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A Model of Concrete Behaviour under Generalised Stress

Modèle de comportement du béton dans des états de contrainte généraux

Modell für das Verhalten von Beton unter allgemeinen Spannungszuständen

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SUMMARY
The paper presents a description of the effect of internal fracture processes on the deformational behaviour of concrete under increasing load and introduces the fundamental concepts which have formed the basis for expressing mathematically the stress-strain relationship of the material under short-term generalised states of stress.

RESUME
L'article présente l'effet des processus de rupture interne sur le comportement du béton sous l'effet d'une augmentation de la charge. Les concepts fondamentaux ici décrits permettent d'exprimer mathématiquement les relations entre les contraintes et les déformations du matériau sous une sollicitation quelconque à court terme.

ZUSAMMENFASSUNG
Der Einfluss innerer Bruchvorgänge auf das Verformungsverhalten von Beton bei Laststeigerung wird beschrieben. Grundlegende Konzepte werden eingeführt, welche die Basis für die mathematische Formulierung der Spannungs-Dehnungs-Beziehung unter kurzzeitig wirkenden allgemeinen Spannungszuständen bilden.

1. INTRODUCTION

It is generally accepted that the nonlinear deformational behaviour of concrete is dictated by internal fracture processes occurring under increasing stress. It is considered, therefore, that any mathematical expression of the deformational properties of concrete must essentially describe the effect of the fracture processes on deformation. Such a description has led to the formulation of a mathematical model describing the deformational behaviour of concrete under generalised stress increasing up to [1,2] and beyond [3] the ultimate strength level.

The present paper presents a detailed description of the effect of the fracture processes on deformation and introduces the fundamental concepts which have formed the basis for modelling the deformational behaviour of concrete under short-term generalised stress states.

3. EFFECT OF FRACTURE PROCESSES ON DEFORMATION

The fracture processes of concrete under increasing stress take the form of crack extension and propagation in the direction of the maximum principal compressive stress (or orthogonal to the direction of the maximum principal tensile stress) [4]. The extension and propagation of cracks reduces the high tensile stress concentrations which exist near crack tips and causes void formation within the body of the material [4].

The reduction of the high tensile stress concentrations may be considered equivalent to the application of an effective internal compressive stress state which tends to reduce the volume of concrete. On the other hand, void formation tends to increase it. The combined effect on deformation, therefore, may be either a decrease or increase of overall volume depending on whether the effect of the internal stress state or that of void formation predominates.

2.1 Effect of Internal Stress State

Each internal stress state may be resolved into a hydrostatic and a deviatoric component.

Under a pure hydrostatic stress applied to the boundaries of an element of concrete, the effect of the hydrostatic component of the internal stress state on deformation is superimposed on that of the applied hydrostatic stress and, for a compressive applied hydrostatic stress, the combined effect is expressed by the variation of the volumetric strain (ε_o) with increasing applied hydrostatic stress (σ_o) shown in Figure 1. In contrast to the hydrostatic component of the internal stress state, the deviatoric component of internal stress is insignificant since the measured deviatoric strain (γ_o) has been found by experiment to be negligible.

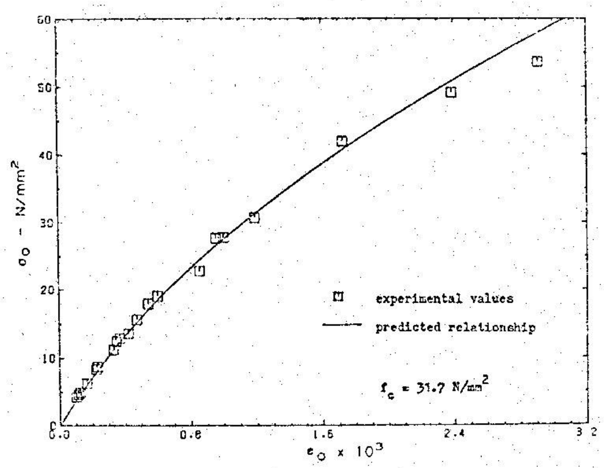


Fig. 1 Typical variation of ε_o with σ_o

Under a pure deviatoric stress applied to the boundaries of an element of concrete, the hydrostatic component of the internal stress state causes a volume decrease up to a stress level which has been termed onset of unstable fracture propagation (OUFP)[4](see Figure 2). Beyond this stress level, which marks the transition from the 'consolidation' to the 'volume dilation' stage, the material behaviour is dictated by the void formation processes and this is discussed in section 2.2.

Nominal values for the hydrostatic component of the internal stress state (σ_{int}) for given levels of applied stress may be evaluated by using the relationships shown in Figures 1 and 2. For a value of volumetric strain (ε_o) corresponding to a given level of applied stress (σ_o,τ_o) below OUFP (see Figure 2), a value of hydrostatic stress which represents a nominal value for the hydrostatic component of the internal stress state (σ_{int}) may be obtained from Figure 1. In this mode , for stress levels up to OUFP, the τ_o-ε_o relationships of Figure 2 may be transformed into the σ_{int}-τ_o relationships of Figure 4.

Contrasting with the effect of the hydrostatic component of the internal stress state on deformation, the effect of the deviatoric component is superimposed on that of the applied deviatoric stress and the combined effect is reflected in the nonlinear variation of the deviatoric strain (γ_o) with increasing applied deviatoric stress (τ_o) shown in Figure 3.

2.2 Effect of Void Formation

Although void formation may start at low levels of applied stress, its effect on deformation becomes significant when OUFP is exceeded. The void formation process dictates both the 'volume dilation' portion of the τ_o-ε_o relationship (see Figure 2) as well as the faster rate of increase of γ_o with τ_o exhibited by the τ_o-γ_o relationship for stress levels beyond OUFP (see Figure 3). Furthermore, void formation increases the total cross-sectional area of the material to such an extent that the overall stress, defined on the basis of the total cross-sectional area, decreases with increasing applied stress whereas the stress defined on the basis of the solid cross-section may, in fact, continue to increase. Such a process eventually leads to disintegration of the element of concrete and complete loss of its load carrying capacity.

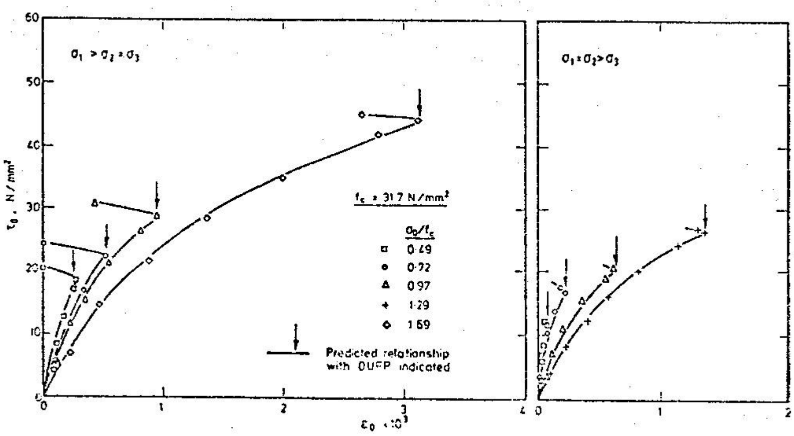


Fig. 2 Typical variation of ε_o with τ_o for concrete under various levels of σ_o

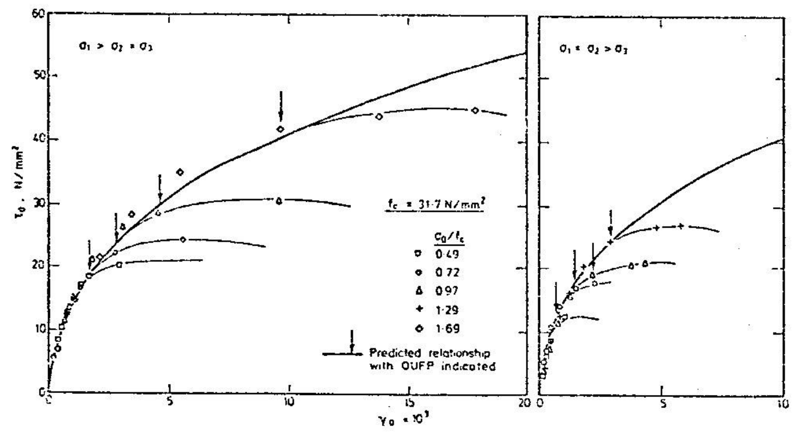


Fig. 3 Typical variation of γ_o with τ_o for concrete under various levels of σ_o

2.3 Components of Deformational Behaviour

Based on the considerations discussed in the preceding sections the deformational behaviour of concrete may be decomposed into the following nonlinear components:-

(a) a component defined by the mechanical properties of concrete assumed as a solid continuum,

(b) a component expressing the effect of the internal stress state caused by the fracture processes, and

(c) a component expressing the effect of void formation.

To quantify the above components using available experimental information appears to be an impossible task since, as discussed in sections 2.1 and 2.2, most of the stress-strain data describe overall material behaviour. It will be shown in the following, however, that the use of these concepts can form a sound basis for the mathematical description of the deformational behaviour.

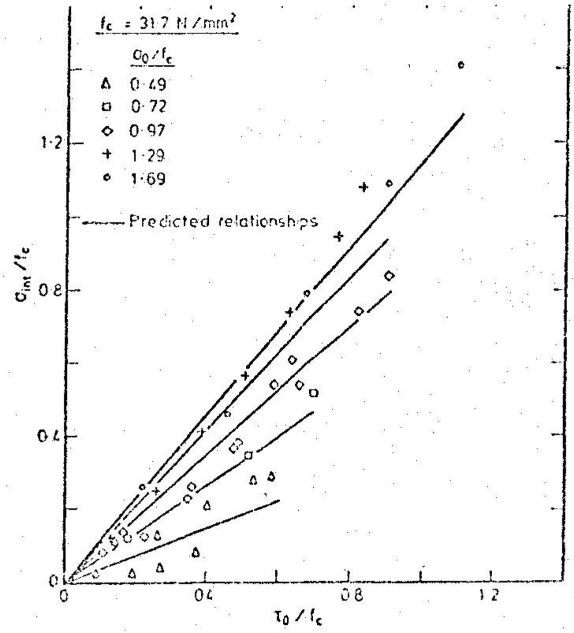


Fig. 4 Typical variation of σ_{int} with increasing applied stress

3. MODELLING OF CONCRETE BEHAVIOUR

For the mathematical description of the deformational behaviour of concrete a model material with the strength properties of concrete has been introduced such that the fracture processes of the model are qualitatively similar to those of concrete [5]. The deformational behaviour of the model has been considered to consist of three nonlinear components similar to those described for concrete i.e.

component A - defined by the mechanical properties of the model assumed as a solid isotropic continuum,

component B - dictated by the internal stress state caused by the fracture processes, and

component C - expressing the effect of void formation occurring during the fracture processes.

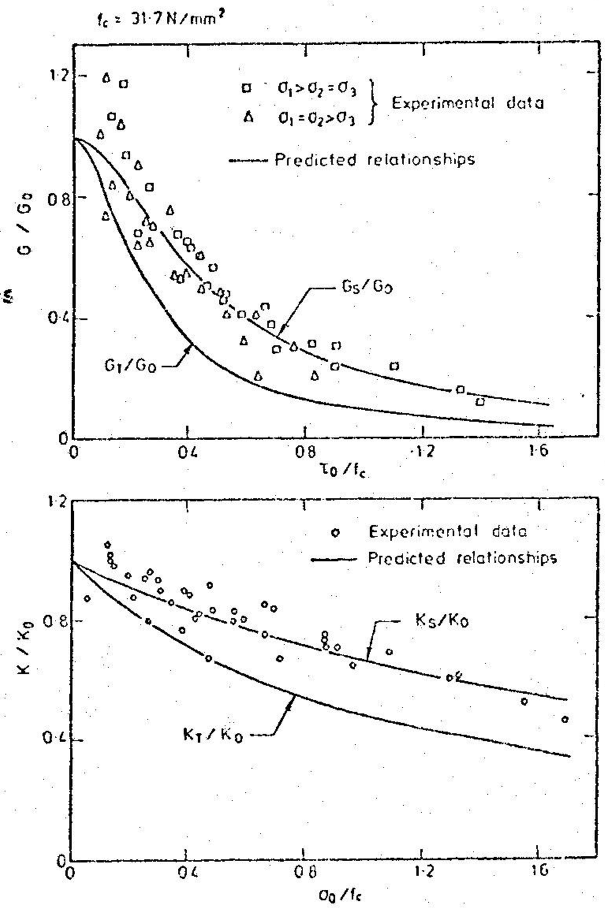


Fig. 5 Typical variations of the secant and tangent values of the bulk and shear moduli with increasing applied stress

3.1 Mechanical Properties of the Model

The model material has been devised such that its mechanical properties, when it is considered as a solid isotropic continuum, are completely defined by the σ_o-ε_o and τ_o-γ_o relationships obtained for concrete under stress increasing up to OUFP (see Figures 1 and 3, respectively). These relationships have been found to be unique and independent of the third stress invariant and stress path effects [1,5]. Independence of the third stress invariant is considered to indicate that the relationships are independent of any 'damage induced' anisotropy. Such behaviour is consistent with the assumption that these relationships can be used to describe the mechanical properties of a solid isotropic continuum. Furthermore, the σ_o-ε_o and τ_o-γ_o relationships have been described mathematically by a regression analysis of available experimental data which have been obtained in previous investigations of the behaviour of concrete under multiaxial stress states [5].

Using the above relationships the tangent and secant values of the bulk (K_t,K_s) and shear (G_t,G_s) moduli can be easily derived as described elsewhere [1]. Graphical representations of the variations of the above moduli with increasing applied stress for a typical concrete are given in Figure 5.

Having established the bulk and shear moduli the tangent and secant values of the modulus of elasticity (E) and Poisson's ration (ν) can be easily onbtained from the well-known formulae of linear elasticity, $K=E/3(1-2\nu)$ and $G=E/2(1+\nu)$.

3.2 Effect of Internal Stresses

The internal stress state caused by the fracture processes of the model is taken to be equivalent to the hydrostatic component of the internal stress state which develops within concrete when subjected to deviatoric stress. Nominal values of the internal stress state σ_{int} can be evaluated as described in section 2.1. A graphical representation of the variation σ_{int} with increasing applied stress for a typical concrete is shown in Figure 4.

By comparing available experimental data obtained from tests using various states of stress and stress paths [6], it has been found that the relationships shown in Figure 4 are effectively independent of the third stress invariant and stress path effects [1]. The independence of σ_{int} on the third stress invariant is expected since σ_{int} represents the hydrostatic component of the internal stress state developing within concrete under pure deviatoric stress.

The effect of σ_{int} on deformation is apparently equivalent to the deformational response of the model subjected to three principal stresses $\sigma_1=\sigma_2=\sigma_3=\sigma_{int}$.

3.3 Effect of Void Formation

The void formation processes occurring within the model are assumed to be similar to those occurring within concrete. They are caused by crack propagation processes which occur in the direction of the maximum principal compressive stress and thus affect predominantly the deformation in the orthogonal directions. Such an effect should be reflected on the ratios of the incremental strains corresponding to the two principal stresses orthogonal to the direction of crack propagation with respect to the incremental strain in the direction of crack propagation.

If it is assumed, therefore, that the volume dilation which begins at the OUFP level is caused entirely by void formation, then the effect of void formation on deformation can be defined by the variations of the volume strain and the above incremental strain ratios with stress changes beyond the OUFP level. A mathematical expression of these changes has been based on theoretical considerations of the fracture mechanism of concrete which have indicated that void formation within concrete is predominantly dependent on the current stress state, material characteristics and, for stress levels beyond ultimate, on the state of stress at the ultimate strength level [2,3,5].

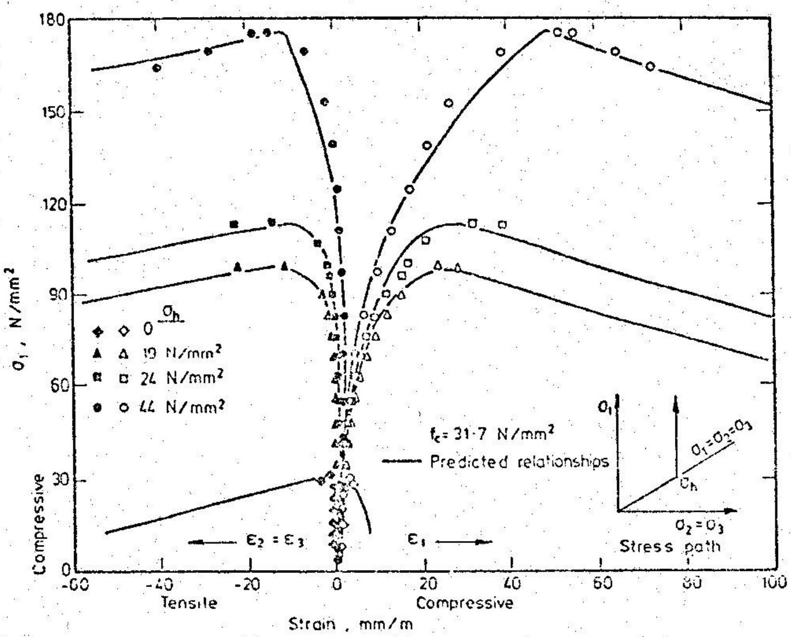
In contrast to the mathematical description of the mechanical properties of the model and the effect of the internal stress state on deformation, the mathematical description of the effect of void formation on deformation is dependent on the third stress invariant and this dependence reflects the 'damage induced' anisotropy which is inherent in concrete.

4. COMPARISON OF PREDICTED AND EXPERIMENTAL STRESS-STRAIN BEHAVIOUR

Figure 6 shows typical representations of the stress-strain relationships predicted for concrete under various states of triaxial axisymmetric stress. The figure also includes experimental values obtained from tests on concrete carried out at Imperial College [5] and show a very close correlation between predicted and experimental relationships.

Figure 7 shows the stress-strain relationships predicted for concrete under biaxial stress conditions together with experimental values obtained at the Technical University of Munich [7]. It is apparent from the figure that the predicted relationships provide a good fit to the data.

The stress-strain relationships predicted for concrete under uniaxial compression has also been found to provide a very close fit to experimental values obtained by other investigators [8]. Furthermore, for concretes with $f_c > 40\ N/mm^2$, they correlate very closely with empirical relationships between applied stress and the corresponding strain in the direction of loading proposed elsewhere for such a stress state [9]. The above relationships are shown in Figure 8.



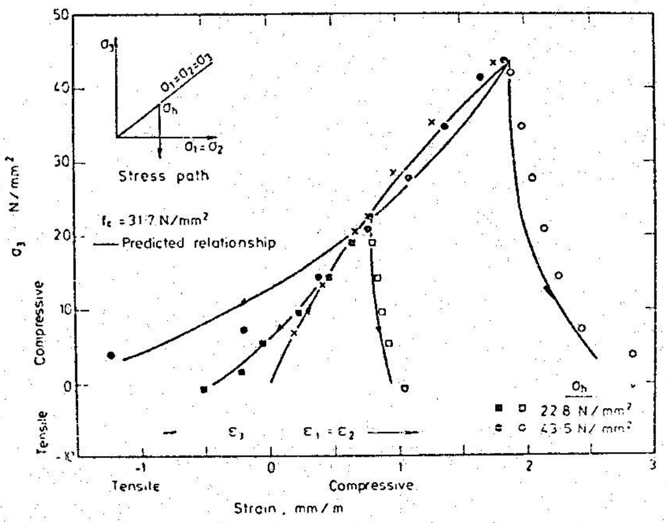


Fig. 6 Typical stress-strain relationships for concrete under triaxial axisymmetric stress states

5. CONCLUSIONS

1. The nonlinear deformational behaviour of concrete is considered to be dictated mainly by internal fracture processes occurring under increasing stress. These processes create voids within the body of concrete and reduce high tensile stress concentrations.

2. For the mathematical description of the deformational behaviour of concrete a model material with the strength properties of concrete has been devised such that its fracture processes are qualitatively similar to those of concrete.

3. The deformational behaviour of the model consists of the following components:-

Component A - defined by the mechanical properties of the model assumed as a solid isotropic continuum.

Component B - dictated by the reduction of the high tensile stress concentrations caused by the fracture processes, and

Component C - expressing the effect of void formation occurring during the fracture processes.

4. Components A and B have been found by experiment to be independent of 'damage induced' anisotropy whereas component C essentially expresses the effect of 'damage-induced' anisotropy on deformation.

5. The mathematical description of components A and B has been based on an analysis of experimental data obtained for concrete under multiaxial stress states whereas that of component C has been based on theoretical considerations of the fracture mechanism of concrete.

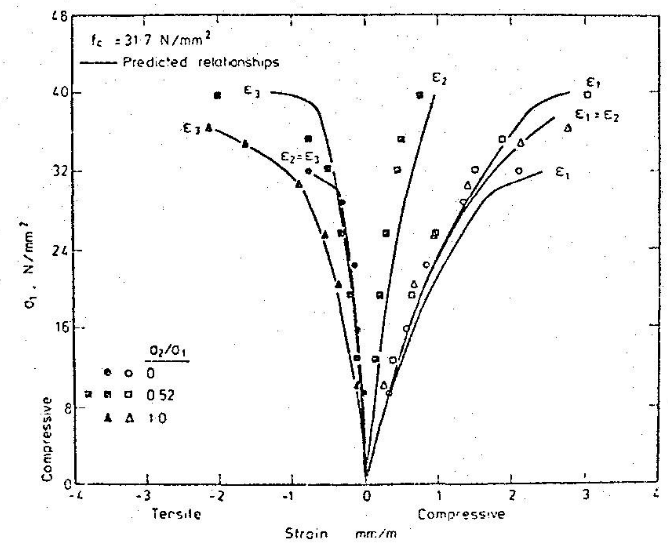


Fig. 7 Typical stress-strain relationships for concrete under biaxial compression

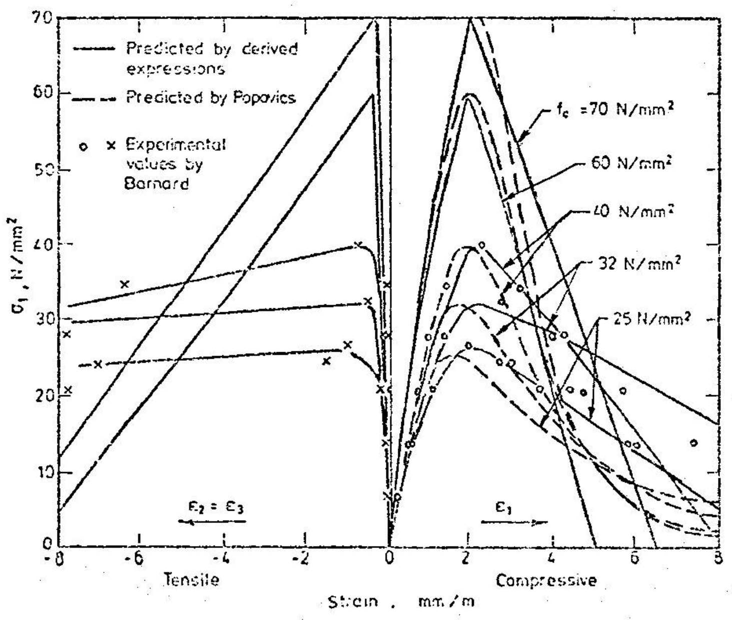


Fig. 8 Typical stress-strain relationships for various concretes under uniaxial compression.

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