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Limit Values of Local Stresses

Valeurs limites des pressions locales

Grenzwerte der örtlichen Pressung

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SUMMARY
On the basis of their own experimental investigations, the authors have determined the actual load when locally loaded concrete elements are brought into a limit state of collapse. Also, applying the condition of concrete collapse they have arrived at the minimum value for the critical load by constructing statically possible stress fields, separated by discontinuity planes. It has been shown that the collapse load, determined in this way, is very near to the collapse load obtained on the tested specimens.

RESUME
Sur la base de leurs essais, les auteurs ont déterminé les pressions locales correspondant à l'état de rupture des éléments en béton chargés localement. Les valeurs inférieures de la charge ultime de tels éléments ont été calculées en construisant des champs discontinus de contraintes et en appliquant un certain critère de rupture pour le béton. Les valeurs théoriques concordent bien avec les valeurs expérimentales.

ZUSAMMENFASSUNG
Auf der Grundlage eigener Versuche haben die Autoren die dem Bruchzustand lokal belasteter Betonelemente entsprechenden örtlichen Pressungen ermittelt. Ebenso wurden durch Konstruktion diskontinuierlicher, statisch zulässiger Spannungsfelder und Anwendung der Bruchbedingung für Beton untere Grenzwerte für die Traglast berechnet. Diese rechnerischen Werte stimmen gut mit den experimentell ermittelten überein.

1. TEST RESULTS AND DISCUSSION

The paper presents the results from experimental and theoretical investigations in the behaviour of locally loaded concrete elements in the state of limit strength.

These investigations are the part of a very large research programme, which is still under way at the Faculty of Civil Engineering in Belgrade, and were undertaken in order to verify some assumptions for the innovation of the actual Yugoslav Code for Reinforced Concrete Structures.

The loading was of static character and was gradually increased, approximately with that velocity of loading which is applied to a standard testing of a cube, the side of which was 20 cm. The load was transmitted to the specimen through a very rigid steel "stamper". The processing of contact surface of the "stamper" and the concrete specimen was the same as reguired in testing standard cubes. The compressive strength of concrete was ranging from 20 to 50 MPa.

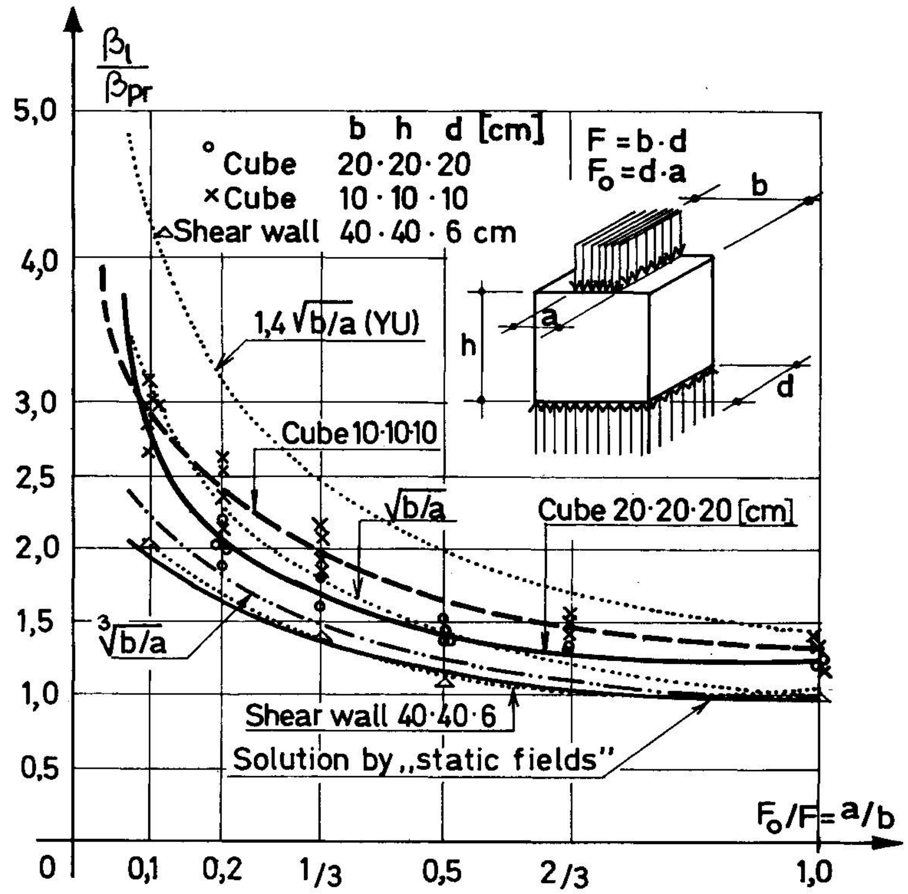
Characteristic results of experimental investigations, statistically processed, are shown in Figs. 1,2 and 3. Fig. 1 shows a case of local loading along the whole thickness of the element d, and Figs. 2 and 3 illustrate the cases when the locally loaded surface F_0 in both directions is of less dimensions than the dimensions of the examined concrete element in the plan.

Instead of the ratio F_0/F, the ratios of the sides a/b for parallelepipeds are applied to the abscissa of the diagram, that is,the ratios d/D for cylinders. The ratios of compressive stresses $\beta_l (=P_l/F_0)$ and the strength of the prism β_{pr} are applied to the ordinate, when P_l is the force under which the specimen collapsed, and F_0 is a locally loaded surface.

On the basis of the results of experimental tests the following observation can be given:

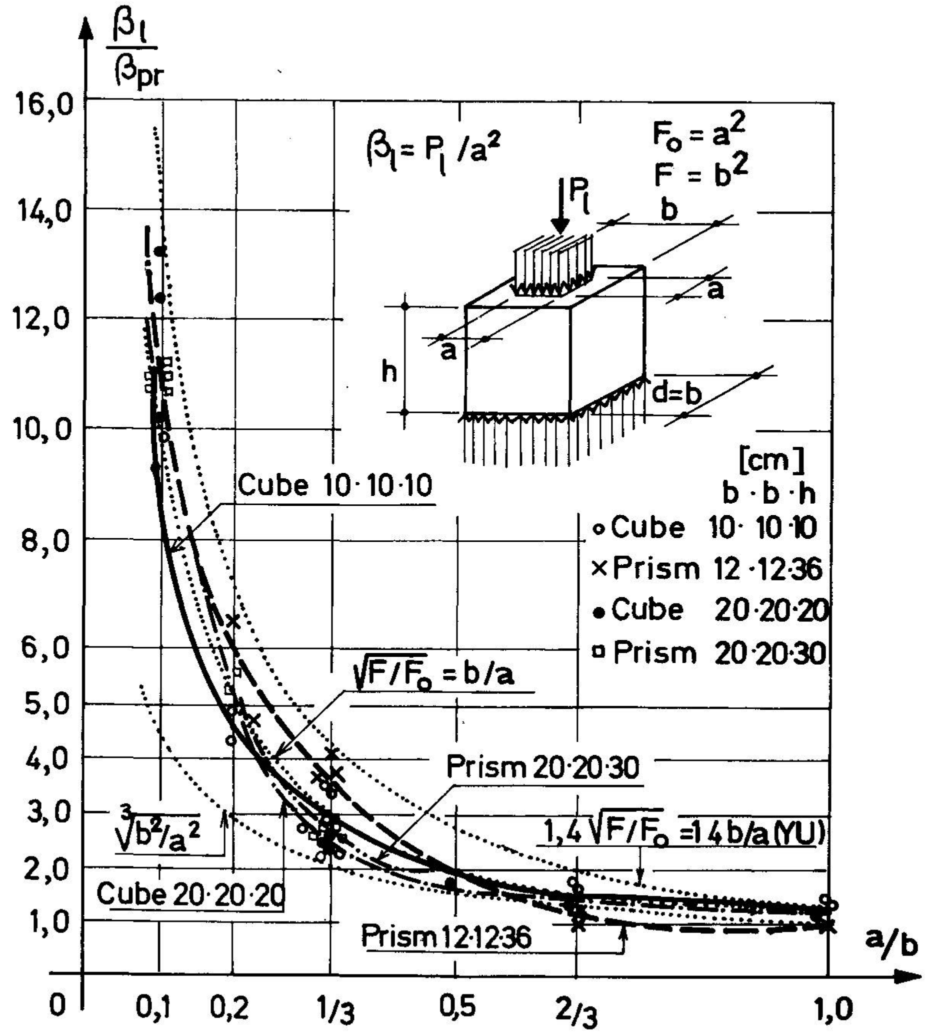
a) Local compressive stresses are grouped around the curved forms of the hyperbola with asymptotes β_l/β_{pr} = 1 and a/b = 0, that is, d/D = 0.

b) The influence of the quantity of locally loaded surface F_0 upon the change of local compression β_{pr} is practically negligible in the interval $2/3 \leqslant a/d \leqslant 1$, that is $2/3 \leqslant d/D \leqslant 1$ and up to the relation $a/b \geqslant 1/2$ .... $(d/D \geqslant 1/2)$ the change is very slight. When the realtion $a/b \approx 1/3$ the stress ranges from 1,5 to 3,0β_{pr}. By further reduction of the relation a/b, that is, d/D, local compressions increase progressively, so that for the realtion 0,10 they may increase from 12 to 15 β_{pr}



β_l / β_{pr}
b h d [cm]
° Cube 20·20·20
× Cube 10·10·10
△ Shear wall 40·40·6 cm
F = b·d
F_o = d·a
1,4 √(b/a) (YU)
Cube 10·10·10
√(b/a)
Cube 20·20·20 [cm]
∛(b/a)
Shear wall 40·40·6
Solution by „static fields"
F_o/F = a/b

Fig. I



β_l / β_{pr}
$\beta_l = P_l / a^2$
$F_o = a^2$
$F = b^2$
Cube 10·10·10
[cm]
b·b·h
∘ Cube 10·10·10
× Prism 12·12·36
• Cube 20·20·20
▫ Prism 20·20·30
√(F/F_o) = b/a
Prism 20·20·30
∛(b²/a²)
1,4 √(F/F_o) = 1,4 b/a (YU)
Cube 20·20·20
Prism 12·12·36
a/b

Fig. 2

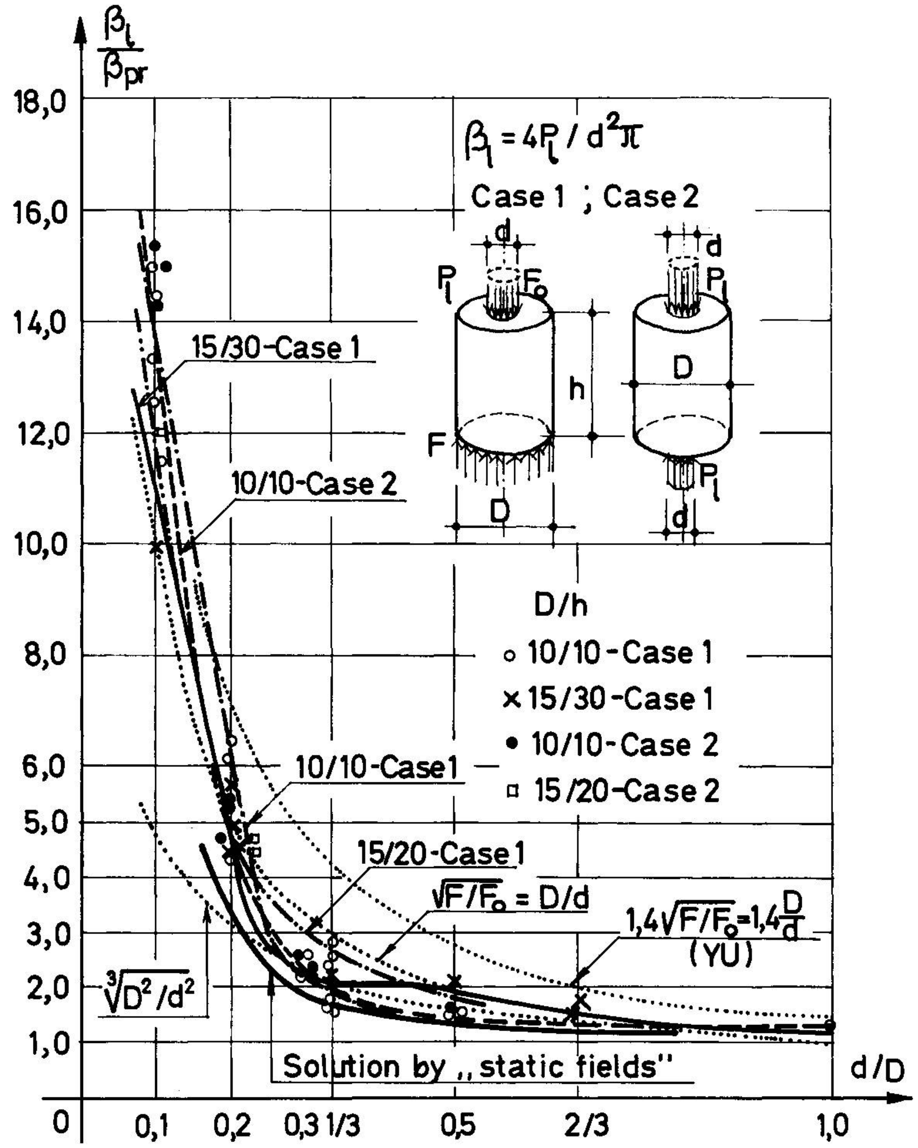


Fig. 3

c) The local compression β_l upon the specimens of the same dimensions and quality of concrete and of the same relation of the sides a/b is very dependent on that whether the loading acts along the whole thickness d of the specimen or only partially. In the first case (Fig. 1) the model is under conditions that are nearer to the biaxial stress state, so that local compressions, even in small relations $F_0/F = a/b$, reach comparatively modest values, Thus, on the tested shear wall of dimensions 40cm, 40cm, 6cm, for which it may be said that it sufficiently correctly - for practical purposes - satisfies the conditions of biaxial stress states, a local compression $\beta_l \simeq 2\,\beta_{pr}$ is obtained, when $F_0/F = a/b = 0{,}10$. In the second case, when the dimensions of locally loaded surface F_0 in both directions are less than the dimensions of the total surface F (Figs. 2 and 3), the specimen is under the conditions o triaxial stress state and it is also understandable why very high stresses occur in the "wedge" that is under the triaxial stresses of compression.

d) A good approximation of experimental results of limit local compressions for very thin elements, that means for the plane state of stress, fives the formula

$$\beta_l = \beta_{pr}\sqrt[3]{F/F_o} \qquad (1)$$

For the elements which are of considerable thickness, loaded along all the thickness, as well as for triaxial stressed elements, (Figs. 2 and 3) a satisfactory approximation of experimental results is achieved by the formula with square root.

$$\beta_l = \beta_{pr}\sqrt{F/F_o} \qquad (2)$$

The formula

$$\beta_l = 1{,}4\,\beta_{pr}\sqrt{F/F_o}\,, \qquad (3)$$

given in the actual Yugoslav Code for Reinforced Concrete, is not proved by these experiments. It gives larger values than the experimental ones.

2. THEORETICAL ANALYSIS OF LOAD CARRYING CAPACITY BY APPLYING EXTREME PRINCIPLES

For the determination of load carrying capacity of locally loaded elements the condition of plasticity collapse of concrete suggested in the paper /3/ has been used. This condition, for the biaxial stress state, in the system $\bar{\sigma}_1-\bar{\sigma}_2$, (where σ_1, σ_2 are principle stresses), is:

$$A_0 + \frac{A_1}{3}(\bar{\sigma}_1+\bar{\sigma}_2) + \frac{A_2}{9}(\bar{\sigma}_1+\bar{\sigma}_2)^2 + \frac{A_3}{27}(\bar{\sigma}_1+\bar{\sigma}_2)^2 - \frac{2}{9}(\bar{\sigma}_1^2+\bar{\sigma}_2^2-\bar{\sigma}_1\bar{\sigma}_2) = 0 \qquad (4)$$

when

$$A_0 = \frac{2}{3}\bar{\beta}_s^2\,; \quad A_1 = 2\bar{\beta}_s^2\,\frac{1-2\mu}{\mu} - \frac{1}{3}\mu\alpha\,; \quad A_2 = 2 - \frac{6\bar{\beta}_s}{\mu} - (1-\mu)\alpha\,; \quad A_3 = 3\alpha$$

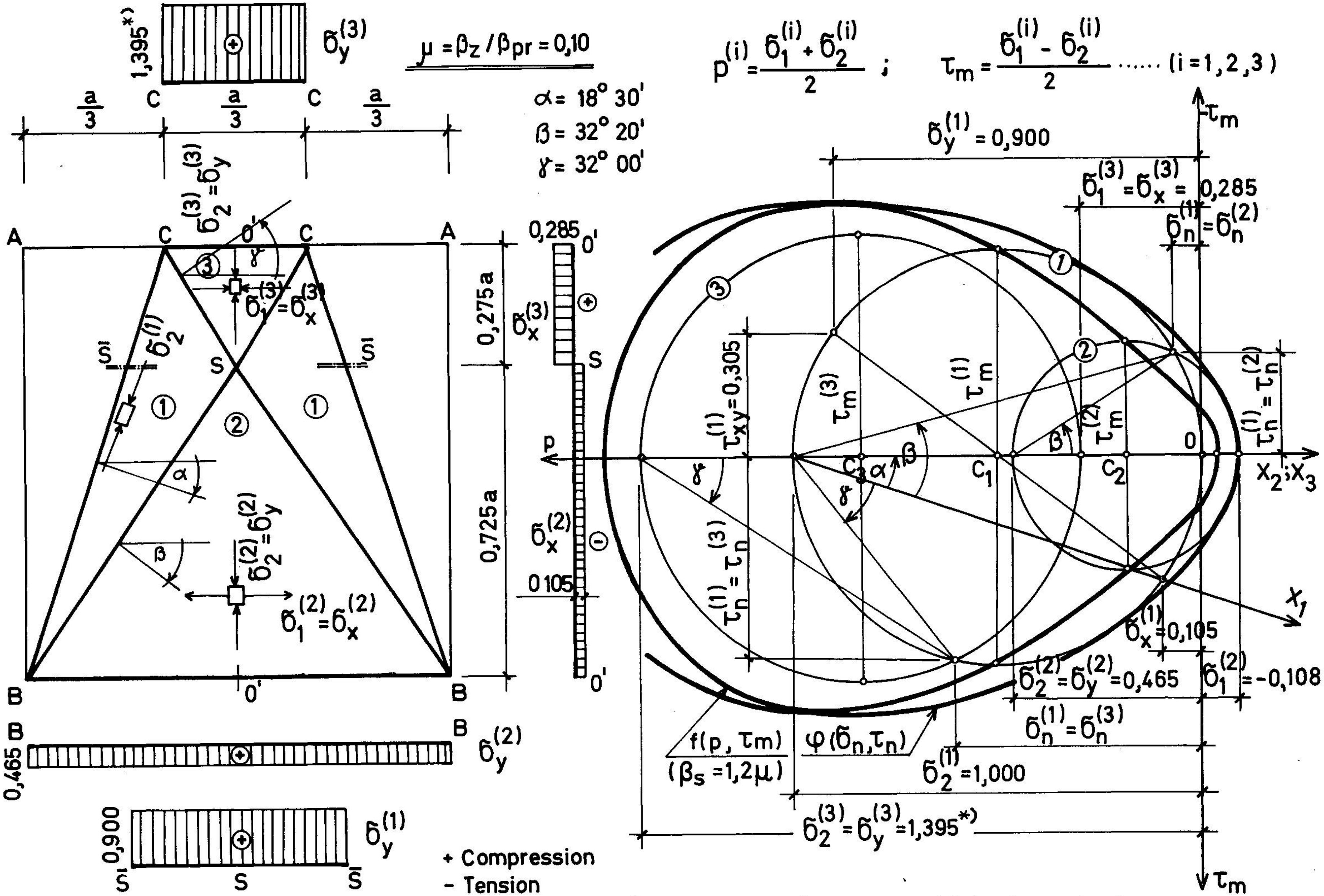
$$\alpha = \frac{\frac{\xi-2}{2\xi-1} - 2}{2\xi+\mu} + 3\,\frac{\bar{\beta}_s^2}{\xi\mu}\,; \quad \bar{\sigma}_1 = \sigma_1/\beta\,; \quad \bar{\sigma}_2 = \sigma_2/\beta_{pr}$$

$$\mu = \beta_z/\beta_{pr}\,; \quad \bar{\beta}_s = \beta_s/\beta_{pr}\,; \quad \xi = \bar{\sigma}_1 - \bar{\sigma}_2$$

A uniaxial tensile strength of concrete is indicated by β_z, the strength of the prism by β_{pr}, where the strength in pure shear is indicated by $\beta_s = \sigma_1 = -\sigma_2$, and the ratio of strengths of concrete in equal biaxial compressions to the strength of prism is indicated by ξ.

For triaxial stress state the condition of plasticity-collapse of concrete /3/ is given by the expression

$$\bar{\tau}_o = A\bar{\sigma}_o + B \qquad (5)$$



*) The values of stresses obtained are to be multiplied by β_{pr}

Fig. 4

where $\bar{\tau}_o = \tau_o/\beta_{pr}$; $\bar{\sigma}_o = \sigma_o/\beta_{pr}$

when $\tau_o = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ $\quad \sigma_o = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$

represent octahedral shear and normal stress. For μ = 0,10, the constants A and B are A = 0,700 and B = 0,238.

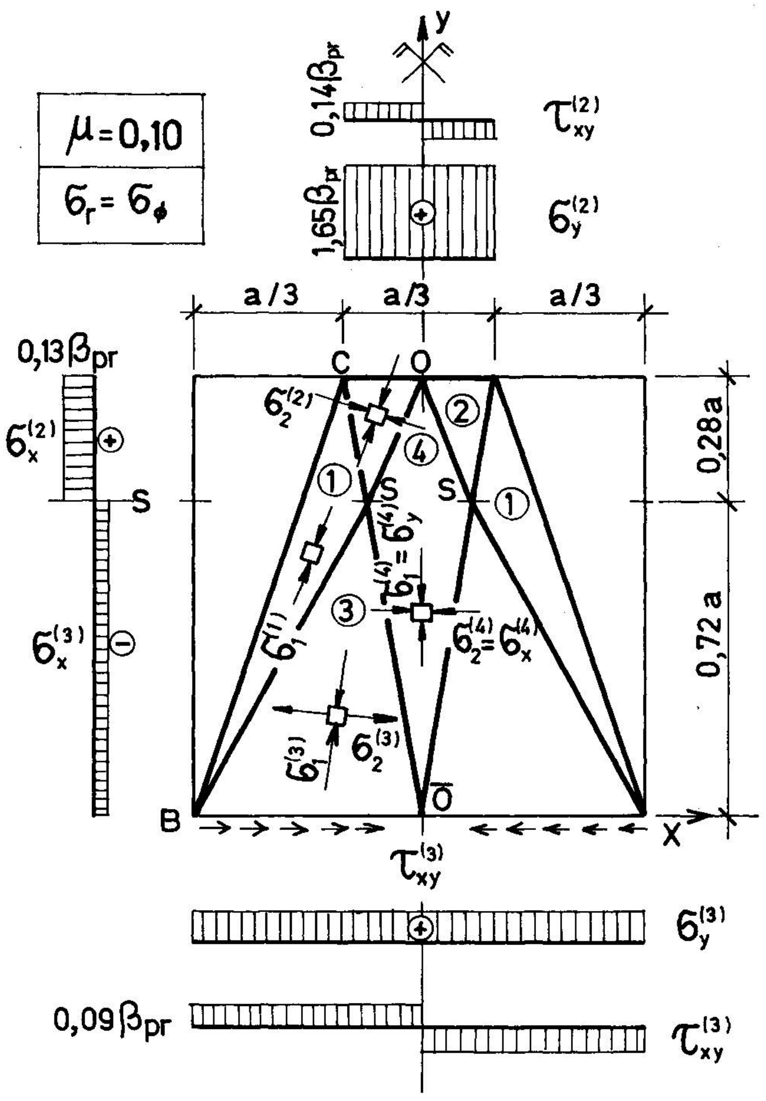


Fig. 5

In this paper the limit load is defined by statically possible fields, constant stresses, associated with the regions of discontinuity, which, as it is known, give the lowest value for loading when the collapse of locally loaded specimen occurs. Figs. 4 and 5 show the constructions of statically possible fields made of fields made of fields of constant stresses connected by plane of discontinuity BS and CS (SO and S$\bar{O}$). Fig. 4 shows such a field for a biaxial stress state in the system p-τ_m $[p = (\sigma_1+\sigma_2)/2$; $\tau_m = (\sigma_1-\sigma_2)/2]$, in locally loaded surface, with the relation a/b = = 1/3. From the diagram, in Fig. 1, one can see that the solutions by means of these fields are in good agreement with results experimentally obtained in testing a shear wall considered to have been under the conditions of biaxial stress state.

Fig. 5 shows statically possible fields of stresses for locally loaded cylindrical specimens in the coordinate system p,τ_m when the collapse condition is used for triaxial stress state given by Eq.(5) for the case of rotational symmetry. It has been shown that the assumptions made about the relations of stress in radial and tangential directions σ_2/σ_3 = 1 and σ_2/σ_3 = 0,5, in well constructed field, have been proved by the tested specimens. Thus, Fig. 3 gives the results of the solution by means of "static fields", when σ_2 = 0,5 $\sigma_3 \ldots (\sigma_2 \equiv \sigma_r, \sigma_3 = \sigma_\phi)$, which, as it is seen, represents the lower value of the actual limit load of rotationally loaded cylindrical bodies. When $\sigma_2 = \sigma_3$, the condition (5) in the system p, τ_m has the form

$$\tau_m = 0{,}597p + 0{,}203 \tag{6}$$

while when $\sigma_2 = 0{,}5 \cdot \sigma_3$, the condition (5) overpasses the hyperbola Fig.5 shows one possible construction of static fields by introducing friction into the contact plate of specimens with massive plates - "stampers".

3. CONCLUSIONS

On the basis of aforementioned theoretical analysis one may observe that by means of statically possible fields, one can prognosticate very simply the least low value of loading, which brings the concrete specimen into the state of collapse. A good agreement of results of such theoretical solutions with the results obtained on concrete specimens shows that the application of static fields has its full justification. In addition, the previous analyses show that the conditions must be precise in that when to apply Eq. (1), and when Eq. (2). The investigations of the authors have shown that Eq. (1) may be applied only when the locally loaded concrete element is in the biaxial stress state, while, Eq. (2) may be applied to the concrete elements with triaxial stress state. Eq. (3), given in Yugoslav Standards, and in the standards of some others countries, is not acceptable for the previous stress states. It gives considerably larger values than the ones obtained by the authors in their tests. Some preliminary investigations of the authors pointed out that Eq. (3) can be used in defining local limit stress in concrete elements which are in the state not far from the plane deformation, However, more precise conclusions, concerning it, may be given only after detailed experimental tests, which the authors have in plan to make.

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