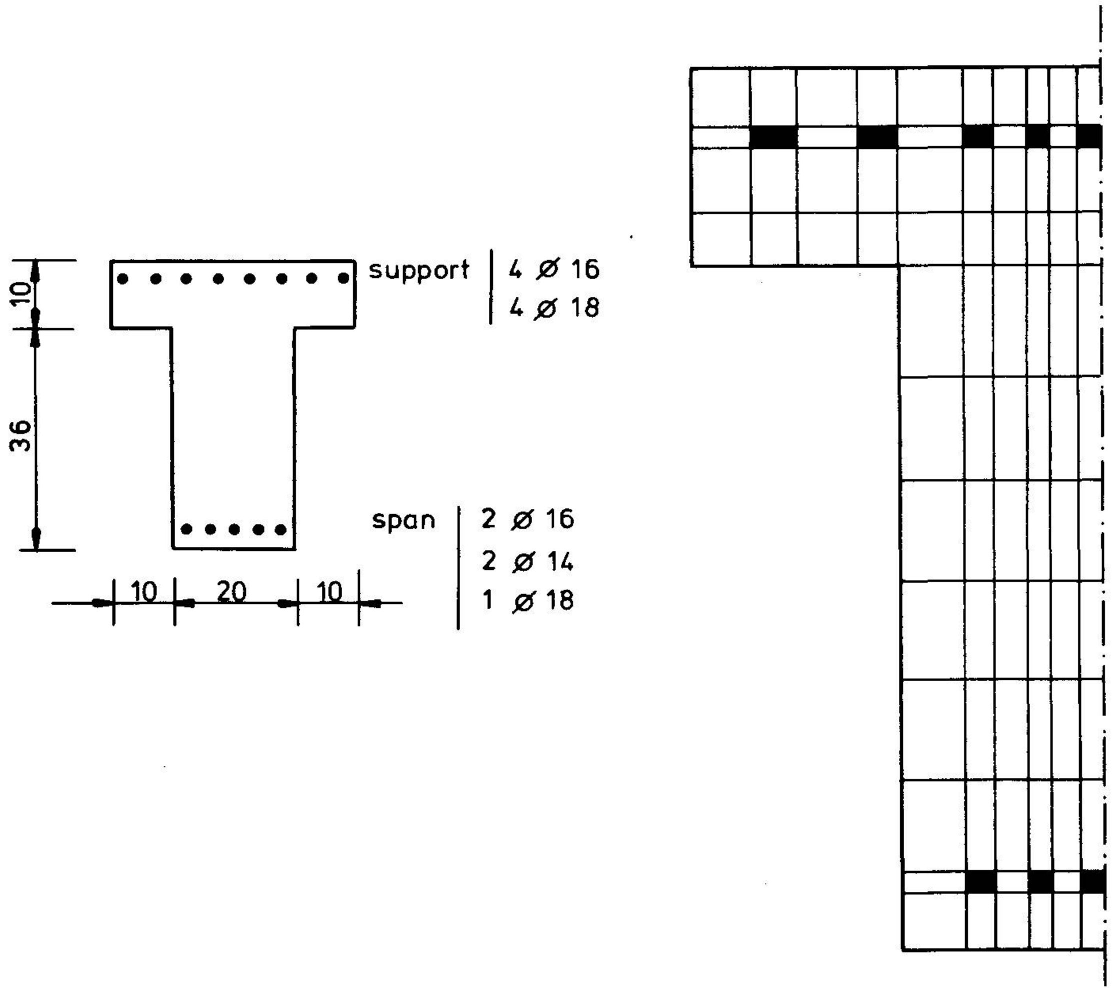

Figure 6: Cross section.
Reinforcement and subdivision in a rectangular mesh

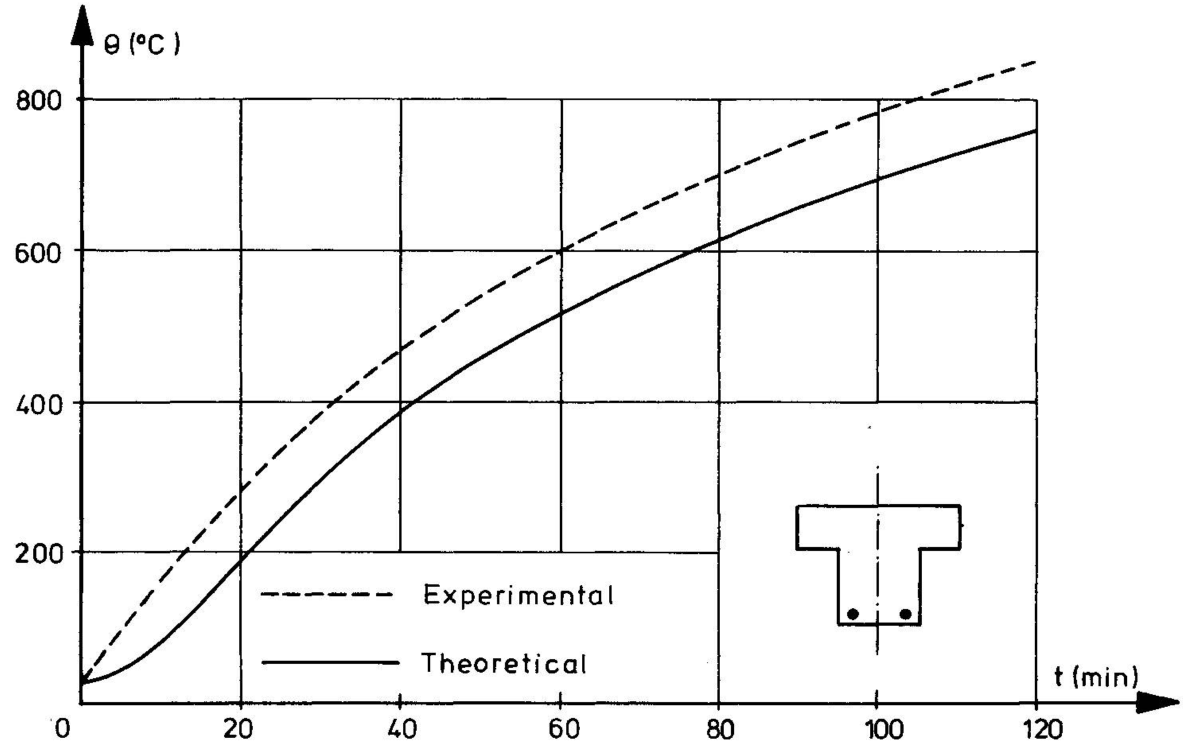

Figure 7: Temperature increase in the reinforcement

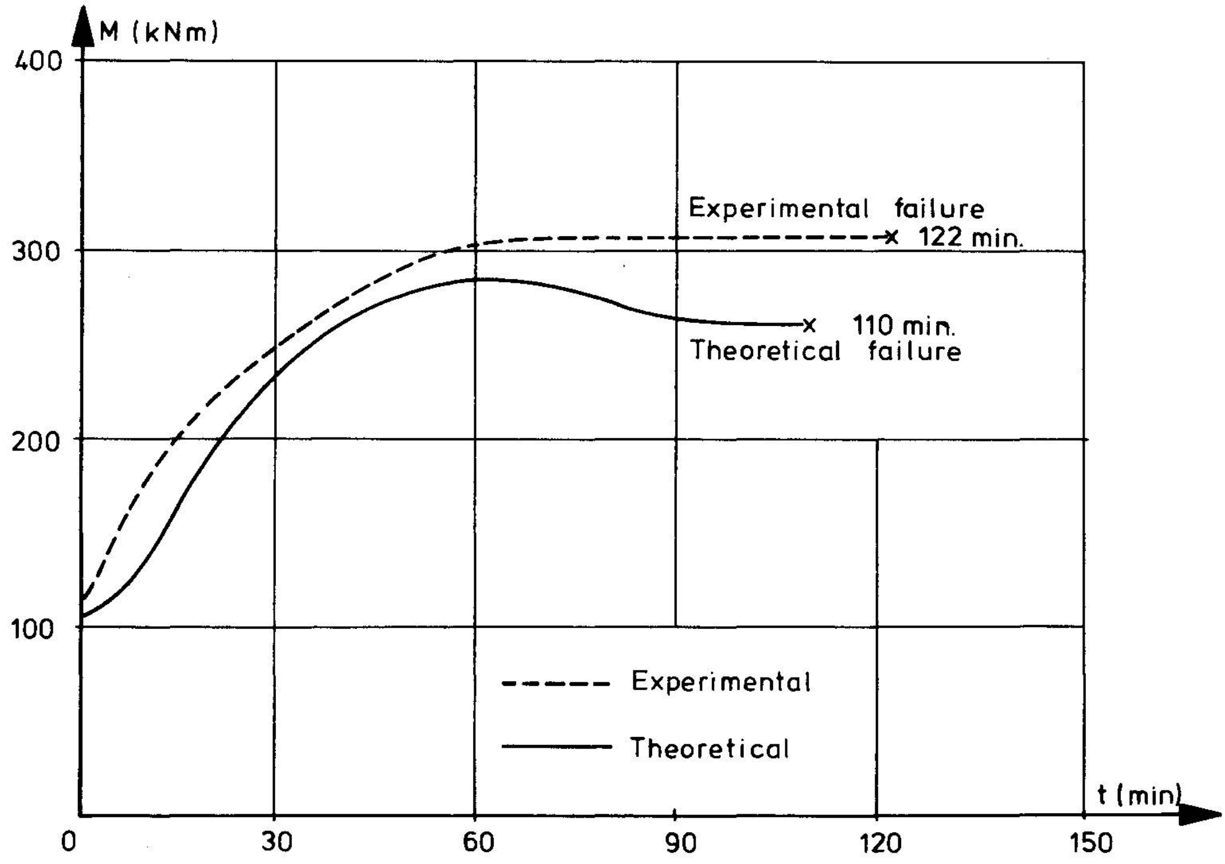

Figure 8 : Variation of the bending moment on the central support

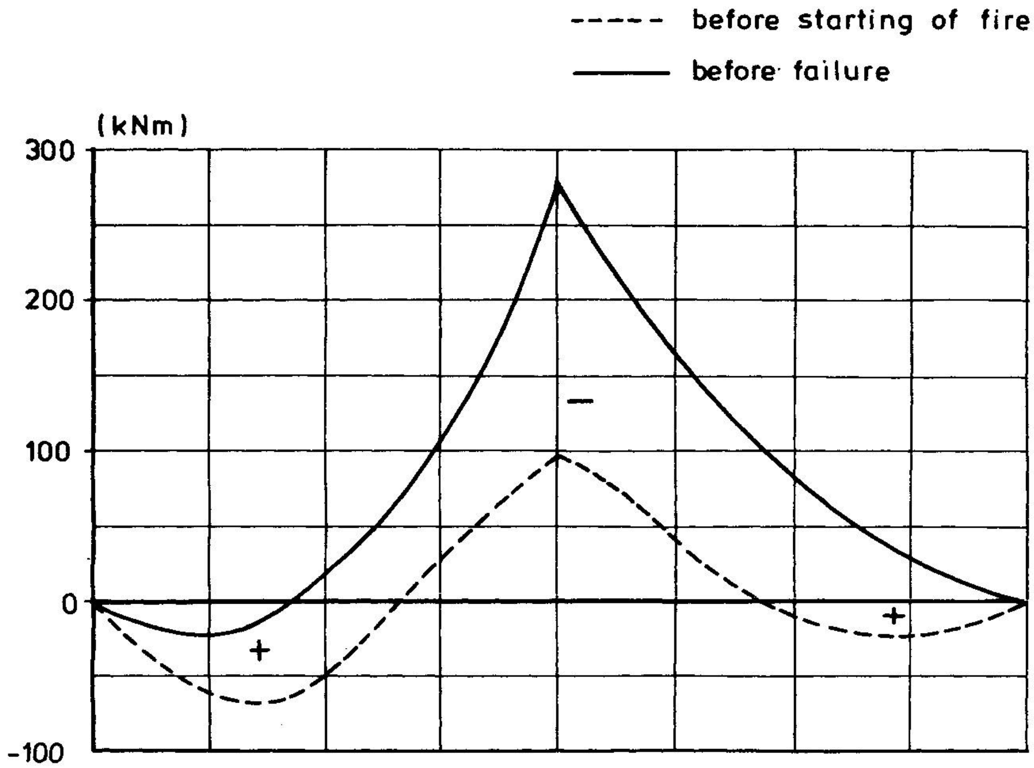

Figure 9 : Variation of the bending moment diagram in the beam

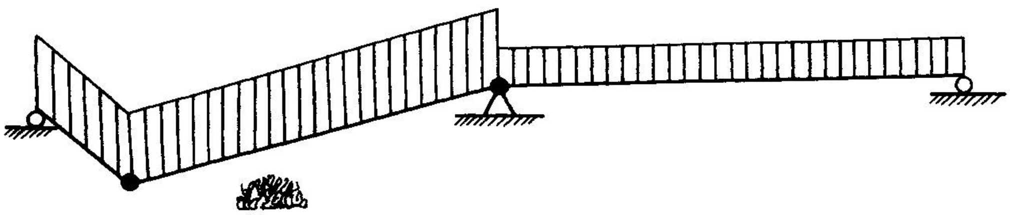

Figure 10 : Failure mechanism

The corresponding evolution of the bending moment diagram in the beam is represented on figure 9. This shows the important redistribution of internal forces in the element due to the thermal gradient. Before failure the diagram has only negative zones, except on the left hand side of the heated and most loaded span.

This explains the occurrence of failure after 2 hours. At this moment, the temperature in the tension reinforcement is greater than 700°C and the beam is no longer able to carry a small positive bending moment. For this reason a second plastic hinge forms where the positive moment is maximum and this leads to the failure mechanism represented on figure 10. The experimental observations confirm the formation of such a mechanism [5][9].

6. CONCLUSIONS

A numerical procedure for the analysis of the structural behaviour of reinforced concrete structures under fire conditions has been presented. It is based on the finite element method with subdivision of the cross section in a rectangular mesh. A particular formulation has been used for the evaluation of thermal effects in a discretized structure and for the step-by-step structural analysis under increasing temperatures. A good agreement has been obtained between theoretical and experimental results.

There is a growing need of theoretical results to provide adequate structural integrity during fires. It is of course not possible, and even not desirable, to replace all laboratory tests by simulation programs or theoretical formulations. Experiment remains essential to understand structural behaviour and to observe almost unpredictable phenomena. Nevertheless numerical procedures such as the method presented in this paper, along with experimental studies, provide a powerful tool for a more thorough understanding of the fire endurance of structures.

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