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Direct Design by Concrete Flow

Dimensionnement direct en considérant la transition des forces dans le béton

Direkte Bemessung durch Betrachtung des Kraftflusses im Beton

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SUMMARY
The paper presents a method of handling the equations for axial force, bending moment, shear force and torsion. The principal stress failure criterion for concrete, which is in mixed force and geometry variables, is re-written as a criterion in force variables only. This enables the longitudinal force requirements for shear and torsion to be simply obtained and incorporated in the beam axial force and flexure equations with which the designer is familiar. The method is general and adaptable to any cross-section shape or reinforcement layout and the equations are in a form suitable for direct design.

RESUME
La méthode présentée traite les équations pour une sollicitation par flexion, par une force axiale, par un effort tranchant et par un moment de torsion. Le critère de rupture pour le béton est exprimé à nouveau en termes de forces. Ainsi on peut incorporer les forces agissantes longitudinalement dues à la torsion et au cisaillement dans les équations bien connues qui décrivent l'action de la flexion et de la force normale. La méthode est générale et on peut l'adapter à une forme de poutre et une disposition de l'armature quelconque. Les équations résultantes permettent un dimensionnement pratique.

ZUSAMMENFASSUNG
Es wird eine Methode zur Handhabung der für eine Beanspruchung durch Biegung, Normalkraft, Querkraft und Torsion geltenden Beziehungen dargestellt. Die Bruchbedingung des Betons wird in Kraftgrössen ausgedrückt. Damit können die Längskräfte infolge Torsion und Querkraft in die üblichen Beziehungen für Biegung und Normalkraft einbezogen werden, mit denen der Ingenieur vertraut ist. Die Methode ist allgemein und kann für beliebige Querschnittsformen und Bewehrungsanordnungen angepasst werden. Die auftretenden Gleichungen eignen sich für eine direkte Anwendung bei der Bemessung.

1. INTRODUCTION

The safety stage comparison of design in structural concrete is, by international concensus, made at the level of member cross-section action. By this choice the design problem is decomposed into two modelling sub-problems, one above and one below the comparison level [1]. The higher level problems which will not be considered in this paper is the mapping of all loadings into the load-effect space. The lower level problem is most appropriately treated as a plasticity problem and consists of mapping the bounds of the safe domain in the same load-effect space. The most general such space is six-dimensional consisting of axial force, two bending moments, two shears and a twisting moment. A definitive stage of modelling in a two-dimensional axial force and bending space was reached some years ago [2] [3] and this led, possibly without adequate appreciation of the consequences, to the adoption of internal action as the comparison level. The most serious consequence has been a forced semi-rational empiricism when it comes to adding the shear dimension [4] and dissatisfaction by the profession with the resulting complexity of rules.

There has also been steady progress toward a consistent rational approach. The state of the art is reviewed by Thürlimann [5]. He presents the assumptions and principles and develops a rigorous approach via the space truss concept in which both upper and lower bound principles are satisfied simultaneously. The starting point is the failure criterion for concrete at a level intermediate between cross-section action and stress, namely force per unit length applied to a concrete sheet. This will be called force flow as a generalisation of the accepted term shear flow. The criterion is a very simple one, a square one in principal force flow space. However complete definition of a principal stress and hence of a principal force flow requires the specification of a geometrical variable, orientation, as well. As a result the crack orientation (or principal compression orientation) plays a major part in the section analysis. The practising designer appears, however, to find such techniques esoteric and unrelated to the models with which he is familiar. He is familiar with the equations for axial force and bending applied to the right cross-section and the purpose of this paper is to show how terms for shear and torsion may be directly included in these familiar equations.

2. THE CONCRETE FORCE FLOW CRITERION

If we define C_x as the longitudinal force per unit length when the shear flow is denoted by C_{xs} (Figure 1) Thürlimann's square criterion restricts C_1 and C_2 to the range $0 \leqslant C_{1,2} \leqslant C^n$ where $C^n = kF_c^{\prime}t$, t being the thickness of the sheet and k a factor modifying the cylinder strength $F_c^{\prime}$.

Force flows transform according to the same transformation of axes laws as stresses. Hence the limit

$0 \leqslant C_{1,2}$ implies that

$$\frac{C_x + C_s}{2} - \sqrt{\left(\frac{C_x - C_s}{2}\right)^2 + C_{xs}^2} \geqslant 0$$

which simplifies to :-

$$C_{xs}^2 - C_x C_s \leqslant 0 \qquad (1)$$

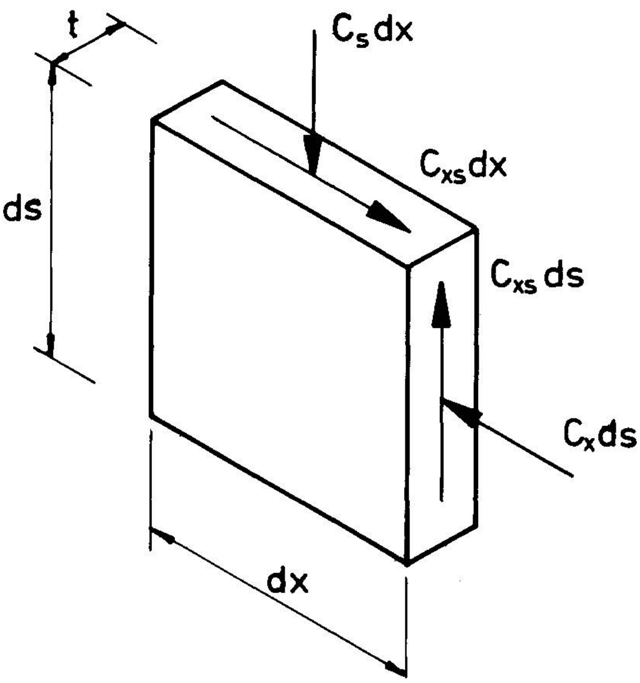


Figure 1. Concrete Sheet Element

Similarly $C_{1,2} \leq C^n$ implies that

$$C_{xs}^2 - (C^n - C_x)(C^n - C_s) \leq 0 \qquad (2)$$

Relationship (1) is the important one for designing under-reinforced beams and will be the key to most important design cases. The boundary where (1) co-incides with (2) is readily shown to be

$$C_x + C_s = C^n \qquad (3)$$

and satisfaction of (3) is the key to certain over-reinforced cases.

For design purposes it will be shown that the very simple inequality (1) is extremely powerful but to understand the physical significance the designer must be aware that failure controlled by (1) is dilatant. Any sheet failing by satisfaction of (1) is dilating in the x and s directions thus imposing on reinforcement in these directions a tensile strain and hence a tensile force. The apparent anomaly of compatibility of concrete under compression and parallel steel under tension is thus resolved.

The analytical demonstration of this dilatancy is achieved by simple application of the flow rule of plasticity theory [6] which yields

$$\dot{\varepsilon}_x = -\lambda C_s \qquad \dot{\varepsilon}_s = -\lambda C_x \qquad \dot{\gamma}_{xs} = 2\lambda C_{xs} \qquad (4)$$

$$\dot{\varepsilon}_x = \lambda (C^n - C_s) \qquad \dot{\varepsilon}_s = \lambda (C^n - C_x) \qquad \dot{\gamma}_{xs} = 2\lambda C_{xs} \qquad (4a)$$

3. APPLICATION TO MODE 1 TORSION, SHEAR AND BENDING

The failure state of a properly reinforced beam subjected to torsion, shear and bending is modelled using the dilatant concrete sheet subsystem by choosing the concrete sheets in the form of a polygonal tube which is restrained from bursting by the tensile stirrup force. The limit on this restraint against dilation is set by the stirrup yield strength but the effectiveness of this limit is set by the effectiveness of the force transfer between sheet and stirrup.

The potential circumferential force flow controlled by the stirrup is given by :

$$C_s = \frac{A_w f_{wy}}{s} \qquad (5)$$

This is the limit which applies in criterion (1) so that if the required shear flow C_{xs} can be determined then the required value of C_x follows from (1) namely

$$C_x = \frac{C_{xs}^2}{C_s} = \frac{C_{xs}^2}{\frac{A_w f_{wy}}{s}} \qquad (6)$$

The statics of determining values of C_{xs} is well established in the space truss theory. If the dilatant sheets circumscribe an area A_o then the shear flow in these sheets due to twisting moment T is given by

$$C_{xs} = \frac{T}{2A_o} \qquad (7)$$

or for a rectangular section

$$C_{xs} = \frac{T}{2y_1 z_1} \qquad (8)$$

For a simple rectangular beam, shear flows due to shear force V are given by :

$$C_{xs} = \frac{V}{2y_1} \qquad (9)$$

The effects of T and V are additive on one face and subtractive on the other so that the faces carry force flows as follows in the x direction :-

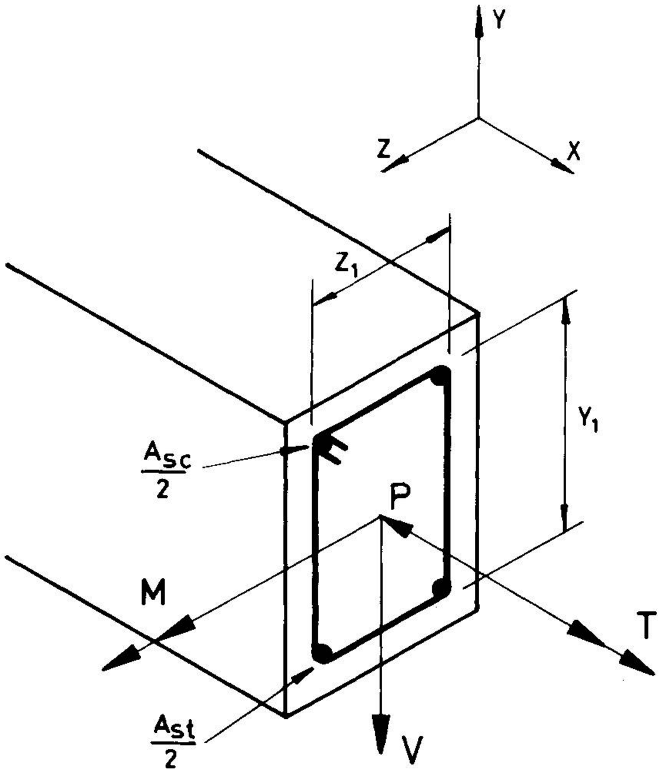
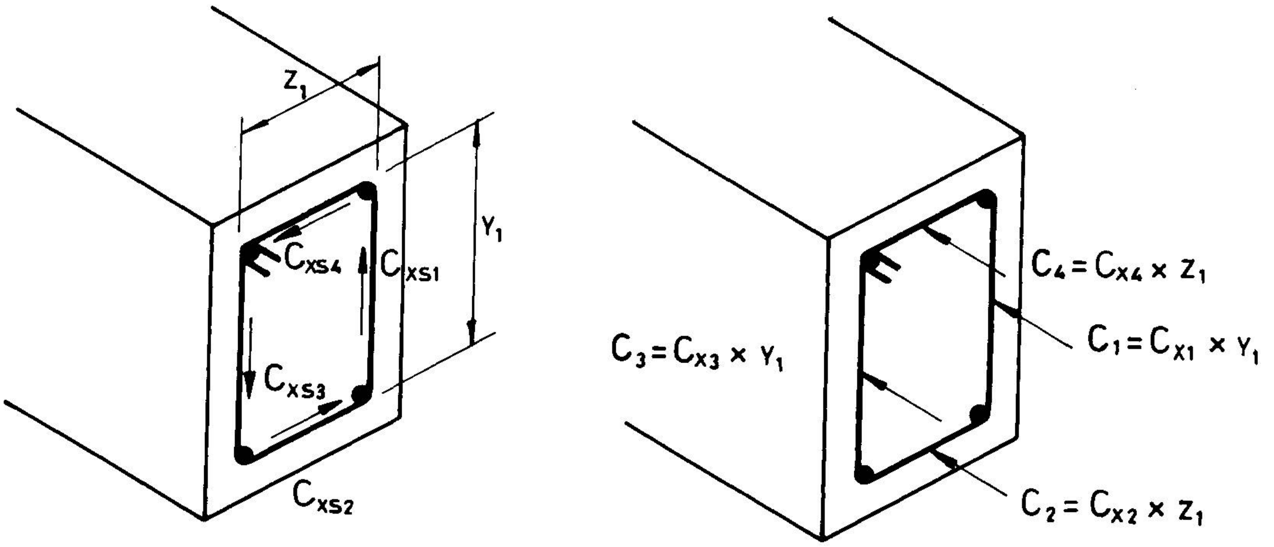


Figure 2. Rectangular Section and Loading



(a) Shear flows (b) Associated Longitudinal Forces

Figure 3. Rectangular Beam.

$$C_{x1} = \frac{\left(\frac{T}{2y_1 z_1} - \frac{V}{2y_1}\right)^2}{\frac{A_w f_{wy}}{s}} \qquad C_{x2} = C_{x4} = \frac{\left(\frac{T}{2y_1 z_1}\right)^2}{\frac{A_w f_{wy}}{s}} \qquad C_{x3} = \frac{\left(\frac{T}{2y_1 z_1} + \frac{V}{2y_1}\right)^2}{\frac{A_w f_{wy}}{s}} \qquad (10)$$

The resultant force C_i on the ith face is the product of C_{xi} and the length of the ith face (Figure 3) and is located at the mid-point. These forces are readily incorporated in the equations for axial force and flexure, if an additional compressive force C_o is assumed on one of the faces as the compressive stress block due to bending. For failure mode 1 (sagging bending) it will act on the C_4 face so that the equations are :-

$$C_1 + C_2 + C_3 + C_4 + C_o - A_{st}f_{sy} = P \qquad (11)$$

$$(A_{st}f_{sy} - C_2)y - (C_1 + C_3)\frac{y_1}{2} = M \qquad (12)$$

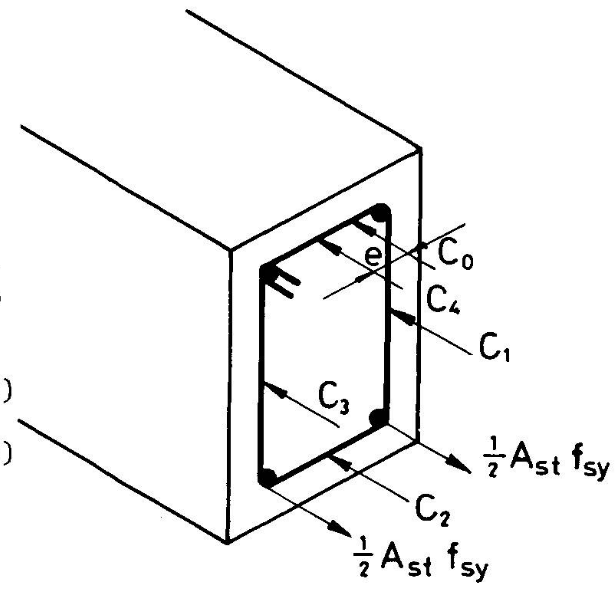


Figure 4. Forces in beam flexure equations.

Equation (11) is the standard one for determining the neutral axis location but the simplifying assumption implicit in all theories such as this is that the location of the compressive stress block resultant is known so that lever arms in [12] are independent of the solution of (11). No new theory would be required to produce more rigorous forms where the magnitude of C_o would determine the depth of the stress block and the lever arm would depend in turn on this. Substituting from (10 in (12) produces

$$M = A_{st}f_{sy}y - \frac{\left(\frac{T}{2y_1z_1}\right)^2}{\frac{A_wf_{wy}}{s}}z_1y_1 - \frac{\left(\frac{T}{2y_1z_1} - \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}}y_1\frac{y_1}{2} - \frac{\left(\frac{T}{2y_1z_1} + \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}}y_1\frac{y_1}{2} \qquad (13)$$

In a design situation no further development is needed. A trial selection of $\frac{A_wf_{wy}}{s}$ would immediately lead to a solution for $A_{st}f_{sy}$ and vice versa since all other terms would be numerical ones at this stage. Hence the designer can obtain design parameters directly by an equation with which he is familiar. The power of the method is that exactly the same procedure may be adopted for more complex cross-sections and the tensile reinforcement need not be treated as equivalent stringers because the moments of individual bars may be included in the moment equation. The practical advantages of relating the analysis to familiar equations are obvious. As an analytical approach it has an important advantage in common with the space truss theory over the author's earlier more complex approach [7] and the skew bending theory [8] because it can analyse the Collins' [9] reductio-ad-absurdum beam with all steel external to the concrete as a standard case.

The analytical extension of (13) is trivial but confirms that the simple steps do in fact produce an interaction relationship identical with the space truss theory and rigorous skew bending theory. Re-arrangement of terms in (13) readily leads to :-

$$\frac{M}{M_o} + \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 \leqslant 1 \qquad \text{where } M_o = A_sf_{sy}d \qquad (14)$$

$$T_o = 2y_1z_1\sqrt{\frac{A_wf_{wy}}{s}\,\frac{A_{st}f_{sy}}{y_1 + z_1}} \qquad V_o = 2y_1\sqrt{\frac{A_wf_{wy}}{s}\,\frac{A_{st}f_{sy}}{y_1}}$$

4. ECCENTRICITY OF COMPRESSIVE FORCE

Derivation of mode 3 (top yielding) and mode 2 (side yielding) equations is equally direct, and the resulting interaction relationships are equally consistent with space truss and skew bending. Of greater significance is the new insight that the method offers into the complete identification of all longitudinal forces and their location so that the statics of the final design is thoroughly understood by the designer.

For the mode 1 case given above, the inequality of the shears in the two y legs means that C_o must be eccentric (Figure 4) for zero moment about the second axis, i.e.

$$C_o\, e = (C_3 - C_1)\, \tfrac{1}{2} z_1 \tag{15}$$

But e has maximum value $\frac{1}{2}z_1$ and when this is satisfied in (15) and C_o is solved from (11) using a value of $A_{st}f_{sy}$ which satisfies (12), (15) becomes :-

$$\tfrac{1}{2} \geq \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 + \frac{2TV}{T_oV_o}\sqrt{\frac{y_1}{y_1 + z_1}} \tag{16}$$

What is significant is that (16) is the mode 2, sideways bending, interaction equation with zero top steel. In fact locating C_o at $\frac{1}{2}z_1$ leads to simultaneous derivation of the mode 1 and mode 2 failure criteria. The equation now becomes:-

$$A_{st}f_{sy} + \tfrac{1}{2}A_{sc}f_{sy} - (C_1 + C_2 + C_3 + C_4 + C_o) = P \tag{17}$$

$$A_{st}f_{sy}\, y_1 - C_2 y_1 - \tfrac{1}{2}(C_1 + C_3)\, y_1 = M \tag{12}$$

$$\tfrac{1}{2}A_{st}f_{sy}\, z_1 + \tfrac{1}{2}A_{sc}f_{sy}\, y_1 - C_3 z_1 - \tfrac{1}{2}(C_2 + C_4) z_1 = 0 \tag{18}$$

Equation (12) is unchanged and leads to the mode 1 equation and equation (18) satisfies equilibrium in lateral flexure and leads directly to the mode 2 interaction relationship

$$\tfrac{1}{2}\left(1 + \frac{A_{sc}f_{sy}}{A_{st}f_{sy}}\right) = \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 + \frac{2TV}{T_oV_o}\sqrt{\frac{y_1}{y_1 + z_1}} \tag{19}$$

Again (17) is not directly involved in the steel design but evaluation of C_o from it would be a step in the establishment of the requirements of the eccentric compression crushing area in a rigorous analysis since C_1 to C_4 are all in a dilatant tensile strain, state. In fact the equations may be seen to be those of a rigid plastic analysis with a skew neutral axis such that C_o acts at the corner position assumed and the rest of the section is in tensile strain state.

It is not proposed to discuss the mode 3 equations since they do not illustrate any new aspect of the method, but they would, of course, be written and utilised in a practical design situation. If they showed that A_{sc} needed an increase to cover the hogging bending situation this would not require any iteration for the rectangular beam, because, although C_o would now be at a bottom corner, equation (18) would apply to this case as well. The advantages of the method do not lie in a particular formula which it can generate but in the understanding it can provide to the designer as to why the reinforcement is required and the control it gives him in decision-making as to how much is required. Some other aspects in which it assists the designer and potential refinements will now be discussed.

5. WEB CRUSHING IN SHEAR

The minimum thickness of the concrete sheet implicit in the solution may be derived if (3) is assumed to hold at failure. In the beam design equations considered, for instance, this leads to :-

$$t = \frac{\dfrac{C_{xs}^2}{\dfrac{A_w f_{wy}}{s}} + \dfrac{A_w f_{wy}}{s}}{kF'_c} \qquad (20)$$

The case of web crushing in shear is readily treated by the method. The equation for under-reinforced design in shear is (13) with T = 0. If the web width is such that t as determined by (20) is greated than $\frac{b}{2}$ then design for web crushing is required. In that case design of the stirrup to yield simultaneously with the crushing is readily achieved. Knowing C_x from (6), (3) becomes :-

$$\frac{(C_{xs})^2}{C_s} + C_s = C^n \quad \text{or} \quad \frac{\left(\dfrac{V}{2y}\right)^2}{\dfrac{A_w f_{wy}}{s}} + \frac{A_w f_{wy}}{s} = kF'_c \frac{b}{2} \qquad (21)$$

Solving for $\frac{V}{2y}$, $$\frac{V}{2y} = \sqrt{\frac{A_w f_{wy}}{s}\left(kF'_c\frac{b}{2} - \frac{A_w f_{wy}}{s}\right)} \qquad (22)$$

This semi-circular relationship between V and C_s is the one that was derived by Nielsen and Braestrup [10] for reinforced concrete web crushing and by Chitnuyanondh et al [11] for prestressed concrete web crushing. However, when coupled with the general theory it goes further than these methods because it then allows C_x to be obtained since C_x and C_s must sum to C^n . Hence the associated longitudinal steel equations may be written. In the resulting design the strain rates will be the sum of (4) and (4a).

6. CONCLUSIONS

A general method has been presented for the organization of the calculations which follow from the basic assumptions of plasticity theory when applied to reinforced concrete members subjected to axial force, bending, shear and torsion. The designer's problem has been transformed into writing the equilibrium equations for the resultants of longitudinal internal stresses, a problem with which he is familiar from the establishment of the interaction equations for axial force and bending. The method is independent of the shape of the cross-section or of the disposition of the reinforcement on the faces of the member. The limits of under-reinforcement can be identified and the web crushing case for beam shear may be designed directly.

NOTATION

b	web thickness
e	eccentricity of C_o
f_{st}, f_{sc}, f_{wy}	yield stress for A_{st}, A_{sc}, A_w steel
k	reduction factor for F'_c
s	(i) perimetral co-ordinate used in x,s co-ordinate system
	(ii) stirrup spacing used in $\frac{A_w f_{wy}}{s}$
t	concrete sheet thickness
x,y,z	beam co-ordinate system
A_o	area bounded by shear flow
A_{st}	steel area on bottom face
A_{sc}	steel area on top face
A_w	stirrup area
C_o	compressive force due to P,M.
C_i	compressive force due to C_{xi} on the ith sheet

C_x, C_s, C_{xs}	forces per unit length, force flows, on concrete sheet
F'_c	concrete cylinder strength
M	bending moment
P	axial force
T	twisting moment
V	shear force
M_o, P_o, T_o V_o	ultimate values of M,P,T,V when acting alone
$\dot{\varepsilon}_x$, $\dot{\varepsilon}_s$, $\dot{\gamma}_{xs}$	strain rate
λ	scalar multiplier

REFERENCES

1. CLYDE, D.H. The safety verification problem in structures. Proc. Fifth Australasian Conference on the Mechanics of Structures and Materials, Melb., 1975, pp 103-112.

2. HOGNESTAD, R. A Study of Combined Bending and Axial Load in Reinforced Concrete Members. Univ. of Illinois Bulletin, No.399. 1951.

3. GRANHOLM, H. A General Flexural Theory of Reinforced Concrete with Particular Emphasis on the Inelastic Behaviour of Concrete and Reinforcement. Stockholm, Almqvist and Wiksell. 1965. 209p.

4. ACI 428-62. Shear and Diagonal Tension. ASCE Joint Committee Report. ACI journal. Proceedings Vol.59, Jan 1962 pp 1-30, Feb pp.277-333, March 1962 pp. 353-395.

5. THÜRLIMANN, B. Plastic analysis of reinforced concrete beams. Introductory Report, IABSE Colloquium, Plasticity in Reinforced Concrete, Copenhagen.

6. PRAGER, W. Introduction to Plasticity. Reading. Addison-Wesley. 1959. 148p.

7. CLYDE, D.H. A general theory for reinforced concrete elements. Sixth Aust. Conference on the Mechanics of Structures and Materials, Christchurch,N.Z. 1977. pp 327-333.

8. ELFGREN, L., KARLSON, I., and LOSBERG, A. Torsion-bending-shear interaction for concrete beams. Journal Structural Div.Proc. ASCE V.100 No.ST8,1974, pp. 1657-1676.

9. MITCHELL, D. and COLLINS, M.P. Diagonal compression field theory - a rational model for structural concrete in torsion. Journal American Concrete Institute. V.71, 1974, pp.396-408.

10. NIELSEN, M.P. and BRAESTRUP, M.W. Plastic Shear Strength of Reinforced Concrete Beams. Bygningsstatiske Meddelelser. V.46,No.3. 1975 pp.61-69.

11. CHITNUYANONDH,L., CAMPBELL, T.I., BATCHELOR, B.del and CSAGALY,P. Shear failure due to web crushing in prestressed concrete I Beams. Proc.American Concrete Institute Symposium, Philadelphia. 1976.