

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen  
**Band:** 34 (1981)  
**Artikel:** The damping behaviour of R/C cantilever elements  
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**DOI:** <https://doi.org/10.5169/seals-26881>

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## **The Damping Behaviour of R/C Cantilever Elements**

Le comportement d'amortissement des éléments en porte à faux en béton armé

Das Dämpfungsverhalten von Stahlbeton-Kragträgererelementen

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### **SUMMARY**

Based on recent works of the author mathematical models for the calculation of the energy dissipated during one cycle of loading by SDOF-systems, consisting of elasto-dissipative R/C members and a mass, are presented. Material damping as well as hysteretic slip damping between steel and concrete is taken into account. The energy dissipated during one cycle of vibration is calculated and equated to that of an equivalent viscous damping mechanism in order to evaluate an equivalent damping ratio.

### **RÉSUMÉ**

Basé sur des travaux antérieurs l'auteur présente des modèles mathématiques pour la détermination de l'énergie d'amortissement pour une période de vibration. Ce cas est traité en utilisant un modèle très simple composé d'une masse concentrée et une poutre en encorbellement en béton armé. On tient compte non seulement de l'amortissement interne mais aussi du frottement entre béton et acier. A l'aide du bilan d'énergie on obtient des formules de coefficient d'amortissement visqueux équivalent.

### **ZUSAMMENFASSUNG**

Basierend auf vorangegangenen Arbeiten des Autors werden mathematische Modelle zur Berechnung der Dämpfungsarbeit pro Schwingungsperiode für Einmassenschwinger, die aus einem Stahlbeton-Kragträger und einer Masse bestehen, aufgestellt. Es werden hierbei die Materialdämpfung sowie die Dämpfung zufolge Reibung zwischen Stahl und Beton berücksichtigt. Nach Aufstellung der Energiebilanz erhält man Gleichungen für äquivalente viskose Dämpfungszahlen.



## 1. INTRODUCTION

During the last years from a lot of dynamic insitu tests equivalent viscous damping ratios of R/C structures were obtained. Moreover, R/C members were tested by cyclic loading tests or resonance tests e.g. [7,8,12] giving in some cases empirical formulas for the energy dissipated e.g. [7, 8]. In the author's opinion too less attempts have been made to model damping mechanism. The stresses in a member (magnitude and distribution) are important parameters of damping, which was mentioned earlier by Lazan [5] and Newmark [9]. Recently several damping mechanism active in R/C members (mainly in R/C tension-compression members) were considered in [10,11].

In the case of uncracked members material damping and hysteretic damping due to slip and counterslip in the lead-in areas at the ends of the member were taken into account. For cracked members an additional hysteretic damping mechanism due to slipping in the lead-in areas at both sides of each main crack (total concrete area cracked) was considered. Material damping was modelled by the formulas of Lazan [5]. For the hysteretic damping four different slip models were developed by the author. The basic idea is, that at least a great part of bond is due to slip, which was found by several authors [1,2,3,4], therefore during reloading counterslip must take place, resulting in hysteretic damping. Formulas for two of these slip mechanism are given in chapter 2.

In the case of smooth bars bond is nearly completely due to slip, therefore the mathematical slip models work very well. In practice only deformed steels are used, arising the question how far the models can be still used. In the authors opinion the models can be adapted by some additional coefficients. Some estimations about this problem can be found in [10,11]. Dynamic tests are planned by the author to improve the models. The problems are discussed in chapter 3 together with restrictions resulting from differences prototype - mathematical model.

In chapter 4 the concept is applied to SDOF-systems consisting of an elasto-dissipative R/C cantilever member and a mass. The bending stiffness is assumed to be linear. For any nonlinear bending stiffness the problem could be linearized by the use of the secant stiffness as it was done for the R/C tension-compression members in [10,11]. The system is excited by sinusoidal forces with the maximum force amplitude  $\bar{p}_0$  at resonance frequency (for a nonlinear system at the resonance frequency of the equivalent linear system). The energy dissipated during one cycle of vibration is calculated and equated to that of an equivalent viscous damping mechanism in order to evaluate an equivalent damping ratio (fraction of critical damping). Formulas for  $\xi$  are given for uncracked as well as cracked members. In the second case a simple cracking mechanism is used giving the number of cracks as a function of the maximum force amplitude  $\bar{p}_0$ .

In chapter 5 the results of calculations are discussed.

## 2. SLIP MECHANISM

Model 4 and Model 3 [10,11] are shortly discussed in what follows. For each of them the energy dissipated during one load cycle with  $F_{max} = F$  and  $F_{min} = r.F$  (for  $1 > r \geq -1$ ) is evaluated where  $F$  is the total spring force. These models are used to simulate the damping in the lead-in areas at the ends of the member as well as at every main crack.

### 2.1 Model 1, used for R/C tension-compression members

The configuration of the model is similar to that of Panovko, Golzev and Strakhov [6]. The bond stress  $\tau$  is assumed to be constant over the lead-in length and independent of the load amplitude over the whole load cycle. The steel force after the lead-in length is given by  $\bar{k}F\alpha$  and the concrete force by  $(1-\bar{k})F\alpha$ , where  $\alpha$  is the loading parameter ( $1 \geq \alpha \geq r$ ). The energy dissipated during one cycle of loading is given by

$$D_v = \frac{(1-\bar{k})^2 (1-r)^3}{6\pi d \tau E_s f_s} F^3 = h_1 F^3 \quad (1)$$

where  $d$  is the sum of all steel diameters,  $f_s$  the sum of all steel areas and  $E_s$  the E-modulus of steel.

### 2.2. Model 3, used for R/C tension-compression members

Based on an investigation of the test results of Kuuskoski [1] model 3 was developed. The bond stress  $\tau$  is assumed to be uniformly distributed over the lead-in length and to depend on the load parameter  $\alpha$ . Depending on the parameter  $c$  of the bondstress - load relationship one can obtain closed as well as open hysteresis loops. Open hysteresis loops will be used in a forthcoming paper. The expression for the energy dissipated during one cycle of loading for arbitrary values of  $c$  and  $r$  is quite lengthy and can be found in [10]. For closed loops and  $r = -1$   $D_v$  is of the form

$$D_v = \frac{(1-\bar{k})^2}{\pi d E_s f_s} \frac{0,9096}{(a+bF)} F^3 = h_3^* \frac{F^3}{(a+bF)} \quad (2)$$

and for  $r = 0$

$$D_v = \frac{(1-\bar{k})^2}{\pi d E_s f_s} \frac{0,1792}{(a+bF)} F^3 = h_3^{*'} \frac{F^3}{(a+bF)} \quad (3)$$

where  $a$  and  $b$  are coefficients of the bond stress - load relationship.

## 3. APPLICABILITY OF THE SLIP MODELS TO DEFORMED BARS

In the case of deformed bars only a part of the force is transmitted by slip. Thus, the question arises, whether it is possible to separate the two force transmission mechanism slip and normal stresses at the ribs especially during unloading and reloading. Counterslipping could be largely influenced by the ribs. Furthermore local separation of steel and concrete surfaces due to the action of the ribs (mainly at the ends of the bars) take place, which was reported e.g. by Hahn [3] and Lutz [4]. Moreover, local shear cracking of concrete at a cylinder surface with a diameter corresponding to the maximum diameter of the bar (including the rib height) will produce slipping or sliding between concrete surfaces. The influence of these facts on the damping mechanism must be studied more detailed.

A first investigation of the applicability of the models to deformed bars was presented in [11], using the test results of Kuuskoski [1]. From two tension-tension loaded samples with the same E-modulus of concrete and smooth steel in the first and deformed steel in the second case the maximum concrete force in the second case is found to be about twice as much as in the first case. Assuming that about half of the force is transmitted by slipping in the case of the deformed bars, it was found that the



permanent steel stresses at the end of each load cycle ( $\alpha = 0$ ) reported in [1] fit the results obtained with model 3 very well. Later, a more detailed investigation of the available data was carried out by the author and was presented in [10]. It is concluded that bond due to slip lies within 30 - 40 % of the total bond. As a first approximation it is assumed that the models can be used for deformed bars. The coefficient  $1-\bar{k}$  as well as the bond stress  $\tau$  is reduced by the factor  $r_d$  ( $0,3 \leq r_d \leq 0,4$ ). It seems that  $r_d$  will be lower for ribbed bars and higher for torsteel bars.

When using the formulas derived in chapter 4 the following facts should be considered for smooth bars as well as for deformed bars:

- the equations for  $\tau$  give probably too high values. In the slip model a free front end of the bar is assumed. In practise there are hooks, bent up's or at least embedments. This fact tends to reduce the hysteretic loops coefficient  $r$  from  $r = -1$  to  $r = 0$ .

- the coefficient  $(1-\bar{k})$  as well as the bond stress  $\tau$  decreases with increasing number of load cycles. If the decrease ratio of  $(1-\bar{k})$  squared is less than the decrease ratio of  $\tau$  then the energy dissipated will decrease with increasing number of load cycles. A decrease of  $\tau$  was considered in model 4 [10,11]. Results of calculations with an improved model will be shown in a forthcoming paper.

- the influence of stirrups on slip and counterslip must be investigated.
- it seems probable that the contribution of slip mechanism to the total damping will decrease if the yield point of steel is approached.

#### 4. DAMPING OF R/C CANTILVER MEMBERS

The concept of [10,11] is now applied to the system shown in Fig. 1. Possible influences of curvature on slip and counterslip are neglected. The equations are derived for symmetrical longitudinal reinforcement. It is assumed that no slip will occur at the fixed end.

##### 4.1. Damping without cracks

The total force in the compression- and the tension area of cross section E-E is given by

$$\alpha F = \frac{p_o l \left[ \frac{1}{4} + \frac{2(\frac{h}{2} - h^x) \bar{n} \mu^*}{h} \right]}{\left[ \frac{\frac{h}{6}}{6} + \frac{4(\frac{h}{2} - h^x)^2 \bar{n} \mu^*}{h} \right]} \alpha = \alpha \frac{p_o l c_1}{c_2} \quad (4)$$

The steel force is of the form

$$\alpha F_s = \frac{2(\frac{h}{2} - h^x) \bar{n} \mu^* p_o l \alpha}{h c_2} \quad (5)$$

The parameters  $h^x$ ,  $\bar{n}$ ,  $h$  and  $\mu^*$  are defined in Fig. 1.

It is assumed that  $F_s$  is transmitted from concrete to steel by bond stress acting over the length  $l$  and that bond due to slip is within 30 - 40 % of the total bond ( $0,3 \leq r_d \leq 0,4$ ). The bond stress  $\tau$  now follows as

$$\tau = \frac{2 \left( \frac{h}{2} - h^x \right) \bar{n} \mu^* p_o r_d \alpha}{h c_2 d \pi} = \bar{b} F \alpha \quad (6)$$

For model 3 in [11]  $\tau$  was of the form

$$\tau = a + \bar{b} F \alpha \quad (7)$$

Therefore now model 3 is used with  $a=0$ . As force is transmitted from concrete to steel,  $(1-\bar{k})$  is given by

$$(1-\bar{k}) = \frac{F_s \alpha}{F \alpha} = \frac{2 \left( \frac{h}{2} - h^x \right) \bar{n} \mu^* r_d}{h c_1} \quad (8)$$

From equ. (4) and equ. (6)  $\bar{b}$  now follows as

$$\bar{b} = \frac{2 \left( \frac{h}{2} - h^x \right) \bar{n} \mu^* r_d}{h d l c_1 \pi} \quad (9)$$

In equ. (2)  $E_s$  is now replaced by the E-modulus of concrete,  $E_c$ , and  $f_s$  by  $b h / 2$ . Then equ. (4), equ. (8) and equ. (9) are inserted in equ. (2). Two mechanism, each equivalent to the half of the slip damping of R/C tension-compression members take place simultaneously (in the compression and the tension area of the member). Thus, the energy dissipated during one cycle of load by slip damping is given by

$$D_v = \frac{3,6384 \left( \frac{h}{2} - h^x \right) \bar{n} \mu^* c_1 p_o^2 l^3 r_d}{c_2^2 h^2 b E_c} = h_o \frac{\bar{p}_o^2}{4 \xi^2} = h_o^* M^2 \quad (10)$$

where  $M$  is the bending moment  $p_o l$ . In another approach the total force in cross section B-E is obtained using the net section of concrete.  $\alpha F$  is given by

$$\alpha F = \frac{3 p_o l}{2 h} \alpha \quad (11)$$

It is assumed that a part of this force is transmitted to steel by slip resulting in the same expression for  $(1-\bar{k})$  than given by equ. (8). Assuming a linear bending stiffness  $K = 3 E_c I / l^3$  with  $I = b h^3 / 12$ , without material damping the equivalent damping ration is given by

$$\xi = \frac{0,4548 \left( \frac{h}{2} - h^x \right) \bar{n} \mu^* c_1 h r_d}{\pi c_2^2} \quad (12)$$

Material damping of concrete is modelled by the formulas of Lazan [5]. It is assumed that material damping of steel can be neglected below the yield point. The energy dissipated during one cycle of load is of the form

$$D_s = \frac{1}{\left[ b h (1-\mu) \right]^{n-1}} \frac{\bar{k}^n F^n}{\alpha} = h_s F^n \quad (13)$$



where  $J$  and  $n$  are material constants which must be evaluated from tests. In this paper only rough estimations of these parameters can be used. The coefficient  $\alpha_L$  ( $\alpha_L \leq 1$ ) depends on the stress distribution in the member and is defined by Lazan [5]. Due to the uncertainties of  $J$  and  $n$   $\alpha_L = 1$  is assumed. The total energy dissipated during one cycle of loading is given by

$$D_T = D_V + D_S \quad (14)$$

The total energy dissipated is equated to that of an equivalent viscous damping mechanism. Thus, the damping ratio is given by

$$\xi^{n-2} - \frac{8\pi l^3}{h_0 E_C b h^3} \xi^{n-1} + \frac{4h_0}{h_0} \left( \frac{c_1 l}{2c_2} \right)^n \bar{p}_0^{n-2} = 0 \quad (15)$$

The coefficient  $n$  will be within  $2 \leq n < 3$ . For  $2 < n < 3$  equ. (15) has to be solved by an iterative procedure.

#### 4.2. Damping with cracks

The situation is more complex than in the case of R/C tension - compression members [10,11]. Therefore some simplifications are necessary. To show the contribution of cracks to total damping only this mechanism is taken into account in what follows. In Fig. 2 a simple cracking mechanism and the coefficients used are shown. When the concrete force  $F_{C,e}$  in cross section E-E has the value

$$F_{C,e} = \frac{p_0 l}{4c_2} = F_{C,c}^x \quad (16)$$

in E-E the first crack will occur.  $F_{C,c}^x$  is the cracking tensile force of concrete which is given by

$$F_{C,c}^x = \frac{\beta_{C,c} b h}{4} \quad (17)$$

where  $\beta_{C,c}$  is the bending tension strength of concrete. Using slip model 3 at the instant of cracking the bond stress is of the form

$$\tau = \bar{b} F \alpha = \bar{b} \frac{p_0 l c_1 \alpha}{c_2} = 4 \bar{b} F_{C,c}^x c_1 \alpha \quad (18)$$

Thus,  $\text{tg} \beta$  is given by

$$\text{tg} \beta = 4 \bar{b} F_{C,c}^x d \pi c_1 \alpha \quad (19)$$

$\text{tg} \delta$  is of the form

$$\text{tg} \delta = \frac{F_{C,c}^x}{1} \quad (20)$$

The locations of the following cracks are now obtained under the simplifying assumption, that on one hand  $\tau$  is constant and is given by equ. (18) with  $\alpha = 1$  and on the other hand the lead-in length is constant. The lead in length is given by

$$l_{e,1} = \frac{1}{4 \pi d \bar{b} c_1 l + 1} \quad (21)$$

The length  $l_e^*$  is of the form

$$l_e^* = \frac{1}{4\pi d b c_1} \quad (22)$$

The  $i$ -th crack will occur at the location  $x_i$  if the concrete force approaches the value  $F_{c,c}^x$  in  $x_i$ .  $x_i$  is given by

$$x_i = l - (i-1)l_e^* \quad (23)$$

The fictive concrete force in cross-section E-E is given by

$$F_{c,i} = \frac{F_{c,c}^x \cdot l}{l - (i-1)l_e^*} \quad (24)$$

The lead-in length is of the form

$$l_{e,i} = \frac{l_i}{4l_i \pi d b c_1 + 1} \quad (25)$$

where  $l_i$  is given by

$$l_i = l - (i-1)l_e^* \quad (26)$$

The maximum concrete strain for a cracked section is given by

$$\epsilon_c = \left[ 2f_s' \bar{n}(\bar{y} - h^x)(h - 2h^x) + b\bar{y}^2(h - h^x - \frac{\bar{y}}{3}) \right]^{-1} \frac{2M(x)\bar{y}}{E_c} \quad (27)$$

The steel tension force of the cracked section is given by

$$F_s^{(II)} = \frac{\epsilon_c(h - \bar{y} - h^x)}{\bar{y}} E_s f_s = u_1 M(x) \quad (28)$$

For the  $i$ -th crack the steel force is obtained from

$$F_{s,i}^{(II)} = u_1 \left[ l - (i-1)l_e^* \right] p_0 = u_{1,i} p_0 \quad (29)$$

where  $i$  is an integer number within  $0 < i \leq n_r$ .  $n_r$  is the actual number of cracks. The uncracked region starts at the location  $x_u$

$$x_u = l - (n_r - 1)l_e^* - l_{e,n_r} \quad (30)$$

At this point the steel force is of the form

$$F_{s,n_r}^{(I)} = \frac{2(\frac{h}{2} - h^x)\bar{n}\mu^*}{hc_2} \left[ l - (n_r - 1)l_e^* - l_{e,n_r} \right] p_0 = u_3 p_0 \quad (31)$$

The concrete force transmitted by slip is obtained from

$$F_{c,n_r}^{(I)} = \frac{rd}{4c_2} \left[ l - (n_r - 1)l_e^* - l_{e,n_r} \right] p_0 = u_4 p_0 \quad (32)$$

Further, for the region between two cracks the steel force is given by

$$\bar{F}_{s,i}^{(I)} = \frac{2(\frac{h}{2} - h^x)\bar{n}\mu^*}{hc_2} \left[ l - (i-1)l_e^* - \frac{l_e^*}{2} \right] p_0 = u_{5,i} p_0 \quad (33)$$

and the concrete force transmitted by slip

$$\bar{F}_{c,i}^{(I)} = \frac{rd}{4c_2} \left[ l - (i-1)l_e^* - \frac{l_e^*}{2} \right] p_0 = u_{6,i} p_0 < \frac{F_{c,c}^x}{2} \quad (34)$$





with  $0 < i \leq (n_r - 1)$ . Equ. (33) and (34) are only valid for  $F_{c,i}^{(x)} < F_{c,c}^x/2$ . The energy dissipated during one cycle of load at each crack is now calculated by equ. (3) with  $a = 0$ . It is assumed that all cracks will close exactly at the instant  $\alpha = 0$ . Thus, the energy dissipated due to slip at the  $i$ -th crack is of the form

$$D_{v,i} = \frac{(1 - \bar{k}_i)^2 \cdot 0.1792 F_{s,i}^{(x)2}}{d E_s f_s b r_d} = \bar{k}_i^2 h_3 F_{s,i}^{(x)2} \quad (35)$$

For the point  $x_u = 1 - (n_r - 1)l_e^* - l_{e,n_r}$  the coefficient  $\bar{k}_{n_r}^*$  is given by

$$\bar{k}_{n_r}^* = \frac{F_{c,n_r}^{(x)} r_d}{F_{s,n_r}^{(x)}} = \frac{u_u}{u_{2,n_r}} \quad (36)$$

For the region between two cracks the coefficient  $\bar{k}^*$  is obtained from

$$\bar{k}_i^* = \frac{F_{c,i}^{(x)}}{F_{s,i}^{(x)}} = \frac{u_{\theta,i}}{u_{2,i}} \quad \text{for } F_{c,i}^{(x)} < \frac{F_{c,c}^x}{2} \quad (37)$$

and from

$$\bar{k}_i^* = \frac{F_{c,c}^x}{2 F_{s,i}^{(x)}} = \frac{F_{c,c}^x}{2 u_{2,i} p_0} \quad \text{for } F_{c,i}^{(x)} > \frac{F_{c,c}^x}{2} \quad (38)$$

During a step by step analysis one could check for each point whether equ. (37) or (38) has to be used. To simplify the procedure the following mean values are used:

$$\bar{k}_i^x = \frac{2 u_{\theta,i} p_0 + F_{c,c}^x}{4 u_{2,i} p_0} \quad (39)$$

and

$$\bar{k}_i^o = \frac{2 u_{\theta,i} p_0 + F_{c,c}^x}{4 u_{2,i+1} p_0} \quad (40)$$

The total energy dissipated during one cycle of loading is given by

$$\sum_i D_{v,i} = 2 \left[ \left( \frac{u_u}{u_{2,n_r}} \right)^2 h_3 F_{s,n_r}^{(x)2} + \sum_{i=1}^{n_r-1} \bar{k}_i^{x2} h_3 F_{s,i}^{(x)2} + \sum_{i=1}^{n_r-1} \bar{k}_i^{o2} h_3 F_{s,i+1}^{(x)2} \right] \quad (41)$$

From the equations (29), (39), (40) and (41) the following equation is obtained

$$\begin{aligned} \sum_i D_{v,i} &= p_0^2 \left[ (2 u_u^2 + \sum_{i=1}^{n_r-1} u_{\theta,i}^2) h_3 \right] + p_0 F_{c,c}^x h_3 \sum_{i=1}^{n_r-1} u_{\theta,i} + \\ &+ \frac{(n_r-1) F_{c,c}^{x2}}{4} h_3 = p_0^2 v_1 + p_0 v_2 + v_3 = \frac{\bar{p}_0^2}{4 \xi^2} v_1 + \frac{\bar{p}_0 v_2}{2 \xi} + v_3 \end{aligned} \quad (42)$$

Thus, from the energy balance the following equation for the equivalent damping ratio  $\xi$  is obtained

$$\xi = \frac{\bar{p}_0}{4 v_3} \left( \frac{\bar{p}_0 \pi}{K} - v_2 \right) - \sqrt{\left[ \frac{\bar{p}_0}{4 v_3} \left( \frac{\bar{p}_0 \pi}{K} - v_2 \right) \right]^2 - \frac{\bar{p}_0^2}{p_0^2} v_1 / v_3} \quad (43)$$

where  $K$  is the stiffness,  $K = 3 EI/l^3$ , and  $I = bh^3/12$ .

Equation (43) is now solved for increasing  $n_r$  starting with  $n_r = 0$  until the following equation is valid

$$\frac{\bar{p}_0 \cdot 1}{8 \xi c_2} < F_{c,i+1} \quad (44)$$

where  $F_{C,i+1}$  is given by equ. (24).

## 5. RESULTS

Results of calculations for cracked as well as uncracked members with deformed bars are given in what follows. First a set of basic dimensions is chosen. The single parameters are varied in Fig. 3-8 for uncracked members and in Fig. 9-17 for cracked members. In some cases  $\xi$  is given as a function of two parameters. As in the case of uncracked members it is assumed that force is transmitted from concrete to steel and vice versa in the case of cracked members, the tendencies are different.

### uncracked members:

- damping decreases slightly with increasing dimensions  $h$  and  $b$  of the cross section. The decrease is less when also mat. damping is considered.
  - damping increases slightly with increasing steel area
  - thus, combining the first two statements, damping increases slightly with increasing percentage of reinforcement
  - damping decreases slightly with increasing E-modulus of concrete
  - $\xi$  is a linear function of the coefficient  $r_d$
  - the coefficient  $J$  was evaluated from Lazan [5] and means only a rough assumption
  - the coefficient  $n$  has a strong influence on damping. In the case of uncracked members it seems that  $n$  will be within  $2 \leq n \leq 2.2$
  - the influence of  $h^x$  on damping can be neglected
  - $\bar{p}_0$  and  $l$  have only an influence on material damping (small for  $2 \leq n \leq 2.2$ )
- It is concluded that  $\xi$  is within  $0.01 \leq \xi \leq 0.02$  for uncracked members.

### cracked members: (slip damping only)

- damping increases strongly with increasing  $h$  and  $b$
  - damping decreases strongly with increasing steel area
  - thus, damping decreases strongly with increasing perc. of reinforcement
- The same tendency was observed by Dieterle [12]
- damping increases with increasing E-modulus of concrete
  - $r_d$  within  $0.3 \leq r_d \leq 0.4$  has only a small influence on the results
  - the influence of  $\beta_{cc}$ ,  $b$ ,  $h^x$  and  $l$  on damping can be neglected
  - the influence of  $\bar{p}_0$  is shown in Fig. 13. The results show a max.  $\xi$  for low force levels which was also observed by Dieterle [12]. In [12] also a strong decrease of  $\xi$  for increasing displacement is reported. This could be due to the decrease of  $\bar{k}_i^*$  with the number of load cycles. In a forthcoming paper an attempt will be made to simulate this behaviour.
- It seems that the values for  $\xi$  obtained by equ. (43) are too high in some cases but that Fig. 9-17 show very well the tendencies.

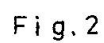
## 6. CONCLUSIONS

In this paper the concept of [10,11] is applied to SDOF-systems consisting of a R/C cantilever element and a mass. As the situation is more complex than in the case of R/C tension-compression members, additional simplifications are necessary. In the author's opinion the slip models can be adapted for ribbed bars and torsteel bars. First estimations are discussed in chapter 3. The author plans tests to improve the modelling for deformed bars as well as to get the parameters of material damping. In chapter 4 equations for equivalent viscous damping ratios  $\xi$  are derived. If these equations are used, the limitations discussed in chapter 3 should be considered. In a forthcoming paper the decrease of bond and of the coefficient  $(1-\bar{k})$  will be taken into account as a function of the number of load cycles.



## 7. REFERENCES

1. KUUSKOSKI, V.: Über die Haftung zwischen Beton und Stahl. Dissertation Technische Hochschule Helsinki, 1950
2. REHM, G.: Über die Grundlage des Verbundes zwischen Stahl und Beton. Deutscher Ausschuß für Stahlbeton, Heft 138, 1961
3. HAHN, V.: Über die Verbundwirkung des Querrippenstahls. Dissertation Technische Hochschule Stuttgart, 1952
4. LUTZ, L.A.: The Mechanics of Bond and Slip of Deformed Reinforcing Bars in Concrete. Dep. of Struct.Eng., Cornell University Report no. 324, 1966
5. LAZAN, B.J.: Damping of Materials and Members in Structural Mechanics. Pergamon Press, New York, 1968
6. GOODMAN, L.E.: A Review of Progress in Analysis of Interfacial Slip Damping. Coll. on Struct. Damping, ASME, 1959
7. HAWKINS, N.M.(editor): Reinforced Concrete Structures in Seismic Zones. ACI Publication SP-53, 1977
8. BERTERO, V.V. (editor): Proceedings of a Workshop on Earthquake Resistant Reinforced Concrete Building Construction. The University of California, Berkeley, 1977
9. NEWMARK, N.M., HALL, W.J.: Seismic Design Criteria for Nuclear Reactor Facilities. Proc. of the 4th WCEE, Santiago, Chile, 1969
10. FLESCHE, R.: Das Dämpfungsverhalten schwingender Stahlbetontragwerke. Dissertation TU-Wien, 1980
11. FLESCHE, R.: Estimating Damping of Reinforced Concrete Members and Structures. Int. Conf. on Recent Advances in Structural Dynamics, ISVR-Southampton, 1980
12. DIETERLE, R., BACHMANN, H.: Einfluß der Rissbildung auf die dynamischen Eigenschaften von Leichtbeton- und Betonbalken. Schweizer Ingenieur und Architekt, Heft 32/1980



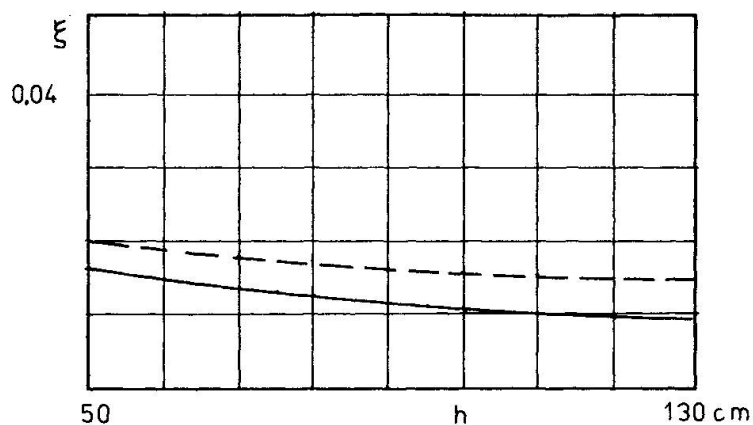


Fig. 3  
variation of h

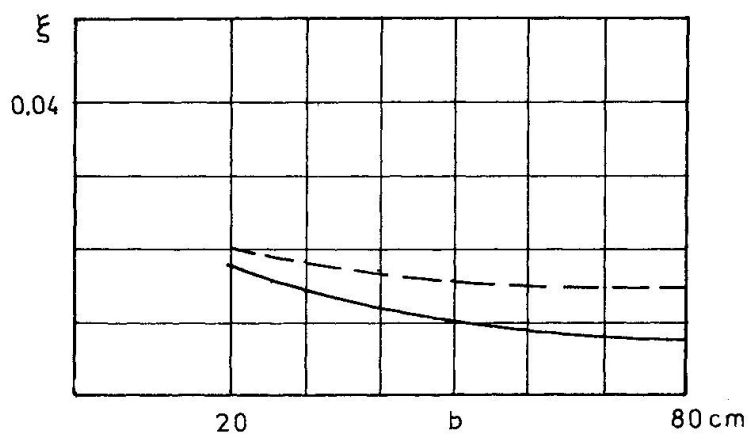


Fig. 4  
variation of b

uncracked member  
basic dimensions:

$h = 0.9 \text{ m}$   
 $b = 0.4 \text{ m}$   
 $E_c = 3000 \text{ kN/cm}^2$   
 $5 \times \phi 0.03 \text{ m}$   
 $r_a = 0.4$   
 $J = 3.351 \cdot 10^{-9}$   
 $n = 2$   
 $h^* = 0.03 \text{ m}$   
 $l = 2.5 \text{ m}$   
 $\bar{p}_0 = 10 \text{ kN}$

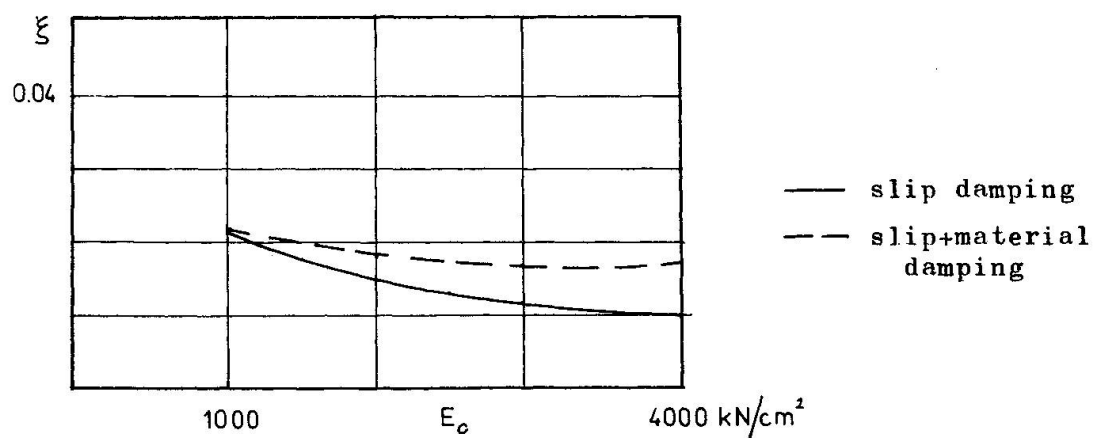


Fig. 5  
variation of  $E_c$

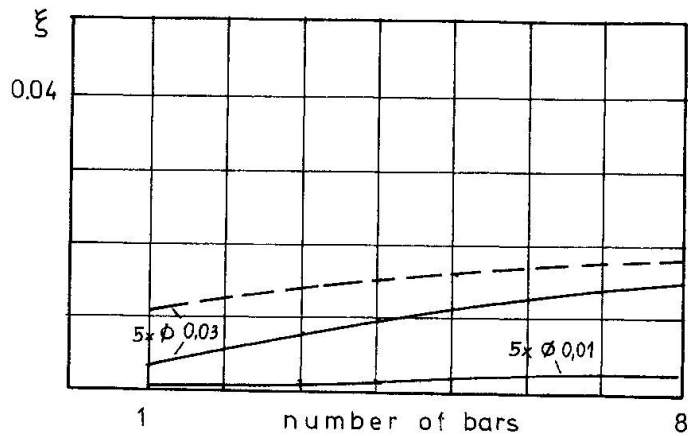


Fig. 6  
variation of number of bars

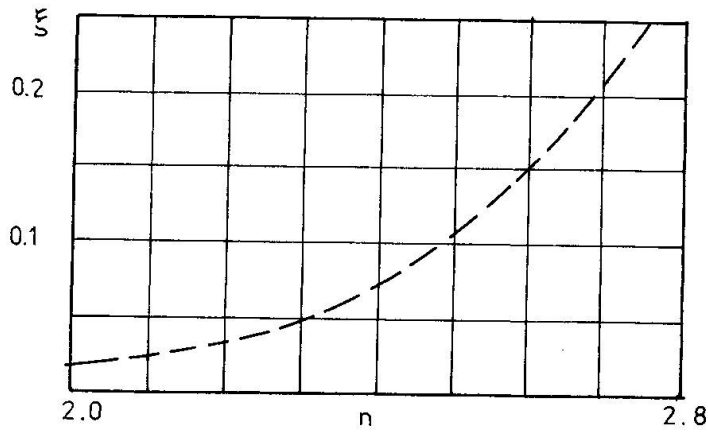


Fig. 7  
variation of coefficient n

uncracked member

basic dimensions:

$h = 0.9$  m

$b = 0.4$  m

$E_c = 3000$  kN/cm<sup>2</sup>

$5 \times \varnothing 0.03$  m

$r_d = 0.4$

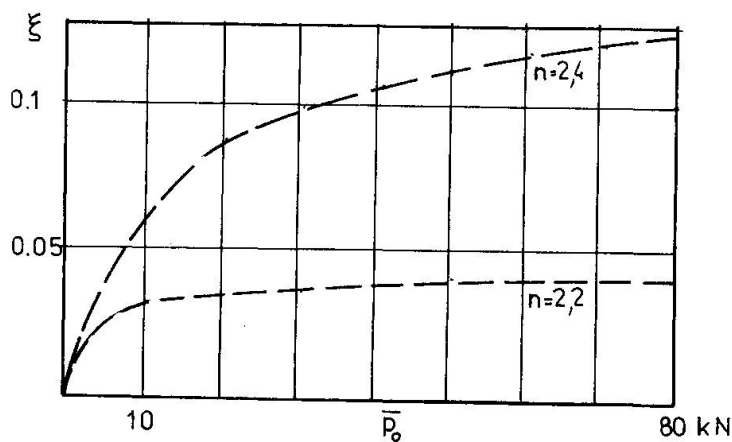
$J = 3.351 \cdot 10^{-9}$

$n = 2$

$h^* = 0.03$  m

$l = 2.5$  m

$\bar{p}_0 = 10$  kN



— slip damping  
-- slip + material damping

Fig. 8  
variation of max. force amplitude

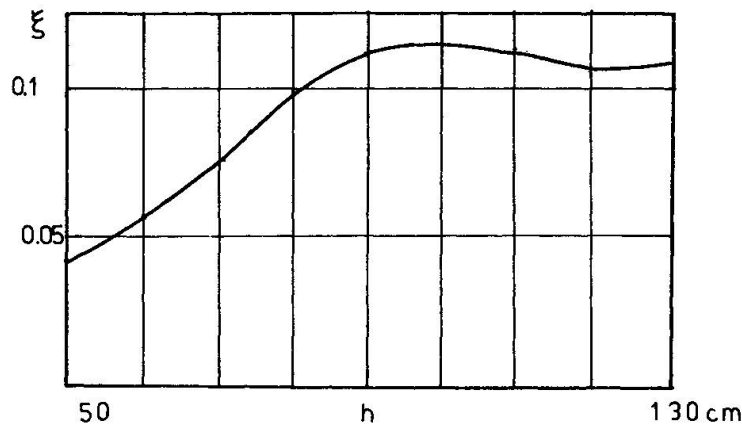


Fig.9  
variation of  $\xi$

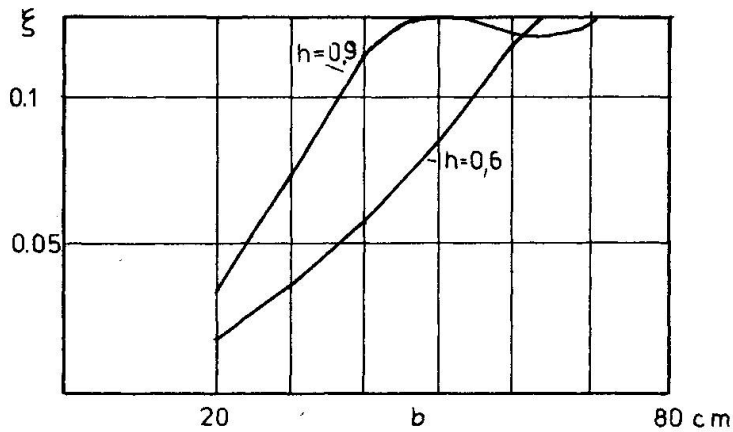


Fig.10  
variation of  $\xi$

cracked member  
basic dimensions:

$$h = 0.6 \text{ m}$$

$$b = 0.4 \text{ m}$$

$$l = 2.5 \text{ m}$$

$$h^* = 0.03 \text{ m}$$

$$E_c = 3000 \text{ kN/cm}^2$$

$$5 \times 0.03$$

$$r_d = 0.4$$

$$\beta_{c,c} = 87.5 \text{ N/cm}^2$$

$$\bar{b} = 60/\text{m}^2$$

$$\bar{p}_0 = 10 \text{ kN}$$

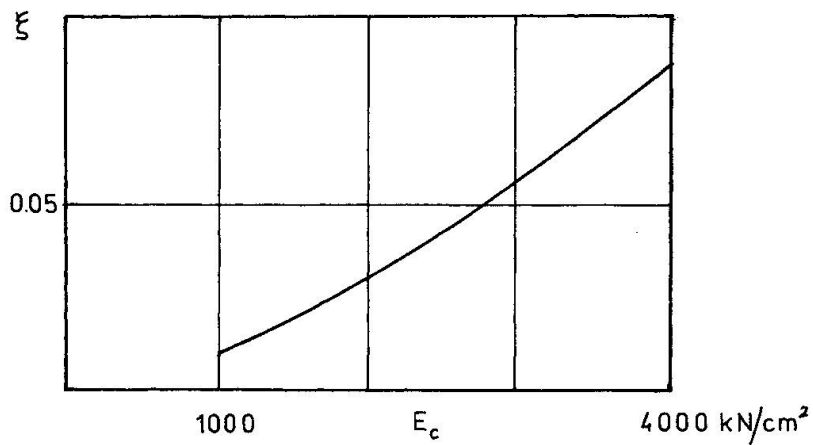


Fig.11  
variation of  $\xi$

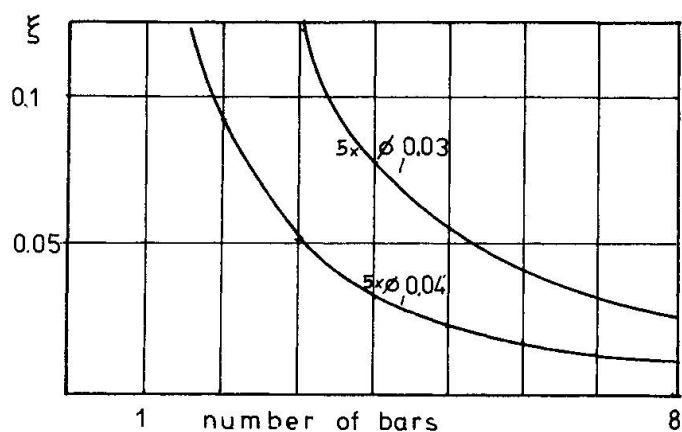


Fig. 12  
variation of the number of bars

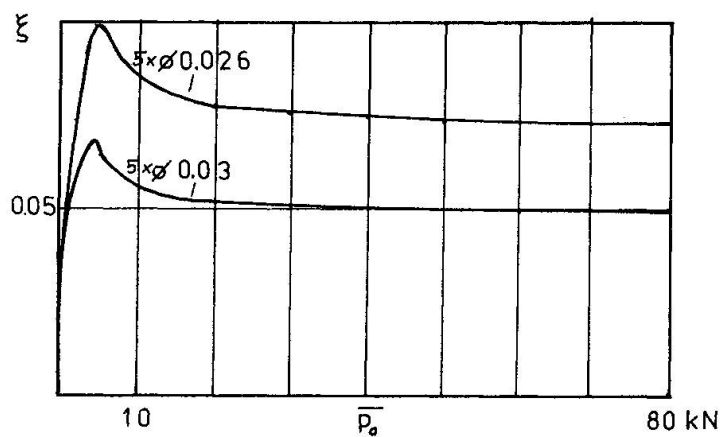


Fig. 13  
variation of max. force amplitude

cracked member  
basic dimensions:

$h = 0.6 \text{ m}$

$b = 0.4 \text{ m}$

$l = 2.5 \text{ m}$

$h^x = 0.03 \text{ m}$

$E_c = 3000 \text{ kN/cm}^2$

$5x\emptyset 0.03$

$r_d = 0.4$

$\beta_{c,c} = 87.5 \text{ N/cm}^2$

$\bar{b} = 60/\text{m}^2$

$\bar{p}_o = 10 \text{ kN}$

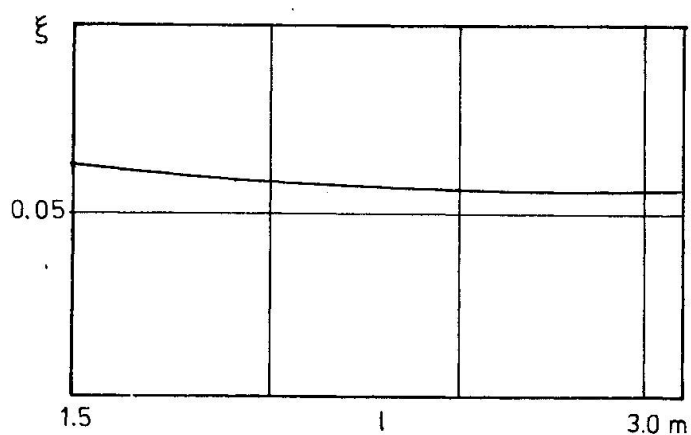


Fig. 14  
variation of  $l$



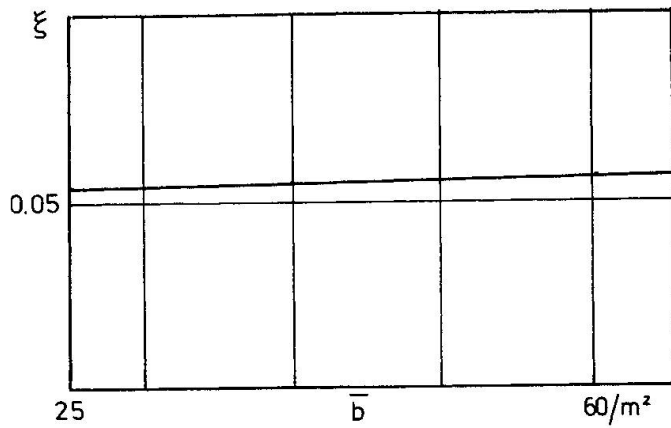


Fig. 15  
variation of coefficient  $\bar{b}$

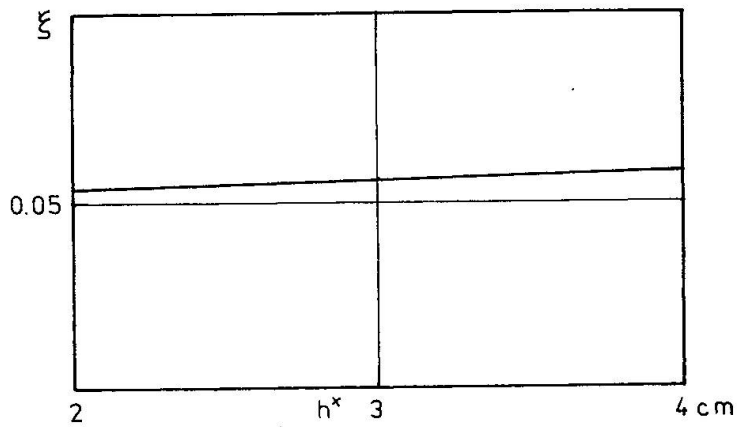


Fig. 16  
variation of  $h^x$

cracked member  
basic dimensions:  
 $h = 0.6$  m  
 $b = 0.4$  m  
 $l = 2.5$  m  
 $h^x = 0.03$  m  
 $E_c = 3000$  kN/cm<sup>2</sup>  
 $5 \times 0.03$   
 $r_d = 0.4$   
 $\beta_{cc} = 87.5$  N/cm<sup>2</sup>  
 $\bar{b} = 60$ /m<sup>2</sup>  
 $\bar{p}_0 = 10$  kN

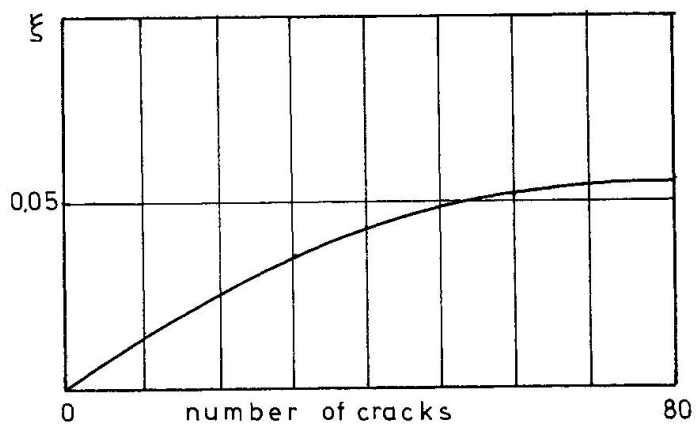


Fig. 17  
increase of  $\xi$  with number of cracks