Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 34 (1981)

Artikel: Plain concrete under load - a new interpretation

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DOI: https://doi.org/10.5169/seals-26884

Plain Concrete Under Load - A New Interpretation

Béton non-armé sous charge - Une nouvelle interprétation

Unbewerhter Beton unter Belastung - Eine neue Interpretation

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SUMMARY

It is propesed that plain concrete in compression suffers a complete loss of load carrying capacity when ultimate strength is exceeded. This hypothesis, which contrasts with the widely held view of post-ultimate behaviour, is supported by experimental evidence and the results of analyses of structural forms. The results of these analyses indicate that the response of the structures investigated is independent of the post-ultimate behaviour of concrete under compressive stress states. It is shown that the large deformations attributed to the post-ultimate characteristics of concrete as a material may be associated with triaxial stress states existing in any structure, but which are usually ignored for purposes of simplicity.

RÉSUMÉ

On avance l'hypothèse qu'en compression, le béton non armé perd toute sa capacité de charge dès que le niveau de résistance ultime est atteint. Bien qu'elle s'inscrive en marge du concept généralement accepté de comportement en régime post-ultime, cette hypothèse réflète néanmoins bien certains résultats d'essais et d'analyse de quelques structures. Les résultats d'analyse suggèrent que le mode d'action des différentes structures soumises à l'étude soit indépendant du régime post-ultime du comportement typique du béton non armé chargé en compression. Les déformations importantes, qui étaient jusqu' à maintenant considérées comme caractéristiques du régime post-ultime du béton, seraient plutôt attribuables à des champs de contraintes triaxiales qui existent dans toute structure, mais qui sont généralement négligés, par souci de simplicité.

ZUSAMMENFASSUNG

Es wird vorgeschlagen, dass unbewehrter Beton die ganze Festigkeit verliert, sobald die Grenzbeanspruchung überschritten wird. Diese Hypothese, die im Wilderspruch zur weitverbreiteten Ansicht des überkritischen Tragverhaltens steht, wurde durch Versuche und analytische Berechnungen erhärtet. Die Resultate dieser Berechnungen weisen darauf hin, dass das Verhalten der untersuchten Tragwerke unabhängig vom überkritischen Bereich der druckbeanspruchten Betonzonen ist. Es wurde gezeigt, dass die g.ossen, den überkritischen Eigenschaften des Betons zugeschriebenen Deformationen mit den dreiachsigen Spannungszuständen in Verbindung gebracht werden können, die in jedem Tragwerk vorhanden sind, aber aus Gründen der Vereinfachung oft vernachlässigt werden.

1. INTRODUCTION

Most constitutive models of concrete behaviour under compressive stress states published to date predict that the deformational response of the material in the direction of the maximum principal compressive stress exhibits a trend similar to that given in Fig. 1 which shows a stress-strain relationship consisting of an ascending and a gradually descending portion. Such predictions result from deriving the models on the basis of the widely held view that, beyond ultimate strength, the load carrying capacity of concrete under a compressive load is progressively reduced with increasing deformation. This view has been reinforced by experimental evidence which has indicated that the deformational response of concrete under uniaxial compression applied by a stiff testing machine is described by a stress-strain curve which exhibits the characteristics of the relationship shown in Fig. 1. Such stress-strain curves have been used to describe the behaviour of concrete in the compression zone of reinforced concrete structural members such as beams, slabs, etc. [1].

However, the validity of the stress-strain data for concrete, as for other materials, depends on the testing techniques adopted and this fact has been recognised by many research workers [2]. As a result techniques have been developed in order to minimise restraint effects due to the interaction between specimen and testing device. However, it is shown in the following that, even when minimal, these secondary effects have a significant effect on material behaviour for stress levels in the region of ultimate strength and beyond. In view of this it is postulated that, if restraint effects could be eliminated completely, concrete would exhibit a complete loss of load carrying capacity when ultimate strength is exceeded. Experimental data which have been produced by tests in which secondary testing effects have been effectively eliminated support this hypothesis.

It is shown that the large deformations attributed to post-ultimate strength stress-strain characteristics could be due to the triaxial state of stress which exists in any structure under load but which is usually ignored for the purposes of simplicity. This consideration is supported by the results obtained by a finite element analysis of plain concrete structural forms under concentrations of load which suggest that the overall behaviour of the structural forms investigated is independent of the post-ultimate strength behaviour of concrete under a compressive state of stress.

2. TESTING METHODS AND STRESS-STRAIN BEHAVIOUR

It is generally accepted that the fundamental deformational response of concrete under increasing stress is realistically described by the stress-strain behaviour measured for laboratory specimens provided definable states of stress are induced. The application of such states of stress is achieved by using testing techniques which have been developed so as to minimise restraint effects due to the interaction between specimen and testing device. The most significant of these effects is the frictional restraint which develops at the specimen-platen interfaces under increasing load.

The use of cylindrical or prismatic specimens with a length to width ratio of between approximately 2 and 2.5 is one method which has been adopted in order to minimise the effect of the above frictional restraint on the specimen behaviour [3]. It has been found by experiment that, while the end zones of such specimens under uniaxial compression are subjected to an indefinable triaxial compressive state of stress caused by this frictional restraint, the central zone of the specimens is effectively free from such effects [3]. Other methods used to minimise frictional effects involve the use of various types of anti-friction

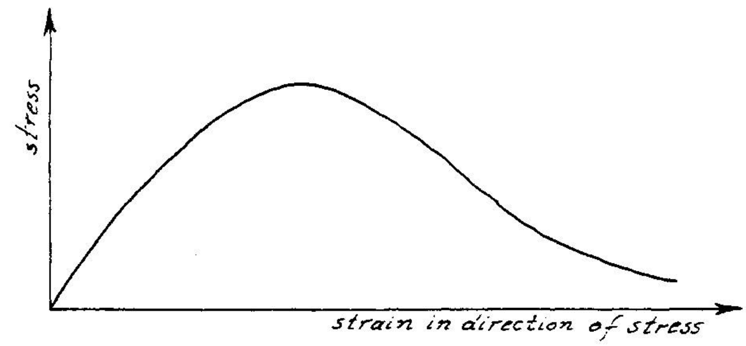


Fig. 1 Generally assumed form of stress-strain relationship for concrete under compressive stress

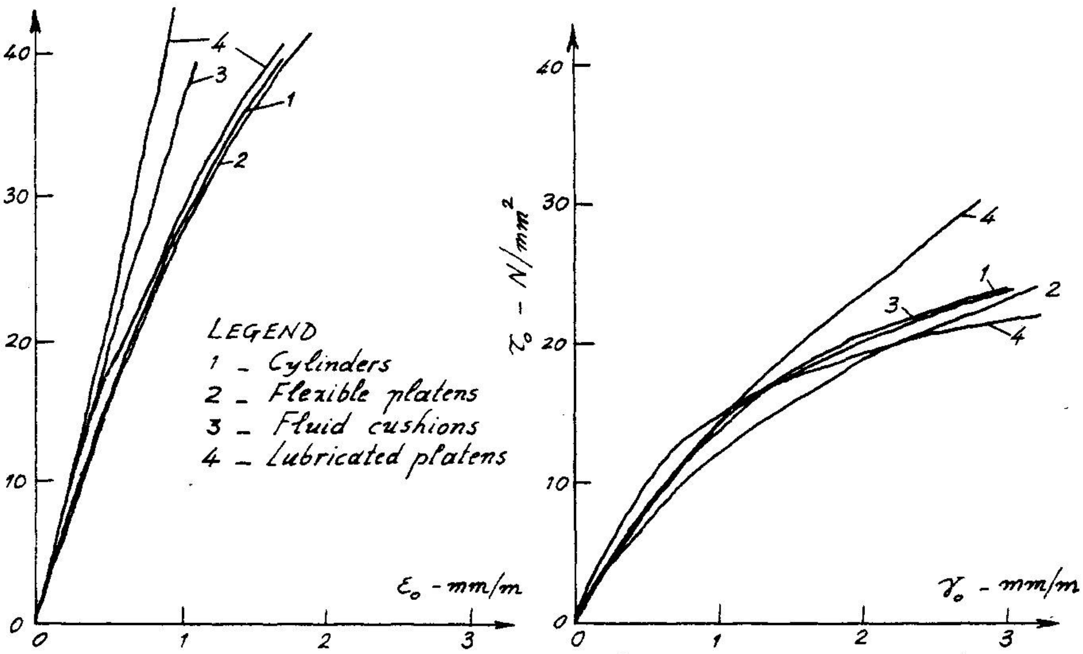


Fig. 2 Volumetric (σ_o-ε_o) and deviatoric (τ_o-γ_o) stress-strain relationships for concrete under triaxial compression obtained from tests using various friction reducing techniques

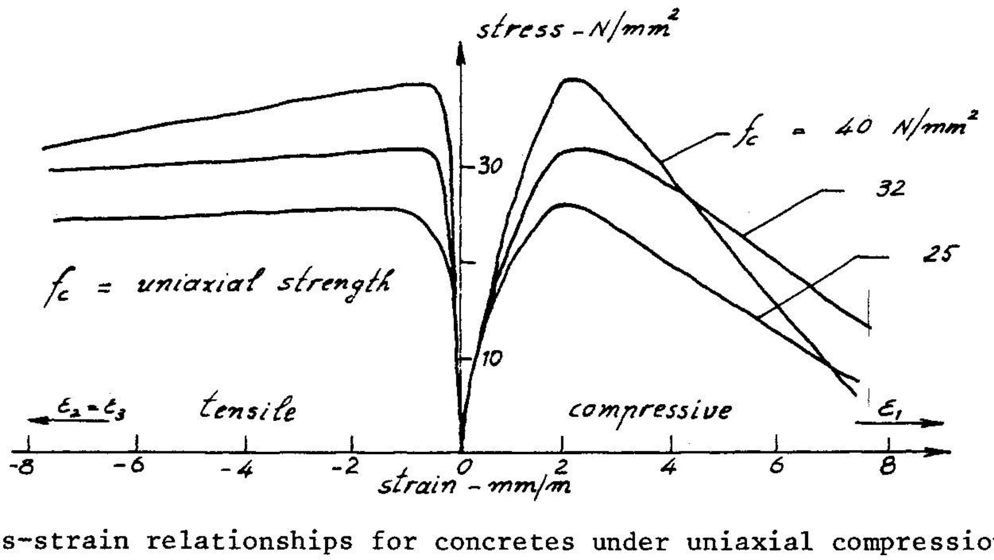


Fig. 3 Stress-strain relationships for concretes under uniaxial compression obtained from tests on cylinders

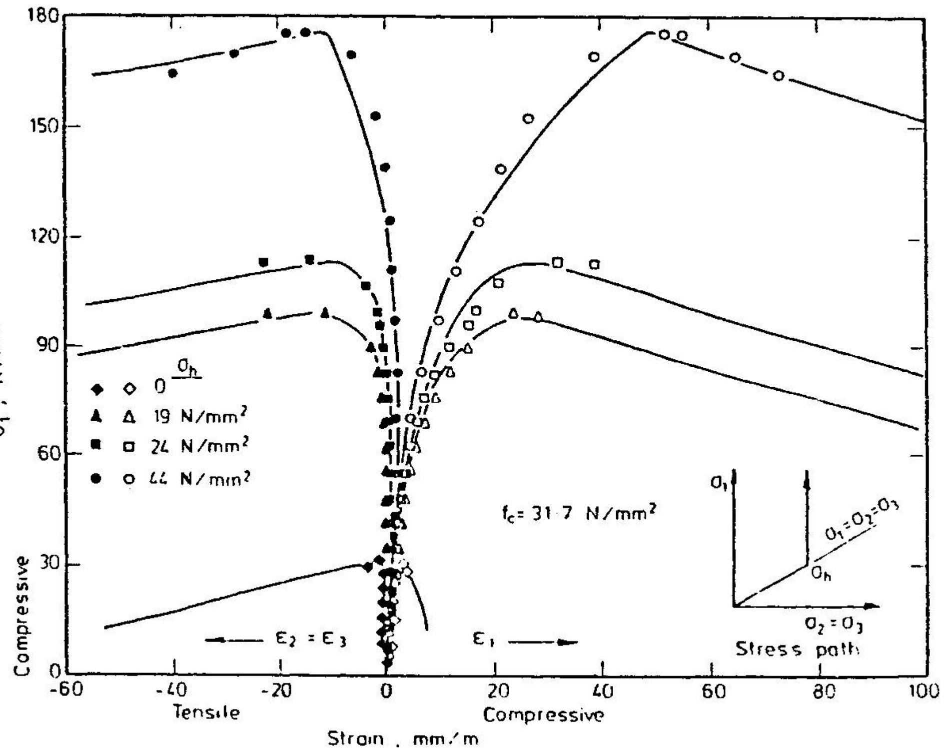


Fig. 4 Stress-strain relationships for concrete under various compressive states of stress obtained from triaxial tests on cylinders

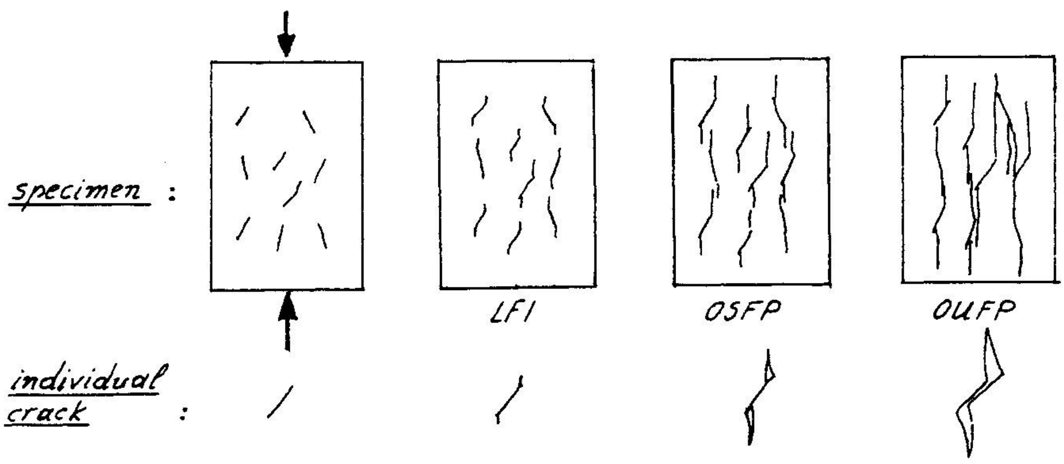


Fig. 5 Stages in the process of crack extension and propagation for concrete under compressive stress

pads [4], brush platens [5], flexible platens [6], fluid cushions [2], etc.

The development of the above testing methods has been based on the assumption that the mechanical properties of concrete during its whole loading history are qualitatively comparable to the mechanical properties of a continuous material. For example, loading devices such as brush and flexible platens are considered to induce a negligible frictional restraint at the specimen-platen interfaces when designed to allow displacements in the direction orthogonal to loading to be compatible with tensile strains in concrete calculated on the basis of Poisson's ratio values up to 0.5 which is the maximum value for a continuous material. As long as this condition is satisfied, most of the above methods have been found to produce stress-strain data which correlate very closely [7] (see Figure 2). It should be noted, however, that most of the above data describe material behaviour under stress levels approaching, but not exceeding, ultimate strength.

Complete stress-strain relationships for concrete (i.e. relationships with both ascending and descending portions) have been obtained mainly from tests on cylinders under uniaxial compression [8] (see Figure 3). There is also some experimental evidence which suggests that similar trends of behaviour are exhibited under triaxial compression [9] (see Figure 4). The description of the specimen behaviour beyond ultimate strength has been achieved by loading the specimens at a constant rate of displacement through a 'stiff' testing machine either by using a loading system capable of releasing rapidly any load in excess of that which can be sustained by the specimen at any time [10] or by loading a steel specimen in parallel with the concrete specimen in a manner such that, as the load carrying capacity of concrete is reduced, the concrete-steel system transfers the excess load from the concrete to the steel specimen to maintain the internal equilibrium of the overall system [11].

It is interesting to note in both Figures 3 and 4 that, for stress levels beyond a level close to ultimate strength, the tensile strain increases at a rate very much higher than that of the compressive strain. Such behaviour can only be attributed to the effect on deformation of void formation caused by the fracture processes of the specimens [12]. Thus it is difficult, if not impossible, to explain the specimen behaviour on the basis of continuum mechanics concepts. Since the development of the testing techniques used to minimise secondary testing procedure effects has been based on assumptions which imply mechanical properties for concrete quantitatively comparable to those of a continuous material, the effectiveness of the testing methods used to induce definable states of stress in specimens for the whole length of a loading path is questionable. The effect, therefore, of the frictional restraint on specimen behaviour must be reconsidered when using all current data.

3. EFFECT OF FRICTIONAL RESTRAINT ON CONCRETE BEHAVIOUR

The nonlinear deformational behaviour of concrete under increasing stress is dictated by internal fracture processes. Therefore, any effect on the deformation due to frictional restraints at the loaded boundaries will cause a modification of the fracture processes.

3.1 Fracture Processes of Concrete

Previous research work [13] has indicated that the fracture processes of concrete under increasing stress take the form of crack extension, due to initiation of crack branches, followed by stable propagation of these branches which eventually becomes unstable and leads to complete disruption. Crack extension and propagation occur in the direction of the maximum principal compressive stress in order to relieve high predominantly tensile stress concentrations which exist near the crack tips. During the above fracture processes voids are created within the body of the material.

Four stages in the process of crack extension and propagation have been identified under increasing stress [14]. The boundaries to those stages have been termed Local Fraction Initiation (LFI), Onset of Stable Fracture Propagation (OSFP) and Onset of Unstable Fracture Propagation (OUFP) and their variation in stress and strain space have been established by experiment. A schematic representation of the stages of crack extension and propagation is shown in Figure 5. The variation of LFI, OSFP, OUFP and ultimate strength in stress and strain space forms the failure envelopes shown in Figure 6.

3.2 Effect of Frictional Restraint on Fracture Processes

A concrete cylinder subjected to uniaxial compression can be considered as exhibiting 2 principal zones of behaviour as shown in Figure 7. Material in the central zone is generally accepted to be subjected to a near-uniaxial compressive stress state whereas the end zones are subjected to a complex and indefinable compressive stress state caused by the frictional restraint which prevents the specimen from expanding in the direction of the specimen-platen interfaces.

As indicated by the failure envelopes of Figure 6, cracking of the concrete cylinder will initiate in the central zone which is subjected to near-uniaxial compression. For stress levels increasing up to OUFP, cracking occurs in the microscopic level in localised regions within the material in this zone [14]. As a result, the overall deformation response of the specimen is comparable to the deformation response of a continuous material and the effect of the boundary frictional restraint is confined within the end zones of the cylinder.

When OUFP is exceeded cracking becomes continuous and propagates towards the end zones. As discussed previously (see section 3.1) cracking propagates in the direction of the maximum principal stress trajectories which, in the central zone of the specimen, coincide with the direction of the applied compressive stress. Outside this zone, both stress trajectories and propagating cracks deviate from the direction of the initial crack branches due to the indefinable multiaxial state of stress which exists.

The above change in the orientation of the crack propagation path increases the energy required for crack extension and a higher load is required for the fracture process to continue. This will result in an overestimate of both the strength and the strain in the direction of the applied load. Furthermore, the above delay in the fracture process will also reduce the rate of void formation which affects predominantly deformation in the direction orthogonal to the crack propagation path. This may result in a serious underestimate of the lateral tensile strain of the cylinder.

As the applied load increases to a level very close to the maximum load that can be sustained by the specimen (ultimate strength), cracking spreads within the end zones and propagates towards the specimen-platen interfaces. It becomes apparent at this stage that the only reason for the central zone responding as a unit under the applied load is the delay in the fracture process within the end zones caused by the change in orientation of the crack propagation path. This suggests, therefore, that if the state of stress was the same throughout the specimen, no such change in the orientation of the crack propagation path would occur and the specimen would completely collapse due to rapid and unstable fracture propagation. A schematic representation of the fracture processes of concrete specimens (with and without frictional restraint) under increasing load is given in Figure 8.

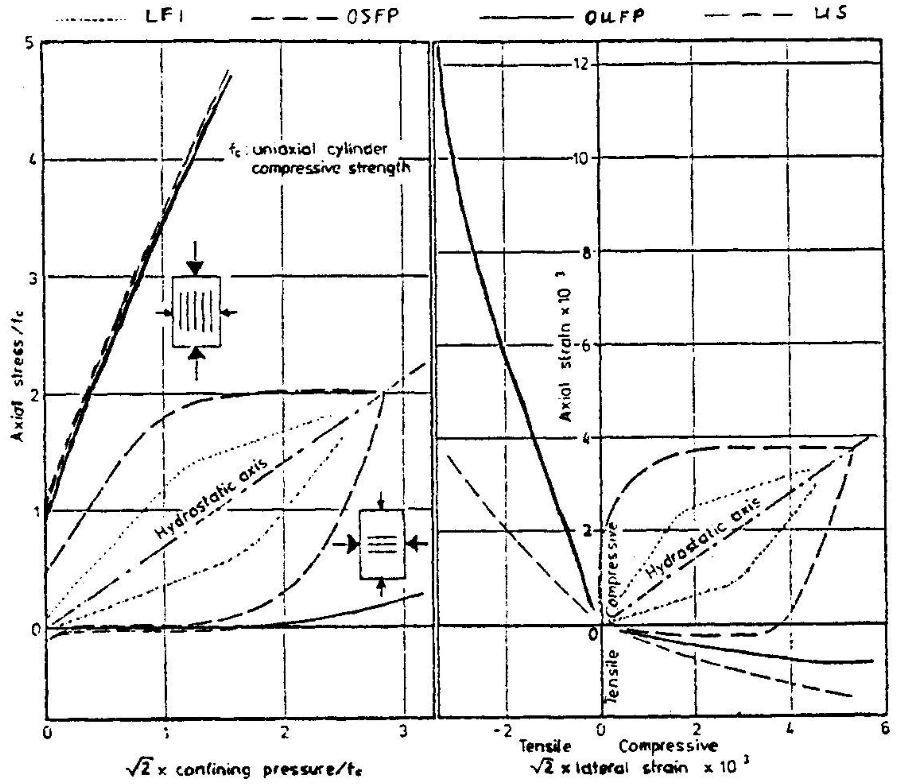


Fig. 6 Failure envelopes for concrete as derived from triaxial tests data for cylinder

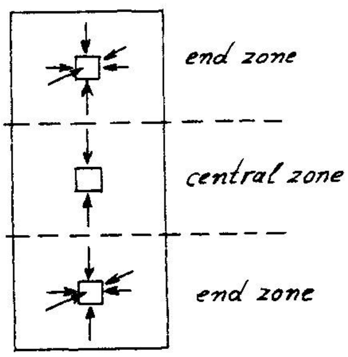
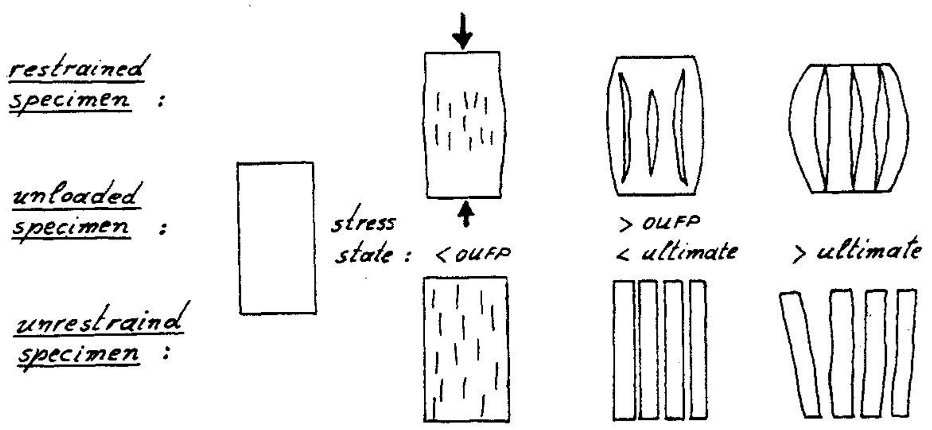


Fig. 7 Schematic representation of the effect of boundary frictional restraint on the state of stress within cylinders

Fig. 8 Schematic representation of the fracture processes for restrained and unrestrained concrete specimens under increasing compressive stress



3.3 Effect of Frictional Restraint on Deformation

On the basis of the above considerations the true deformational behaviour of concrete under compression with no secondary effects may realistically be described by the stress-strain relationships proposed in Figure 9. These are characterised by a complete loss of load carrying capacity as soon as the ultimate strength level is exceeded. For comparison the figure also includes a typical form of experimental relationship obtained under monotonically applied compression for concrete cylinders with a height to diameter ratio of 2.5 loaded through rigid platens.

For stress levels up to OUFP, proposed and experimental relationships coincide since, for such levels, the specimen behaviour has been shown previously (see section 3.2) to be essentially unaffected by the frictional restraint which exists at the specimen-platen interfaces. For stress levels between OUFP and ultimate strength, there will be a significant deviation of the two relationships due to the frictional restraint on deformation. The proposal that complete loss of load carrying capacity will occur when ultimate strength is attained, is considered to provide a realistic description of the post-ultimate strength behaviour of an 'unrestrained' material.

A measure of the effect of the frictional restraint on the deformational behaviour of the specimen is given by the deviation of the proposed from the experimental relationships shown in Figure 9.

4. EVIDENCE FOR SUGGESTED NEAR- AND POST-ULTIMATE BEHAVIOUR

An unequivocal experimental proof of the validity of the trends of concrete behaviour proposed in section 3.3 can only be obtained by using testing techniques which eliminate completely any frictional restraint on the specimen-platen interfaces. However, on the basis of the considerations discussed in section 3, such a proof is unlikely to be obtained by using any of the existing testing techniques in isolation, although the fluid cushion technique [2] appears in theory to eliminate frictional restraints. An investigation of the post-ultimate behaviour of concrete under uniaxial compression could be based on a comparative study of the trends of post-ultimate response of concrete specimens tested by using testing methods employing different techniques to reduce friction. Such an investigation forms part of an ongoing programme of research being carried out by the Concrete Materials Research Group (CMRG), which is concerned with the derivation of a constitutive model of concrete and its use with computer-based techniques of structural analysis.

There is, however, published experimental evidence which suggests that under certain types of triaxial compressive stress states concrete suffers a complete loss of load carrying capacity when the ultimate strength is exceeded [9]. Furthermore, an analytical study of the fracture processes of plain concrete structural forms subjected to a wide range of boundary conditions has produced some interesting results regarding the transfer of load from elements which suffer a partial loss of load carrying capacity due to cracking to elements subjected to stress states below ultimate strength [15,16]. These results indicate that the overall behaviour of the structural forms investigated is <u>independent</u> of the post-ultimate strength behaviour of the material under compressive states of stress. A brief description of the above experimental and analytical information is given in the following.

4.1 Information from Experiments

The experimental data which are discussed in this section have been obtained during a comprehensive programme of research into the behaviour of concrete under multiaxial stress carried out by the CMRG. The testing techniques used in this programme have been fully described elsewhere [17] and the obtained experimental data have formed the basis of a number of publications concerned with the fracture processes [18,19], strength [20], and deformational response [9,12,21, 22] of concrete under generalised stress.

4.1.1. Test Procedure

Concrete cylinders with a height to diameter ratio of 2.5 have been subjected to an axial compression (σ_α) and a lateral confining pressure (σ_c) combined in such a way that the state of stress within the specimen has been always either triaxial 'compression' ($\sigma_\alpha > \sigma_c$) or triaxial 'extension ($\sigma_c > \sigma_\alpha$)(see Figure 10). The axial compression has been applied by using a loading method similar to that described in section 2, whereas the confining pressure has been hydrostatic resulting in the curved surface of the specimen being essentially friction-free. For both triaxial 'compression' and triaxial 'extension' tests the specimens were initially subjected to a given hydrostatic pressure and then the axial compression was either increased (triaxial 'compression') or decreased (triaxial 'extension') to failure (see Figure 10).

4.1.2. Test Results

Typical stress-strain relationships obtained from the above tests are shown in Figure 11 which indicates that while under triaxial 'compression' concrete exhibits a gradual reduction of load carrying capacity for stress levels beyond ultimate strength, under triaxial 'extension' concrete suffers an immediate and complete loss of load carrying capacity. This difference in material behaviour is considered to reflect the effect of frictional restraint on the fracture processes of the specimens. According to the fracture mechanism of concrete discussed in section 3.1, under triaxial 'compression' crack propagation occurs in the axial direction and thus, when cracking spreads in the end zones of the specimen, the frictional restraint will affect the specimen behaviour as discussed in section 3.2. On the other hand, under triaxial 'extension' crack propagation occurs in the lateral direction and thus the fracture processes which take place in the central zone of the specimen are not affected by the frictional restraint existing at the specimen-platen interfaces.

However, in contrast to the complete loss of load carrying capacity exhibited by concrete when ultimate strength is exceeded in an 'extension' test, experimental stress-strain curves obtained for concrete under equal biaxial compression applied through brush platens, which is a particular case of triaxial 'extension' with $\sigma_\alpha = 0$, exhibit a gradually descending portion [5] similar to that obtained from triaxial 'compression' tests (see Figure 12). The difference between the two types of tests are considered to reflect frictional restraint effects. It should be noted, however, that in contrast to the sequential loading path used for the triaxial tests, the above biaxial tests have been performed by using a proportional loading path. Since it has been shown that concrete behaviour is essentially independent of stress path effects for stress levels up to OUFP only [21], the dependence of the material behaviour on stress path for higher stress levels may also reflect the effect of the frictional restraint.

In view of the above, the complete loss of load carrying capacity when ultimate strength is exceeded in a triaxial 'extension' test is considered to be characteristic of material behaviour under <u>any</u> state of stress.

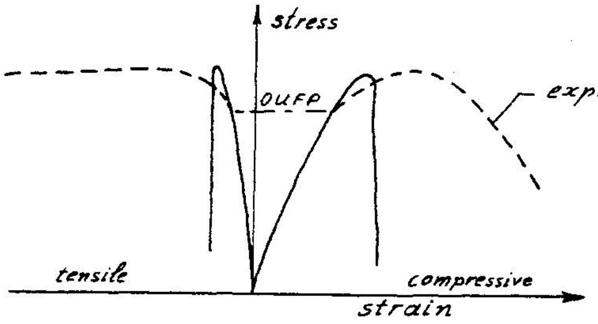


Fig. 9 Proposed form of stress-strain relationships for concrete under compressive stress

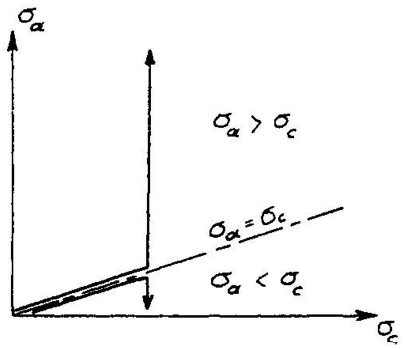


Fig. 10 Stress path used in triaxial tests

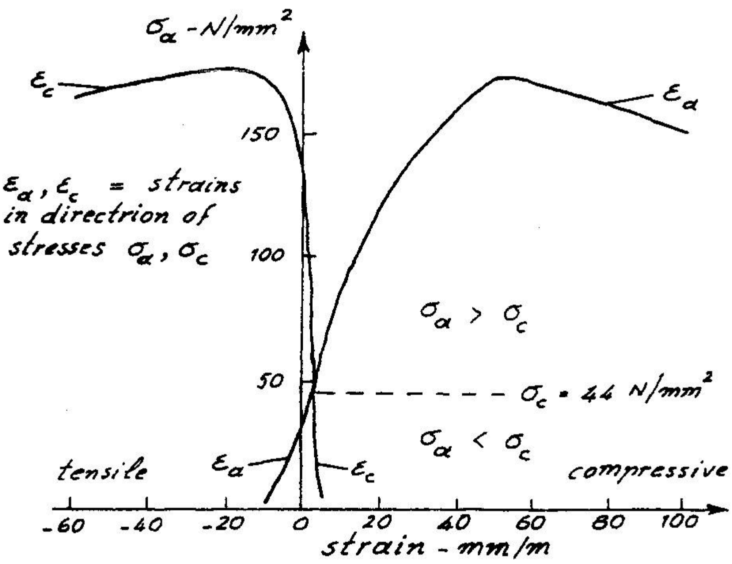


Fig. 11 Typical stress-strain relationships obtained from triaxial 'compression' and 'extension' tests on cylinders

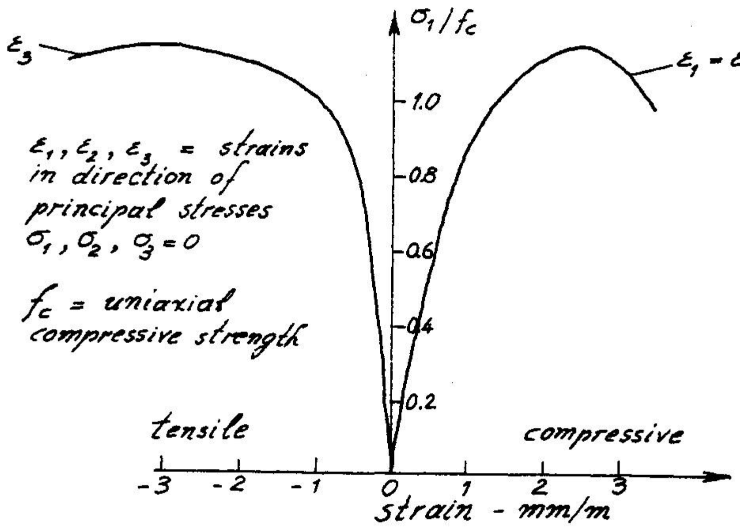


Fig. 12 Typical stress-strain relationships for concrete under equal biaxial compression ($\sigma_1 = \sigma_2$) obtained from tests using 'brush' platens

4.2 Information from Analysis

The analytical data which are discussed in this section have been obtained as part of an investigation by the CMRG into the use of constitutive models of concrete when coupled with computer-based methods of analysis. Nonlinear finite element techniques incorporating the constitutive model of concrete behaviour described in references [12] and [20] have been used to analyse plain concrete structural forms under concentrations of load induced by a wide range of boundary conditions (see Figure 13). The analysis has been found to yield realistic predictions of structural response which correlate very closely with published experimental data. The following is concerned only with a brief description of the fracture mechanism of the structural forms investigated predicted by the analysis. A full description of the results obtained has been given elsewhere [15,16].

4.2.1. Constitutive Model

The constitutive model used has been devised so as to describe the effect of the fracture processes of concrete on the material deformation [12]. Its formulation has been based on an analysis of triaxial experimental data [22-24] and as a result it suffers, like any other model, from the inherent disadvantage that it may describe specimen rather than material behaviour. However, unlike other models, it has been expressed in a modular form which allows improvements to be made in the light of any new information regarding the material behaviour without changing the theoretical basis of the model.

The above model describes completely the deformational behaviour of concrete under generalised stress increasing up to and, for 'compressive' states of stress, beyond ultimate strength. When the ultimate strength of concrete under a state of stress with at least one principal stress component tensile is exceeded, the maximum principal tensile stress is set to zero and the material behaviour is defined by the model on the basis of the principal stresses in the orthogonal directions.

4.2.2. Results of Analysis

It has been very interesting to find that for all cases investigated collapse of the structure occurs without the compressive strength of concrete being exceeded anywhere within the structure. The analysis predicts cracking will occur in regions subjected to a state of stress with at least one of the principal stress components tensile. For the cases investigated, the most critical state of stress is that which causes cracking to occur in the region marked B in Figure 14a. With increasing load, this cracking only propagates into other regions subjected to similar states of stress, i.e. with at least one of the principal stress components tensile, up to the formation of a crack pattern corresponding to the collapse stage (see Figure 14b).

Figure 14 also shows that region A, which is subjected to a wholly compressive state of stress, reduces in size as the applied load increases above the level which causes crack initiation. This is due to stress redistribution which transforms the state of stress at the periphery of region A from a wholly compressive state of stress to a state of stress in which at least one of the principal stress components is tensile. When the strength of concrete under this latter state of stress is exceeded cracking occurs and the size of region A is further reduced (see Figure 14b). In all cases investigated collapse of the structure occurs before the strength of concrete in region A is exceeded.

If the fracture process described above is typical of that of any concrete structure, then the post-ultimate strength behaviour of concrete (i.e. the

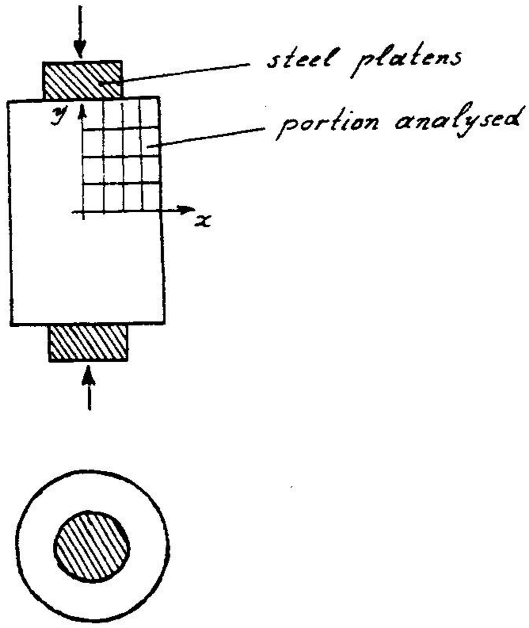


Fig. 13 Typical structural unit investigated

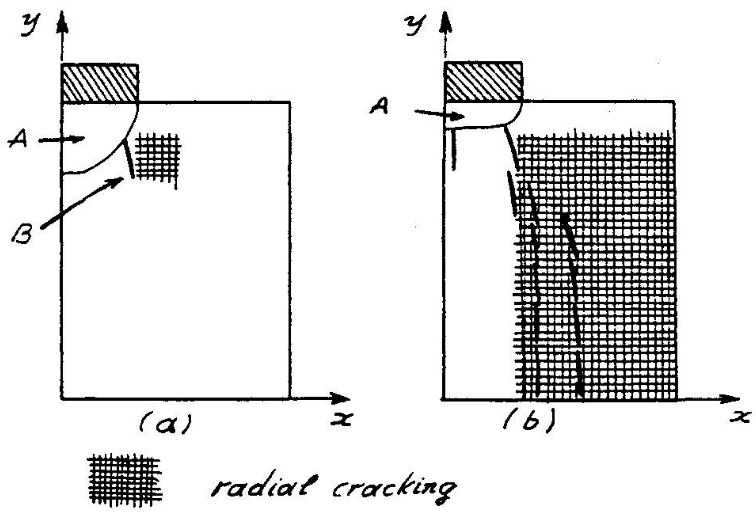


Fig. 14 Typical crack patterns at (a) crack initiation and (b) collapse as predicted by analysis

descending portion of the stress-strain relationship) under a compressive state of stress has no apparent effect on the overall behaviour of the structure. The large deformations attributed to post-ultimate strength stress-strain characteristics may be associated with the triaxial state of stress which exists in any structure but which is usually ignored for the purposes of simplicity. For example, the compression zone of reinforced concrete beams is actually subjected to a triaxial compressive state of stress due to the restraints imposed on the transverse expansion of the beams by the reinforcement, surrounding concrete, etc., but this is not accounted for in the design calculations.

A simplified description of the effect of hoop reinforcement on the strength and deformation of linear concrete structural members subjected to axial compression may be obtained by a finite element analysis in which the hoop reinforcement is simulated as a spring support with a stiffness equivalent to the stiffness of the reinforcement. The results obtained from such an analysis are shown in Figures 15a to 15c.

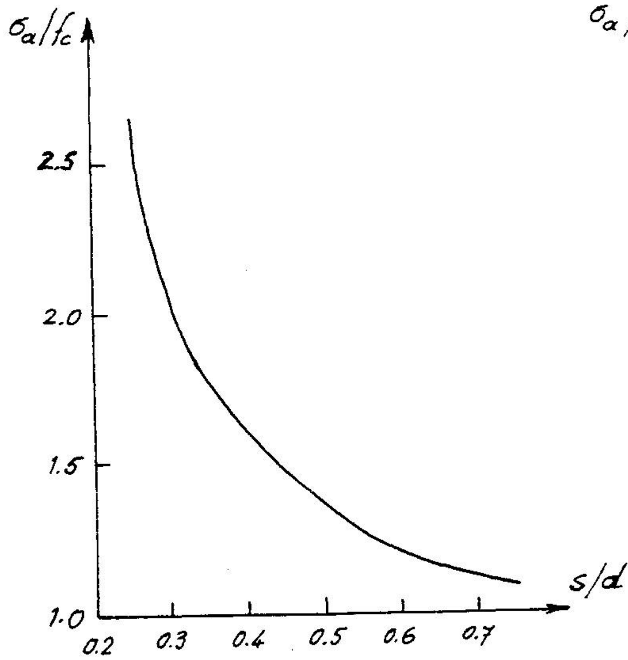
Figure 15a shows that the load carrying capacity of concrete increases with decreasing the spacing of the hoop reinforcement. The cause of such behaviour is demonstrated in Figure 15b which shows the effect of hoop reinforcement spacing on the stress path to which concrete under increasing load is subjected. Figure 15b also includes the ultimate strength envelope for plain concrete and indicates that the confining pressure (σ_c) induced by the restraint which the hoop reinforcement imposes on the transverse expansion of concrete with increasing applied axial load increases with decreasing the spacing of the reinforcement and thus a higher axial stress (σ_α) is required for the ultimate strength level to be exceeded.

Figure 15c shows the variations of axial and lateral strains with increasing axial stress for various values of reinforcement spacing. The stress-strain relationships obtained from tests on plain concrete under uniaxial compression are also shown in the figure for purposes of comparison. It should be noted that, for values of the reinforcement spacing up to about 0.5 x the diameter of the concrete member, the axial compressive strains corresponding at ultimate strength are comparable, or higher than, the maximum axial strain exhibited by plain concrete under uniaxial compression.

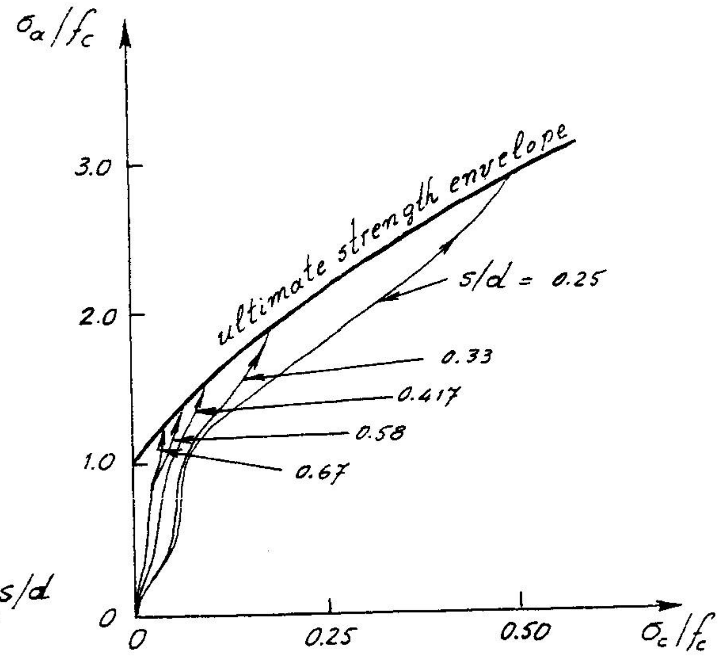
5. CONCLUSIONS

Based on a discussion of secondary testing procedure effects on the fracture processes of concrete specimens under compressive stress states, it is shown that experimental stress-strain relationships may realistically describe material response only for stress levels up to a region close to the ultimate strength level. Beyond this level, the hypothesis that concrete suffers a complete loss of load carrying capacity is considered to provide the most realistic prediction of concrete behaviour.

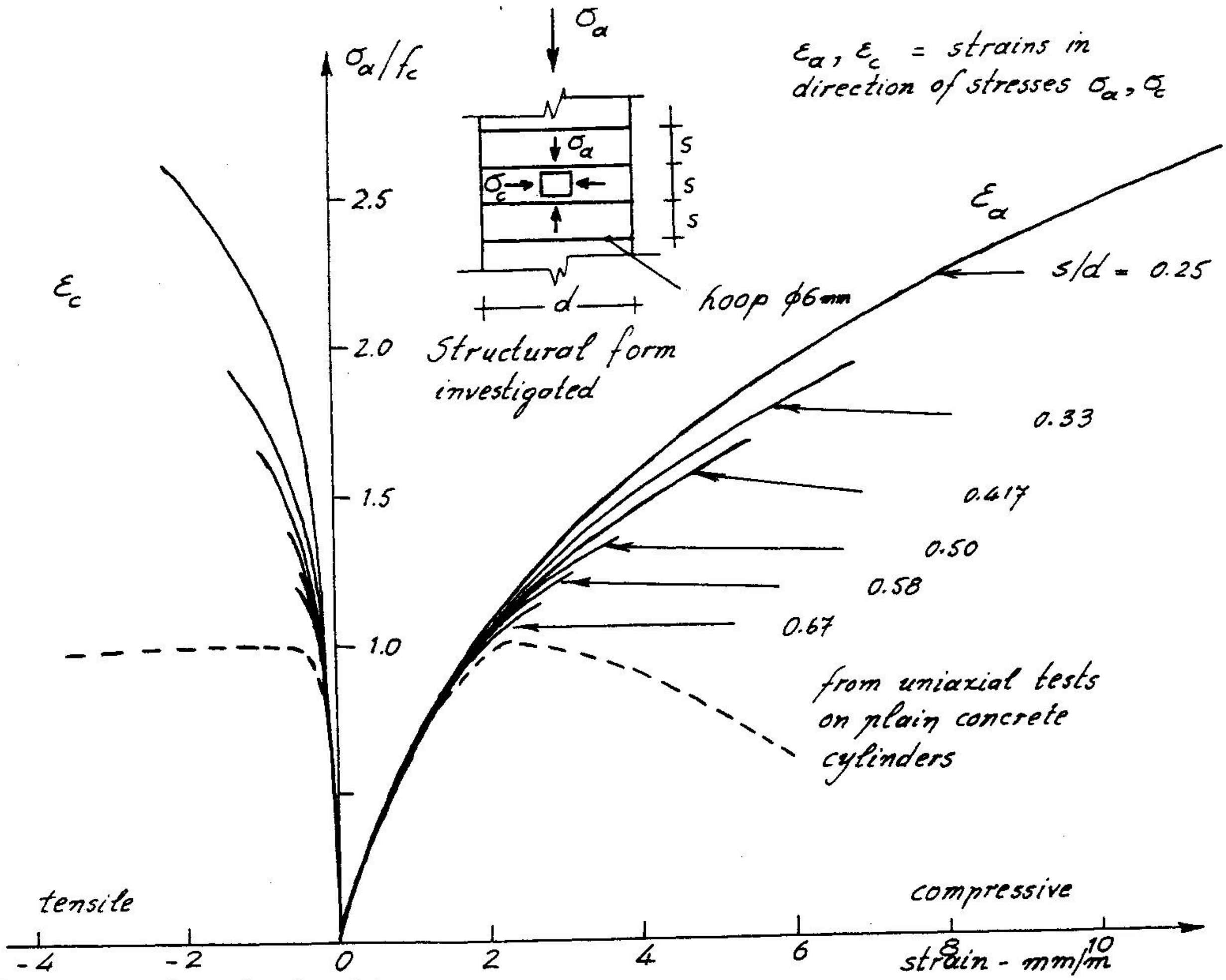
The above hypothesis is supported by experimental evidence obtained from triaxial tests on concrete subjected to wholly compressive states of stress which indicates that when the fracture processes are not affected by secondary testing effects then the specimens suffer a complete loss of load carrying capacity when ultimate strength is exceeded. Furthermore, results obtained by finite element analysis indicate that the behaviour of plain concrete structural forms subjected to load is independent of the behaviour of concrete under compressive stress states beyond ultimate strength. If such behaviour is typical of any concrete structure then the large deformations attributed to post-ultimate stress-strain characteristics may, in fact, be associated with the triaxial state of stress which exists in any structure but is usually ignored for purposes of simplicity.



(a) relationship between strength and reinforcement spacing



(b) Effect of reinforcement spacing on strength and stress path



(c) Stress-strain relationships

Fig. 15 Relationships as predicted by analysis for the structural form investigated

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