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The Anchorage Strength of Reinforcement Bars at Supports

L'ancrage des armatures à l'appui

Verankerung der Bewehrungsstäbe bei Auflagern

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SUMMARY
The theory of plasticity is used to attempt a solution of anchorage and splicing problems for deformed reinforcing bars.
The paper reports calculations on anchoring of one and two bars in the support zone of a beam.

RESUME
La théorie de la plasticité est appliquée à l'étude des problèmes de l'ancrage et du recouvrement des armatures. Des calculs sont présentés pour l'ancrage d'une et de deux barres d'armature dans la zone d'appui d'une poutre.

ZUSAMMENFASSUNG
Die Plastizitätstheorie wird auf Probleme der Verankerung und der Überlappung von Bewehrungsstäben angewendet. Berechnungen für die Verankerung von einem oder von zwei Bewehrungsstäben im Auflagerbereich eines Balkens werden dargestellt.

1. INTRODUCTION

Until quite recently, only a few rational theories concerning the anchorage strength of reinforcement bars and the strength of splices have been formulated. Tepfers made an attempt in his dissertation [1], and some investigators have tried to solve the problem using the finite element method (see for instance Tepfers [1] , Lutz [2], etc.).

Since the mid sixties, a research group in Denmark has been investigating the application of the theory of plasticity on concrete structures. Research was earlier mainly centered on the problems of shear in beams, shear in joints and punching shear (see Nielsen et al. [3]).

It thus seemed natural to examine whether the same theory and methods could be used to solve the problems of anchoring and splicing of reinforcement bars.

Such an investigation has now come so far that it is possible to report the first results.

2. BASIC ASSUMPTIONS

The work is based on the following assumptions:

1) The concrete is considered a rigid-plastic material with the modified Coulomb failure criterion as yield condition and the angle of friction $\varphi = 37^o$. As it is well known, concrete is not perfectly plastic. This is taken into account by reducing the concrete cylinder strength σ_c with an empirical effectiveness factor ν .

2) Deformations in the concrete are determined by the normality condition (the associated flow rule).

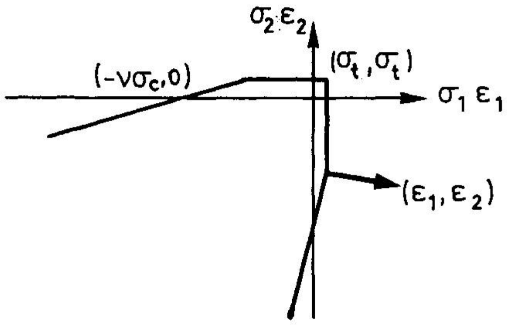

Figure 1: Modified Coulomb Failure Criterion for plane strain with the Associated Flow Rule.

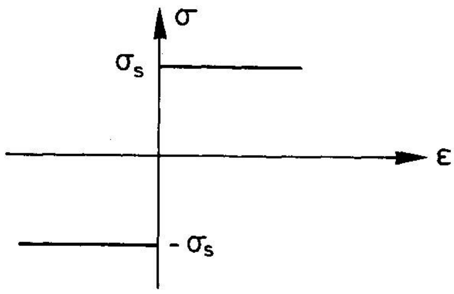

Figure 2: Rigid-plastic Stress-strain Relation valid for the Reinforcement.

3) The reinforcement bars are assumed being rigid plastic, and only able to carry longitudinal stresses.

In the following calculations the upper bound theorem will be used.

The upper bound theorem can be defined as:

> A load found by the work equation, for a geometrically possible failure mechanism, is greater than or equal to the yield load.

3. THE LOAL FAILURE MECHANISM

Concerning the failure along a bar it is assumed that there is no adhesion between the concrete and the reinforcement bar.

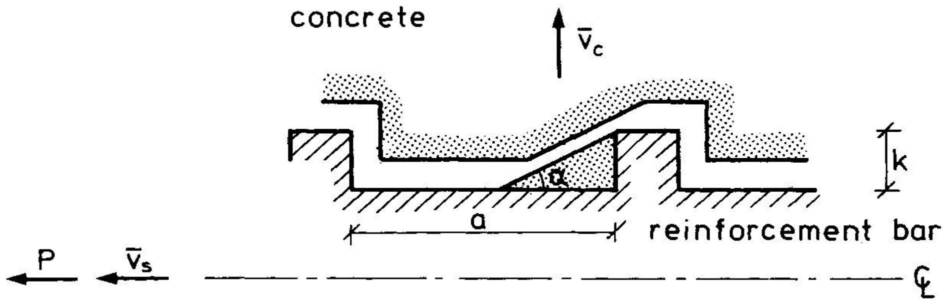

Figure 3: The Local Failure Mechanism

Consider an idealized reinforcement bar with idealized ribs. If the bar is moved in the direction $\bar{v}_s$ (the same direction as the force P , see figure 3) , the concrete in front of the ribs will yield and the yield plane form an angle γ with the bar axis. The surrounding concrete is then pushed axisymmetrically away from the bar.

The displacement of the surrounding concrete relative to the reinforcement bar is:

$$\bar{v}_{cs} = \bar{v}_c - \bar{v}_s$$

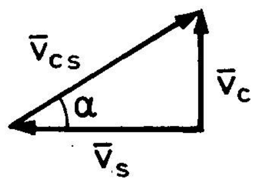

Figure 4

If we denote the angle between $\bar{v}_c$ and $\bar{v}_{cs}$ as α , the external work is $W_e = P||\bar{v}_{cs}|| \cos\alpha$. The internal work W_i is:

If $\gamma \geq \text{Arctan}\,(\frac{k}{a})$:

$$W_i = ||v_{cs}||\ (d+k)\pi\ \frac{k\ \ell}{a\ \sin\gamma}\ \{\frac{\nu\sigma_c}{2}(1-\sin(\alpha-\gamma)) +$$

$$+\ \sigma_t\ \frac{\sin(\alpha-\gamma)\sin\varphi}{1-\sin\varphi}$$

where ℓ is the length of the anchorage and d the diameter of the reinforcement bar.

If $0 \leq \gamma \leq \text{Arctan}\,(\frac{k}{a})$:

$$W_i = ||\bar{v}_{cs}||\ (d+2k - a\tan\gamma)\pi\ \frac{a\ \ell}{a\ \cos\gamma}\ \cdot$$

$$\cdot\ \{\frac{\nu\sigma_c}{2}(1-\sin(\alpha-\gamma)) + \sigma_t\ \frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\}$$

The expressions have been calculated for plane strain condition. Concerning the basic expressions, see for instance Nielsen et. al. [3] or Jensen [4].

Putting $W_e = W_i$ we obtain the work equation:

If $\gamma \geq \text{Arctan}\ (\frac{k}{a})$:

$$\frac{P}{\ell d\pi\sigma_c} = \frac{\tau}{\sigma_c} = \frac{(d+k)k}{da\ \cos\alpha\sin\gamma}\left\{\nu\ \frac{1-\sin(\alpha-\gamma)}{2} + \rho\ \frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\right\}$$

where $\rho = \sigma_t/\sigma_c$

If $0 \leq \gamma \leq \text{Arctan}\ (\frac{k}{a})$:

$$\frac{P}{\ell d\pi\sigma_c} = \frac{\tau}{\sigma_c} = \frac{(d+2k-a\tan\gamma)}{d\ \cos\gamma\ \cos a}\left\{\nu\ \frac{1-\sin(\alpha-\gamma)}{2} + \rho\ \frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\right\}$$

In the above expressions we have not taken into account the internal work carried out in the surrounding concrete and reinforcement (for instance work carried out in stirrups).

The expressions are only strictly correct if the surrounding concrete is displaced axisymmetrically to the bar axis, i.e. the concrete has to crack along an infinite number of radii.

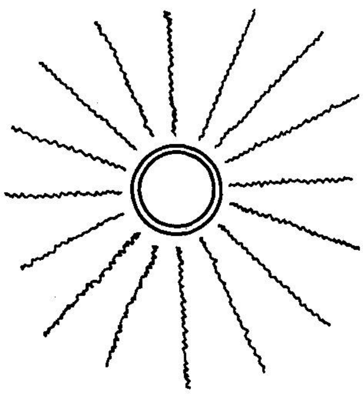

Figure 5: Yield Lines around a Bar.

This is not always so in practice. For instance, the surrounding concrete at a normal anchorage or splice in a beam is not displaced axisymmetrically.

Despite this fact, the expressions are used without modifications in the following, assuming the error to be without importance.

4. ANCHORAGE OF ONE REINFORCEMENT BAR

Consider first the case where a single bar is anchored along a beam support, having a uniformly distributed support reaction r. The failure pattern is assumed to be as shown in figure 6.

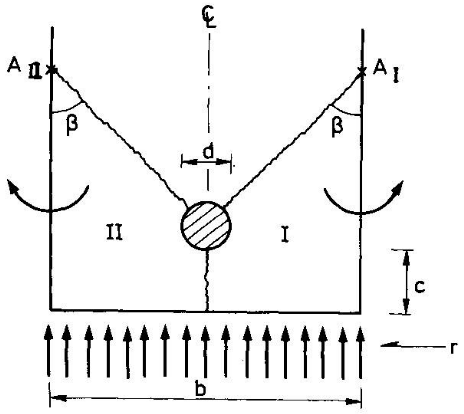

Figure 6: Yield Lines at the Anchorage of One Bar

When the reinforcement bar is displaced longitudinally the concrete will yield in front of the ribs and force the parts I and II to rotate about A_I and A_{II} respectively.

The bearing capacity τ/σ_c of such an anchorage becomes a function of the angles α, γ and β, the geometry and the dimensionless stress r/σ_c from the reaction. As the upper bound theorem is used, τ/σ_c is to be minimized with respect to α, γ and β.

Minimum has been found numerically by a computer. The result is shown in figure 7, compared with tests made by Rathkjen [5].

In the calculations the effectivity factor ν has been put equal to 0.50 and the dimensionless tensile strength $\rho = 0.025$.

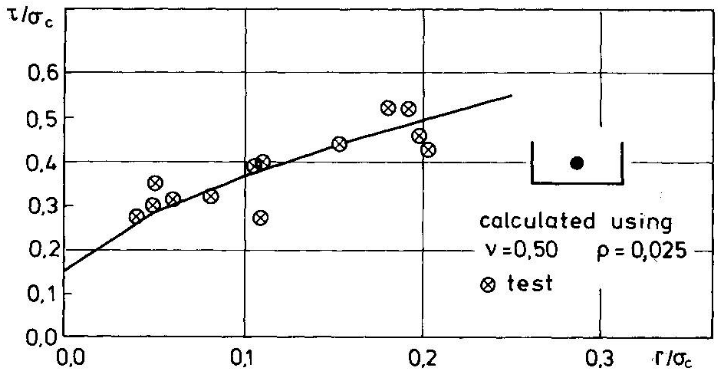

Figure 7: Comparison between Tests and Calculations on the Anchoring of Kamsteel 42 ø14

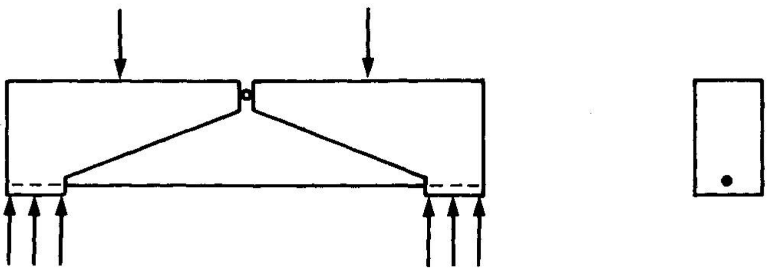

Figure 8: The Testbeams used by Rathkjen [5]

5. ANCHORAGE OF TWO REINFORCEMENT BARS

If two bars are anchored, there are several possible failure patterns. In the calculations referred to here, 4 failure patterns have been studied.

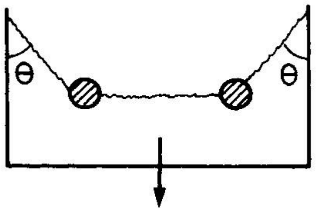

Figure 9: Failure 1, the Cover Splits into One Piece.

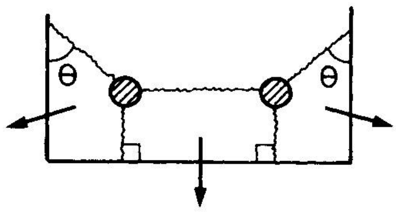

Figure 10: Failure 2, the Cover Splits into Three Pieces.

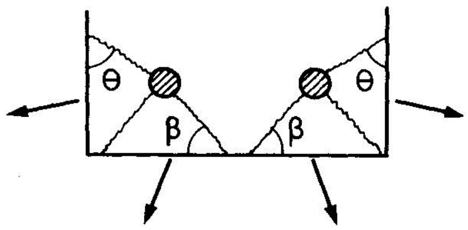

Figure 11: Failure 3, the Corners Split into Two Pieces

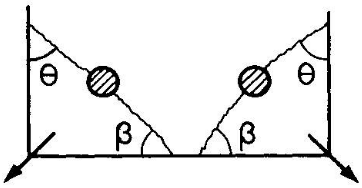

Figure 12: Failure 4, the Corners Split into One Piece.

One more mechanism could be studied. A failure where triangle formed bodies are pressed out below the reinforcement often occurs. But since the beams used for comparison did not have the wide bar spacings necessary for this mechanism to occur, this has been left out of consideration.

The expressions to be minimized are very extensive and the minimizations have to be made numerically on a computer.

The following figures show some of the results. The calculations are again compared with tests made by Rathkjen [5].

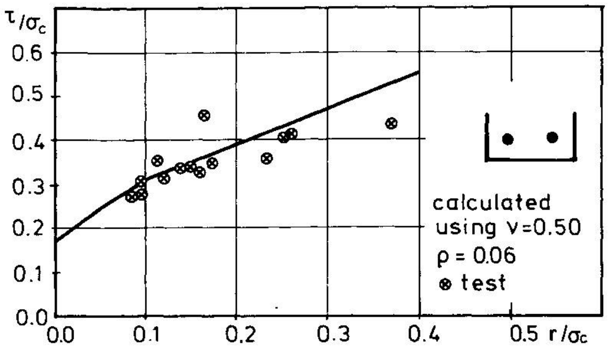

Figure 13: Kamsteel 42 ⌀14. Comparison between Tests and Calculations.

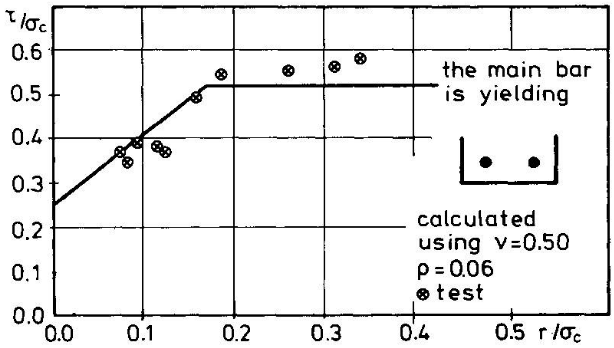

Figure 14: Kamsteel 42 ⌀ 10. Comparison between Tests and Calculations.

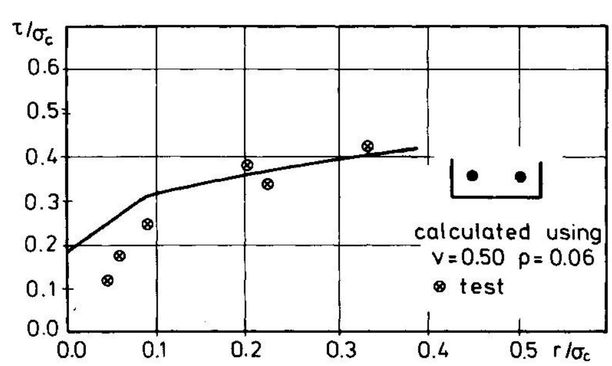

Figure 15: Tentor-steel 56 ⌀14. Comparison between Tests and Calculations.

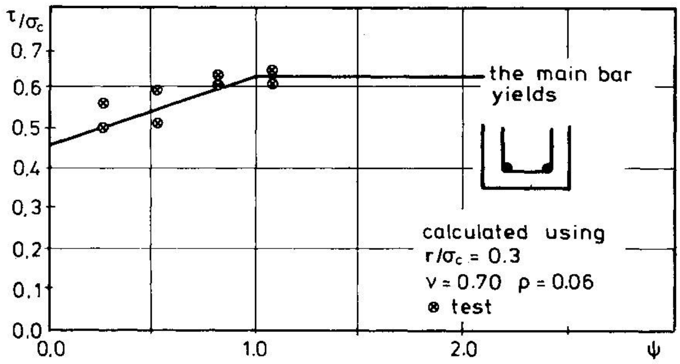

Figure 16: Comparison between Tests and Calculations of the Anchoring of Kamsteel 42 ⌀14 with Stirrups at the Supports.

Figures 13, 14 and 15 show the anchorage strength τ/σ_c as a function of the dimensionless support stress r/σ_c

The calculations have been carried out with data corresponding to Danish Kamsteel 42 ⌀14 mm and ⌀10 mm and Tentor-steel 56 ⌀14 mm.

The effectiveness factor ν was 0.50 and the dimensionless tensile strength $\rho = \sigma_t/\sigma_c$ was 0.06.

Figures 16 and 17 show the anchorage strength τ/σ_c as a function of the stirrup strength in the anchorage zone.

The parameter $\psi = \dfrac{2A_s\,\sigma_s}{\ell\, d\, \sigma_c}$ is used as a dimensionless stirrup strength.

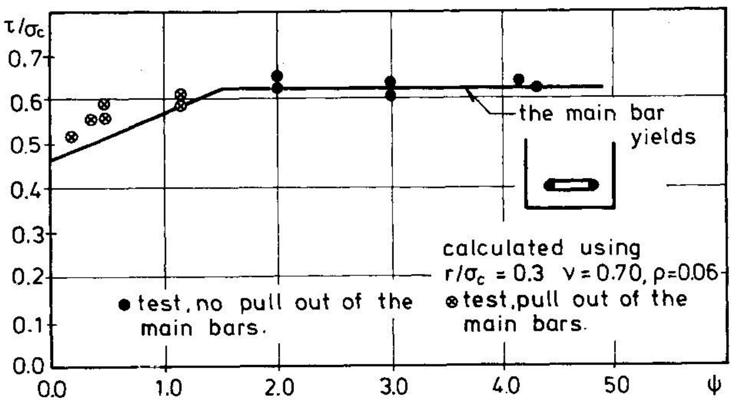

Figure 17: Comparison between Tests and Calculations of the Anchoring of Kamsteel 42 ø14 with Stirrups at the Supports.

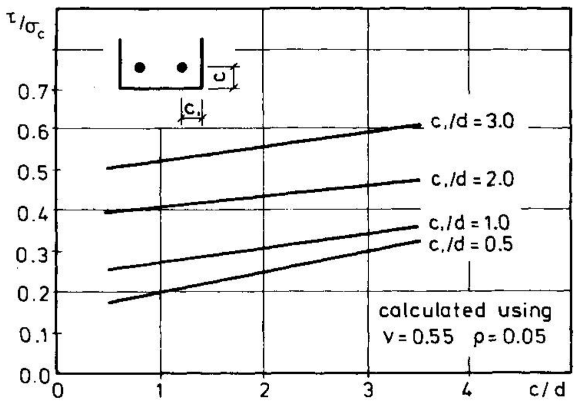

Figure 18: Calcultated Strength τ/σ_c as a Function of the Covers c and C_1 .

Figure 18 shows the anchorage strength τ/σ_c as a function of the bottom cover c and the side cover c_1 , both c and c_1 made dimensionless by division by d. No tests have been found to compare with, but the variation of the covers gives the effect we would expect, i.e. in principle the same as described by Tepfers [1].

6. CONCLUSION

As we have seen, the theory of plasticity is able to explain the influences of parameters as the support stress r/σ_c , and the stirrup strength ψ in an acceptable way.

There are some shortcomings in the ν-values, i.e. we have different values for, for instance, anchoring of one or two bars and anchoring with or without stirrups. Some of these difficulties are due to shortcomings in the model and others are due to the well known difficulties with plastic calculations of unreinforced concrete (see for example Jensen [4]).

Future work will have to be centered on eliminating these shortcomings and extending the theory to, for instance, the splicing problem or the anchoring of three or more bars.

7. REFERENCES

[1] Tepfers,R.: A Theory of Bond applied to Overlapped Tensile Reinforcement Splices for Deformed Bars. Publ. 73:12, Division of Concrete Structures, Chalmers Tekniska Högskola, Göteborg 1973.

[2] Lutz,L .A.: The Mechanics of Bond and Slip of Deformed Reinforcing Bars in Concrete. Report No. 324, Department of Structural Engineering, School of Civil Engineering, Cornell University, August 1966.

[3] Nielsen, M.P., Bræstrup, M.W., Jensen, B.C., Bach, Finn: Concrete Plasticity. Special Publication, Dansk Selskab for Bygningsstatik, Lyngby, Oktober 1978.

[4] Jensen, B.C.: Some Applications of Plastic Analysis to Plain and Reinforced Concrete, Report No. 123, 1977. Technical University of Denmark, Institute of Building Design, Copenhagen.

[5] Rathkjen, A.: Forankringsstyrke af armeringsjern ved bjælkeunderstøtninger. Rapport 7203, DIAB Aalborg 1972, Ren & Anvendt Mekanik.