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Failure Criteria for Concrete under Multiaxial Stress States

Un critère de rupture du béton sous l'effet de contraintes multiaxiales

Bruchbedingungen für Beton unter mehrachsigen Spannungszuständen

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SUMMARY
Various failure criteria for multiaxial compression available in literature are compared with selected experimental data. It appears that the biaxial degenerations of some criteria proposed for triaxial compression yield an unexpected shape, while the results obtained in the use of other criterial accord with expectations. Since none of the investigated criteria corresponds completely to the experimental data in the studied triaxial stress range, a failure criterion for multiaxial stress states is proposed.

RESUME
Différents critères de rupture du béton sous l'effet d'une compression multiaxiale proposés dans la littérature sont comparés à des résultats expérimentaux. Il s'avère que les dégénérations biaxiales de certains critères pour compression triaxiale présentent une forme inattendue, en désaccord avec les résultats expérimentaux, tandis que d'autres sont satisfaisantes. Parce qu'aucun des critères considérés ne concorde pour tous les états de contraintes triaxiales étudiés avec les résultats expérimentaux, un nouveau critère de rupture du béton sous l'effet de contraintes multiaxiales est proposé.

ZUSAMMENFASSUNG
Verschiedene in der Literatur für mehrachsige Druckbeanspruchung vorgeschlagene Bruchbedingungen werden mit ausgelesenen Versuchsergebnissen verglichen. Es zeigt sich, dass gewisse für dreiachsigen Druck aufgestellte Bruchbedingungen für den ebenen Spannungszustand zu einer unerwartet schlechten Übereinstimmung mit Versuchsergebnissen führen. Da keine der untersuchten Bedingungen für die betrachteten dreiachsigen Spannungszustände völlig mit den Versuchsergebnissen übereinstimmt, wird eine neue Bruchbedingung für mehrachsige Spannungszustände vorgeschlagen.

1. EXPERIMENTAL DATA

Since the beginning of the century, a lot of experimental investigations were performed to determine the strength of concrete under multiaxial stress states. However, the behaviour of concrete being very complex [1], the experimental conditions often are not those one wants or supposes to obtain. The occuring problems, lying outside the subject of this paper, are treated profoundly by HILSDORF [2] and KUPFER [3]. Only recently these problems were solved in a more or less satisfying manner. A critical attitude towards experimental data is indispensable, and leads to elimination of many of them. A comprehensive study of experimental investigations available in literature was made in [4], without claiming completeness. The retained compressive data, represented on the figures, were extracted from references [5] to [9].

2. EXISTING CRITERIA

Following conventions are used : $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ are the principal stresses, compression being negative. Three invariants of the stress tensor are used : the octahedral normal stress σ_{oct}, the octahedral shear stress τ_{oct}, and the Lode angle θ, defined as follows :

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\tau_{oct} = \frac{1}{3}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]^{1/2} \qquad (1)$$

$$\cos\theta = (2\sigma_1 - \sigma_2 - \sigma_3) / (3\sqrt{2}\,\tau_{oct}) \qquad 0° \leqslant \theta \leqslant 60°$$

The stress ratios are defined as $k_1 = \sigma_1/\sigma_3$ and $k_2 = \sigma_2/\sigma_3$.
The investigated failure criteria were drawn using a proportional stress increase (constant k_1 and k_2) up to the point where failure is predicted by the criterion. This technique seemed the most logical one : it is the only way of obtaining a unique solution for all criteria and for all stress states. Moreover, the criteria in question are almost exclusively used in finite element applications, where an incremental force (or displacement) technique is used in nonlinear computations, so that the stress increase is roughly proportional. The loading path of a proportional stress increase is represented both in principal stress and in octahedral stress coordinates by a straight line through the origin, the slope being a function of k_1 and k_2, as follows from (1).

2.1. Biaxial compression

Seventeen criteria are drawn on figures 1 and 2, together with the biaxial experimental data from refs. [5] to [9], in principal stress coordinates normalised by R'_{br}, the absolute value of the uniaxial compressive strength. When parameters need to be determined, it is assumed that the uniaxial strength equals $-R'_{br}$, that in compression the equibiaxial strength equals $\beta = 1.16$ times the uniaxial strength and that the ratio between R'_{br} and the tensile strength R_{br} equals 10. The comparison of a criterion to the experimental data is done through the values MRSD (Mean Relative Square Deviation) and RDEV (Relative DEViation) :

$$MRSD = \frac{1}{n}\sum_{i=1}^{m} n_i \frac{(OP-OQ)^2}{OQ^2}$$

$$RDEV = \frac{1}{n}\sum_{i=1}^{m} n_i \frac{OP-OQ}{OQ} \qquad (2)$$

where m is the number of different values of k_2 in the experiments, n_i is the number of experiments for one value of k_2, and $n = \sum_{i=1}^{m} n_i$ is the total number of experiments. The point P represents failure during the experiment, while Q, laying on the straight line OP through the origin O, represents the failure predicted by the criterion. It can be seen that P and Q share the same stress ratio k_2. A criterion gives a good approximation to the experimental data when MRSD and RDEV are small, and a safe one when RDEV is positive. The values of MRSD and RDEV are given in Table 1.

Criterion	$MRSD.10^3$	$RDEV.10^3$
[10]	1.34	+ 10
[11]	3.57	+ 33
[7]	0.99	- 2.2
[12]	1.18	+ 12
[13]	1.29	+ 10
[14]	2.30	+ 19
[15]	2.15	- 18
[17]	3.92	+ 41
[18] (3p)	3.28	- 26
[18] (5p)	1.61	- 14
[19]	37.10	+ 186
[20]	75.10	- 197
[22]	79.36	- 200
[23]	143.03	- 208
Author	0.89	+ 1.4

Table 1 Correspondence between experimental data and criteria for biaxial compression.

The conclusion can be drawn that all the criteria presented on figure 1 behave in a more or less satisfying manner, with [19] on the safe side (the definition of admissible stress is more stringent), and [15] and [18] on the unsafe side. The choice of a criterion can thus be guided by additional requirements, such as continuity of the slope for equibiaxial stresses, or simplicity of the analytical formulation. The criterion of DRUCKER - PRAGER [10] with adapted parameters offers a good and simple approximation.

The four criteria represented on figure 2 are striking by their strong deviation from the experimental data. For [21] the reason herefore is evident : it is based upon not retained (because unreliable) experimental data. For the remaining three, which were proposed for triaxial compression, the reason can be found in the ambiguity of representing a triaxial stress state in octahedral coordinates, as will be mentioned in next section. On the other hand, one can easily check that wen these criteria are computed in a different way, the correspondence to the experimental data is much better : when the experimental values of σ_{oct} and θ at failure during an experiment are substituted in the criterion, the value of τ_{oct} for which the criterion predicts failure is considerably nearer to the experimental one than the value which would be obtained by proportional stress increase. Unfortunately, this way of reasoning is completely erroneous : indeed, the stress state for which failure is predicted is not a biaxial one, as can be easily computed by inverting (1), and comparison to experiment has no sense.

2.2. Triaxial compression

Inspection of the experimental data leads to the conclusion that the failure criterion must be dependent upon all three stress invariants of (1). Representing the criterion in octahedral axes implies the use of θ as a parameter. On figures 3 to 9, the criteria are drawn for the values $\theta = 0°$, $30°$ and $60°$ (from the bottom upwards), together with the corresponding experimental data, respectively marked by squares, circles and asterisks. The stress region studied is limited to $|\sigma_{oct}| \leqslant 1.5\ R'_{br}$. Following conclusions can be drawn :

- criterion [13] is represented on figure 3 by a unique curve, since it contains only two invariants. It becomes more and more conservative with increasing values of σ_{oct} and θ.

- the other criteria all give a good approximation for $\theta = 0$ (lower curve). Criterion [19] lies below the experiments since the definition of admissible stress is more stringent.

- for $\theta = 60°$ however (upper curve), only a few criteria do correspond to the experiments, namely [19] (figure 4) and [18] (5 parameters, figure 5 - the parameters given by the authors were however changed). All the other criteria are lying above the experimental data. Although the difference does not seem to be critical, it really is. This is demonstrated on figure 10 : suppose a proportional loading path OA with a certain value of θ, and the corresponding trace in octahedral stresses of the failure criterion for that value of θ. Consider three proportional stress states A, B and C of different magnitude, as indicated on figure 10, where B is the failure stress predicted by the criterion. When the slope of the criterion is nearly equal to that of the loading path, as is the case in figure 10, and also for $\theta = 60°$ for most criteria, the "corresponding" points A' and C' do not differ much in ordinate from a respectively C. Suppose now that A is the failure stress state experimentally obtained, instead of B. The one would conclude from the fact of A' being near to A, that the criterion describes well failure, although it only predicts failure at the double magnitude (point B). It can thus be concluded, that the presented experimental results for $\theta = 60°$ (asterisks) are much more "distant" from the failure criteria [15], [18] (3 parameters), [20], [22] and [23] than would be expected from the figures. As an example, [18] (3p) and [23] do <u>not</u> predict failure for $\theta = 60°$ for a proportional stress increase with $k_1=k_2 \geqslant 0.099$, [20] and [22] for $k_1=k_2 \geqslant 0.184$, while experimentally [9] failure occurs already at $\sigma_{oct} = -1.5\ R'_{br}$ for $k_1=k_2=0.205$.

- for $\theta = 30°$, the corresponding of the criteria to experimental data is somewhat intermediate between the corresponding for $\theta = 0$ and $\theta = 60°$. This means that the dependence of the criteria upon θ is good, but also for [15] and [18] (3p) that the predicted failure stresses are too high, as can be seen on figures 7 and 6. Exceptions to this are criteria [20], [22] and [23], for which the interpolation function is not satisfying. Besides the fact that the predicted failure stresses are too high, the meridian is only represented for $|\sigma_{oct}| > 1.44\ R'_{br}$ on figure 8, ([20] and [22] share the same meridians for $\theta = 0$, 30° and 60°) and it is traced in dotted line on figure 9 for [23]. The reason for this is that the region in octahedral coordinates, where triaxial stresses can exist, is limited, so that the criterion for triaxial compression is a <u>bounded</u> curve. This is a fact that is often overlooked. Space limitations are preventing a complete analysis, but it can be said that the region where triaxial compression with a certain value of θ can exist is limited by a straight line through the origin, the slope of which is depending upon θ. The intersection of this line with the meridian of the criterion for that value of θ is a point of biaxial compression, and the section of the meridian with $|\sigma_{oct}|$ lower than this "limit point" represents stress states of compression-compression-tension ! Now if the criterion is not supposed to be valid for other stress states than triaxial compression, this section has no sense. The problem is that the "limit point" is not apparent in octahedral coordinates, as it would be in principal stress coordinates. The extremity on the right of the meridians represented on the figures is precisely this "limit point". On figure 8, the abscis of the "limit point" for $\theta = 30°$ is approximately - 1.44, and the meridian is limited to that point since criteria [20] and [22] are not assumed to be valid for tension-compression stress states. This is well assumed for [23], but the abscis of the "limit point" for $\theta = 30°$ is approximately - 2.57, so that the complete section on figure 9 represents tension-compression stress states, and is traced in dotted line. Anyhow, it is lying well above the experimental data. It is hoped that a more comprehensive study of these phenomena will be published in the near future.

- the locus of the "limit points" represents the failure criterion for biaxial compression. It can be suspected from figures 8 and 9 that for the criteria in question, this will have a bulged shape in octahedral coordinates (and also in principal stress coordinates). This is confirmed by figure 2.

3. PROPOSED CRITERION FOR MULTIAXIAL STRESS STATES

Since none of the investigated criteria correspond to the experimental data for the complete stress region studied, a criterion is proposed, based on the experimental data [5] to [9] and taking into account following conditions :

- a regression is made of the experimental octahedral stresses, for $\theta = 0$ and $\theta = 60°$. A linear regression seems appropriate, due to the good linearity of the data in the studied stress region ($|\sigma_{oct}| \leq 1.5\ R'_{br}$). The regression needs to be performed in polar coordinates, with the angle as independent coordinate (cfr. the dangers mentioned in 2.2).
- the meridians must pass through the points of uniaxial and equibiaxial compressive strength. The latter equals $\beta = 1.16$ times the former.
- the interpolation function in θ must be continuous and convex. The elliptic function proposed by WILLAM and WARNKE [18] is used. Considering this, following criterion is proposed for multiaxial compression :

$$\frac{\tau_{oct}}{R'_{br}} = \frac{2C(C^2-T^2)\cos\theta + C(2T-C)\left[4(C^2-T^2)\cos^2\theta + 5T^2 - 4TC\right]^{1/2}}{4(C^2-T^2)\cos^2\theta + (C-2T)^2}$$

$$\text{with}\quad \begin{aligned} T &= 0.12051 - 0.55128\ \sigma_{oct}/R'_{br} \\ C &= 0.25834 - 0.63917\ \sigma_{oct}/R'_{br} \\ |\sigma_{oct}| &\leq 1.5\ R'_{br} \end{aligned} \tag{3}$$

C and T, the dimensionless octahedral shear stress values for the compressive ($\theta = 60°$) and tensile ($\theta = 0°$) meridians of the criterion, are determined by regression of the experimental data. This criterion has the same analytical formulation as [19], although it was drawn up independently. However, the parameters are not the same, thus yielding different results. The criterion is represented on figure 11. It can be noticed that also for $\theta = 30°$ the criterion corresponds very well to the experimental data, though no use of this data was made to construct the criterion. Its trace for biaxial compression is represented on figure 1, together with the other criteria. The correspondence to the experimental data is better than that of all other criteria, as appears in table 1, though no use of the biaxial data was made to construct the criterion, except uniaxial and equibiaxial strength. This proves that the elliptic interpolation function proposed in [18] is really the best one can choose, and suggests that also for triaxial compression there is good agreement with experimental results for all values of θ.
For compression-tension stress states, the same interpolation function as (3) is proposed, but T and C are replaced by :

$$\begin{aligned} T &= \frac{\sqrt{2}\,\alpha\beta}{2\beta+\alpha} - \frac{\sqrt{2}\,(\beta-\alpha)}{2\beta+\alpha}\,\sigma_{oct}/R'_{br} \\ C &= \frac{\sqrt{2}\,\alpha\beta}{3\alpha\beta+\beta-\alpha} - \frac{\sqrt{2}\,(\beta-\alpha)}{3\alpha\beta+\beta-\alpha}\,\sigma_{oct}/R'_{br} \end{aligned} \tag{4}$$

with $\alpha = R_{br}/R'_{br}$ (= 0.1 on figure 11) ; $\beta = 1.16$.
These meridians satisfy continuity with the triaxial compression criterion,

share the same intersection with the octahedral stress axis, and the first one satisfies the tensile strength R_{br}. Comparison to experimental data is difficult to check, due to the scarcity of reliable experimental data. Agreement with [8] is good for low tensile stresses, but not so well for low compressive stresses. Tensile strength R_{br} was however not determined experimentally in [8], which may possibly explain the deviation.

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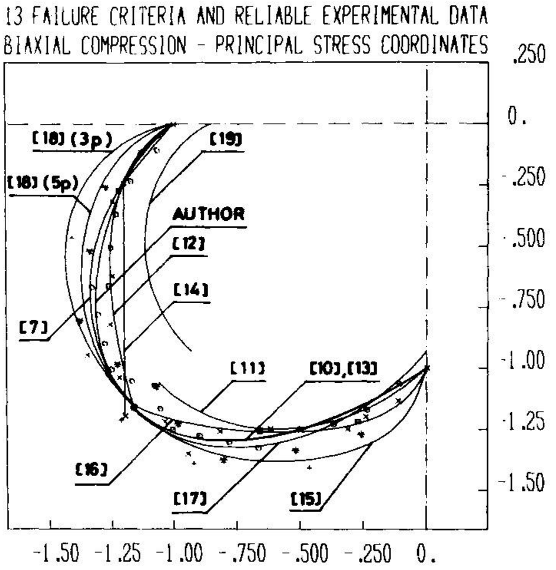
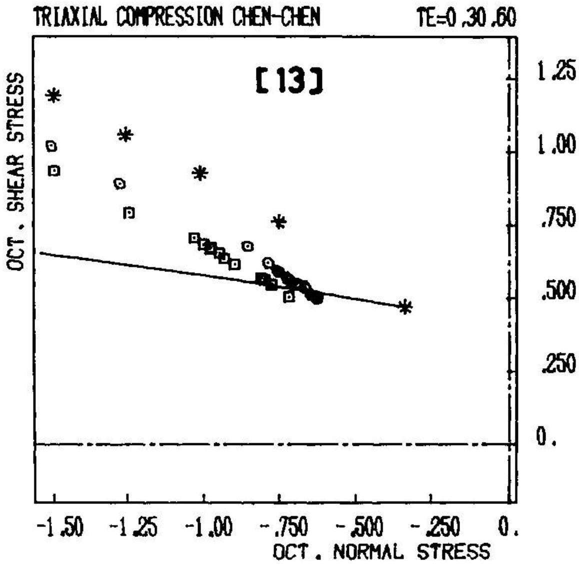


figure 1



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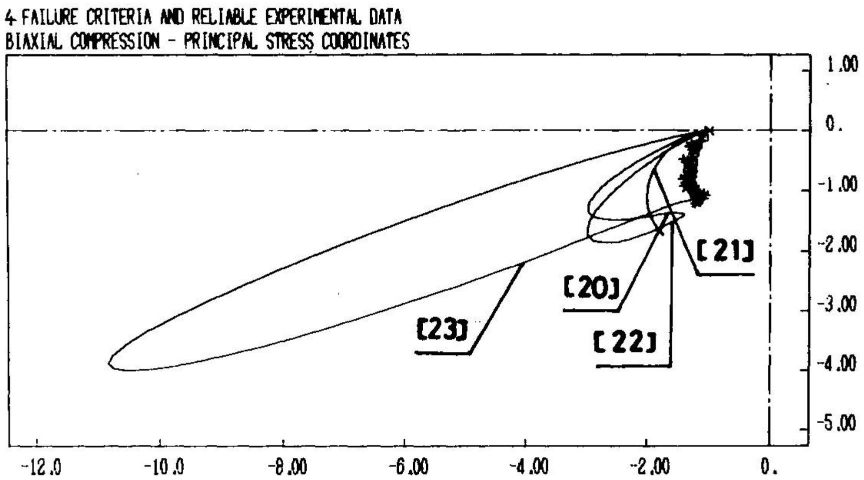

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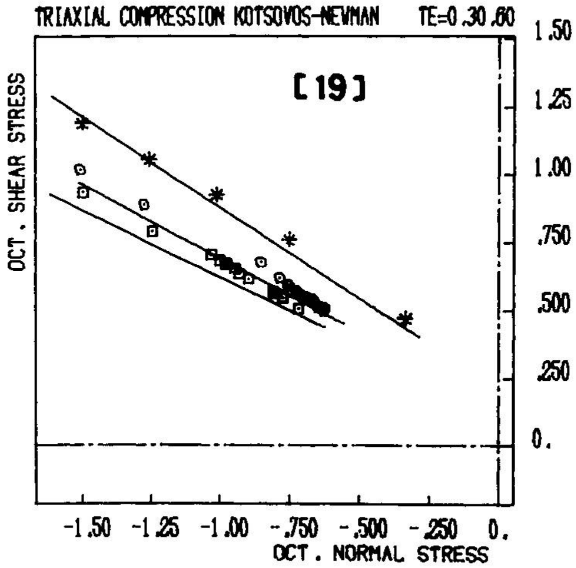

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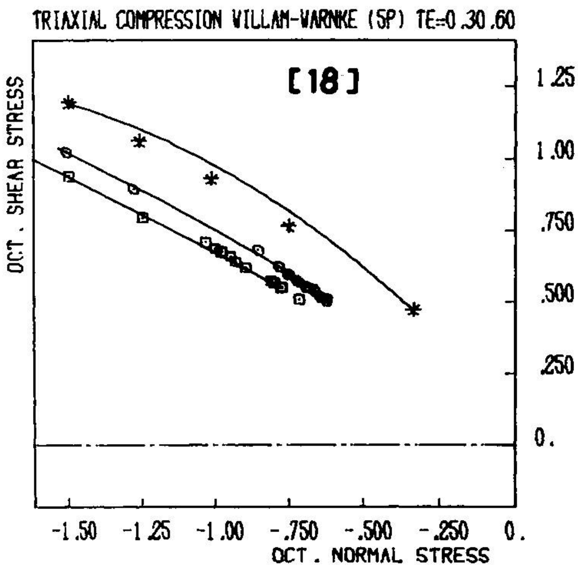

figure 5

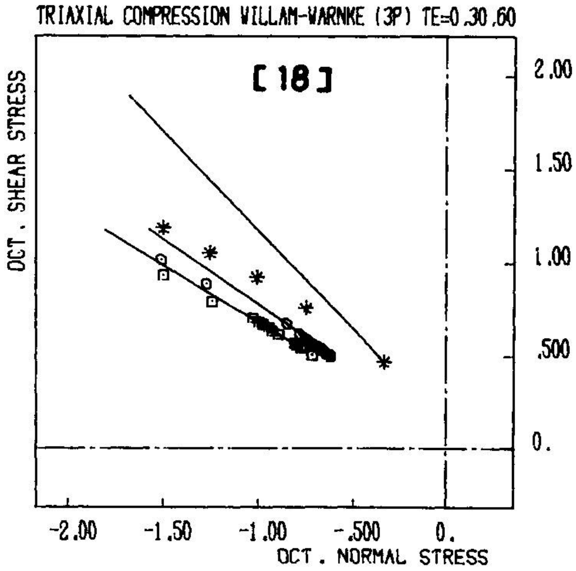

figure 6

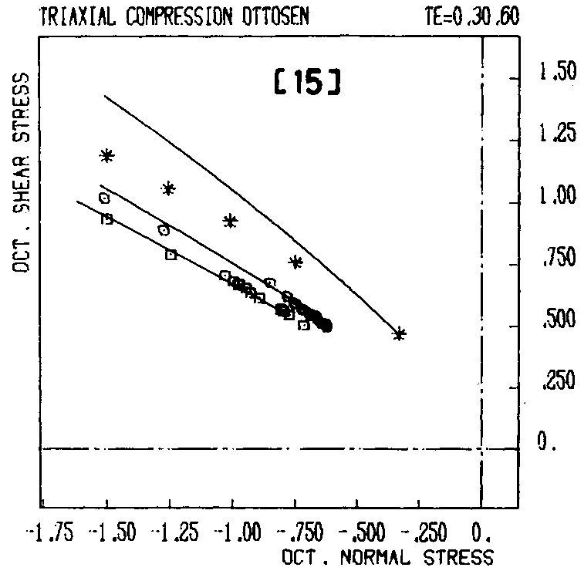

figure 7

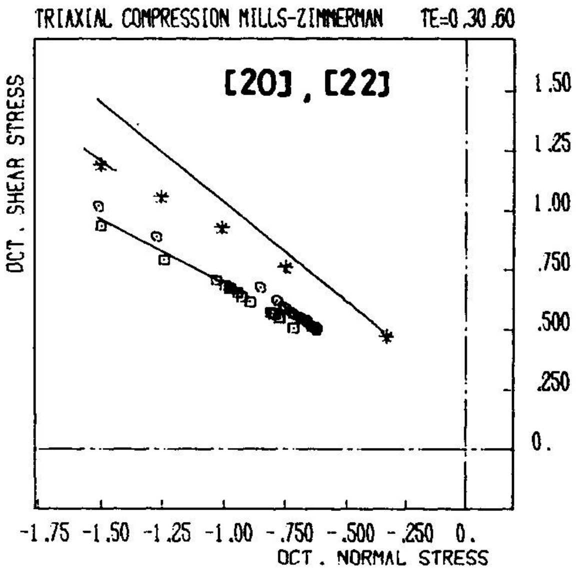

figure 8

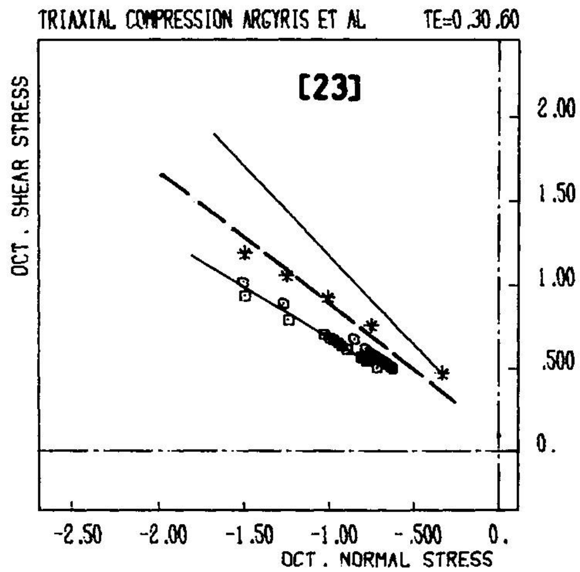

figure 9

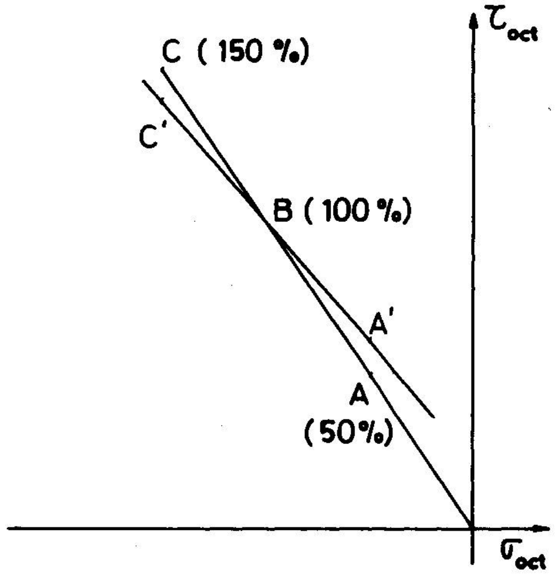

figure 10

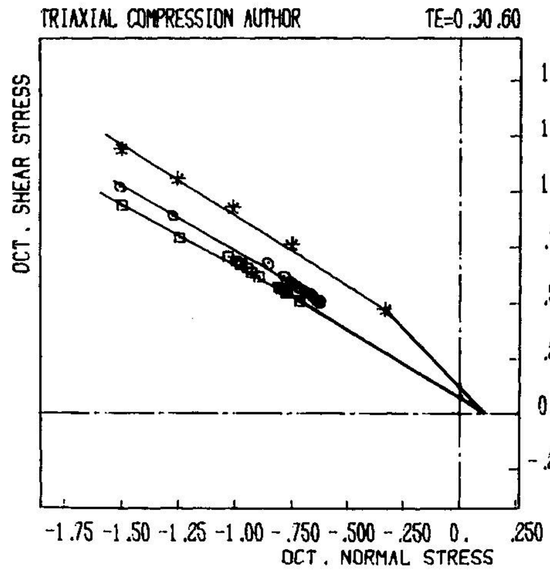

figure 11