V

Optimal Shear Design of Beams with CEB-FIP Model Code

Dimensionnement optimal à l'effort tranchant des poutres à l'aide du code modèle CEB-FIP

Optimale Schubbemessung von Balken nach der CEB-FIP Mustervorschrift

R. MAQUOI
Senior Research Associate
National Foundation for Scientific Research
Liège, Belgium

J. RONDAL
Senior Lecturer
University of Liège
Liège, Belgium

SUMMARY
The contribution presents a simpler and more convenient formulation of C.E.B.-F.I.P. Model Code. The findings of an extensive numerical investigation are given for the economical choice of shear and longitudinal tensile reinforcement, based on an inclination of the compression field $\theta = 31^{\circ}$ which is shown to belong in any case to the optimal solution.

RESUME
La contribution présente une formulation plus simple et plus commode des Recommandations C.E.B.-F.I.P. relatives à l'effort tranchant. On y expose les résultats d'une vaste recherche numérique quant au choix économique des armatures d'effort tranchant et des armatures longitudinales, en ayant adopté au préalable un angle d'inclinaison du champ de compression dans l'âme $\theta = 31^{\circ}$, valeur qui appartient en tout cas à la solution optimale.

ZUSAMMENFASSUNG
Eine einfache Formulierung der CEB-FIP Empfehlungen für die Schubbemessung wird gegeben. Resultate einer ausgedehnten numerischen Untersuchung im Hinblick auf die wirtschaftliche Auslegung von Schub- und Längsbewehrung werden dargestellt. Es wird gezeigt, dass die angenommene Druckfeldneigung $\theta = 31^{\circ}$ in jedem Fall zur optimalen Lösung führt.

INTRODUCTION.

The 1978 issue of the C.E.B.-F.I.P. Model Code contains |1| a realistic proposal for the shear design of concrete beams which is still based on the truss analogy but is generalized and completed by a set of limitations and improvements drawn from a lot of theoretical and experimental results |2|. In its refined presentation, called *accurate method,* it leaves to the designer the choice of the Θ crack inclination instead of the usual value $\Theta = 45°$, according to the original concept of Ritter and Mörsch |5|. On the value of Θ, depend the amount of shear reinforcement and the increase of the longitudinal tensile reinforcement. Both types of reinforcement are varying in the opposite direction : if Θ is chosen so that the amount of shear reinforcement increases, the supplement of longitudinal reinforcement decreases. Thus, the question is : does an optimum inclination angle Θ exist, that minimizes the total amount of reinforcement required by shear loading ? The purpose of present paper is to bring an answer to such a question, when case of concentrated loads near the supports is excluded.

The notations used in present paper are similar to those of the C.E.B.-F.I.P. Model Code ; the main of them are listed at the end.

C.E.B. - F.I.P. RECOMMENDATIONS.

The shear limit state can be reached either by diagonal compression in the concrete, causing crushing of the web, either by tension in the web reinforcement which reaches its design strength . The applied design shear V_{sd} must fulfil the following conditions on the resistant shear forces V_{Rd2} and V_{Rd3} for shear reinforcement and web concrete respectively :

$$V_{sd} \leqslant V_{Rd2} \quad (1)$$

$$V_{sd} \leqslant V_{Rd3} \quad (2)$$

The truss analogy can be remarkably improved by taking account in a restricted range, in addition to the usual shear force V_{wd} carried by truss action, of a contribution V_{cd} corresponding to the shear force carried by the compression flange and other effects, so that the resistant design shear force for web concrete reads :

$$V_{Rd3} = V_{cd} + V_{wd} \quad (3)$$

with : $V_{cd} = 2{,}5\ V_{Rd}$ (if $V_{sd} \leqslant 2{,}5\ V_{Rd}$) (4a)

$= 0$ (if $V_{sd} \geqslant 7{,}5\ V_{R}d$) (4b)

$$V_{Rd} = A_{sw}(0{,}9\ d/s)\ f_{ywd}(\text{cotg}\ \alpha + \text{cotg}\ \Theta)\ \sin\alpha \quad (5)$$

V_{Rd} is a codified resistant design shear.

The resistant design shear force for reinforcement, is given below, and has an upper bound :

$$V_{Rd2} = 0,6\ f_{cd}\ b_w\ d(\text{cotg}\ \alpha + \text{cotg}\ \theta)\ \sin^2\theta \leqslant 0,45\ f_{cd} b_w\ d\ \sin 2\ \theta \qquad (6)$$

In any case, the shear reinforcement must comply with a specified minimum amount :

$$\rho_w = A_{sw}/sb_w\ \sin\alpha \geqslant \rho_{w\ min} \qquad (7)$$

Last, in order to control the crack width for the serviceability state, the value of the inclination θ is bounded as follows :

$$3/5 \leqslant \text{cotg}\ \theta \leqslant 5/3 \qquad (8)$$

The longitudinal tensile reinforcement should be increased to resist the following additional tensile force :

$$\Delta F_{t\ell} = \Delta A_{s\ell}\ f_{y\ell d} = V_{sd}^2\ s\ /\ 2\ A_{sw} f_{ywd}\ d\ \sin\alpha - V_{sd}\ \text{cotg}\ \alpha \qquad (9)$$

In the authors' opinion, it is more convenient to develop the explicit form of above expressions and to substitute the shear stresses τ to the shear forces V and the characteristic values to design values by using a load faction $\gamma_c = 1,5$. It is shown elsewhere |4| that the factors τ_{Rd}, τ_{cd} and $\rho_{w,min}$ can be written by means of analytical expressions, which thus adequately replace tables of numerical values. Taking account of these facts, the whole set of design requirements becomes :

a) minimum shear reinforcement :

$$A_{sw}/sb_w \geqslant (0,01\ f_{ck} + 0,2)\ \sin\alpha\ /f_{ywk} \qquad (10)$$

b) web concrete strength :

$$\tau_{sd} \leqslant \text{MIN}\ \left[0,4\ f_{ck}(\text{cotg}\ \theta + \text{cotg}\ \alpha)\ \sin^2\theta\ ;\ 0,3\ f_{ck}\ \sin 2\theta\right] \qquad (11)$$

c) shear reinforcement strength :

$$A_{sw}/sb_w \geqslant 1,278\ \left[(\tau_{sd} - \tau_{cd})/f_{ywk}\right]\ /\ (\text{cotg}\ \theta + \text{cotg}\ \alpha)\ \sin\alpha \qquad (12)$$

with $\tau_{cd} = \text{MAX}\ (0\ ;\ 0,03\ f_{ck} + 0,375 - 0,5\ \tau_{sd})$ (13)

d) inclination of the diagonal concrete compression field :

$$3/5 \leqslant \text{cotg}\ \theta \leqslant 5/3 \qquad (14)$$

The requirement on the concrete strength is governed by a relation between the applied design shear stress and the characteristic concrete strength, and is without influence on the reinforcement design. It may thus be canceled from the design equations, under the condition that it be checked independently.

The bounds of the design, drawn in a figure $(A_{sw}/sb_w) = f(\text{tg}\theta)$, are two vertical lines for (14), and horizontal line for (10) and an hyperbola for (12). The feasible domain is hatched on figure 1 ; it may take several configurations with respect to the relative position of the hyperbola and the horizontal line (figure 2).

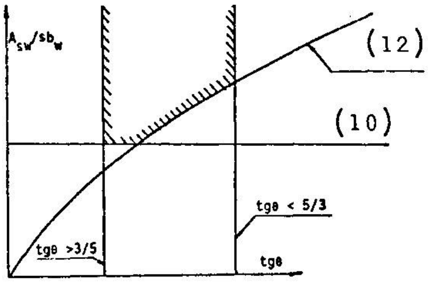
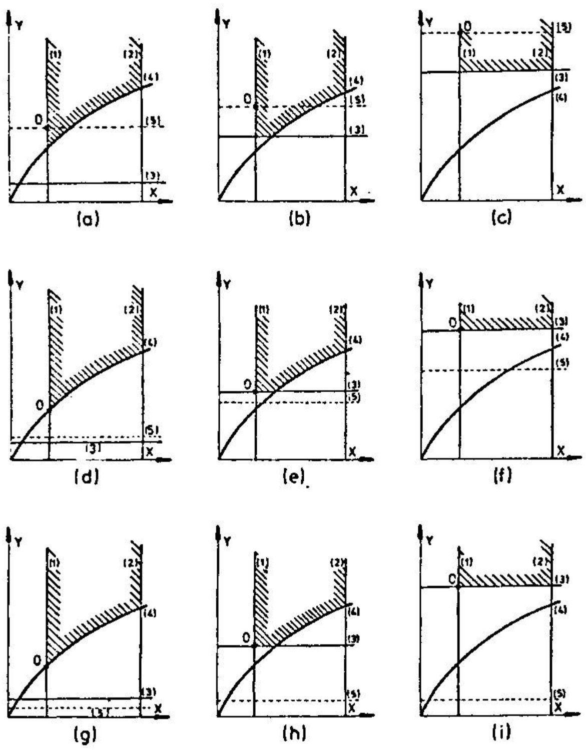


Figure 1 - Feasible domain



$X = tg\ \theta$

$Y = A_{sw}/sb_w$

(1) et (2) : bounds of θ

(3) : minimum shear reinforcement

(4) : shear reinforcement strength

(5) : unconstrained minimum.

Figure 2 - Configurations of the feasible domain.

OPTIMUM DESIGN.

The objective to optimize is the total amount of reinforcement needed by the shear design ; in fact, it would apply the cost of these reinforcements. If factor ρ is the cost ratio of shear - to tensile longitudinal reinforcement, the unit cost may be written :

$$(F/b_w d) = \rho(A_{sw}/sb_w)\ \lambda\ (1 + \varepsilon b_w/d) + (\Delta A_{s\ell}\ /b_w d) \qquad (15)$$

whilst the unit supplementary longitudinal reinforcement is :

$$(\Delta A_{s\ell}/b_w d) = \xi \tau_{sd}^2/(A_{sw}/sb_w)\, f_{ywk}\, f_{y\ell k} - \eta \cdot \tau_{sd}/f_{y\ell k} \quad (16)$$

where λ, ε, ξ and μ are numerical coefficients.

A specific value of (A_{sw}/sb_w) minimizes the objective function and corresponds to the annulment of the first derivative of this function. It is found to be :

$$(A_{sw}/sb_w)_{min} = \tau_{sd} \sqrt{\xi/\rho\lambda\ (1 + \varepsilon b_w/d)\ f_{ywk}\ f_{y\ell k}} \quad (17)$$

This minimum is called *unconstrained* because it does not interfere with the limits of the problem, and is represented by an horizontal line in figure 2. When this latter line does not intersect the feasible domain, the optimum value of (A_{sw}/sb_w) is obtained from the nearest apex of the feasible domain ; if the contrary is true, it is derived from the unconstrained minimum and a certain variation range of tgΘ is associated with the optimum value of (A_{sw}/sb_w), (see cases e, f, h, i of figure 2) so that this latter does not correspond to a unique value of Θ. It must however be observed that the lower bound tgΘ = 3/5 belongs in any case to the optimal solution. It is the reason why this value, which corresponds to $\Theta = 31°$, is selected for the further optimization process ; it is in complete agreement with experimental results obtained for beams subject to rather distributed loading |2|.

It is said above that the objective function depends on a cost factor ρ, the value of which is generally comprised between 1 and 1,5 as pointed out by THURLIMANN |3|. A numerical investigation proved that the influence of this factor is small ; as, in addition, the value of ρ is likely to vary with respect to the factory, the country and to the labor-to material cost ratio, it is reasonable to put $\rho = 1$.

Finally, the design procedure, based on the recommendations of the C.E.B.-F.I.P. Model Code, can be summarized as follows :

To check the web concrete :

$$(\tau_{sd}/f_{ck}) \leqslant \upsilon\ (\%) \quad (18)$$

To design the shear reinforcement :

by selecting for (A_{sw}/sb_w) the largest value of the three following expressions :

- minimum shear reinforcement : $\beta(0{,}01\ f_{ck} + 0{,}2)/f_{ywk}$ (19)
- shear reinforcement strength

$$\delta \left[\tau_{sd} - \mathrm{MAX}\ (0\ ;\ 0{,}03\ f_{ck} + 0{,}375 - 0{,}5\ \tau_{sd}) \right] / f_{ywk} \quad (20)$$

- unconstrained minimum :

$$\tau_{sd} \sqrt{\xi / (1 + \varepsilon b_w/d)\ f_{ywk}\ f_{y\ell k}} \quad (21)$$

To design the additional longitunal tensile reinforcement :

$$(\Delta A_{s\ell}/b_w d) = \xi \, \tau^2_{sd} \; / \; (A_{sw}/sb_w) \, f_{ywk} \, f_{y\ell k} - \eta \, \tau_{sd}/f_{y\ell k} \tag{22}$$

The numerical coefficients β, δ, ξ, η, ν, ε and λ depend on the inclination Θ of cracks and α of the stirrups, and on the configuration of the shear reinforcement in the cross-section of the beam. They are given in references |4| and |6| for different types of reinforcement.

Let us insist on the fact that the above formulation remains general and does not yet at all depend on the value Θ = 31°. Except for the unconstrained minimum, it represents, in the author's opinion, thus a more simple convenient presentation of the C.E.B.-F.I.P. Recommendations.

To make easier the design procedure for shear, a lot of charts can be drawn, each of them being specific of the configuration of the stirrups and of the yield stress of shear - and longitudinal reinforcement respectively. A full set of charts drawn for Θ = 31° are available and can be provided by the authors.

ECONOMICAL CHOICE OF SHEAR REINFORCEMENT.

On base of the C.E.B.-F.I.P. Recommendations for shear design and of an inclination Θ = 31° for the compression field, as discussed above, a lot of numerical simulations have been performed.

Several parameters are investigated :

a) the geometrical configurations of the shear reinforcement (figure 3) : closed stirrups with inclination α = 45°, 59° and 90°, and single and closed nets with resultant inclination α = 45° and 59° ;
b) the steel grades : S 220, S 400 and S 500.;
c) the compressive concrete strength : C 20,.C 30, C 40 and C 50 ;
d) the aspect ratio fo the cross-sectional dimensions : d/b_w = 1, 2, 3, 4 and 5.

From this extensive work |6|, it can be concluded that :

1. the economical classification of the shear reinforcement does not depend on the value of the applied design stress τ_{sd}, except for small values of τ_{sd} for which the minimum amount of shear reinforcement is governing ;
2. the type of optimal reinforcement does not depend on the compressive concrete strength ;
3. the aspect ratio d/b_w only influences the choice of the type of shear reinforcement if both shear - and longitudinal reinforcement are made of high strength steel ;

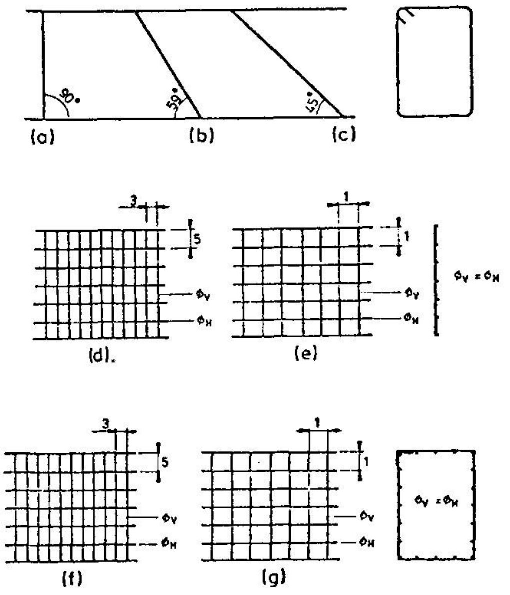


Figure 3 : Geometrical configurations of reinforcements.

A simple practical proposal for the choice of an economic shear reinforcement can be recommended as follows :

- shear reinforcement S 220 : stirrups with $\alpha = 45°$;
- shear reinforcement S 400 or S 500 :

  $f_{y\ell k} \geqslant f_{ywk}$: stirrups with $\alpha = 45°$;

  $f_{y\ell k} < f_{ywk}$: single net with resultant inclination $\alpha = 45°$.

It leads to values of the objective function which only differ of 3 to 5 % from these obtained with effective optimal configurations.

A comparison between vertical and inclined stirrups shows that the use of inclined shear reinforcement allows an economy of 20 to 25 % of the total amount of reinforcement required by shear, the reference being the solution with vertical stirrups. Both inclinations $\alpha = 45°$ and $\alpha = 59°$ lead to nearly the same economy ; for practical reasons, inclined stirrups with $\alpha = 45°$ is highly recommended.

Investigation on the value of Θ shows that the choice of $\Theta = 31°$ instead of $\Theta = 45°$ leads to a decrease of the amount of shear reinforcement and to an increase of the longitudinal tensile reinforcement, so that a global economy results, which reaches 20 to 13 %, if τ_{sd} exceeds 7,5 % of f_{ck}. If the contrary, the economy decreases until zero when the minimum shear reinforcement becomes governing.

Usually, the steel grade for longitudinal tensile reinforcement is decided prior to that of shear reinforcement ; then the maximum economy requires to use for shear reinforcement the highest steel grade and the following configuration : net and 45° stirrups when longitudinal reinforcement is of type S 220 - S 400 and S 500 respectively.

CONCLUSIONS.

The formulation of the C.E.B.-F.I.P. Recommendations for shear design is improved and used for a numerical simulation with $\theta = 31°$, which belongs in any case to the economical solution. It is shown that inclined stirrups with $\alpha = 45°$ and single net with resultant inclination $\alpha = 45°$, are the most economical configurations. Inclined stirrups are about 20 % more economical than vertical ones, whilst with the choice of $\theta = 31°$, an economy of 10 to 20 % can be expected with respect to $\theta = 45°$. Last, one shows how to choose the configuration and the steel grade of shear reinforcement, when the steel grade of longitudinal reinforcement is specified, in order to obtain the least amount of both types of reinforcements.

REFERENCES.

1. C.E.B.-F.I.P. : Code-Modèle pour les structures en béton. 3d Edition of the International Recommendations C.E.B.-F.I.P., 1978.
2. C.E.B.-F.I.P. : Bulletin d'information n°126 : Shear and Torsion, June 1978.
3. GROB, J. and THURLIMANN, B. : Ultimate strength and Design of Reinforced Concrete Beams under Bending and Shear. Proceedings of the I.A.B.S.E., 36-II, 1976, pp. 105-120.
4. MAQUOI, R. and RONDAL, J. : L'armature optimale d'effort tranchant dans les poutres en béton. To be published in Annales des Travaux Publics de Belgique.
5. MORSCH, E. : Die Schubsicherung der Eisenbetonbalken - Beton und Eisen, 1927, H.2, pp.27-35.
6. RONDAL, J. and MAQUOI, R. : Considérations économiques sur le choix d'une armature d'effort tranchant. I.A.B.S.E. Proceedings P-22/79, May 1979.

NOTATIONS.

A_{sw} : cross-sectional area of shear reinforcement ;

$\Delta A_{s\ell}$: additional longitudinal tensile reinforcement ;

F : objective function ;

$V_{cd}(\tau_{cd})$: shear force (stress) carried by compression flange ;

V_{Rd2}, V_{Rd3} : resistant shear for shear reinforcement, for web concrete ;

$V_{Rd}(\tau_{Rd})$: codified resistant design shear force (stress) ;

$V_{sd}(\tau_{sd})$: applied design shear force (stress) ;

V_{wd} : shear carried by truss action ;

b_w : web breadth ;

d : effective depth ;

$f_{cd}(f_{ck})$: design (characteristic) compressive strength of concrete ;

$f_{ywd}(f_{ywk})$, $f_{y\ell d}(f_{y\ell k})$: design (characteristic) yield strength for shear reinforcement, for longitudinal reinforcement ;

s : stirrup spacing ;

α : inclination of stirrups ;

θ : inclination of compression field ;

ρ_w : minimum geometrical percentage of shear reinforcement ;

ρ : cost factor ;

β, δ, ϵ, λ, ξ, η, ν : numerical values depending on the configuration of shear reinforcement and/or on the inclinations θ and α .