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Autor: Grob, J.

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Design of Reinforced Concrete Beams Based on Directive 34 of the Swiss Code SIA 162

Dimensionnement des poutres en béton armé sur la base de la directive 34 des normes suisses SIA 162

Bemessung von Stahlbetonträgern nach der Richtlinie 34 der schweizerischen Norm SIA 162

J. GROB
Dr. sc. techn.
Ing. Büro Schneller-Schmidhalter-Ritz
Brig, Switzerland

SUMMARY
In April 1976 the new Directive No 34 of the Swiss Code SIA 162 was introduced for the design of reinforced and prestressed concrete beams. The design rules for beams under bending, shear, torsion and combined actions are based on plastic solutions for strength using a truss model with variable inclination of the concrete compression struts.

RESUME
En avril 1976 la nouvelle directive 34 des normes suisses SIA 162 a été introduite pour le dimensionnement pratique des poutres en béton armé et précontraint. Les règles pour le dimensionnement des poutres soumises à la flexion, à l'effort tranchant, à la torsion et à des efforts combinés sont basées sur des solutions plastiques pour la résistance en utilisant un modèle de treillis avec inclinaison variable des bielles en béton.

ZUSAMMENFASSUNG
Im April 1976 wurde die neue Richtlinie 34 der schweizerischen Norm SIA 162 für die Bemessung von Stahlbeton- und Spannbetonträgern eingeführt. Die Bemessungsregeln für Balken unter Biegung, Querkraft, Torsion und kombinierten Beanspruchungen stützen sich auf plastische Lösungen für den Bruchwiderstand unter Verwendung eines Fachwerkmodells mit variabler Neigung der Betondruckdiagonalen.

1. INTRODUCTION

Structural design requires a suitable structural behavior under service loads and an acceptable factor of safety against failure. The Directive 34, observing these design principles given in the Swiss Code SIA 162, unifies the design of reinforced and prestressed concrete structures. Hence an adequate behavior of a structure under service loads with regard to cracking and deflections must be ensured. Since the safety of a structure depends directly on the ultimate strength the determination of the strength is of primary importance. In the Directive 34 the same truss model is assumed as failure model for all actions as bending, shear, torsion and combined.

The Directive 34 of the Swiss Code SIA 162 offers two different design procedures. First, for the design of a member cross-section, an adequate factor of safety against failure has to be applied to the internal actions (bending moments, shear forces etc.) determined elastically.

$$\frac{W_r}{s_W} \geq S'(s_{Li} \cdot L_i) \quad \text{or} \quad W_r \geq S^* = s_W \cdot S'(s_{Li} \cdot L_i) \qquad (1)$$

with

L_i	Design Loads	W_r	Theoretical Resistance of Member Cross-Section
S'	Internal Actions due to $(s_{Li} \cdot L_i)$	s_{Li}	Load Factors $(0.8 \leq s_{Li} \leq 1.4)$
S^*	Internal Actions for Required Minimum Resistance	s_W	Resistance Factor $(s_W = 1.3)$

Additionally a design method is employed based on an adequate carrying capacity of a structure with respect to standard design load configurations.

2. TRUSS MODEL

To describe the static behavior of reinforced and prestressed concrete beams a truss model is used with the longitudinal bars acting as stringers, the stirrups as vertical ties and the concrete diagonals as inclined struts. In order to simplify the mathematical treatment the longitudinal reinforcement is first assumed to be concentrated into corner stringers.

If a beam element is subjected to a bending moment M^*, an axial force N^* and a shear force Q^*, the equilibrium conditions lead to the following stringer and stirrup forces:

$$B = Q^* \cdot \frac{s_B}{h_o} \cdot \tan\gamma$$

$$Z_o = -\frac{M^*}{h_o} + \frac{N^*}{2} + \frac{Q^*}{2} \cdot \cot\gamma \qquad (2)$$

$$Z_u = \frac{M^*}{h_o} + \frac{N^*}{2} + \frac{Q^*}{2} \cdot \cot\gamma$$

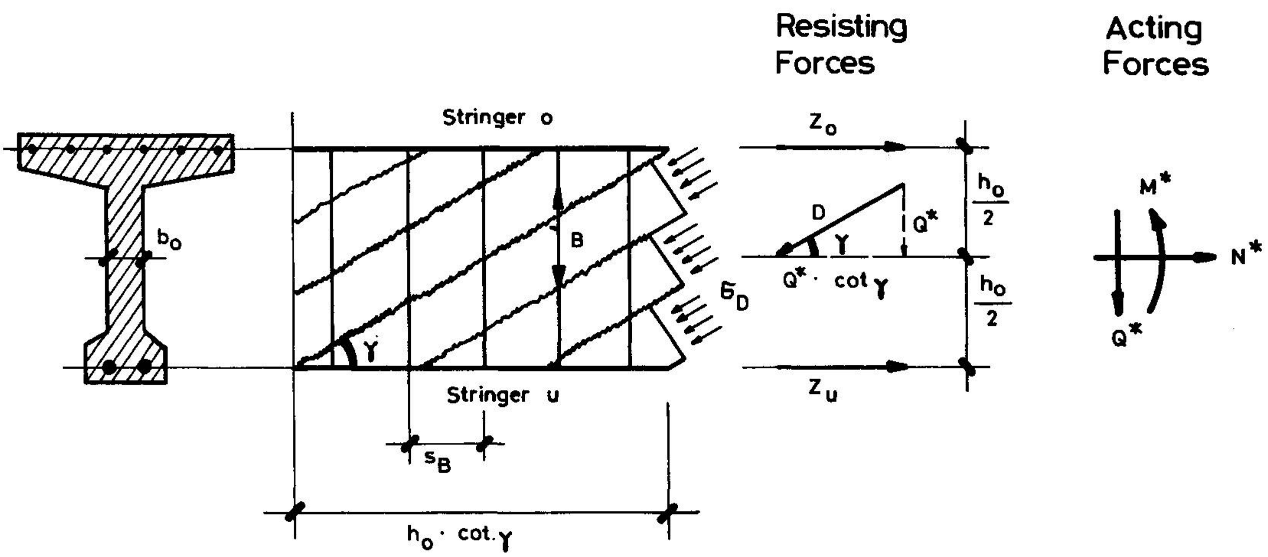

Fig. 1 Truss Model

The stresses in the reinforcement and the concrete have to be limited. The stringer and stirrup forces must not violate the yield conditions of the steel, and the concrete stresses must not produce a concrete failure. Concrete failure can be caused by crushing of the concrete compression zone or of the concrete compression struts.

If the concrete stress in the compression zone is limited, the extension of the compression zone must be considered. Thus the assumption of concentrated stringer forces in the corners does no longer hold. The distance y between compression and tension resultant gets smaller than h_o, Fig. 1. This fact can easily be taken into account by using the well-known bending theory. The tensile force due to shear is then added to the resultant forces due to bending and axial force.

$$Z_o = Z_o\,(M^*,\ N^*) + \frac{Q^*}{2} \cdot \cot\gamma$$
$$Z_u = Z_u\,(M^*,\ N^*) + \frac{Q^*}{2} \cdot \cot\gamma \qquad (3)$$

The stresses σ_D in the concrete compression struts are caused by the diagonal force D, Fig. 1.

$$\sigma_D = \frac{Q^*}{b_o \cdot h_o} \cdot \frac{1}{\cos\gamma \cdot \sin\gamma} = \tau^* \cdot \frac{1}{\cos\gamma \cdot \sin\gamma} \qquad (4)$$

From tests, Ref. [3], the following bounds have been chosen for the inclination of the concrete compression struts.

$$0.5 \leq |\tan\gamma| \leq 2.0 \qquad (5)$$

Further explanations are given in Refs. [1], [2] and [6].

3. DESIGN OF REINFORCED CONCRETE BEAMS

3.1 Bending and Axial Force

The design of beams subjected to a bending moment and an axial force is based on the well-known bending theory.

For an advantageous design procedure all material strength properties must be put on the same basis. The Directive 34 of the Swiss Code SIA 162 generally uses nominal strength values according to the 5%-fractile for both steel and concrete.

$$\text{Concrete}: \beta_r = 0.60 \cdot \beta_w \simeq 0.75 \cdot \beta_c$$
$$\text{Steel}: \sigma_f = \sigma_{2.0} \qquad (6)$$

with

β_r Nominal Compressive Strength of Concrete

β_w Cube Strength (16%-Factile)

β_c Cylinder Strength (16%-Fractile)

σ_f Nominal Yield Stress of Steel

$\sigma_{2.0}$ Yield Stress for 2 ‰ Permanent Strain (5%-Fractile)

If prestressing steel is present, then the initial strain in the prestressing steel is taken into account in calculating the ultimate strength. This formulation leads to an unified approach for any possible choice of prestressing between reinforced and fully prestressed concrete.

3.2 Shear

The nominal shear stress is a measure of the concrete stress in the web:

$$\tau^* = \frac{Q^*}{b_o \cdot h_o} \qquad (7)$$

No shear cracks occur below the value τ_r. Therefore the shear is carried by the concrete alone and none or only a nominal shear reinforcement is required for $\tau^* \leqslant \tau_r$.

The values of τ_r given in the current Swiss Code SIA 162 and in the new Directive 34, depend on the concrete strength β_w (cube strength).

Cube Strength β_w (N/mm2)	:	20	30	40	⩾	50
Approx. Cylinder Strength β_c	:	16	24	32	⩾	40
Design Strength β_r	:	12	18	24	⩾	30
Design Shear Stress τ_r	:	0.8	1.0	1.2		1.4

(8)

On the other side the nominal shear stress τ^* should not exceed the maximum value τ_{max} which depends on the concrete strength and the maximum stirrup spacing s_B.

$$\text{Normal Spacing}: \tau_{max} = 5\,\tau_r \text{ for } s_B \leqslant \frac{h_o}{2} \text{ but } s_B \leqslant 30 \text{ cm} \qquad (9)$$

$$\text{Close Spacing : } \tau_{max} = 6\,\tau_r \text{ for } s_B \leq \frac{h_o}{3} \text{ but } s_B \leq 20 \text{ cm}$$

Between the uncracked state, $\tau^* \leq \tau_r$, and the fully developed truss action, about $\tau^* \geq 3\,\tau_r$, a transition takes place. Observations on test beams and recent theoretical investigations, Refs. [6] and [7], show that the initial shear strength of the uncracked concrete is reduced progressively. For design purposes the following approximation is used:

$$\begin{aligned} &\text{For } \tau_r < \tau^* < 3\,\tau_r : Q_b = \frac{1}{2} \cdot (3\,\tau_r - \tau^*) \cdot b_o \cdot h_o \\ &\text{For } \tau^* \geq 3\,\tau_r \quad : Q_b = 0 \end{aligned} \qquad (10)$$

This additional contribution is taken into account for the design of the stirrup reinforcement by deducting Q_b from the applied shear force Q^*. Thus the final design equations are obtained.

Stirrup Reinforcement F_B (Q):

$$\begin{aligned} &F_B\,(Q) \cdot \sigma_{fB} \geq (Q^* - Q_b) \cdot \frac{s_B}{h_o} \cdot \tan\gamma \\ &\text{but} \\ &F_B\,(Q) \cdot \sigma_{fB} \geq \frac{1}{2} \cdot \tau_r \cdot b_o \cdot s_B \text{ (min. stirrup reinf.)} \end{aligned} \qquad (11)$$

Longitudinal Reinforcement F_L (Q) due to Shear:

$$F_L\,(Q) \cdot \sigma_{fL} \geq \frac{1}{2} \cdot Q^* \cdot \cot\gamma \qquad (12)$$

In the Directive 34, closer limits than given by Eq. (5) are prescribed for the inclination γ of the concrete compression struts.

$$\frac{3}{5} \leq |\tan\gamma| \leq \frac{5}{3} \qquad (13)$$

The cracking evaluation of test beams, Ref. [2], showed an adequate behavior under service loads, if the limiting values from Eq. (13) are used.

For design purposes the Eqs. (11) and (12) are put in a non-dimensional form. In Fig. 2 the corresponding design diagrams are given for the practical inclination range of the compression struts, e.g. from about 30^o to 45^o.

As shown in Fig. 2 smaller inclinations γ lead to a reduction of the stirrup reinforcement but to an increase of the longitudinal reinforcement. Usually the longitudinal reinforcement is such, that the inclination $\tan\gamma$ can be fixed to the value 3/5. Furthermore, as shown in Ref. [2], the selection of an angle γ approaching 30^o gives the most economical solution.

So far the influences of a variable depth of the beam and of inclined prestressing tendons have not been considered. They can be taken into account by substituting the applied shear force Q^* by the effective web shear force Q^*_{eff}. In order to simplify the design procedure the force Z_s, Fig. 3, in the tendon is taken equal to the prestressing force V of the tendon.

$$Q^*_{eff} = Q^* - \frac{M^*}{h_o} \cdot 2 \tan\frac{\delta}{2} - V \cdot (\sin\alpha - \frac{y_s}{h_o} \cdot \cos\alpha \cdot 2 \tan\frac{\delta}{2}) \qquad (14)$$

The design convention for the forces and the angles are indicated in Fig. 3.

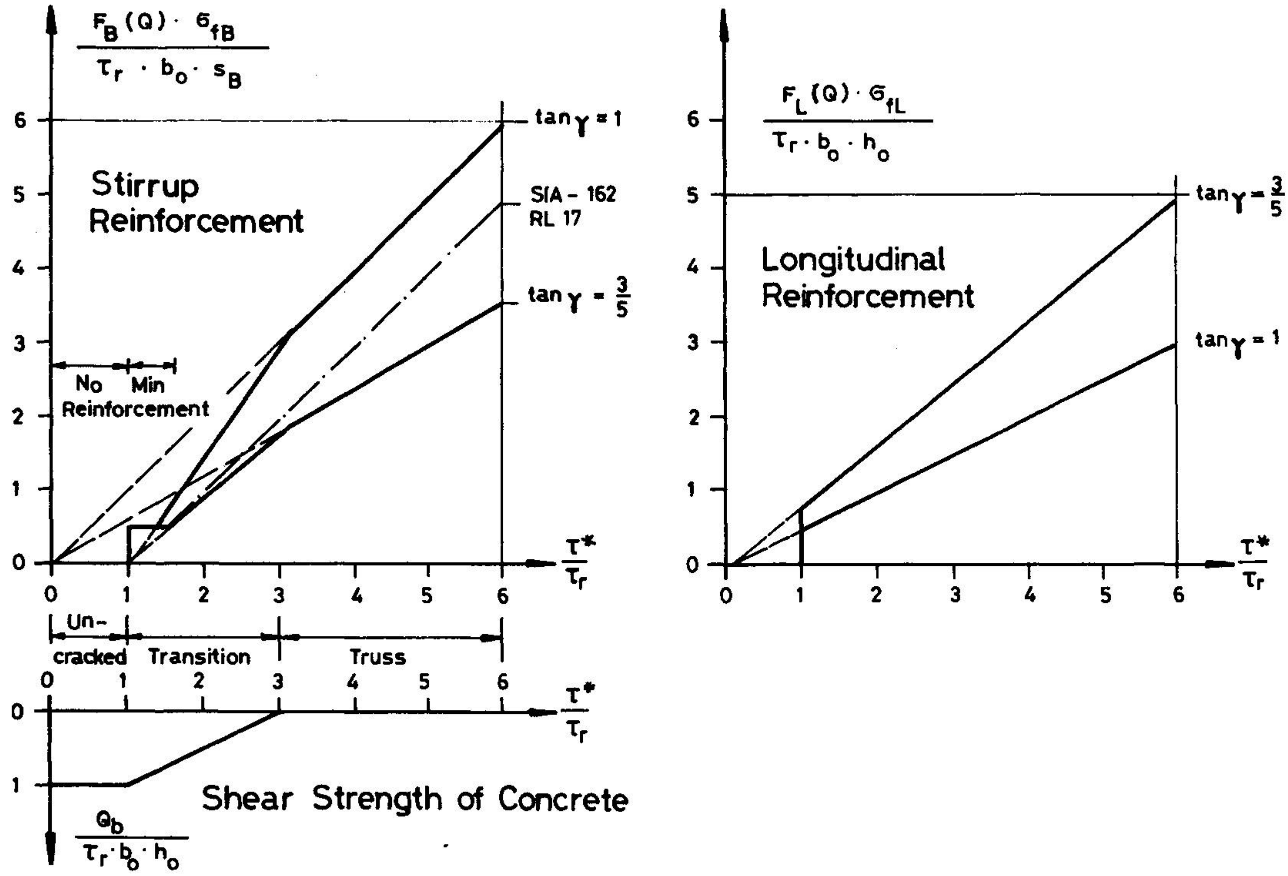

Fig. 2 Design of Shear Reinforcement

3.3 Torsion and Combined Actions

In a box beam torsion produces a constant shear flow S^* around the perimeter u_o.

$$S^* = \tau^* \cdot t_o = \frac{T^*}{2\,F_o} \tag{15}$$

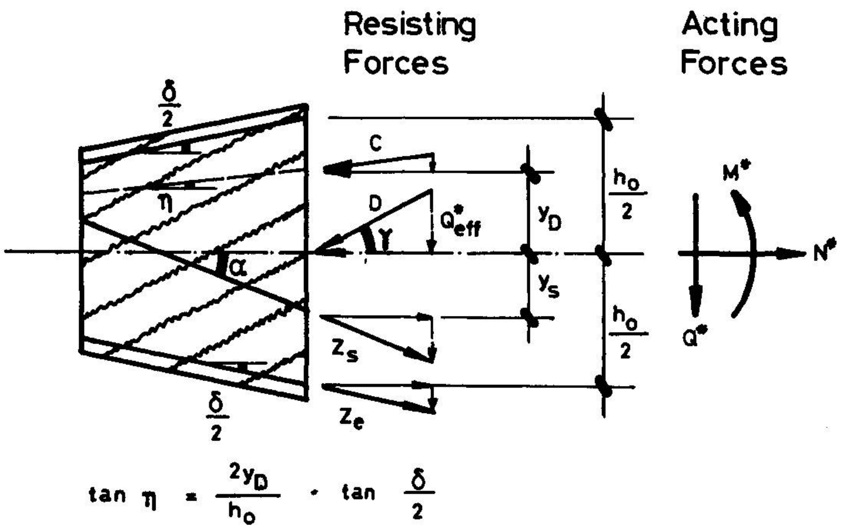

Fig. 3 Variable Depth and Inclined Tendons

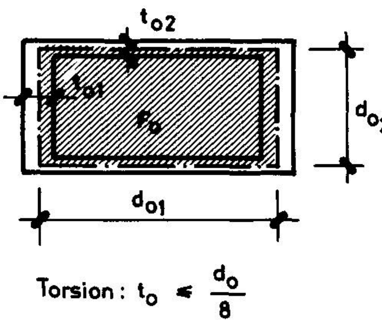

Fig. 4 Effective Wall Thickness for Torsion

Assuming the same static model for both shear and torsion, the design equations for shear (shear flow Q^*/h_o) can easily be transformed for torsion (shear flow $T^*/2\,F_o$).

$$\frac{Q^*}{h_o} \rightarrow \frac{T^*}{2F_o} \qquad \frac{Q_b}{h_o} \rightarrow \frac{T_b}{2F_o} \qquad b_o \rightarrow t_o \qquad h_o \rightarrow u_o \tag{16}$$

Torsion produces an additional effect due to the distortion of the side walls of a box beam. The concrete stresses in the compression struts caused by the truss action are superimposed by secondary bending stresses, Ref. [3]. In the Directive 34 these bending stresses are limited in a pragmatic manner by adopting a cautious value for the effective wall thickness t_o, Fig. 4.

$$t_o \leqslant \frac{d_o}{8} \tag{17}$$

If the nominal shear stress due to torsion is calculated by considering Eq. (17), the bounds for the nominal shear stress are the same for both torsion and shear.

The design of beams subjected to combined actions can be made by superposition of the stresses and reinforcements respectively due to bending, shear and torsion. However in practical design a more efficient procedure, pointed out next, is recommended.

The design of the stirrup reinforcement and the control of the concrete compression struts can be based upon the calculation of the shear resultants in the side walls of a beam. The design rules given in Sec. 3.2 for shear are then applied to these shear resultants. For the example shown in Fig. 5 the following shear resultants R^* are obtained in the webs:

$$R^* = \frac{Q^*}{2} \pm \frac{T^*}{2F_o} \cdot h_o \tag{18}$$

The design of the longitudinal reinforcement and the control of the compression zone can be based upon the effective axial force Z^* due to the applied axial force, the shear force and the torque, Fig. 5.

$$Z^* = N^* + |\cot\gamma| \cdot (|Q^*| + |T^*| \cdot \frac{u_o}{2F_o}) \tag{19}$$

The tensile force due to shear acts at the centroid of the two webs, whereas the tensile force due to torsion acts at the centroid of the perimeter u_o around the area F_o.

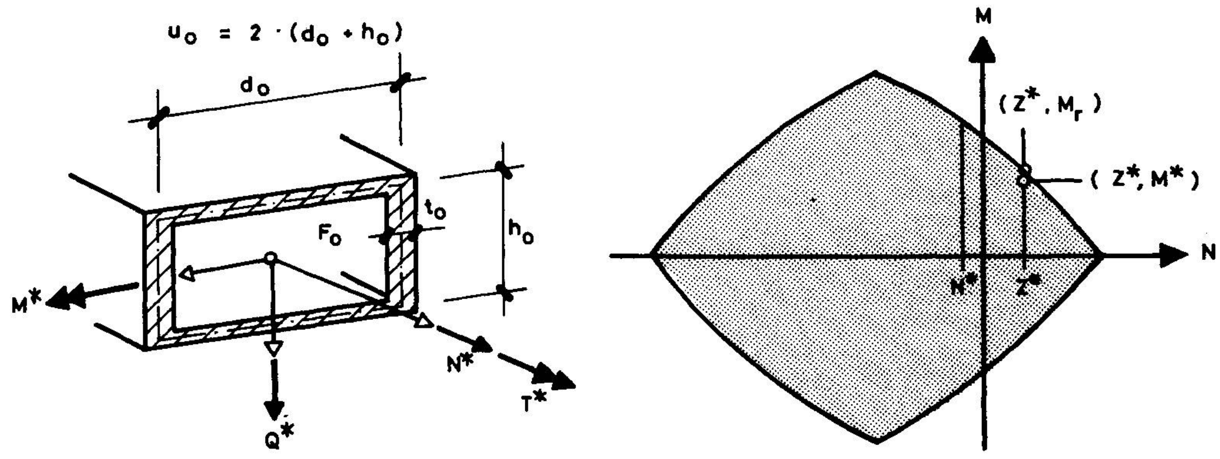

Fig. 5 Combined Actions

A sufficient resistance of the member cross-section is obtained, if the stress point (Z^*, M^*) lies inside the interaction curve, Fig. 5, determined by application of the bending theory.

CONCLUSION

As generally can be stated, design methods are better the simpler they are. A simple and unified design approach including a clear safety concept leads to safe structures, because the designer gets a sound idea of the static behavior and of the carrying capacity. It is felt that plastic analysis, especially the static method, helps to develop a simple, unified and reasonably accurate design concept.

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