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A Kinematic and Isotropic Hardening Plasticity Model for Plain Concrete under General Triaxial Stress Conditions

Modèle à écrouissage cinématique et isotrope du béton, dans des états de contraintes triaxiales

Kinematisch und Isotrop verfestigend-plastisches Modell für Beton unter allgemeinen räumlichen Spannungszuständen

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SUMMARY
A new plastic model valid under general triaxial conditions is formulated and comparisons with experimental data are carried out. The model requires knowledge of only the elastic modulus, Poisson's ratio, and the compressive and tensile strengths.

RESUME
Un nouveau modèle valable pour des états de contrainte généraux est décrit et comparé avec des résultats expérimentaux. Le modèle ne nécessite que la connaissance de quatre paramètres: le module d'élasticité, le coefficient de Poisson et les résistances à la compression et à la traction maximales.

ZUSAMMENFASSUNG
Ein neues plastizitätstheoretisches Modell für allgemeine räumliche Spannungszustände wird beschrieben und mit Versuchsresultaten verglichen. Nur vier Parameter müssen bekannt sein, nämlich Elastizitätsmodul und Querdehnungszahl sowie einachsige Druck- und Zugfestigkeit.

1. INTRODUCTION

The purpose of this paper is to investigate the possibilities and limitations of a plasticity model based on Drucker's postulates [1] for plain concrete under general trixial conditions. This is done by proposing a new model with two sets of yield surfaces. The model involves both isotropic and kinematic work-hardening, but it requires knowledge only of the elastic modulus E, Poisson's ratio ν, the uniaxial compressive strength f_c, and tensile strength f_t. The results of the model are compared to experimental data.

2. THE NEW PLASTICITY MODEL

The concrete is assumed to be isotropic in the initial state as well as after deformation, and therefore the formulation of the analytical equations will be given in principal strains ε_i and stresses σ_i only. The invariants are designated

$$\left.\begin{aligned} & I_1 = \sigma_1+\sigma_2+\sigma_3, \quad I_2 = \frac{1}{2}(\sigma_1^2+\sigma_2^2+\sigma_3^2), \quad I_3 = \frac{1}{3}(\sigma_1^3+\sigma_2^3+\sigma_3^3) \\ & I_2' = I_2 - \frac{1}{6} I_1^2 \text{ , the 2. deviatoric invariant} \end{aligned}\right\} \tag{1}$$

and

$$J_1 = \varepsilon_1+\varepsilon_2+\varepsilon_3 \text{ , } J_2 = \frac{1}{2}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2), \quad J_3 = \frac{1}{3}(\varepsilon_1^3+\varepsilon_2^3+\varepsilon_3^3) \tag{2}$$

respectively.

The model is closely related to a failure criterion, and in this formulation a "parabolic" criterion proposed by N.S.Ottosen [2] is chosen.

The reason for this can be seen from the comparisons made in Figure 1. In this figure the intersection curves between some analytical failure surfaces and a plane containing the hydrostatic axis and one principal stress-axis in the principal stress space are shown. Also some results from experimental tests are shown.

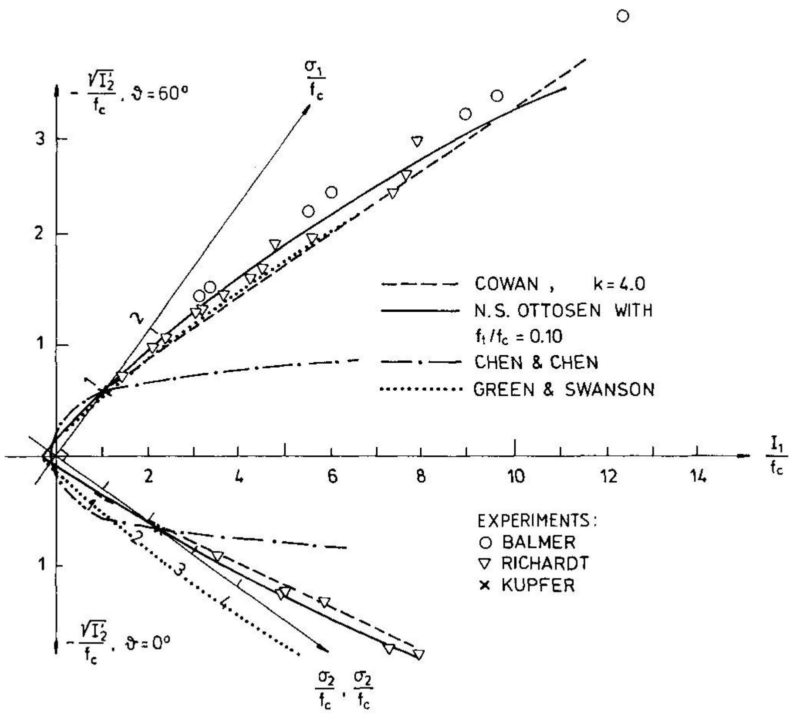


Fig. 1.

A test series performed by Launay et al. [3] indicates that the failure surface is not axisymmetric about the hydrostatic axis. Analytically this means that the third stress-invariant enters into the equation for the failure surface. The abovementioned analytical failure surface takes account on this dependence. See [2].

It is now assumed that in the principal stress-space the yield surfaces are of the shape indicated in Figure 2. One of the main characteristics of this figure is that the yield surfaces are all cut by the hydrostatic axis. The extension of the linear-elastic domain (the shaded area in Figure 2) implies that hydrostatic tension is

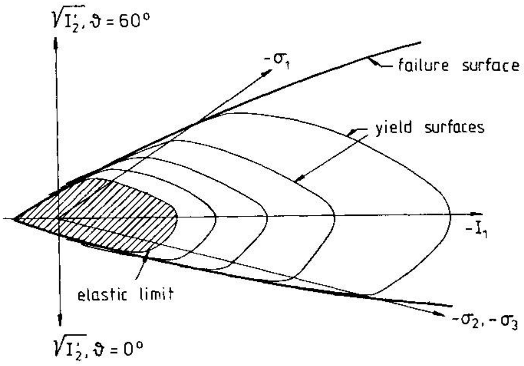


Fig. 2.

assumed to be linear-elastic until failure. Further it can be observed that the yield surfaces have relatively "sharp corners" in the vicinity of the failure surface in order to reflect the sudden volume-expansion that occurs in a compression test just before failure. This gives rise to a mathematical inconvenience that can be circumvented by using two different sets of yield surfaces as shown in Figure 3.

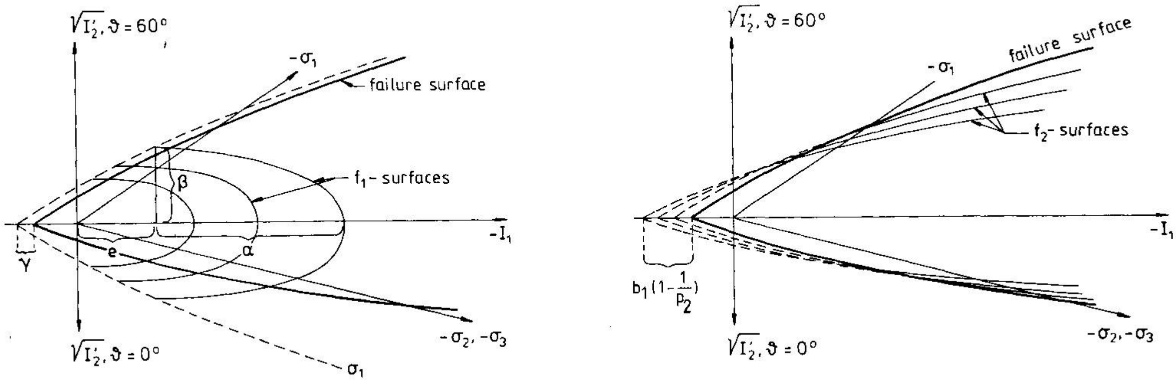


Fig. 3.

Analytically the surfaces can be expressed as

$$f_1 = \frac{(I_1 + p_1 - \alpha)^2}{\alpha^2} + \frac{I_2'}{\beta^2} - 1 = 0 \tag{3}$$

and

$$f_2 = \frac{A}{f_c^2} \cdot \frac{I_2'}{p_2^2} - \frac{\lambda(\theta)}{f_c} \cdot \frac{I_2'}{p_2} + B \cdot (b_1 \cdot (1 - \frac{1}{p_2}) - \frac{I_1}{f_c}) - 1 = 0 \tag{4}$$

The entering constants and the function $\lambda(\theta)$ are explained in the following.

θ is used instead of I_3 . The relationship is

$$\theta = \frac{1}{3} \text{ Arccos } (\frac{3\sqrt{3}}{2} \cdot \frac{I_3 - \frac{2}{3} I_1 I_2 + \frac{2}{27} I_1^3}{(I_2')^{3/2}}) \tag{5}$$

p_1 and p_2 are the two parameters whereby the instantaneous yield surfaces are uniquely determined. b_1 and the ratio $\frac{\alpha}{\beta}$ are chosen to be constants that must be determined empirically. For each value of p_1 eq. (3) is the equation of an ellipsoid. α is now determined so that the circle of toppoints of the ellipsoid are common with the surface

$$g_1 = \frac{A}{f_c^2} \cdot I_2' - \frac{\lambda_c}{f_c} \cdot \sqrt{I_2'} + B \cdot (\gamma - \frac{I_1}{f_c}) - 1 = 0 \tag{6}$$

A, B, λ_c and $\lambda(\theta)$ are determined as for the failure criterion adopted herein. For each value of p_2 and constant θ eq. (3) is the equation of a parabola.

The two sets of yield surfaces are treated entirely separate. The contributions to the plastic strain-increments when the stress-point is moving outside the f_1-surface respectively the f_2-surface are superposed, i.e.

$$d\varepsilon_i^{pl} = d\varepsilon_i^{pl(1)} + d\varepsilon_i^{pl(2)} \tag{7}$$

When the stress-point moves outside or on the instantaneous f_1-surface p_1 is determined so that the stress-point is always lying on the instantaneous f_1-surface which implies $dp_1 \geq 0$.

When the stress-point moves inside the instantaneous f_1-surface $dp_1 = 0$. p_2 is determined in a similar way. The initial values of p_1 and p_2 are designated p_1' and p_2' .

The total plastic strain-increments can now be written as

$$d\varepsilon_i^{pl} = d\varepsilon_i^{pl(1)} + d\varepsilon_i^{pl(2)} = G_1 \cdot \frac{\partial f_1}{\partial \sigma_i} \cdot dp_1 + G_2 \cdot \frac{\partial f_2}{\partial \sigma_i} \cdot dp_2 \text{ ,} \tag{8}$$

where G_1 and G_2 are functions of the stresses, the strain-history and the hydrostatic plastic work $W_{e(hyd)}^{pl}$. This amount of work can be determined from hydrostatic compression tests, but because of lack of data from such experiments it is proposed to put

$$G_1 = \begin{cases} \dfrac{\dfrac{dW_{e(hyd)}^{pl}}{dp_1}}{\sigma_i \dfrac{\partial f_1}{\partial \sigma_i}} & \text{for } dp_1 > 0 \\ 0 & \text{for } dp_1 = 0 \end{cases} \tag{9}$$

with

$$W_{e(hyd)}^{pl} = \begin{cases} -\dfrac{p_1}{3} \cdot c_1 \cdot \dfrac{E}{f_c} \cdot \dfrac{a_1^{t_1}}{\left(\dfrac{p_1 - p_1'}{3}\right)^{t_1} + a_1^{t_1}} & \text{for } -I_1 = p_1 > p_1' \\ 0 & \text{for } -I_1 \leqq p_1 = p_1' \end{cases} \tag{10}$$

a_1, c_1 and t_1 are constants to be determined empirically. In an analogous way it is proposed to put

$$G_2 = \frac{H_2(I_2', p_2)}{\sigma_i \frac{\partial f_2}{\partial \sigma_i}} \tag{11}$$

with

$$H_2 = \begin{cases} a_2 \cdot \frac{E}{f_c} \cdot I_2' \cdot \left(c_2 \cdot \frac{p_2 - p_2'}{1 - p_2}\right)^{t_2} & \text{for } dp_2 > 0 \\ 0 & \text{for } dp_2 = 0 \end{cases} \tag{12}$$

a_2, c_2 and t_2 are constants to be determined empirically.

Now it is possible to give most of the constants, that remain to be determined, a physical interpretation. For further details see [4].

After having studied some available experimental data it is suggested to put

$$\frac{\alpha}{\beta} = 2.4 \;,\; \gamma = -0.15\, f_c \;,\; p_1' = -0.57\, f_c \;,\; p_2' = 0.70 \;,\; a_1 = -6.5\, f_c \;,\; b_1 = 1.1 \;,$$

$$c_1 = 7 \times 10^{-5} \;,\; t_1 = 1.8 \;,\; a_2 = -3.5 \times 10^{-6}/f_c \;,\; c_2 = 0.8 \;,\; t_2 = 1.6$$

With these values for the constants the results of the model are compared to experimental data in Figures 4-14 reproduced from [4]. The only variables in these comparisons are E, ν, f_c and f_t. The first eight figures show comparisons with plane stress-experiments performed by Kupfer et al. The following ones show comparisons with various triaxial experimental results in which two principal stresses are kept at constant values different from zero. Even though the correspond-

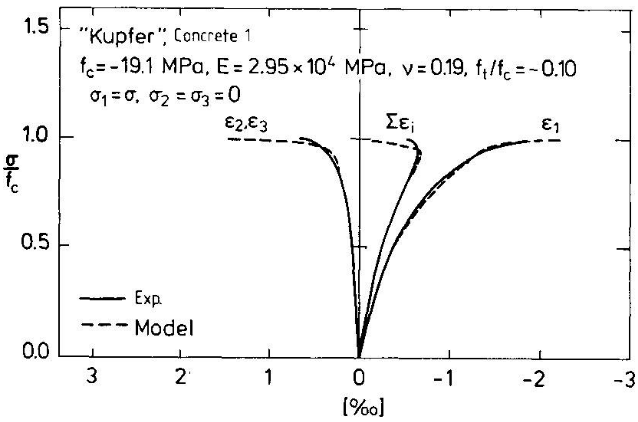


Fig. 4.

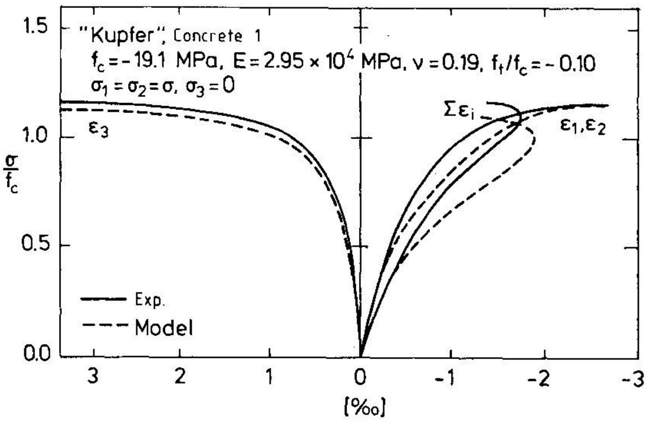
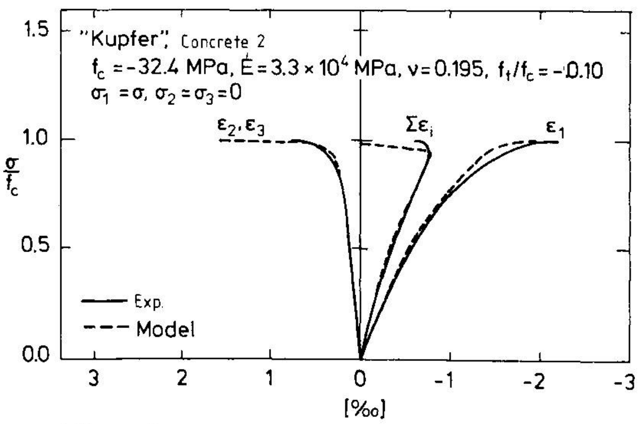


Fig. 5.



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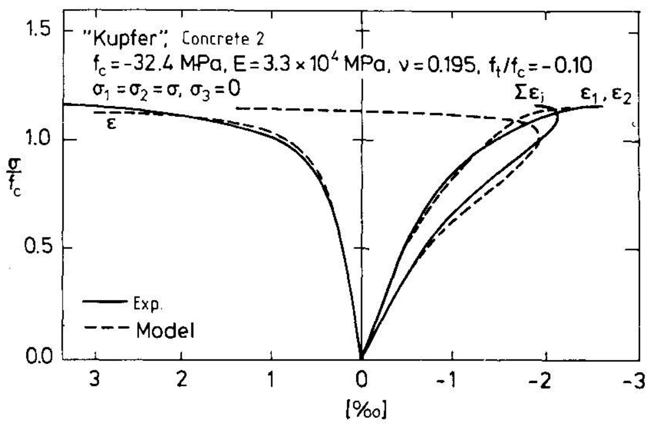
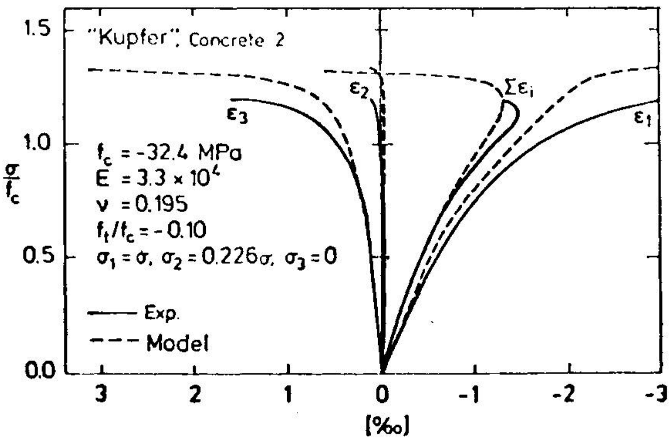


Fig. 7.



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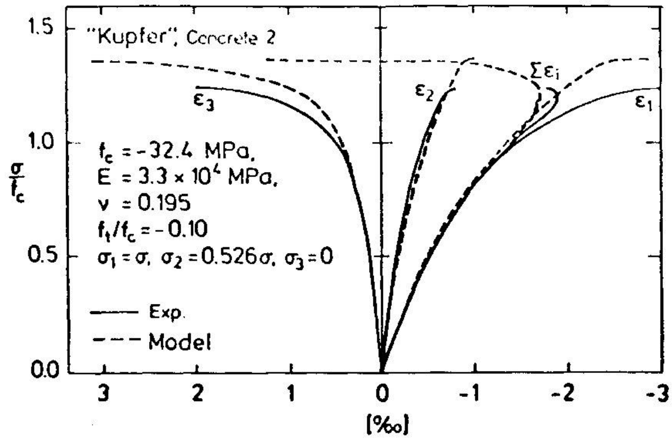

Fig. 9.

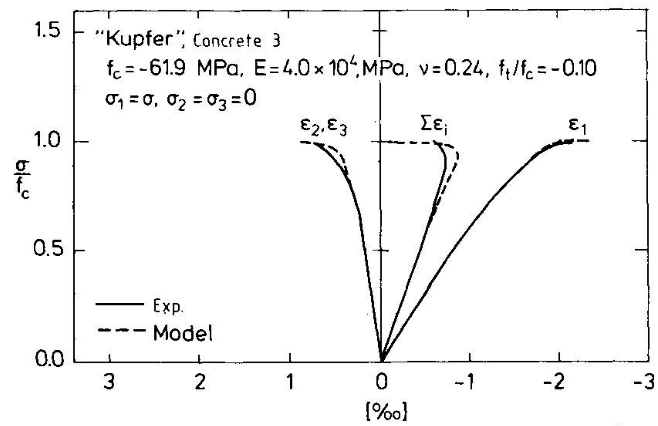

Fig. 10

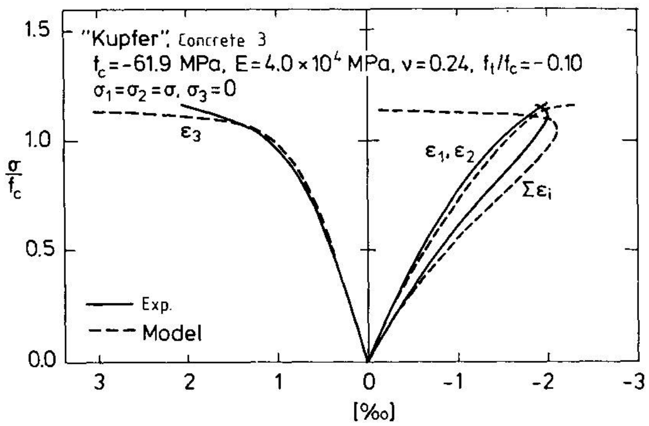

Fig. 11.

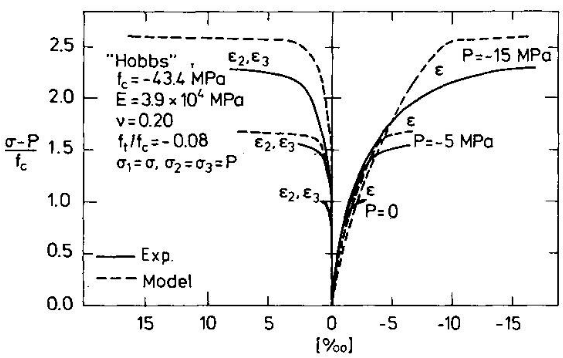

Fig. 12.

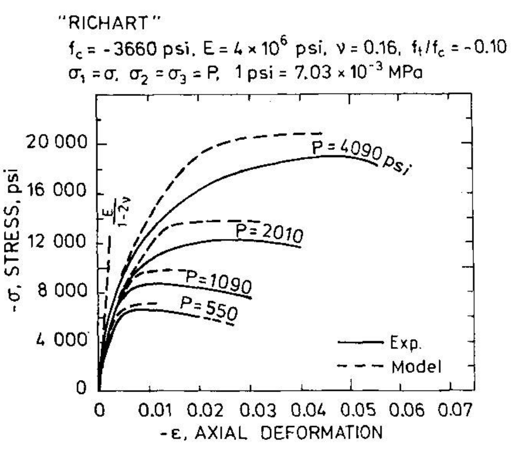

Fig. 13.

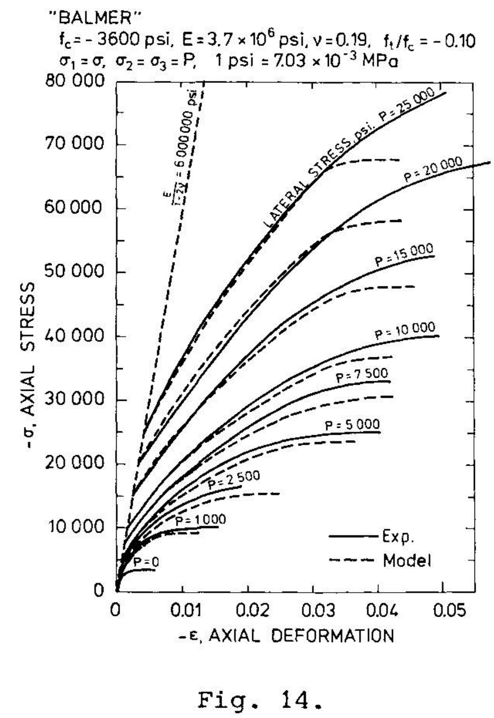

Fig. 14.

ance between model and experiments is quite fair attention should be drawn towards two inherent limitations in traditional plasticity models like the one just outlined above.

3. TWO IMPORTANT LIMITATIONS IN A TRADITIONAL PLASTICITY MODEL

The first limitation to be pointed out in connection with a traditional plasticity model based on Drucker's postulates concerns the normality condition. Both the normality and the convexity conditions are derived by Drucker under the assumption that the elastic properties of the material remain constant under plastic deformation. Experimental tests indicate that for plain concrete this condition is fairly well fulfilled as long as the stress-state is not too near failure. But when the stress-state is near failure then the condition is no longer fulfilled. See Figure 15.

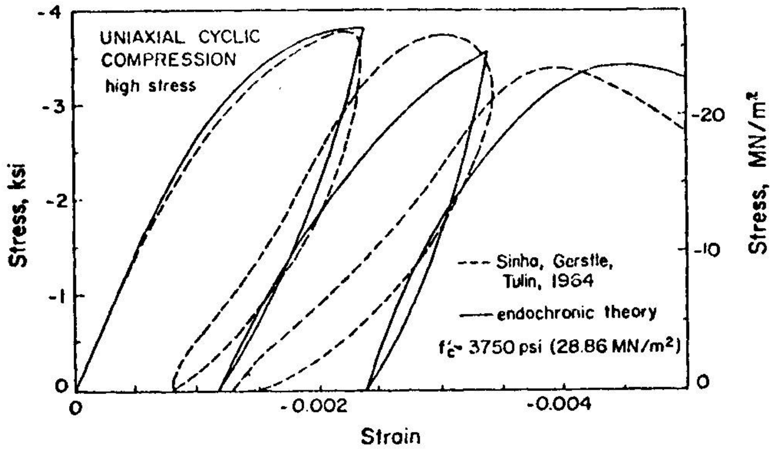

Fig. 15. (Reproduced after [8]).

The second limitation concerns the softening after failure. If, in an uniaxial compression test, the strain is controlled, a stress-strain relationship as shown on Figure 16 can be obtained. For failure calculations this must be of importance in statically indeterminate structures. The importance of this effect has among others been examined by N.S.Ottosen and S.I. Andersen (see [5]. A plasticity model

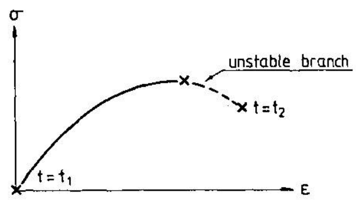

Fig. 16.

based on Drucker's postulates is not able to reflect this softening-effect, simply because one of the postulates requires that the material is not unstable in this way.

On this background it seems natural to look for other theoretical bases for the constitutive equations to be used especially for failure calculations.

4. OTHER THEORETICAL BASES FOR CONSTITUTIVE EQUATIONS FOR CONCRETE

Two other kinds of models that are both closely related to a traditional plasticity theory shall be mentioned. The first kind of model is based on Il'iushin's postulate of plasticity. See [6]. When startpoint is taken in this postulate it is possible to model softening.

The constitutive equations in an endochronic theory are derived from thermodynamic conditions. The basic equations are the 1. and 2. laws of thermodynamics with the usual time "t" measured with a clock replaced by an endochronic time measure "z" depending both on time "t" and a strain measure "ζ".

"ζ" is related to the arc-length of the path followed by the strain-point in a six-dimensional strain-space during deformation. With this kind of model it is possible to reflect softening as well as non-linearity at unloading. A formulation of the theory is given by Valanis [7]. Bazant et al. [8] and Argyris et al. [9] have formulated specific constitutive equations for plain concrete based on the principles suggested by Valanis.

5. CONCLUSIONS

As a conclusion it can be said that although a traditional plasticity model can be a better approximation than a linear elastic model, it seems probable that other theoretical bases are even better suited for describing the stress-strain relationship for plain concrete. This seems especially to be true in the vicinity of failure i.e. for failure calculations. For reinforced concrete members the situation is different because of the interaction with the steel.

REFERENCES

[1] Drucker, D.C.: "A more fundamental approach to plastic stress-strain relations". First U.S. National Cong. of Appl. Mech. ASME 1951.

[2] Ottosen, N.S.: "A failure criterion for concrete". Journ. of the Eng. Mechn. Div. EM4 Aug. 1977.

[3] Launay, P., Gachon, H., Poitevin, P.: "Déformation et résistance ultime du béton sous étreinte triaxiale". Annales de l'Institut technique du Batiment et du Travaux Publics. No. 269. Mai 1970.

[4] Herrmann, B.: "Konstitutive ligninger for beton ved fleraksede spændingstilstande". Risø M-2121. Risø National Laboratory. Denmark. Aug. 1978.

[5] Ottosen, N.S., Andersen, S.I.: "Structural failure of thick-walled concrete elements". Conf. on structural mech. in reactor technology. San Francisco. Aug. 1977.

[6] Dafalias, Y.F.: "Il'iushin's postulate and resulting thermodynamic conditions on elasto-plastic coupling". Int. Journ. Solid Structures. Vol. 13. 1977.

[7] Valanis, K.C.: "A theory of viscoplasticity without a yield surface". Archives of mech. 23,4. Warzawa 1971.

[8] Bazant, Z.P., Bhat, P.D., Shieh, C.L.: "Endochronic theory for inelasticity and failure analysis of concrete structures". Structural Eng. Report No. 1976-12/259. Northwestern Univ. Evanston, Ill. 60201. Dec. 1976.

[9] Argyris, J.H., Pister, K.S., Szimmat, J., Willam, K.J.: "Unified concepts of constitutive modelling and numerical solution methods for concrete creep problems". Computer methods in Appl. Mech. and Eng. 10. North-Holland Publishing Company. 1977.