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DISCUSSION

Session 1, part 1: Modelling of Material Behaviour

Introductory Report by Bazant, U.S.A.

Braestrup (Denmark): Why should be the mesh size at least 3 times the aggregate size?

Bazant: I meant it should not be less. It can be larger (e.g. 5 times the aggregate size). However, if the mesh size is much larger, you need fracture mechanics in your computational model.

Paper by Vecchio/Collins, Canada

Gambarova (Italy): Looking at the crack patterns in your tests, most of the cracks are extended over the whole plate. Did you measure for these cracks the crack width and slip?

Collins: No, this would have taken too much time. We measured the local strains and occasionally the width of some characteristic cracks.

Bazant (U.S.A.): How closely did you achieve a uniform strain, and did you try to keep the stresses or the strains constant on the boundary?

Collins: We kept the stresses constant on the boundary. We tried to apply pure shear. The strains were surprisingly uniform as appeared from our measurements.

Blaauwendraad (The Netherlands): Is it basically a good point of view that you make a superposition of cracks in three directions, which in fact presupposes that the cracks form an orthogonal system?

Schnobrich (U.S.A.): It is stated in the paper and I do not like it.

General discussion

Blaauwendraad (The Netherlands): Is it possible to make some new arguments which put you to promote the approach of plasticity in reinforced concrete?

Braestrup (Denmark): The main argument is that it works; it is a simple model and it can be used as a basis for sound consistent design rules.

König (F.R.G.): Referring to fig. 4 and fig. 5 of prof. Reinhardt's paper, there must be an asymptotic line with increasing depth of the beams. The test results show this asymptotic line, but not your theory.

Reinhardt (The Netherlands): I looked for a theoretical background in order to explain the size effect. I started with a linear fracture mechanics approach, knowing that this is not optimal. In fig. 5 for beams with greater depths the dotted line represents a lower bound.

Because there are no experimental results available it cannot be proved whether the theory is correct or there exists an asymptote. More FEM-calculations are necessary in order to find out if this linear fracture mechanics approach is to be justified.

Schäfer (F.R.G.): In Stuttgart a very thick foundation slab, measuring 3,00 x 3,00 x 0,80 m, was tested recently and its punching load compared with others. It turned out that the relative ultimate shear stress is governed by the span-depth ratio rather than the absolute thickness. Is it possible that because of not equal span/depth-ratio's the tests mentioned in Reinhardt's paper cannot be compared?

Reinhardt (The Netherlands): I compared similar beams with the same shear span.

Braestrup: I refer to Section 4 of the Introductory Report of Prof. Bazant. I think this would be a very appropriate opportunity to try and clarify the relationship between your slip-free limit design and the classical approach, which you insist on calling frictionless design.
Assuming a finite coefficient of friction in the cracks, you arrive at this yield criterion:

$$\left[(N_x^s - N_x) - \beta_1 (N_y^s - N_y)\right] \left[(N_y^s - N_y) - \beta_1 (N_x^s - N_x)\right] = (2\,\beta_2 N_{xy})^2,$$

which is equation (31) of your paper. Here N_x^s and N_y^s are the required yield forces in the orthotropic reinforcement in the x- and y-directions, and N_x, N_y and N_{xy} are the applied membrane forces with regard to the x, y-system.
The parameters β_1 and β_2 are defined as:

$$\beta_1 = \{\tan(\tfrac{\pi}{4} - \tfrac{\beta}{2})\}^2 \;;\; \beta_2 = \tfrac{1}{2}\{\cos(\tfrac{\pi}{4} - \tfrac{\beta}{2})\}^{-2}$$

which is equation (4) of Ref. [68];
where β is the friction angle, i.e. the angle between the cracks and the displacement rate vector.
In the Introductory Paper the definition of β_2 is slightly different, but that must be a misprint.
These expressions are from the paper by Bazant, Tsubaki and Belytschko (Ref. [68]).
Now in the classical approach the yield criterion is simply this:

$(N_x^s - N_x)(N_y^s - N_y) = N_{xy}^2$, if equal notations are used.

In your paper you state that the slip-free design equals the classical case if we put $\beta = \beta_1 = \beta_2 = 0$. This is obviously not correct.

For one thing, $\beta = 0$ will make neither β_1 nor $\beta_2 = 0$.
No, the fact is that we obtain the classical criterion for $\beta = \frac{\pi}{2}$, which gives $\beta_1 = 0$ and $\beta_2 = \frac{1}{2}$.

Thus the classical approach is certainly not frictionless; on the contrary, the coefficient of friction is infinite.

Or rather, friction is not an issue at all, because the classical analysis, which is only valid for underreinforced elements, assumes that the collapse cracks form in the sections of least resistance, which means that the deformation rate is always perpendicular to the discontinuity.
Generally the structure will have more cracks in other directions as well, which are formed during earlier stages of the loading history.
But it is assumed that no tangential slip occurs in these, i.e. that friction or shear transfer is unlimited, or at least sufficient to prevent slip.
It is important to note that the tensile strength of concrete is neglected, i.e. no shear can be transferred on any section unless there is a compressive stress delivered by the reinforcement.
This is described by the Coulomb criterion with a zero tension cut-off (see figure 1).

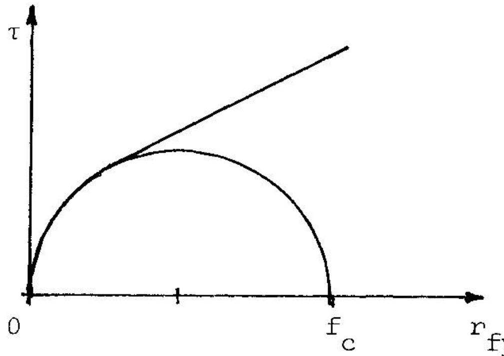


Fig. 1 Modified Coulomb criterion

r_{fy} = compressive strength delivered by reinforcement

Here τ is the shear strength and r_{fy} is the compressive stress, r being the reinforcement ratio.
The slope of the curve may be interpreted as the coefficient of friction, which is infinite at the origin, eventually decreasing to 0,75.
This description is very similar to the classical shear-friction theory, only slightly more sophisticated.
As mentioned, the classical analysis only considered the underreinforced case, where there is always a possibility for the structure to avoid slip in the discontinuities by failing through simple opening of the collapse cracks.
Lately, the analysis has been extended to cases where the failure is constrained by strong reinforcement or for other reasons, in such a way that the optimal failure mechnism involves tangential slip in the discontinuities.
Then this yield criterion is no longer valid and the dissipation is calculated using the modified Coulomb criterion.

To summarize:

The classical limit design does not assume no friction in cracks; on the contrary, it assumes no slip, because it consumes less enery for the structure to let the reinforcement yield, rather than overcome the resistance of the concrete to tangential deformations.
If the reinforcement becomes sufficiently strong, this is no longer true, and tangential slip must be considered.
This is done by the modified Coulomb criterion, which appears to give an adequate description of the shear strength of concrete.

Bazant: Although time does not permit me to deal with your comments in full detail, I wish to summarize at least the following.
Your remarks are useful, but I cannot quite agree with them.
You seem to have a different concept of friction. Apparantly, if the yield surface depends on the mean stress σ, you call that friction.

A proper definition of frictional force, in the context with thermodynamics, is different: it is a force which affects the response, but does no work in the response (see, e.g., Hill and Rice, VMPS, early 1970's, and Bazant, IASS, 1980). Thus, it is the salient feature of friction that it cannot be derived from a work expression by differentiation, i.e., no potential exists. In this sense, the slip-dilatancy model Tsubaki and I developed is a frictional model, which yours is not because you assume normality!

Now, within your point of view, we were well aware that the classical design corresponds to infinite friction coefficient on the crack and we stated so in our first paper (Bazant-Tsubaki, ASCE, Struct.Div. 1979, p.327).

As a consequence of this fact, however, the critical crack (which you curiously call "discontinuity") and actually the only possible crack is that on which there is no slip (no tangential displacement), and since there is no slip there is no shear stress on the crack, i.e. no friction.
So, classical design is a design based on the assumption of no shear stress (no friction) on the critical crack.
Thus I just cannot see how anyone can object to the word "frictionless".
Your arguments are based strictly on the assumption of a plastic yield surface and the type of the yield surface you need to assume to get these results. But that is clearly secondary.

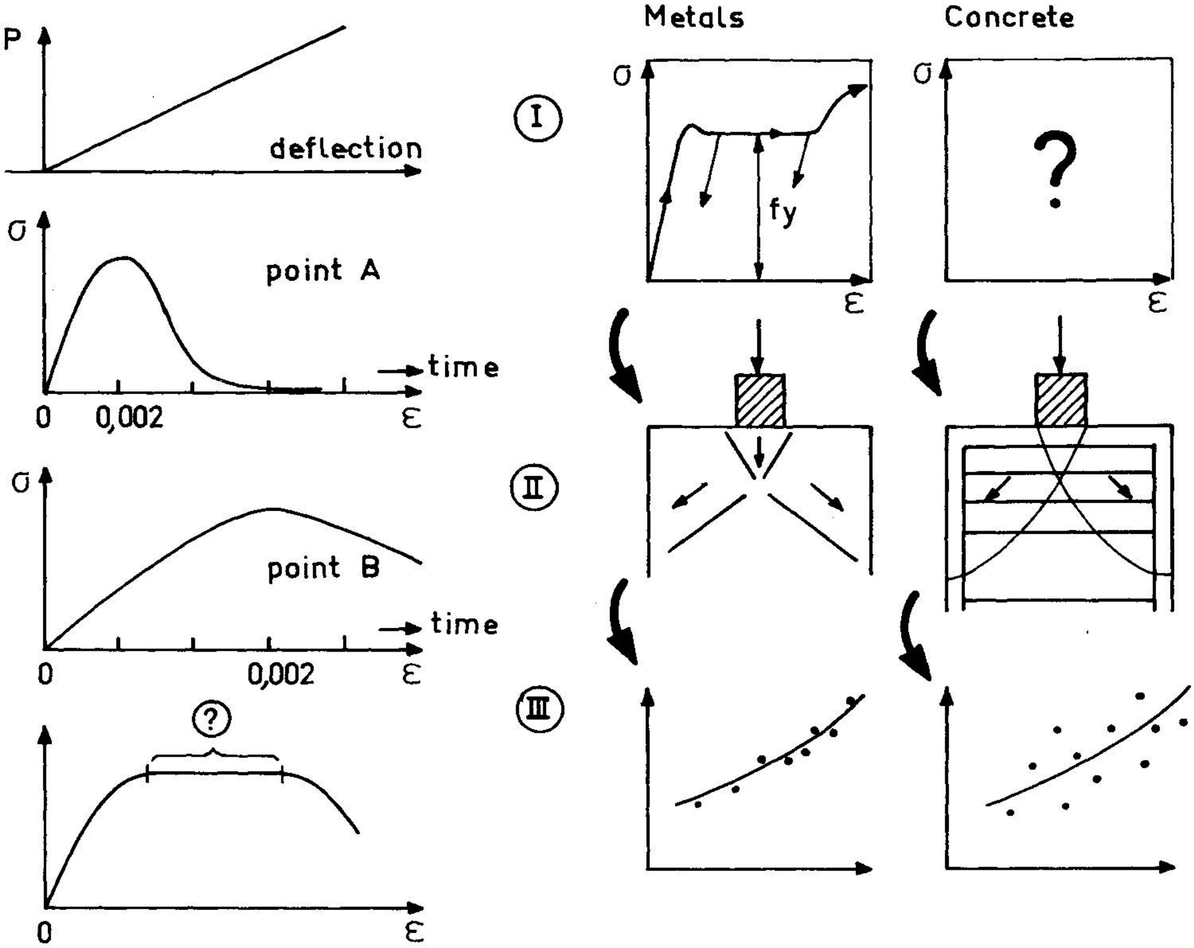
Anyhow, I do not favour predicting the behaviour of concrete on the basis of plasticity, even though plasticity solutions may be adapted (after the fact) to fit reasonably well certain test data for concrete structures.

Prof. Bazant outlined some conceptual problems with applying plasticity to concrete. This discussion can be summarized as follows (see figure).

a. The lack of ductility, i.e. lack of a horizontal plateau in the σ-ε-diagram, means that all points of a postulated collapse surface cannot reach the maximum stress (or any stress value known in advance) simultaneously. So we do not a priori know the stresses at the failure surface at the moment of collapse.

b. The literature on plasticity of concrete strikes us by one difference from the literature on plasticity of metals. In the latter, one always starts with the σ-ε-relation, and being satisfied that it is close enough to ideal plasticity, one uses plasticity to solve a structural problem and compares the calculation results with tests.
In the literature on plasticity of concrete, people also compare the results of the analysis with tests of structures (or adjust them to fit the tests of structures), but suspiciously omit the first stage; they never start by showing the stress-strain relation. If they did, there would be of course no resemblance of ideal plasticity.

Problem of Ductility of Concrete



Concrete is <u>not</u> plastic