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SESSION V

Specifications Based on Plastic Analysis

Normes basées sur le calcul plastique

Anwendung plastischer Lösungen in Normen

Chairman: Prof. T. Brøndum-Nielsen, Denmark

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Reinforced Concrete Corbels – Some Exact Solutions

Consoles en béton armé – quelques solutions complètes

Stahlbetonkonsolen – einige vollständige Lösungen

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SUMMARY
The paper presents some equations for the load carrying capacity of reinforced concrete corbels. The solutions are exact solutions based on the classical theory of plasticity.

RESUME
La résistance ultime des consoles en béton armé est examinée en appliquant la théorie classique de la plasticité. Quelques solutions complètes sont obtenues.

ZUSAMMENFASSUNG
Die Tragfähigkeit von Stahlbetonkonsolen wird mit Hilfe der klassischen Plastizitätstheorie untersucht. Einige vollständige Lösungen werden angegeben.

1. INTRODUCTION

In this paper some lower and upper bounds for the load carrying capacity of reinforced concrete corbels will be presented.

The solutions are based on the assumptions of zero tensile strength in concrete and plane stresse field. Concrete and reinforcement are idealized to rigid-perfectly plastic materials. The bars carry forces in the axial directions only. The yield criterion of concrete is the well known square yield criterion, see fig. 1. From the work on shear problems carryed out in Denmark, it is known that the strength of concrete cannot be expected to be the normal uniaxial compressive strength f_c, see f.ex. [1]. The same condition is well known from normal bending calculations. To take this into account we introduce the effective compressive strength νf_c, where $\nu \leq 1$.

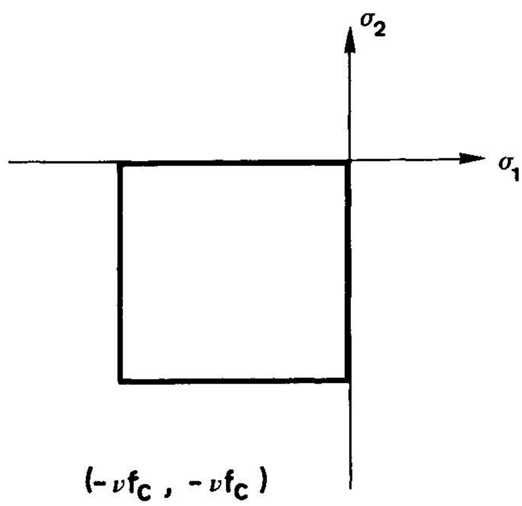

Fig. 1. Square yield criterion for concrete.

2. HORIZONTAL CONCENTRATED REINFORCEMENT

2.1 Lower bound solutions

Consider the stress destribution shown in fig. 2. The force T in the reinforcement is transferred to the concrete at the length EF. In the regions ABC and DEF there are hydrostatic compression. In the region BCDF there is unaxial compression. The hydrostatic stress is equal to the unaxial stress, and both are equal to the concrete strength νf_c.

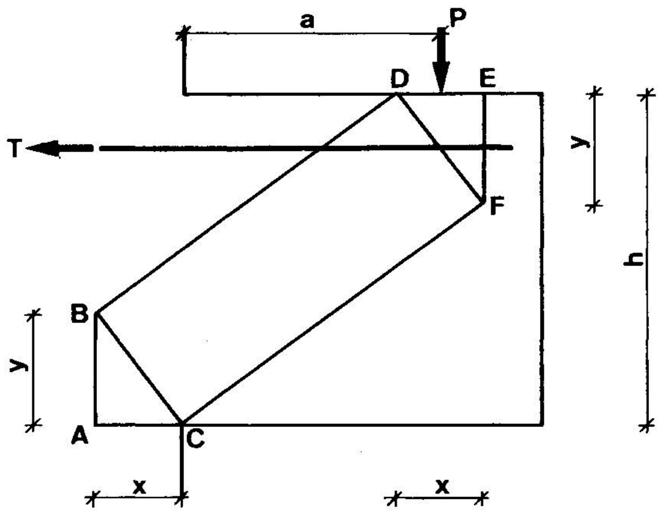

Fig. 2. Stress destribution.

From right-angled triangles in fig. 2 we get

$$(a + \frac{x}{2})^2 + (h - y)^2 = (a - \frac{x}{2})^2 + h^2 - (x^2 + y^2) \tag{1}$$

From this equation we get

$$x = -a + \sqrt{a^2 + 2y(h - y)} \tag{2}$$

The lower bound solution is thus

$$P = x\,\nu\,\sigma_c \tag{3}$$

Using $\tau = {}^{P}/h$ we get

$$\frac{\tau}{\sigma_c} = -\nu\,\frac{a}{h} + \sqrt{\left(\nu\frac{a}{h}\right)^2 + 2\,\nu\frac{y}{h}\left(\nu - \nu\,\frac{y}{h}\right)} \tag{4}$$

Maximum for (4) is found for ${}^{y}/h = \frac{1}{2}$. However, the tensile force T is equal to the compression force at the region EF, i.e.

$$T = y\,\nu\,f_c \le A\,f_y \tag{5}$$

Here is A the area of the reinforcement and f_y is the yield strength. We introduce the degree of reinforcement $\Phi = \frac{A\,f_y}{h\,f_c}$. From (5) we then get $y = \Phi\,{}^{h}/\nu$ but with maximum ${}^{y}/h = \frac{1}{2}$.

Inserting in (4) we get

$$\frac{\tau}{\sigma_c} = \begin{cases} -\nu\,\frac{a}{h} + \sqrt{\left(\nu\frac{a}{h}\right)^2 + 2\,\Phi(\nu - \Phi)} & \Phi \le \frac{\nu}{2} \quad (6a) \\ -\nu\,\frac{a}{h} + \nu\sqrt{\left(\frac{a}{h}\right) + \frac{1}{2}} & \Phi \ge \frac{\nu}{2} \quad (6b) \end{cases}$$

These equations are lower bound solutions if the reinforcement is placed with an effective height $h_e = h - \frac{1}{2}\,y$ or

$$h_e = \begin{cases} h\left(1 - \frac{1}{2}\frac{\Phi}{\nu}\right) & \Phi \le \frac{\nu}{2} \quad (7a) \\ \frac{3}{4}h & \Phi \ge \frac{\nu}{2} \quad (7b) \end{cases}$$

If the effective height is less than given in equation 7 the stress destribution in fig. 3 can be used.

The stress destribution is in fact the same as in fig. 2, except the region GHDE. In this region we have uniaxial stress equal to the concrete strength νf_c.

The lower bound solutions are found in the same way as (6), and we get

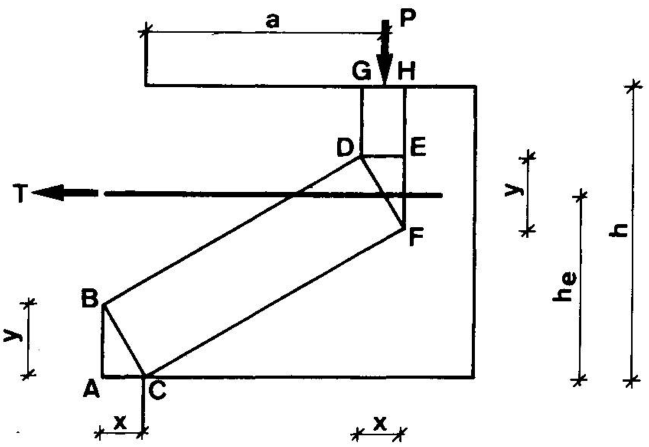

Fig. 3. Stress destribution

$$\frac{\tau}{\sigma_c} = \begin{cases} -\nu\,\frac{a}{h} + \sqrt{\left(\nu\frac{a}{h}\right)^2 + \Phi\left(2\,\nu\,\frac{h_e}{h} - \Phi\right)} & \Phi \le \nu\frac{h_e}{h} \quad (8a) \\ -\nu\,\frac{a}{h} + \nu\sqrt{\left(\frac{a}{h}\right)^2 + \left(\frac{h_e}{h}\right)^2} & \Phi \ge \nu\frac{h_e}{h} \quad (8b) \end{cases}$$

These lower bound solutions are valid when h_e is less than given in (7). If h_e is greater we cannot use this consideration because the tensile force in the reinforcement now is determined by the distance $h - h_e$. At the limit $h = h_e$ the lower bound in this way will be zero.

If h_e is greater than given in (7) we can use the stress destribution in fig. 4. In this case the tensile force is transferred to the concrete as a shear stress $\tau = T/z$. In the region ABC there is hydrostatic compression and in region BCDF there is uniaxial compression. All stresses are equal to the concrete strength νf_c.

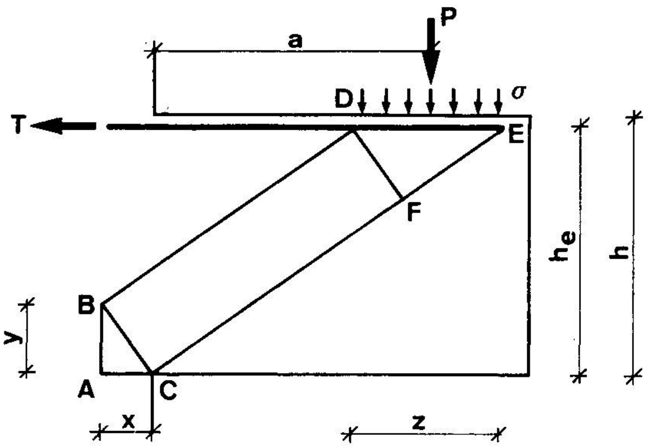

Fig. 4. Stress destribution.

From right-angled triangles in fig. 4 we get

$$(a - \frac{x}{2})^2 + (h_e - \frac{y}{2})^2 - x^2 - y^2 = (a + \frac{x}{2})^2 + (h_e - \frac{y}{2})^2 \tag{9}$$

From (9) we find

$$x = -a + \sqrt{a^2 + y(2h_e - y)} \tag{10}$$

Also from geometrical considerations we get

$$z = \frac{h_e\sqrt{a^2 + y(2h_e - y)} + y\,a - h_e\,a}{h_e - \frac{y}{2}} \tag{11}$$

Now we regard the equlibirum of the triangle DEF. The moment equation gives

$$\sigma = \nu\sigma_c \frac{x^2 + y^2}{z^2} \tag{12}$$

And then the load P turns out to be

$$P = \sigma z = \nu\sigma_c(-a + \sqrt{a^2 + y(2h_e - y)}\;) \tag{13}$$

This force is equal to the vertical force on AC.

Maximum for (13) is found for $y = h_e$. However, horizontal projektion gives

$$T = y\,\nu\,\sigma_c = \tau\; z \leq A\,f_y \tag{14}$$

This means that $y = \Phi\; h/\nu$, but with maximum $y = h_e$. Inserting in (13) we get the same lower bound solutions as in (8), which then i a general lower bound solution. The stress destribution in fig. 4 can of course be used instead of the stress destribution in fig. 2 and fig. 3. However, the way of transferring the tensile force to the concrete is quite different.

2.2 Upper bound solutions

The failure mechanism is shown in fig. 5. Part II rotates the angle α about A. The yield line AB has pure compression and AC is a tensile crack. The tensile crack reaches the upper side of the corbel at an arbitrary place inside the load. The work equation consist of 3 contributions W_E, W_{IR}, W_{IC} from the external force, the reinforcement and the concrete.

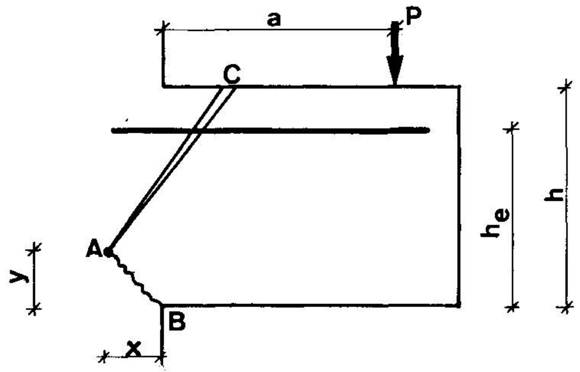

Fig. 5. Failure mechanism.

The contributions are

$$W_E = P(a + x)\alpha \tag{15}$$

$$W_{IR} = A f_y (h_e - y)\alpha \tag{16}$$

$$W_{IC} = \tfrac{1}{2} \nu f_c (x^2 + y^2)\alpha \tag{17}$$

The work equation $W_E = W_{IR} + W_{IC}$ then yields

$$\frac{P}{f_c} = \frac{\frac{1}{2}\nu(x^2 + y^2) + \Phi h(h_e - y)}{a + x} \tag{18}$$

where $\Phi = (A f_y) / (h f_c)$

Minimizing with respect to x and y we get

$$y = \frac{\Phi}{\nu} h \tag{19}$$

$$x = -a + \sqrt{a^2 + \frac{\Phi h}{\nu}(2 h_e - \frac{\Phi h}{\nu})} \tag{20}$$

Inserting in (18) we get the minimum

$$\frac{\tau}{\sigma_c} = -\nu \frac{a}{h} + \sqrt{(\nu \frac{a}{h})^2 + \Phi(2 \nu \frac{h_e}{h} - \Phi)} \tag{21a}$$

The upper limit for the validity of (21a) is the contribution 0 from the reinforcement, that is $h_e = y$ or $\Phi = (h_e/h)$. If the degree of reinforcement is greater we get

$$\frac{\tau}{\sigma_c} = -\nu\frac{a}{h} + \nu\sqrt{(\frac{a}{h})^2 + (\frac{h_e}{h})^2} \qquad \Phi \geq \nu\frac{h_e}{h} \tag{21b}$$

The upper bound solutions (21) are identical with the lower bound solutions (8). The solutions therefore are exact.

3. INCLINED REINFORCEMENT

3.1 Lower bound solutions

In case of inclined reinforcement. The stress destribution is shown in fig. 6. In the regions IKFD, KLGF and CDEB there are uniaxial compression and in the regions ABC, DFE and FGE there are hydrostatic compression. All stresses are equal to the concrete strength. The side EF is perpendicular to the reinforcement and the reinforcement is passing through the middle of EF.

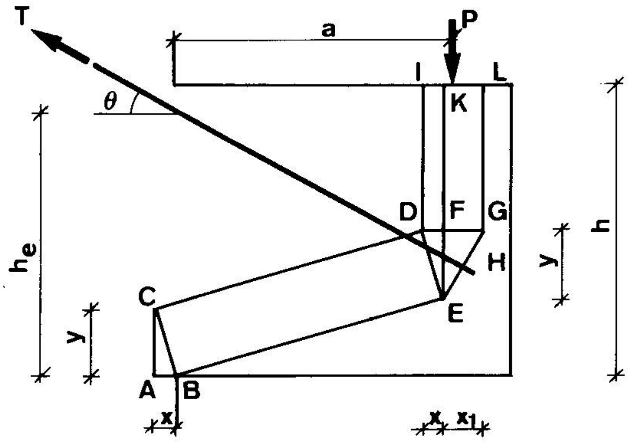

Fig. 6. Stress destribution.

From the geometri we can find two different expressions for the diagonal BD. Then we get

$$(a - \tfrac{1}{2}(x + x_1))^2 + (h_e - (a + \tfrac{1}{2}x)\tan\theta + \tfrac{1}{2}y)^2 - (x^2 + y^2)$$

$$= (a + \tfrac{1}{2}x - \tfrac{1}{2}x_1)^2 + (h_e - (a + \tfrac{1}{2}x)\tan\theta - \tfrac{1}{2}y)^2 \tag{22}$$

In this we have $x_1 = y\tan\theta$ and from (22) we find

$$x = -a + \sqrt{a^2 + y(2h_e - 2a\tan\theta - y)} \tag{23}$$

The lower bound solutions is thus

$$P = \nu\,\sigma_c(x + x_1) \tag{24}$$

and we get

$$\frac{P}{\sigma_c} = \nu\,y\tan\theta + \nu\sqrt{a^2 + y(2h_e - 2a\tan\theta - y)} - \nu a \tag{25}$$

Maximum for (25) is found for

$$y = h_e - a\tan\theta + \sin\theta\sqrt{h_e^2 + (\frac{a}{\cos\theta})^2 - 2a\,h_e\tan\theta} \tag{26}$$

However the tensile force must be equal to the compression force on the side EF, which yields

$$y \nu f_c \leq A f_y \cos\theta \tag{27}$$

This means that

$$y = \frac{\Phi}{\nu} h \cos\theta \tag{28}$$

and with the maximum given by (26).

Inserting in (25), and using $\tau = P/h$, we get the lower bound solutions

$$\frac{\tau}{\sigma_c} = \begin{cases} \Phi \sin\theta - \nu \frac{a}{h} + \sqrt{(\nu \frac{a}{h})^2 + \Phi \cos\theta (2 \frac{h_e}{h} \nu - 2 \frac{a}{h} \nu \tan\theta - \Phi \cos\theta)} & \text{(29a)} \\ \nu \frac{h_e}{h} \tan\theta - \nu \frac{a}{h}(1 + \tan^2\theta) + \frac{\nu}{\cos\theta} \sqrt{(\frac{h_e}{h})^2 + (\frac{a}{h \cos\theta})^2 - \frac{2 a h_e}{h^2} \tan\theta} & \text{(29b)} \end{cases}$$

(29a) is valid when

$$\Phi \cos\theta \leq \nu \frac{h_e}{h} - \nu \frac{a}{h} \tan\theta + \frac{\nu}{h} \sin\theta \sqrt{h_e^2 + (\frac{a}{\cos\theta})^2 - 2 a h_e \tan\theta} \tag{30}$$

3.2 Upper bound solutions

The failure mechanism is the same as in fig. 5, see fig. 7.

The contributions to the work equation from concrete and from the external load are given by (17) and (16). The contribution from the reinforcement is determined by the distance from A perpendicular to the reinforcement. This distance is $(h_e - y)\cos\theta + x \sin\theta$, and the work equation then is

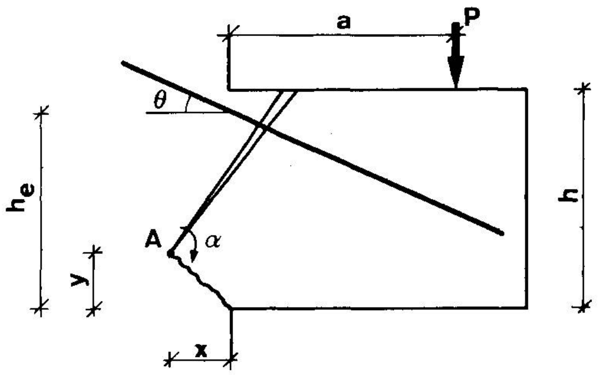

Fig. 7. Failure mechanism.

$$\frac{P}{\sigma_c} = \frac{\frac{1}{2}\nu(x^2 + y^2) + \Phi h((h_e - y)\cos\theta + x \sin\theta)}{a + x} \tag{31}$$

Minimum is found for

$$y = \frac{\Phi}{\nu} h \cos\theta \tag{32}$$

$$x = -a + \sqrt{a^2 + \frac{\Phi}{\nu} h \cos\theta (2 h_e - 2 a \tan\theta - \frac{\Phi}{\nu} h \cos\theta)} \tag{33}$$

Inserting in (31) we find an upper bound solution equal to (29a). The upper limit for the validity of this equation is the contribution 0 from the reinforcement, that is

$$(h_e - y)\cos\theta + x \sin\theta = 0 \tag{34}$$

Inserting (33) in (31) we get (26), and then we get the same limit as in (30). If (30) is not fulfilled, the upper bound solution is (29b).

4. CONCLUSION

The developed upper bound solutions are identical with lower bound solutions. Therefore the solutions (8) and (29) are exact solutions.

In the same way as here, the load carrying capacity of corbels with destributed reinforcement and with combined horizontal and inclined reinforcement can be found. Fig. 8 shows the stress destribution with horizontal and inclined reinforcement.

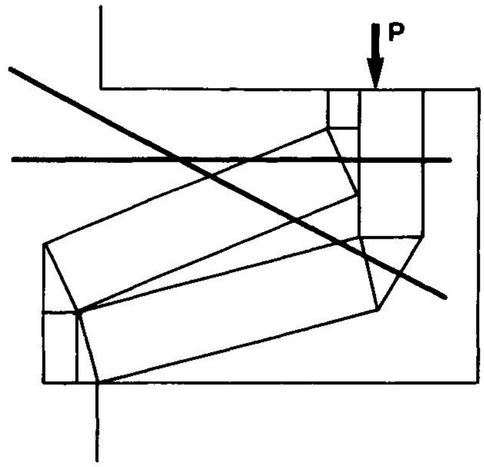

Fig. 8. Stress destribution.

It is noteworthy that the failure mechanism in all cases is a sort of bending-shear failure. Only in a very special case, a sliding failure gives the right upper bound solution. The sliding failure mechanism corresponds to the stress destribution in fig. 2, where the yield line will appear between C and D. The load carrying capacity for corbels presented in [1] is therefore only exact when (7) is fulfilled.

The results presented here can of course be transferred to the shear capacity of beams without shear reinforcement.

Just now we carry out a comparesion of the equations with tests.

5. REFERENCES

[1] Nielsen, M.P., M.W. Bræstrup, B.C. Jensen, & F. Bach: Concrete Plasticity, Special Publication, Danish Society of Structural Science and Engineering, Copenhagen 1978, pp 129.

V

Limit Values of Local Stresses

Valeurs limites des pressions locales

Grenzwerte der örtlichen Pressung

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SUMMARY
On the basis of their own experimental investigations, the authors have determined the actual load when locally loaded concrete elements are brought into a limit state of collapse. Also, applying the condition of concrete collapse they have arrived at the minimum value for the critical load by constructing statically possible stress fields, separated by discontinuity planes. It has been shown that the collapse load, determined in this way, is very near to the collapse load obtained on the tested specimens.

RESUME
Sur la base de leurs essais, les auteurs ont déterminé les pressions locales correspondant à l'état de rupture des éléments en béton chargés localement. Les valeurs inférieures de la charge ultime de tels éléments ont été calculées en construisant des champs discontinus de contraintes et en appliquant un certain critère de rupture pour le béton. Les valeurs théoriques concordent bien avec les valeurs expérimentales.

ZUSAMMENFASSUNG
Auf der Grundlage eigener Versuche haben die Autoren die dem Bruchzustand lokal belasteter Betonelemente entsprechenden örtlichen Pressungen ermittelt. Ebenso wurden durch Konstruktion diskontinuierlicher, statisch zulässiger Spannungsfelder und Anwendung der Bruchbedingung für Beton untere Grenzwerte für die Traglast berechnet. Diese rechnerischen Werte stimmen gut mit den experimentell ermittelten überein.

1. TEST RESULTS AND DISCUSSION

The paper presents the results from experimental and theoretical investigations in the behaviour of locally loaded concrete elements in the state of limit strength.

These investigations are the part of a very large research programme, which is still under way at the Faculty of Civil Engineering in Belgrade, and were undertaken in order to verify some assumptions for the innovation of the actual Yugoslav Code for Reinforced Concrete Structures.

The loading was of static character and was gradually increased, approximately with that velocity of loading which is applied to a standard testing of a cube, the side of which was 20 cm. The load was transmitted to the specimen through a very rigid steel "stamper". The processing of contact surface of the "stamper" and the concrete specimen was the same as reguired in testing standard cubes. The compressive strength of concrete was ranging from 20 to 50 MPa.

Characteristic results of experimental investigations, statistically processed, are shown in Figs. 1,2 and 3. Fig. 1 shows a case of local loading along the whole thickness of the element d, and Figs. 2 and 3 illustrate the cases when the locally loaded surface F_0 in both directions is of less dimensions than the dimensions of the examined concrete element in the plan.

Instead of the ratio F_0/F, the ratios of the sides a/b for parallelepipeds are applied to the abscissa of the diagram, that is,the ratios d/D for cylinders. The ratios of compressive stresses $\beta_l (=P_l/F_0)$ and the strength of the prism β_{pr} are applied to the ordinate, when P_l is the force under which the specimen collapsed, and F_0 is a locally loaded surface.

On the basis of the results of experimental tests the following observation can be given:

a) Local compressive stresses are grouped around the curved forms of the hyperbola with asymptotes $\beta_l/\beta_{pr} = 1$ and a/b = 0, that is, d/D = 0.

b) The influence of the quantity of locally loaded surface F_0 upon the change of local compression β_{pr} is practically negligible in the interval $2/3 \leqslant a/d \leqslant 1$, that is $2/3 \leqslant d/D \leqslant 1$ and up to the relation $a/b \geqslant 1/2$ .... ($d/D \geqslant 1/2$) the change is very slight. When the realtion $a/b \approx 1/3$ the stress ranges from 1,5 to 3,0 β_{pr}. By further reduction of the relation a/b, that is, d/D, local compressions increase progressively, so that for the realtion 0,10 they may increase from 12 to 15 β_{pr}

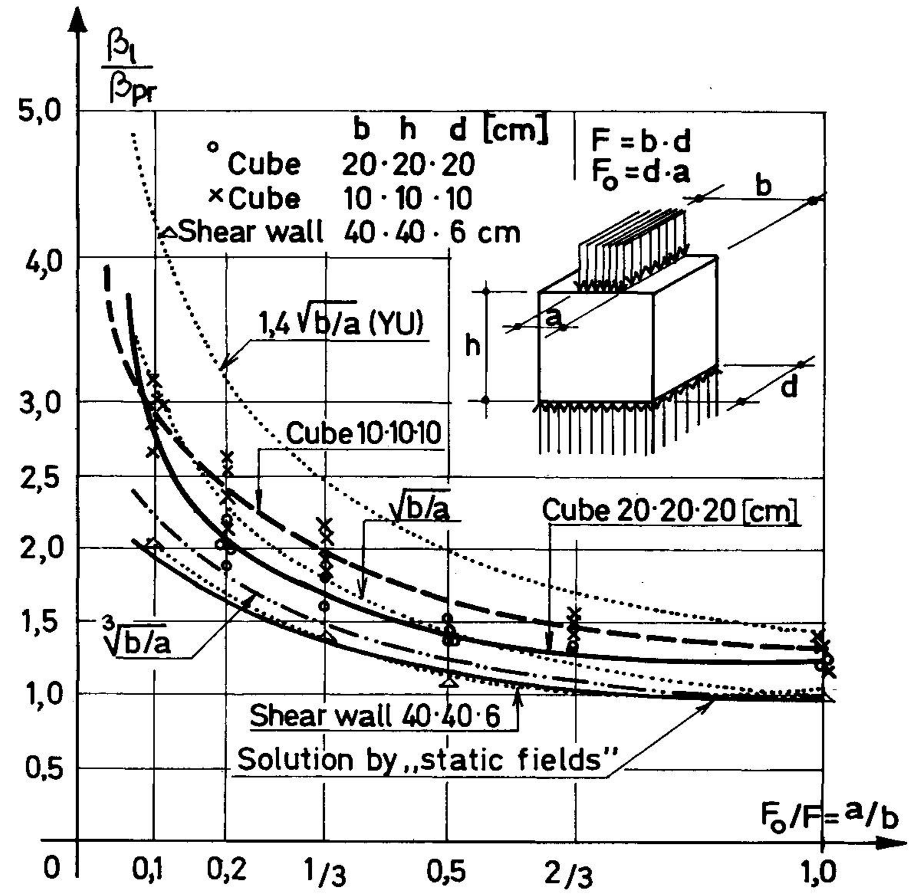
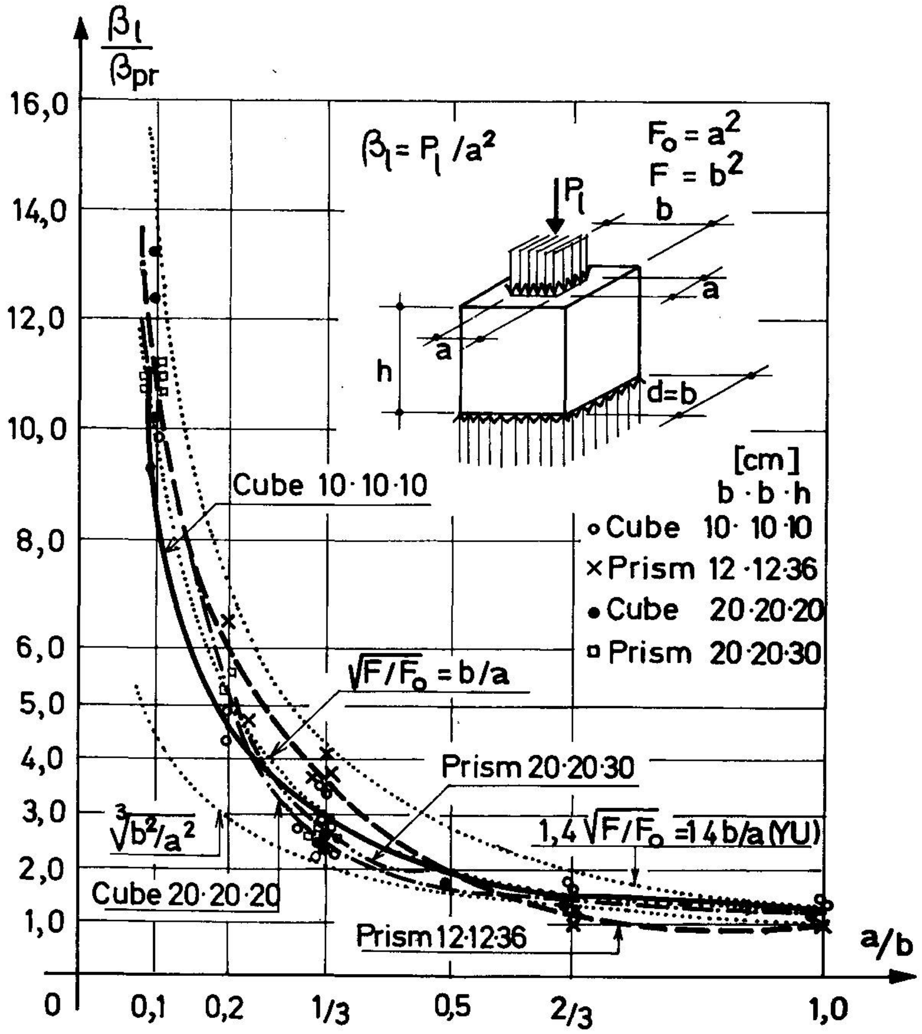

β_l / β_{pr}
b h d [cm]
∘ Cube 20·20·20
× Cube 10·10·10
△ Shear wall 40·40·6 cm
F = b·d
F_o = d·a
1,4 √b/a (YU)
Cube 10·10·10
√b/a
Cube 20·20·20 [cm]
∛b/a
Shear wall 40·40·6
Solution by „static fields"
F_o/F = a/b
5,0
4,0
3,0
2,5
2,0
1,5
1,0
0,5
0
0,1
0,2
1/3
0,5
2/3
1,0

Fig. I


β_l / β_{pr}
$\beta_l = P_l / a^2$
$F_o = a^2$
$F = b^2$
P_l
b
a
h
d=b
[cm]
b · b · h
∘ Cube 10·10·10
× Prism 12·12·36
• Cube 20·20·20
▫ Prism 20·20·30
Cube 10·10·10
√F/F_o = b/a
Prism 20·20·30
∛b²/a²
1,4 √F/F_o = 1,4 b/a (YU)
Cube 20·20·20
Prism 12·12·36
a/b
16,0
14,0
12,0
10,0
8,0
6,0
5,0
4,0
3,0
2,0
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0
0,1
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0,5
2/3
1,0

Fig. 2

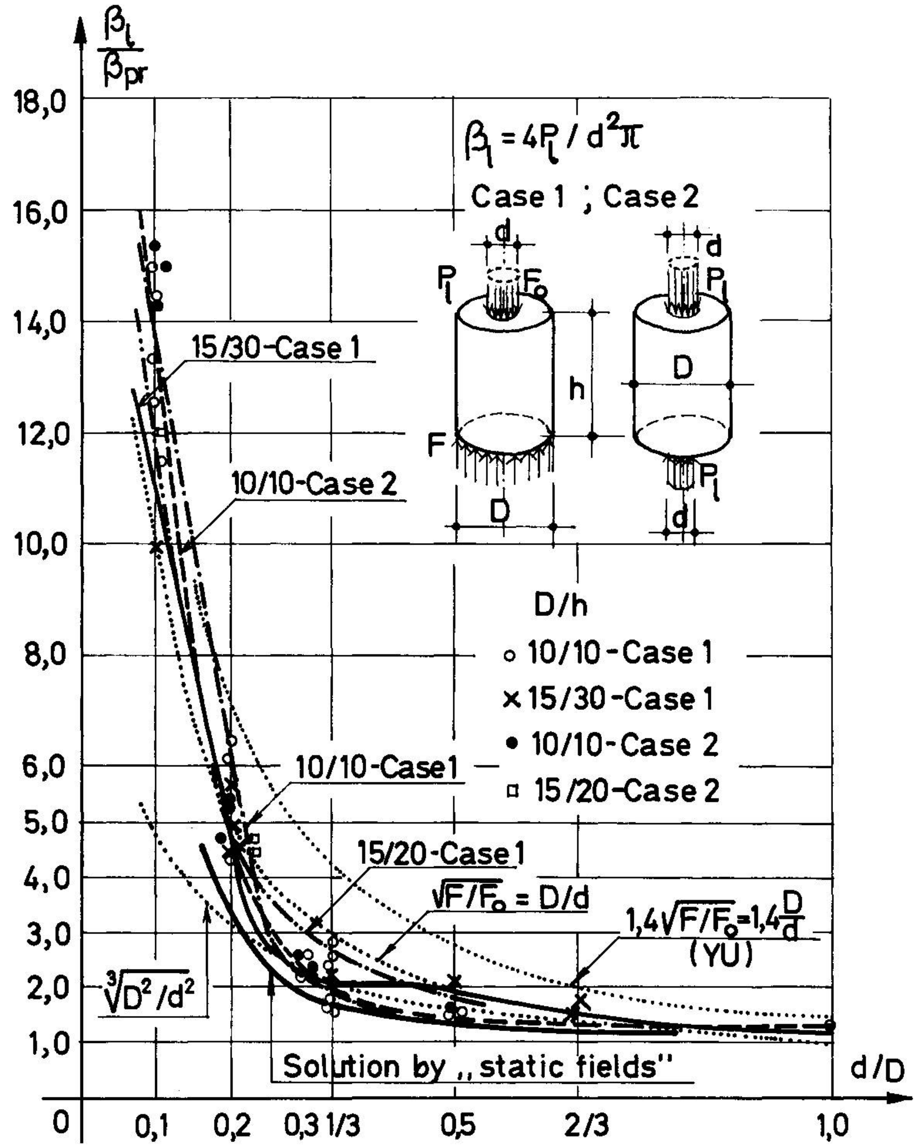


Fig. 3

c) The local compression β_l upon the specimens of the same dimensions and quality of concrete and of the same relation of the sides a/b is very dependent on that whether the loading acts along the whole thickness d of the specimen or only partially. In the first case (Fig. 1) the model is under conditions that are nearer to the biaxial stress state, so that local compressions, even in small relations $F_0/F = a/b$, reach comparatively modest values, Thus, on the tested shear wall of dimensions 40cm, 40cm, 6cm, for which it may be said that it sufficiently correctly - for practical purposes - satisfies the conditions of biaxial stress states, a local compression $\beta_l \simeq 2\,\beta_{pr}$ is obtained, when $F_0/F = a/b = 0{,}10$. In the second case, when the dimensions of locally loaded surface F_0 in both directions are less than the dimensions of the total surface F (Figs. 2 and 3), the specimen is under the conditions o triaxial stress state and it is also understandable why very high stresses occur in the "wedge" that is under the triaxial stresses of compression.

d) A good approximation of experimental results of limit local compressions for very thin elements, that means for the plane state of stress, fives the formula

$$\beta_l = \beta_{pr}\sqrt[3]{F/F_o} \tag{1}$$

For the elements which are of considerable thickness, loaded along all the thickness, as well as for triaxial stressed elements, (Figs. 2 and 3) a satisfactory approximation of experimental results is achieved by the formula with square root.

$$\beta_l = \beta_{pr}\sqrt{F/F_o} \tag{2}$$

The formula

$$\beta_l = 1{,}4\,\beta_{pr}\sqrt{F/F_o}\,, \tag{3}$$

given in the actual Yugoslav Code for Reinforced Concrete, is not proved by these experiments. It gives larger values than the experimental ones.

2. THEORETICAL ANALYSIS OF LOAD CARRYING CAPACITY BY APPLYING EXTREME PRINCIPLES

For the determination of load carrying capacity of locally loaded elements the condition of plasticity collapse of concrete suggested in the paper /3/ has been used. This condition, for the biaxial stress state, in the system $\bar{\sigma}_1 - \bar{\sigma}_2$, (where σ_1, σ_2 are principle stresses), is:

$$A_o + \frac{A_1}{3}(\bar{\sigma}_1+\bar{\sigma}_2) + \frac{A_2}{9}(\bar{\sigma}_1+\bar{\sigma}_2)^2 + \frac{A_3}{27}(\bar{\sigma}_1+\bar{\sigma}_2)^2 - \frac{2}{9}(\bar{\sigma}_1^2+\bar{\sigma}_2^2-\bar{\sigma}_1\bar{\sigma}_2) = 0 \tag{4}$$

when

$$A_o = \frac{2}{3}\bar{\beta}_s^2\,; \quad A_1 = 2\bar{\beta}_s^2\,\frac{1-2\mu}{\mu} - \frac{1}{3}\mu\alpha\,; \quad A_2 = 2 - \frac{6\bar{\beta}_s}{\mu} - (1-\mu)\alpha\,; \quad A_3 = 3\alpha$$

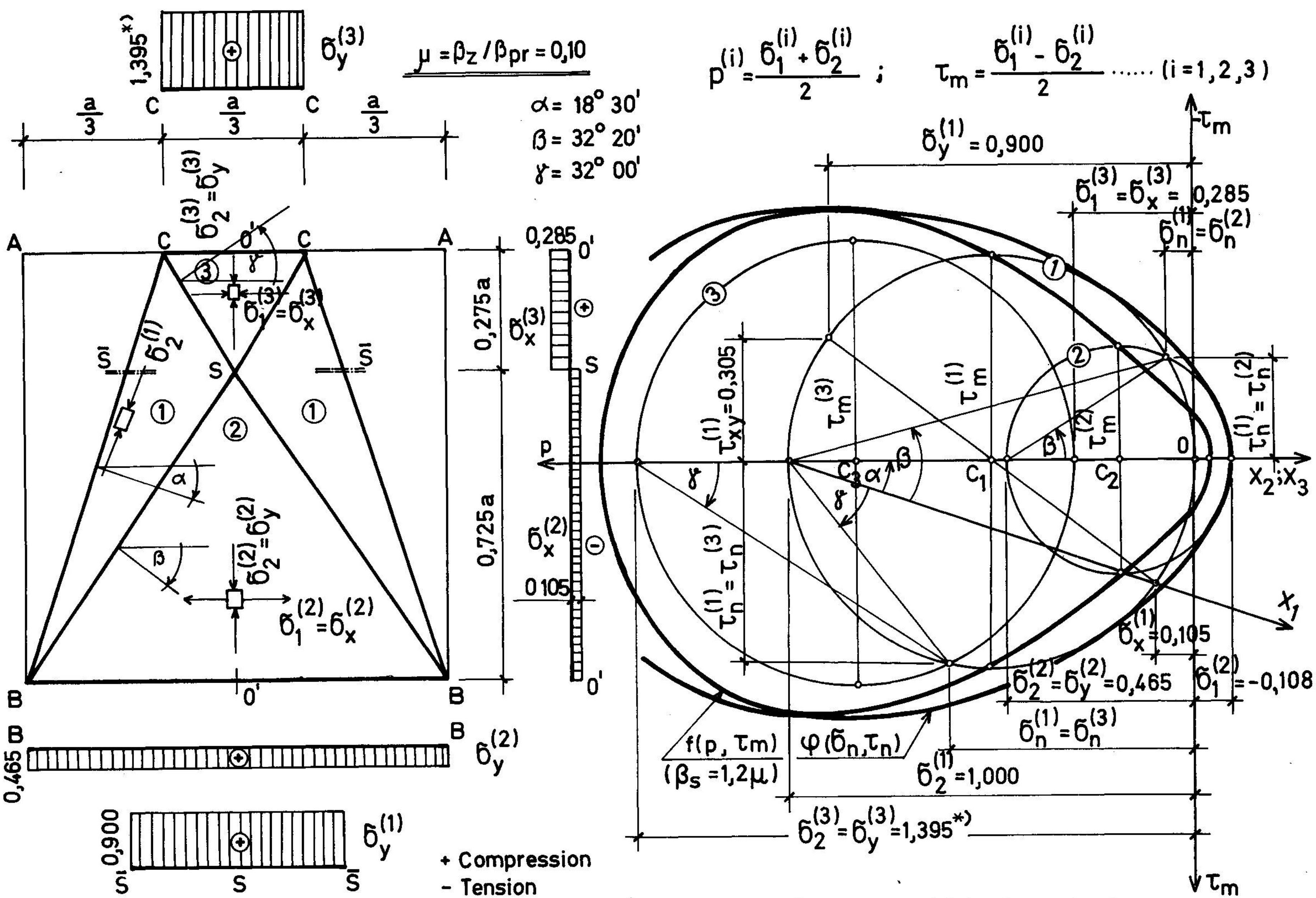
$$\alpha = \frac{\frac{\xi-2}{2\xi-1} - 2}{2\xi+\mu} + 3\,\frac{\bar{\beta}_s^2}{\xi\mu}\,; \quad \bar{\sigma}_1 = \sigma_1/\beta\,; \quad \bar{\sigma}_2 = \sigma_2/\beta_{pr}$$

$$\mu = \beta_z/\beta_{pr}\,; \quad \bar{\beta}_s = \beta_s/\beta_{pr}\,; \quad \xi = \bar{\sigma}_1 - \bar{\sigma}_2$$

A uniaxial tensile strength of concrete is indicated by β_z, the strength of the prism by β_{pr}, where the strength in pure shear is indicated by $\beta_s = \sigma_1 = -\sigma_2$, and the ratio of strengths of concrete in equal biaxial compressions to the strength of prism is indicated by ξ.

For triaxial stress state the condition of plasticity-collapse of concrete /3/ is given by the expression

$$\bar{\tau}_o = A\bar{\sigma}_o + B \tag{5}$$



*) The values of stresses obtained are to be multiplied by β_{pr}

Fig. 4

where $\bar{\tau}_o = \tau_o / \beta_{pr}$; $\bar{\sigma}_o = \sigma_o / \beta_{pr}$

when $\tau_o = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ $\quad \sigma_o = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$

represent octahedral shear and normal stress. For μ = 0,10, the constants A and B are A = 0,700 and B = 0,238.

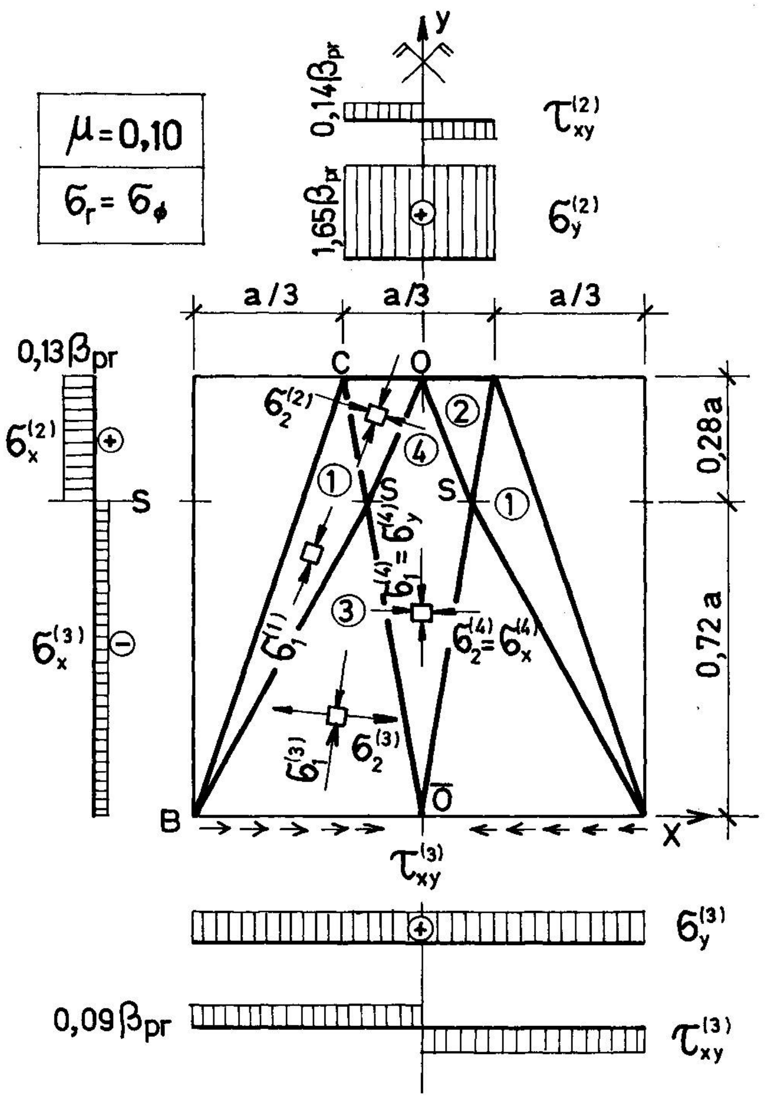


Fig. 5

In this paper the limit load is defined by statically possible fields, constant stresses, associated with the regions of discontinuity, which, as it is known, give the lowest value for loading when the collapse of locally loaded specimen occurs. Figs. 4 and 5 show the constructions of statically possible fields made of fields made of fields of constant stresses connected by plane of discontinuity BS and CS (SO and S$\bar{O}$). Fig. 4 shows such a field for a biaxial stress state in the system p-τ_m $[p=(\sigma_1+\sigma_2)/2$; $\tau_m=(\sigma_1-\sigma_2)/2]$, in locally loaded surface, with the relation a/b = = 1/3. From the diagram, in Fig. 1, one can see that the solutions by means of these fields are in good agreement with results experimentally obtained in testing a shear wall considered to have been under the conditions of biaxial stress state.

Fig. 5 shows statically possible fields of stresses for locally loaded cylindrical specimens in the coordinate system p,τ_m when the collapse condition is used for triaxial stress state given by Eq.(5) for the case of rotational symmetry. It has been shown that the assumptions made about the relations of stress in radial and tangential directions σ_2/σ_3 = 1 and σ_2/σ_3 = 0,5, in well constructed field, have been proved by the tested specimens. Thus, Fig. 3 gives the results of the solution by means of "static fields", when σ_2 = 0,5 $\sigma_3 \ldots (\sigma_2 = \sigma_r, \sigma_3 = \sigma_\phi)$, which, as it is seen, represents the lower value of the actual limit load of rotationally loaded cylindrical bodies. When $\sigma_2 = \sigma_3$, the condition (5) in the system p, τ_m has the form

$$\tau_m = 0{,}597p + 0{,}203 \qquad (6)$$

while when $\sigma_2 = 0{,}5 \cdot \sigma_3$, the condition (5) overpasses the hyperbola Fig.5 showsone possible construction of static fields by introducing friction into the contact plate of specimens with massive plates - "stampers".

3. CONCLUSIONS

On the basis of aforementioned theoretical analysis one may observe that by means of statically possible fields, one can prognosticate very simply the least low value of loading, which brings the concrete specimen into the state of collapse. A good agreement of results of such theoretical solutions with the results obtained on concrete specimens shows that the application of static fields has its full justification. In addition, the previous analyses show that the conditions must be precise in that when to apply Eq. (1), and when Eq. (2). The investigations of the authors have shown that Eq. (1) may be applied only when the locally loaded concrete element is in the biaxial stress state, while, Eq. (2) may be applied to the concrete elements with triaxial stress state. Eq. (3), given in Yugoslav Standards, and in the standards of some others countries, is not acceptable for the previous stress states. It gives considerably larger values than the ones obtained by the authors in their tests. Some preliminary investigations of the authors pointed out that Eq. (3) can be used in defining local limit stress in concrete elements which are in the state not far from the plane deformation, However, more precise conclusions, concerning it, may be given only after detailed experimental tests, which the authors have in plan to make.

R E F E R E N C E S

1. *Aćić M: Granične vrednosti napona u betonu za ravno stanje napona. Materijali XVI Kongresa Saveza jugoslovenskih laboratorija, Vrnjačka Banja, oktobar 1978. (Limit Values of Stress in Concrete for the Biaxial Stress State, Materials for 16th Congress of the Association of Yugoslav Laboratories, Vrnjačka Banja, October, 1978.*

2. *Ivković M: Ponašanje betona u oblasti granične ravnoteže (Doktorska disertacija), Gradjevinski fakultet, Beograd, 1962. (The Behaviour of Concrete in the Domain of Limit State (Doctoral dissertation), Faculty of Civil Engineering University of Belgarade.*

3. *Ivković M, Aćić M: Uslovi plastičnosti-loma betona pri složenim naponskim stanjima. Saopštenja XII Jugoslovenskog kongresa racionalne i primenjene mehanike, sveska C5-6, Ohrid, 1974. (Conditions of Plasticity-Collapse of Concrete in Combined Stress States, Transactions of 12th Yugoslav Congress of Rational and Applied Mechanics, Volume C5-6, Ohrid, 1974).*

4. *Kupfer H.B, Hisdorf H.K, Rüsch H: Behaviour of concrete under biaxial stresses. Proceeding ACI Vol 66, No8 8/1969.*

V

The Anchorage Strength of Reinforcement Bars at Supports

L'ancrage des armatures à l'appui

Verankerung der Bewehrungsstäbe bei Auflagern

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SUMMARY
The theory of plasticity is used to attempt a solution of anchorage and splicing problems for deformed reinforcing bars.
The paper reports calculations on anchoring of one and two bars in the support zone of a beam.

RESUME
La théorie de la plasticité est appliquée à l'étude des problèmes de l'ancrage et du recouvrement des armatures. Des calculs sont présentés pour l'ancrage d'une et de deux barres d'armature dans la zone d'appui d'une poutre.

ZUSAMMENFASSUNG
Die Plastizitätstheorie wird auf Probleme der Verankerung und der Überlappung von Bewehrungsstäben angewendet. Berechnungen für die Verankerung von einem oder von zwei Bewehrungsstäben im Auflagerbereich eines Balkens werden dargestellt.

1. INTRODUCTION

Until quite recently, only a few rational theories concerning the anchorage strength of reinforcement bars and the strength of splices have been formulated. Tepfers made an attempt in his dissertation [1], and some investigators have tried to solve the problem using the finite element method (see for instance Tepfers [1] , Lutz [2], etc.).

Since the mid sixties, a research group in Denmark has been investigating the application of the theory of plasticity on concrete structures. Research was earlier mainly centered on the problems of shear in beams, shear in joints and punching shear (see Nielsen et al. [3]).

It thus seemed natural to examine whether the same theory and methods could be used to solve the problems of anchoring and splicing of reinforcement bars.

Such an investigation has now come so far that it is possible to report the first results.

2. BASIC ASSUMPTIONS

The work is based on the following assumptions:

1) The concrete is considered a rigid-plastic material with the modified Coulomb failure criterion as yield condition and the angle of friction $\varphi = 37^{\circ}$. As it is well known, concrete is not perfectly plastic. This is taken into account by reducing the concrete cylinder strength σ_c with an empirical effectiveness factor ν .

2) Deformations in the concrete are determined by the normality condition (the associated flow rule).

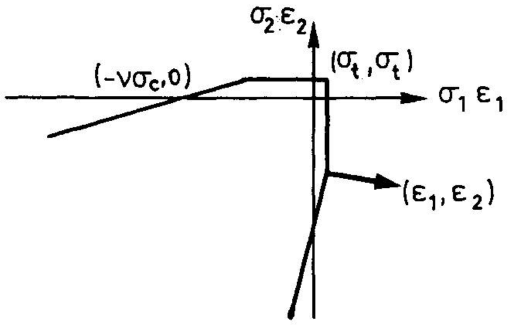

Figure 1: Modified Coulomb Failure Criterion for plane strain with the Associated Flow Rule.

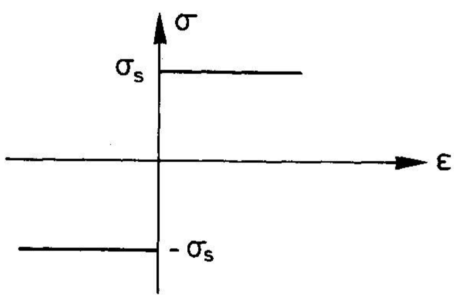

Figure 2: Rigid-plastic Stress-strain Relation valid for the Reinforcement.

3) The reinforcement bars are assumed being rigid plastic, and only able to carry longitudinal stresses.

In the following calculations the upper bound theorem will be used.

The upper bound theorem can be defined as:

A load found by the work equation, for a geometrically possible failure mechanism, is greater than or equal to the yield load.

3. THE LOAL FAILURE MECHANISM

Concerning the failure along a bar it is assumed that there is no adhesion between the concrete and the reinforcement bar.

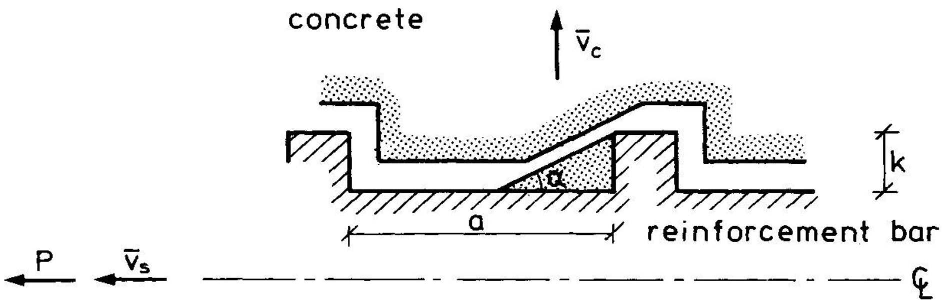

Figure 3: The Local Failure Mechanism

Consider an idealized reinforcement bar with idealized ribs. If the bar is moved in the direction $\bar{v}_s$ (the same direction as the force P , see figure 3) , the concrete in front of the ribs will yield and the yield plane form an angle γ with the bar axis. The surrounding concrete is then pushed axisymmetrically away from the bar.

The displacement of the surrounding concrete relative to the reinforcement bar is:

$$\bar{v}_{cs} = \bar{v}_c - \bar{v}_s$$

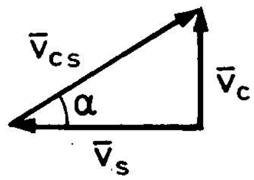

Figure 4

If we denote the angle between $\bar{v}_c$ and $\bar{v}_{cs}$ as α , the external work is $W_e = P||\bar{v}_{cs}|| \cos\alpha$. The internal work W_i is:

If $\quad \gamma \geq \text{Arctan}\,(\frac{k}{a})$:

$$W_i = ||v_{cs}||\,(d+k)\pi\,\frac{k\,\ell}{a\,\sin\gamma}\,\{\frac{\nu\sigma_c}{2}(1-\sin(\alpha-\gamma)) +$$

$$+ \sigma_t\,\frac{\sin(\alpha-\gamma)\sin\varphi}{1-\sin\varphi}$$

where ℓ is the length of the anchorage and d the diameter of the reinforcement bar.

If $\quad 0 \leq \gamma \leq \text{Arctan}\,(\frac{k}{a})$:

$$W_i = ||\bar{v}_{cs}||\,(d+2k - a\tan\gamma)\pi\,\frac{a\,\ell}{a\,\cos\gamma}\,\cdot$$

$$\cdot\,\{\frac{\nu\sigma_c}{2}(1-\sin(\alpha-\gamma)) + \sigma_t\,\frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\}$$

The expressions have been calculated for plane strain condition. Concerning the basic expressions, see for instance Nielsen et. al. [3] or Jensen [4].

Putting $W_e = W_i$ we obtain the work equation:

If $\gamma \geq \text{Arctan}\,(\frac{k}{a})$:

$$\frac{P}{\ell d\pi\sigma_c} = \frac{\tau}{\sigma_c} = \frac{(d+k)k}{da\ \cos\alpha\sin\gamma}\{\nu\ \frac{1-\sin(\alpha-\gamma)}{2} + \rho\ \frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\}$$

where $\rho = \sigma_t/\sigma_c$

If $0 \leq \gamma \leq \text{Arctan}\,(\frac{k}{a})$:

$$\frac{P}{\ell d\pi\sigma_c} = \frac{\tau}{\sigma_c} = \frac{(d+2k-a\tan\gamma)}{d\ \cos\gamma\ \cos a}\{\nu\ \frac{1-\sin(\alpha-\gamma)}{2} + \rho\ \frac{\sin(\alpha-\gamma)-\sin\varphi}{1-\sin\varphi}\}$$

In the above expressions we have not taken into account the internal work carried out in the surrounding concrete and reinforcement (for instance work carried out in stirrups).

The expressions are only strictly correct if the surrounding concrete is displaced axisymmetrically to the bar axis, i.e. the concrete has to crack along an infinite number of radii.

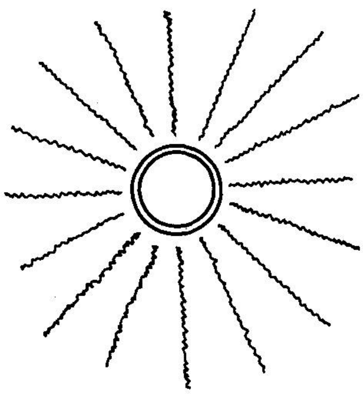

Figure 5: Yield Lines around a Bar.

This is not always so in practice. For instance, the surrounding concrete at a normal anchorage or splice in a beam is not displaced axisymmetrically.

Despite this fact, the expressions are used without modifications in the following, assuming the error to be without importance.

4. ANCHORAGE OF ONE REINFORCEMENT BAR

Consider first the case where a single bar is anchored along a beam support, having a uniformly distributed support reaction r. The failure pattern is assumed to be as shown in figure 6.

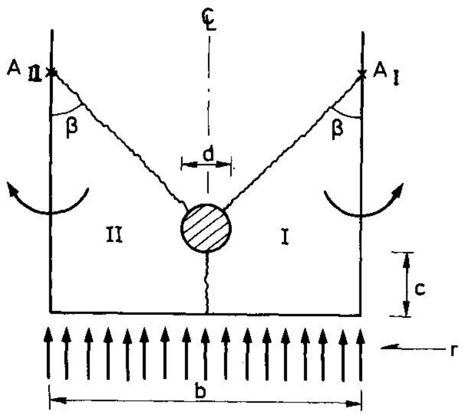

Figure 6: Yield Lines at the Anchorage of One Bar

When the reinforcement bar is displaced longitudinally the concrete will yield in front of the ribs and force the parts I and II to rotate about A_I and A_{II} respectively.

The bearing capacity τ/σ_c of such an anchorage becomes a function of the angles α, γ and β, the geometry and the dimensionless stress r/σ_c from the reaction. As the upper bound theorem is used, τ/σ_c is to be minimized with respect to α, γ and β.

Minimum has been found numerically by a computer. The result is shown in figure 7, compared with tests made by Rathkjen [5].

In the calculations the effectivity factor ν has been put equal to 0.50 and the dimensionless tensile strength $\rho = 0.025$.

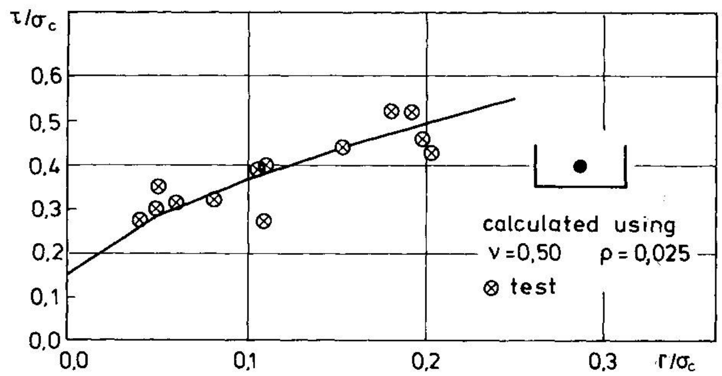

Figure 7: Comparison between Tests and Calculations on the Anchoring of Kamsteel 42 ϕ14

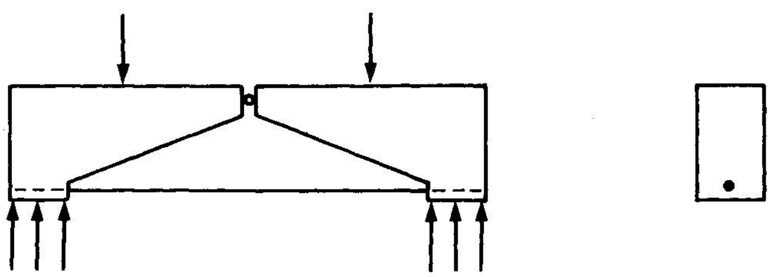

Figure 8: The Testbeams used by Rathkjen [5]

5. ANCHORAGE OF TWO REINFORCEMENT BARS

If two bars are anchored, there are several possible failure patterns. In the calculations referred to here, 4 failure patterns have been studied.

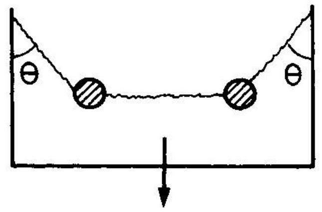

Figure 9: Failure 1, the Cover Splits into One Piece.

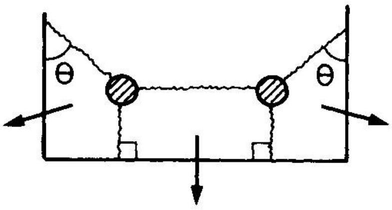

Figure 10: Failure 2, the Cover Splits into Three Pieces.

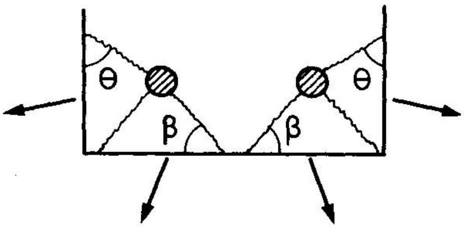

Figure 11: Failure 3, the Corners Split into Two Pieces

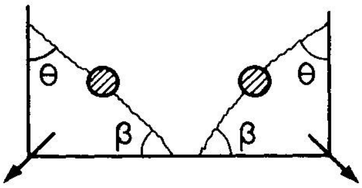

Figure 12: Failure 4, the Corners Split into One Piece.

One more mechanism could be studied. A failure where triangle formed bodies are pressed out below the reinforcement often occurs. But since the beams used for comparison did not have the wide bar spacings necessary for this mechanism to occur, this has been left out of consideration.

The expressions to be minimized are very extensive and the minimizations have to be made numerically on a computer.

The following figures show some of the results. The calculations are again compared with tests made by Rathkjen [5].

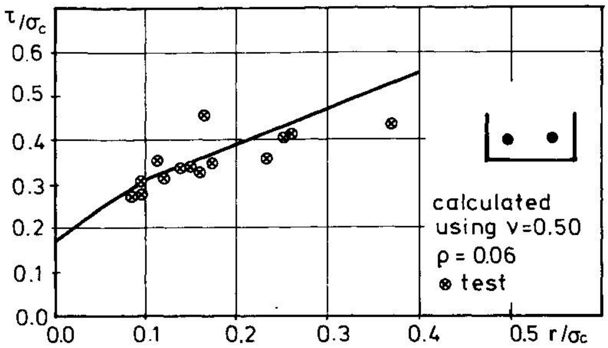

Figure 13: Kamsteel 42 ⌀14. Comparison between Tests and Calculations.

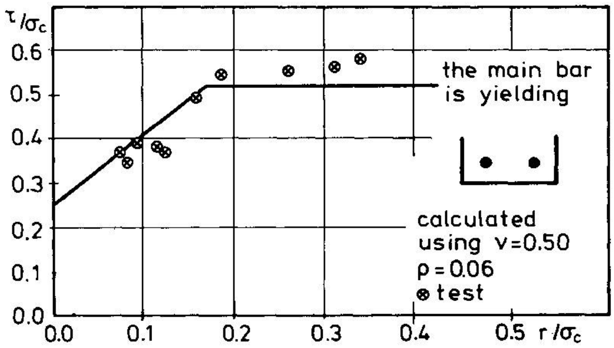

Figure 14: Kamsteel 42 ⌀ 10. Comparison between Tests and Calculations.

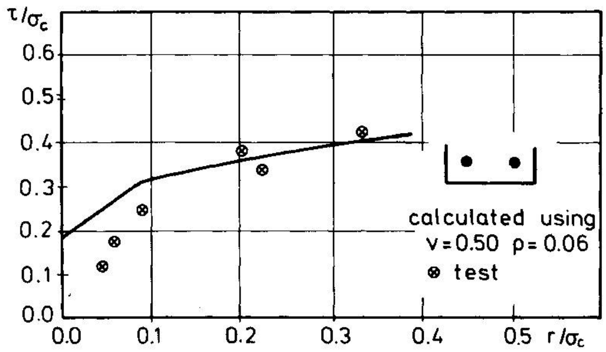

Figure 15: Tentor-steel 56 ⌀14. Comparison between Tests and Calculations.

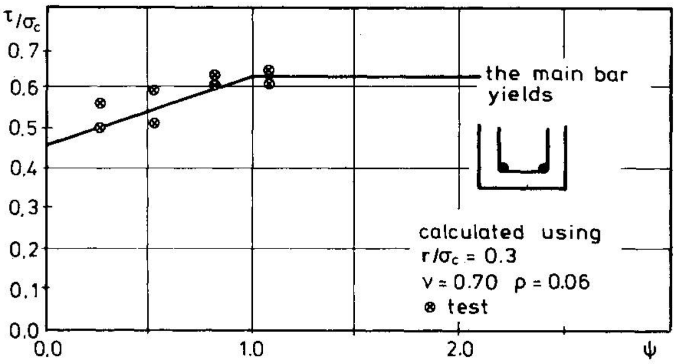

Figure 16: Comparison between Tests and Calculations of the Anchoring of Kamsteel 42 ⌀14 with Stirrups at the Supports.

Figures 13, 14 and 15 show the anchorage strength τ/σ_c as a function of the dimensionless support stress r/σ_c

The calculations have been carried out with data corresponding to Danish Kamsteel 42 ⌀14 mm and ⌀10 mm and Tentor-steel 56 ⌀14 mm.

The effectiveness factor ν was 0.50 and the dimensionless tensile strength $\rho = \sigma_t/\sigma_c$ was 0.06.

Figures 16 and 17 show the anchorage strength τ/σ_c as a function of the stirrup strength in the anchorage zone.

The parameter $\psi = \dfrac{2A_s\,\sigma_s}{\ell\, d\, \sigma_c}$ is used as a dimensionless stirrup strength.

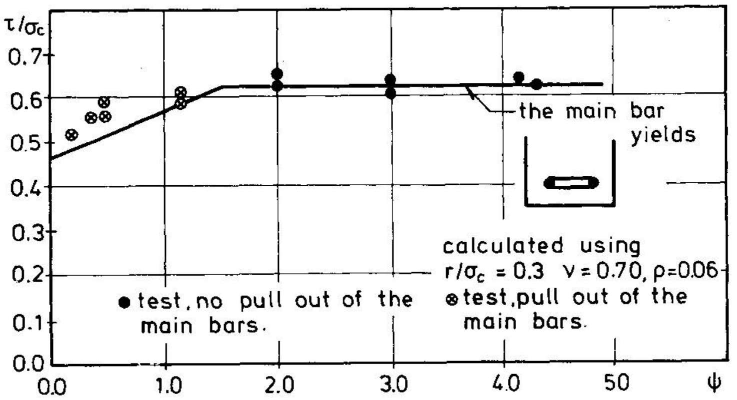

Figure 17: Comparison between Tests and Calculations of the Anchoring of Kamsteel 42 ø14 with Stirrups at the Supports.

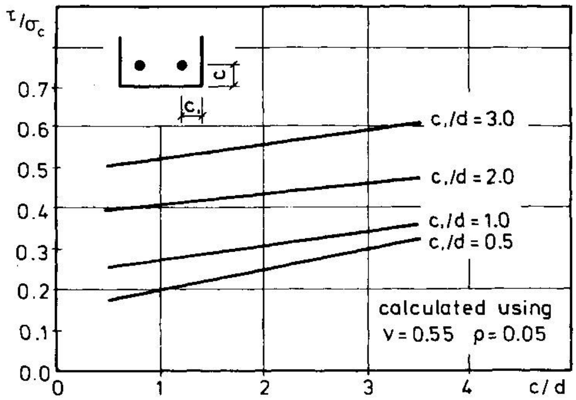

Figure 18: Calcultated Strength τ/σ_c as a Function of the Covers c and C_1.

Figure 18 shows the anchorage strength τ/σ_c as a function of the bottom cover c and the side cover c_1, both c and c_1 made dimensionless by division by d. No tests have been found to compare with, but the variation of the covers gives the effect we would expect, i.e. in principle the same as described by Tepfers [1].

6. CONCLUSION

As we have seen, the theory of plasticity is able to explain the influences of parameters as the support stress r/σ_c, and the stirrup strength ψ in an acceptable way.

There are some shortcomings in the ν-values, i.e. we have different values for, for instance, anchoring of one or two bars and anchoring with or without stirrups. Some of these difficulties are due to shortcomings in the model and others are due to the well known difficulties with plastic calculations of unreinforced concrete (see for example Jensen [4]).

Future work will have to be centered on eliminating these shortcomings and extending the theory to, for instance, the splicing problem or the anchoring of three or more bars.

7. REFERENCES

[1] Tepfers,R.: A Theory of Bond applied to Overlapped Tensile Reinforcement Splices for Deformed Bars. Publ. 73:12, Division of Concrete Structures, Chalmers Tekniska Högskola, Göteborg 1973.

[2] Lutz,L .A.: The Mechanics of Bond and Slip of Deformed Reinforcing Bars in Concrete. Report No. 324, Department of Structural Engineering, School of Civil Engineering, Cornell University, August 1966.

[3] Nielsen, M.P., Bræstrup, M.W., Jensen, B.C., Bach, Finn: Concrete Plasticity. Special Publication, Dansk Selskab for Bygningsstatik, Lyngby, Oktober 1978.

[4] Jensen, B.C.: Some Applications of Plastic Analysis to Plain and Reinforced Concrete, Report No. 123, 1977. Technical University of Denmark, Institute of Building Design, Copenhagen.

[5] Rathkjen, A.: Forankringsstyrke af armeringsjern ved bjælkeunderstøtninger. Rapport 7203, DIAB Aalborg 1972, Ren & Anvendt Mekanik.

V

Some Examples of Lower-Bound Design of Reinforcement in Plane Stress Problems

Dimensionnement de l'armature et problèmes d'états plans de contrainte, en appliquant la méthode statique

Beispiele zur Bemessung der Bewehrung nach der statischen Methode bei Problemen mit ebenen Spannungszuständen

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SUMMARY
The paper demonstrates how simple statically admissible stress fields can be used for the determination of reinforcement in plane stress problems, and presents some standard formulae for the necessary amount of reinforcement as a function of stresses. The problems treated are shear, torsion and combined bending, torsion and shear in beams.

RESUME
On montre comment on peut utiliser des champs de contraintes statiquement admissibles et simples pour l'analyse des problèmes avec états plans de contrainte. Des formules sont données pour dimensionner l'armature en fonction des contraintes. On traite les problèmes du cisaillement et de la torsion dans une poutre ainsi que l'action combinée de la flexion, de la torsion et du cisaillement.

ZUSAMMENFASSUNG
Es wird gezeigt, wie für Probleme mit ebenen Spannungszuständen einfache statisch zulässige Spannungsfelder angewendet werden können. Zur Ermittlung der erforderlichen Bewehrung in Abhängigkeit der Spannungen werden Formeln angegeben. Behandelt werden die Beanspruchung von Balken durch Querkraft und Torsion sowie die kombinierte Beanspruchung durch Biegung, Torsion und Querkraft.

1. INTRODUCTION

It is the purpose of this paper to demonstrate that reinforcement design according to the theory of plasticity in many plane stress problems can be done with advantage by using simple lower bound solutions in connection with the author's reinforcement formulas, [1],[4],[5] and [6]. The methods were originally proposed in [6], in an internal report. Alternative methods dealing with the same problems have been developed by Thürlimann and his associates,[8] and [9].

We shall begin by giving a short summary of the reinforcement formulas and the theory on which they are based. The basic assumption are that both concrete and steel can be treated as rigid, perfectly plastic materials. The yield condition for the concrete is assumed to be the usual square yield locus $-\sigma_c \leqq \sigma_1 \leqq 0$, $-\sigma_c \leqq \sigma_2 \leqq 0$, where σ_1 and σ_2 are principal stresses (tension positive) and σ_c the compressive strength. The tensile strength is thus assumed to be zero. The reinforcement bars are assumed to be able to carry only forces in their longitudinal direction. While the assumption for the reinforcement can be easily justified, the assumption concerning the concrete has a more doubtful connection to reality. The basic reason for this is that although the ductility of the concrete is rather high in compression, the stress falls drastically, when the peak of the stress-strain curve has been reached. Therefore, the theoretical results often have to be modified. One way of doing this is to introduce an effective compressive strength $\nu\sigma_c$, where ν is a so-called effectivity factor, a lower bound of which is representing some kind of average stress in the actual strain region. The effectivity factor, however, also has to take care of other defects of the theory, see [10].

Considering for simplicity only orthogonal reinforcement in the directions x and y , see fig.1.1, and letting A_{ax} represent the reinforcement area in the x-direction per unit length measured in the y-direction, σ_{fx} the yield stress of the reinforcement in the x-direction, σ_x, σ_y and τ_{xy} , the stresses which have to be carried, and finally letting t being the thickness, the reinforcement formulas can, using similar notation for the y-direction, be written, [6]:

<u>Case 1</u>: $\sigma_x \geqq -|\tau_{xy}|\sqrt{\alpha}$ $\qquad$ $\sigma_y \geqq -|\tau_{xy}|/\sqrt{\alpha}$

$$\sigma_{tx} = \frac{A_{ax}\,\sigma_{fx}}{t} = \sigma_x + |\tau_{xy}|\sqrt{\alpha} \qquad \sigma_{ty} = \frac{A_{ay}\,\sigma_{fy}}{t} = \sigma_y + |\tau_{xy}|/\sqrt{\alpha} \qquad (1.1),\ (1.2)$$

$$\sigma_b = |\tau_{xy}|\,(\sqrt{\alpha} + 1/\sqrt{\alpha}) \qquad (1.3)$$

<u>Case 2</u>: $\sigma_x \leqq \sigma_y\alpha$ $\qquad$ $\sigma_x < -|\tau_{xy}|\sqrt{\alpha}$

Reinforcement is only necessary if $\sigma_x\sigma_y \leqq \tau_{xy}^2$

$$\sigma_{tx} = 0 \qquad \sigma_{ty} = \sigma_y + \frac{\tau_{xy}^2}{|\sigma_x|} \qquad (1.4),\ (1.5)$$

$$\sigma_b = |\sigma_x|\left(1 + \left(\frac{\tau_{xy}}{\sigma_x}\right)^2\right) \qquad (1.6)$$

<u>Case 3</u>: $\sigma_x \geqq \sigma_y\,\alpha$ $\qquad$ $\sigma_y < -|\tau_{xy}|/\sqrt{\alpha}$

Reinforcement is only necessary if $\sigma_x\sigma_y \leqq \tau_{xy}^2$

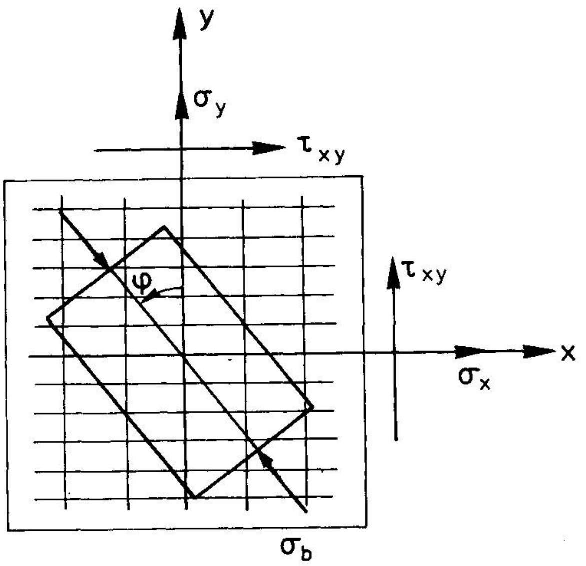
$$\sigma_{tx} = \sigma_x + \frac{\tau_{xy}^2}{|\sigma_y|} \qquad \sigma_{ty} = 0 \qquad (1.7),\ (1.8)$$

$$\sigma_b = |\sigma_y|\left(1 + \left(\frac{\tau_{xy}}{\sigma_y}\right)^2\right) \qquad (1.9)$$

The quantities σ_{tx} and σ_{ty} defined by (1.1) and (1.2) are the yield forces of the reinforcement bars per unit area of the concrete (equivalent reinforcement stresses). They represent the tensile strength of the reinforced material in the x- and y-directions, respectively. The quantity α is given by

$$\alpha = \frac{\sigma_{fx}}{\sigma_{fy}} \qquad (1.10)$$

and σ_b is the compression stress in the concrete. The above formulas determine the optimum value of $A_{ax} + A_{ay}$.



If for some reason the reinforcement in one direction is known, or if the optimum value of the total reinforcement of the structure is found for other inclinations of the compressive stresses than found above, see section 2, the following formulas determine the necessary amount of reinforcement:

$$\sigma_{tx} = \sigma_x + \gamma|\tau_{xy}| \qquad (1.11)$$

$$\sigma_{ty} = \sigma_y + \frac{1}{\gamma}|\tau_{xy}| \qquad (1.12)$$

$$\sigma_b = |\tau_{xy}|\left(\gamma + \frac{1}{\gamma}\right) \qquad (1.13)$$

Figure 1.1

The quantity $\gamma > 0$ can be determined if σ_{tx} or σ_{ty} is known. The relation between γ and the angle φ determining the direction of the concrete stress is $\gamma = \mathrm{tg}\,\varphi$, see fig.1.1. The formulas (1.11) - (1.13) can also be used instead of (1.1) - (1.9) if minimum reinforcement is not looked for. In this case, γ can theoretically be arbitrarily chosen. However, in order to avoid yielding of the reinforcement for service loads, limitations should be put on the choice of γ , see the following.

Having determined the necessary reinforcement by means of the above formulas, it is often found advantageous to use another, perhaps more economical or practical distribution of the reinforcement. In the case of homogeneous stress fields, it should be noted that the reinforcement theoretically might be distributed in any other way resulting in the same statical equivalence of the reinforcement forces. Sometimes, for instance in slabs and shells, such a transformation changes the compression forces in the concrete and, if so, the concrete stresses of course have to be calculated taking account of these extra forces. Also the complete equilibrium of the whole transformed stress field at the boundaries should be considered. Examples of reinforcement transformations are given in the following.

2. SHEAR IN BEAMS

Consider a stringer beam, i.e. a beam with the tensile and the compression zone concentrated in stringers, see fig.2.1. The distance between the stringers is h. Let us determine the shear reinforcement in a zone with constant shear force Q. If the shear zone has the thickness b, the reinforcement may be determined using the homogeneous statically admissible stress field

$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = \tau = \frac{Q}{bh} \tag{2.1}$$

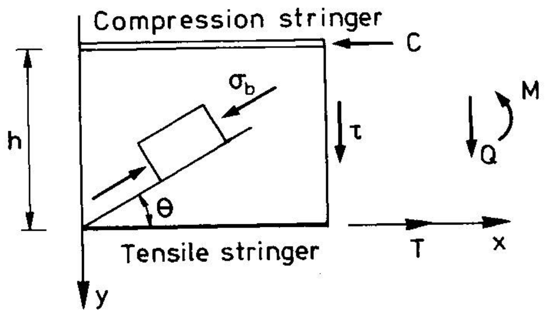


Figure 2.1

The necessary reinforcement in the x- and the y-direction and the concrete stress σ_b is determined by (1.11)-(1.13). We find

$$\sigma_{tx} = \tau \cot\theta \tag{2.2}$$

$$\sigma_{ty} = \tau \operatorname{tg}\theta \tag{2.3}$$

$$\sigma_b = \tau(\cot\theta + \operatorname{tg}\theta) \tag{2.4}$$

where the meaning of θ is shown in fig.2.1. If the shear zone is reinforced accordingly, i.e. reinforced in both directions x and y, the stringers have to carry the forces

$$T = C = \frac{M}{h} \tag{2.5}$$

However, such a shear reinforcement is more expensive than necessary, although it has the advantage of giving zero stringer force at a simple support, a fact which may facilitate the design of this part of the beam because of the small anchorage length of the tensile reinforcement required. The shear reinforcement in the x-direction can be avoided since the total force in this direction $Q\cot\theta$ can be carried by the stringers. This means that the stringers have to carry the forces

$$T = \frac{M}{h} + \frac{1}{2}Q\cot\theta \qquad C = \frac{M}{h} - \frac{1}{2}Q\cot\theta \qquad (2.6),\ (2.7)$$

The result implies that we have to reinforce for a tensile stress in the y-direction determined by (2.3), to secure that the stringers can carry the forces (2.6) and (2.7) and that the concrete stress σ_b determined by (2.4) can be carried by the concrete. Formula (2.6) shows that at a simple support, the tensile stringer must be able to carry half the reaction times $\cot\theta$, i.e. proper care must be taken to anchorage the reinforcement at an end section.

The same result would, of course, be found if the bending moment was assumed to be carried by the stringers and a uniformly distributed compressive stress $\tau\cot\theta$ in the shear zone. The above results can be obtained even in other ways, see for instance [10].

The total amount of reinforcement can, of course, be minimized with respect to θ. The optimum value θ is different from that corresponding to the formulas in section 1 because of the special way, in which the x-reinforcement is arranged. The result of the optimization shall not be given here since other requirements often determine the most practical θ-value . The reader is referred to [3].

If reinforcement in other orthogonal directions is preferred for som reason, the stress field (2.1) in the shear zone just have to be transformed to these directions. An extremely simple result is found when the shear zone is reinforced in the principal directions where only reinforcement in one direction is required.

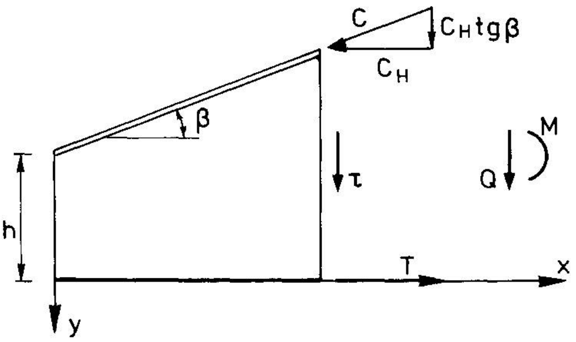


Figure 2.2

If the beam has variable depth, see fig.2.2, we may reinforce for the stress field:

$$\sigma_x = 0 \; ; \quad \sigma_y = 2\tau\frac{y}{h}\,tg\,\beta \; ;$$

$$\tau_{xy} = \tau = \frac{1}{bh}(Q-T\;tg\,\beta) \qquad (2.8)$$

which is easily seen to satisfy the equilibrium equations and the boundary conditions. The σ_y-stress generally is small and can be neglected. Doing so, the formulas (2.2)-(2.4) are still valid. If in (2.7) C is replaced by the horizontal component C_H , see fig.2.2, the formulas (2.6) and (2.7) are valid too.

If a beam has stronger flanges than necessary to carry the stringer forces, it would be natural to utilize the bending and the shearing strength of the flanges too. The most simple way of doing this, when applying a lower bound method, is to superimpose the above stress fields on a stress field, corresponding to ordinary beam action in a frame system composed by the flanges, the end sections and, if necessary, some compressive struts in the shear zone. Fig.2.3 illustrates how such a stress field can be determined in a simple example. The bending moments are here chosen in accordance with the yield mechanism shown in the figure. When the ratio between the bending moments in the hinges have been selected, the values of the bending moments can be determined by the work equation. Having done this, the normal forces and the shear forces can be determined by equilibrium equations. Superimposing the stress fields mentioned, the reinforcement in the flanges and the shear zone can be calculated.

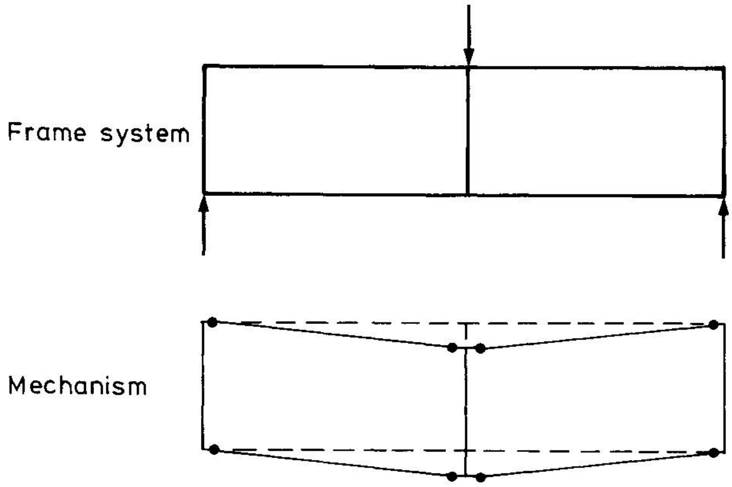


Figure 2.3

The effectivity factor ν of the concrete is rather well known in the case of beam shear. Also limitations to be put on $\cot\theta$ in order to secure satisfactory behaviour for the working load have been proposed. The reader is referred to [10],[12], [8] and [14].

The stress fields treated in this section can as an approximation be used for other loading systems than those giving constant shear force, see [10]. The stress fields are not applicable for deep beams or beams with large concentrated loads near the supports, where arching action is capable of carrying all, or a significant part, of the load. A number of simple statically admissible stress fields for this case have been developed in [4] and [5]. An upper bound method is described in [10].

3. TORSION

An immediate application of the reinforcement formulas of section 1 to torsion problems is possible for a thin walled, closed section. For such a section a pure shear field

$$\tau_{xs} = \frac{V}{2 A_0 t} \tag{3.1}$$

V being the torsional moment, A_0 the area within the center line of the section, s the arc length along the center line, and t the thickness, is statically admissible, see fig.3.1. The formula (3.1) is Bredt's formula. The area of the longitudinal bars and the area of the bars along the center line is determined by (1.1) and (1.2) or (1.11) and (1.12), the first mentioned formulas giving minimum amount of reinforcement. The concrete stress is determined by (1.3) or (1.13). The concrete stress of course has to satisfy the condition $\sigma_b \leqq \nu \sigma_c$.

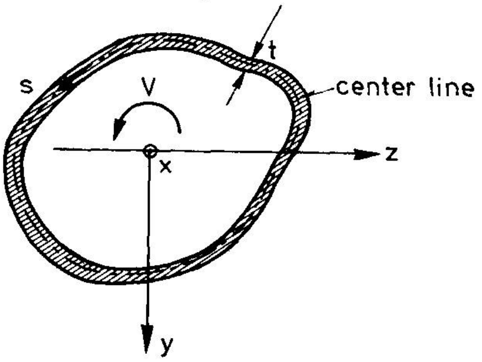


Figure 3.1

The same simple stress field is statically admissible in any solid section if (3.1) is applied to a thin walled closed section lying within the concrete area of the section. The thickness t of the thin walled section of course has to be so large as to render it possible to satisfy the condition $\sigma_b \leqq \nu \sigma_c$.

In many cases, the reinforcement does not have to be placed in the center line of the closed section. A statically equivalent reinforcement lay-out can be used if proper care is taken to design the end sections as in the case of shear in beams.

Consider, as an example, a rectangular section, see fig.3.2.

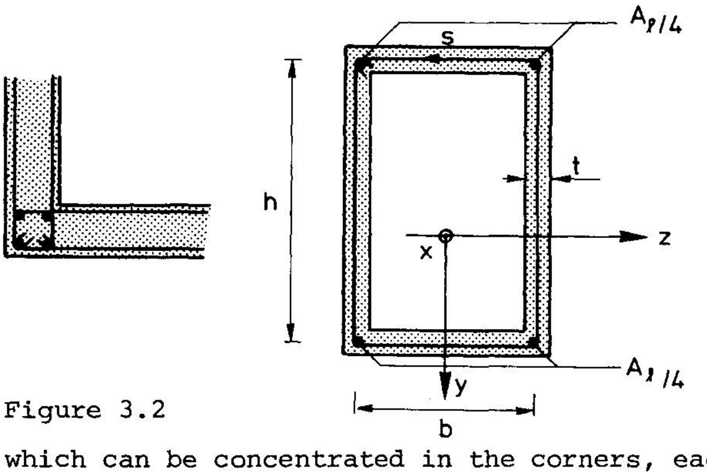


Figure 3.2

If the yield stresses in both reinforcement directions are equal to σ_f, we get by means of (1.1) and (1.2) the following reinforcement areas per unit length:

$$A_{ax} = A_{as} = \frac{V}{2h\, b\, \sigma_f} \tag{3.2}$$

The total amount of longitudinal reinforcement is thus

$$A_\ell = \frac{V(h+b)}{hb\, \sigma_f} \tag{3.3}$$

which can be concentrated in the corners, each of the corner bars having an area of $A_\ell/4$. The reinforcement along the center line can for small sections be chosen as closed stirrups as shown in fig.3.2 to the right. For large sections, closed stirrups in the individual wall sections can be used as illustrated in fig. 3.2 to the left.

For a long rectangular section, the bars in both reinforcement directions can be placed outside the thin walled section as long as both reinforcement layers are placed symmetrically with respect to the middle plane, see fig.3.3. In this case, we are in fact concerned with pure torsion in a slab, the action of which has been

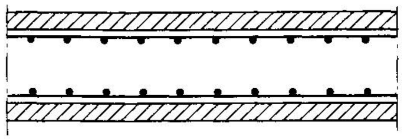


Figure 3.3

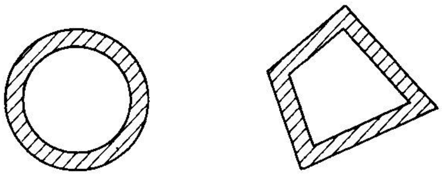


Figure 3.4

more deeply studied in [2], to which the reader is referred.

In fig.3.4, thin walled sections which can be used for the reinforcement calculation in some other cases of solid sections have been illustrated.

For circular sections, the concrete cover is in danger of spalling off, which has actually been observed in tests, see [7].

Only very little is known about the effectivity factor ν of the concrete in the case of torsion, but results are under way *. Until more refined results are available one has to rely upon crude but generally conservative code rules, see for instance [14]. Limits on γ have been studied in [11], see also [9] and [14].

4. COMBINED BENDING, SHEAR AND TORSION

In the case of combined bending, shear and torsion in a thin walled closed section, the necessary amount of reinforcement can be determined by means of the reinforcement formulas of section 1 too. Considering as a simple example a box section acted upon by a bending moment M_z, a shear force Q_y and a torsional moment V, see fig.4.1, one statically admissible stress field can be found using the Navier distribution of the normal stresses σ_x from the bending moment, the corresponding Grashof distribution of the shear stresses τ_{xs} from the shear force Q_y and the Bredt distribution (3.1) of the shear stresses τ_{xs} from the torsional moment V. However, a more suitable statically admissible stress distribution is found by distributing the normal stresses from M_z uniformly, for instance, along the top and bottom flanges. The corresponding shear stress diagram then is linear in the individual walls. Having determined the stress distribution, the reinforcement formulas immediately give the necessary amount of reinforcement. Other thin walled closed sections can be treated in a similar way. For a solid section, the same method as described for pure torsion can be used, i.e. a thin walled section, lying within the concrete area, is selected for carrying the stresses. Consider as an example a solid rectangular section. If reinforcement is supplied in the longitudinal direction and a circumferential direction, perpendicular to this, and if the yield stress of the steel is the same in both directions, then for a section acted upon by a torsional moment V the reinforcement formulas (1.1) and (1.2) require the total longitudinal reinforcement to carry a force $P_\ell = A_\ell \sigma_f$, which can be calculated by means of (3.3). The corresponding reinforcement area can be placed as one fourth of the total area in each corner.

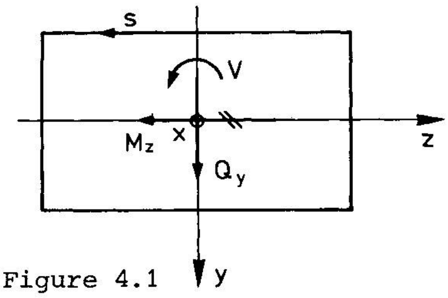


Figure 4.1

Small bending moments M_z, i.e. $M_z \leq \frac{1}{2} V (h+b)/b$ can be carried by moving a part of the reinforcement in the compression zone to the tensile zone, i.e. the force in the longitudinal reinforcement in the top flange can be reduced by M_z/h and the force in the longitudinal reinforcement in the bottom flange has to be increased by M_z/h.

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* Current research seems to show that empirical formulas for ν can be given very similar forms for bending and torsion problems, see [13].

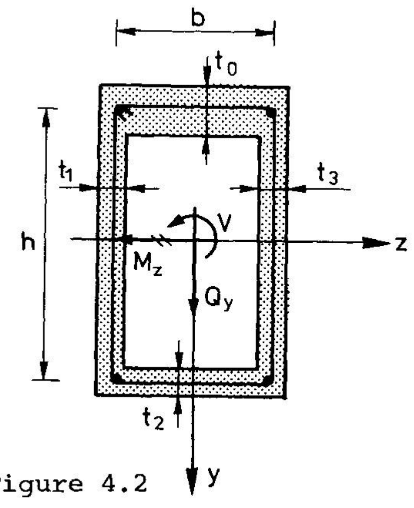

Figure 4.2

If $M_z > \frac{1}{2} V(h+b)/b$ the longitudinal reinforcement in the top flange can be chosen to be zero, while the force in the longitudinal reinforcement in the bottom flange still has to be increased by M_z/h.

If the section is acted upon by a shear force Q_y too, the reinforcement can be determined by adding the shear stress in the thin walled section from Q_y to the stresses from V and M_z. In one of the vertical walls, the shear stresses from Q_y add to the shear stresses from V and in the other one, they subtract to the shear stresses from V. The concrete stress in the individual walls can be determined by means of the formulas of section 1. The quantities t_o, t_1, t_2 and t_3, the meaning of which is shown in fig.4.2, of course have to be fixed at values making it possible to satisfy the condition $\sigma_b \leq \nu\sigma_c$ in each wall. Other solid sections may be treated in a similar way.

As in the case of pure torsion, reliable information about the effectivity factor ν of the concrete is still missing in the case of combined bending, shear and torsion.

5. OTHER PLANE STRESS PROBLEMS

The reinforcement formulas can be applied to several other plane stress problems. We shall, however, not be able to treat other cases in more detail here. A number of statically admissible stress fields for deep beams, to which the reinforcement formulas immediately apply, have been developed by the author, see [4] and [5]. The formulas also apply to the determination of reinforcement in slabs and shells, see [2],[4] and [5].

6. REFERENCES

[1] Nielsen, M.P.: Yield Conditions for Reinforced Concrete Shells in the Membrane State. Proc. IASS Symposium on Non-Classical Shell Problems, Warsaw 1963.

[2] Nielsen, M.P.: Limit Analysis of Reinforced Concrete Slabs, Acta Polytechnica Scandinavica,Ci 26, Copenhagen 1964.

[3] Nielsen, M.P.: Om forskydningsarmering af jernbetonbjælker (On Shear Reinforcement of Reinforced Concrete Beams), Bygningsstatiske Meddelelser,Vol.38, No.2, 1967, pp.33-58.

[4] Nielsen, M.P.: Om jernbetonskivers styrke, København 1969.

[5] Nielsen, M.P.: On the Strength of Reinforced Concrete Discs, Acta Polytechnica Scandinavica,Ci 70, Copenhagen 1971.

[6] Nielsen, M.P.: Kombineret bøjning og vridning af jernbetonbjælker (Combined bending and torsion of reinforced concrete beams), R 7103, Danmarks Ingeniørakademi, Bygningsafdelingen,Ren & Anvendt Mekanik, Aalborg 1971.

[7] Lenschow, R. & M.Bergholdt: Tests on reinforced concrete beams with circular section, Danmarks Ingeniørakademi, Bygningsafdelingen, Lyngby, Report under preparation.

[8] Thürlimann, B.: Shear Strength of Reinforced and Prestressed Concrete Beams - CEB-Approach, ACI-Symposium, Philadelphia 1976.

[9] Thürlimann, B.: Torsional Strength of Reinforced and Prestressed Concrete Beams - CEB-Approach, ACI-Symposium, Philadelphia 1976.

[10] Nielsen, M.P., M.W.Braestrup, B.C.Jensen & Finn Bach: Concrete Plasticity, Beam Shear - Shear in Joints - Punching Shear, Special Publication, Danish Society for Structural Science and Engineering, Draft, Lyngby 1976, Final edition, Lyngby 1978.

[11] Jensen, Verner: Revnet jernbetons stivhedsforhold (Stiffness of reinforced concrete), MSc-Thesis, Structural Research Laboratory, Technical University of Denmark, Lyngby 1976.

[12] Jensen, J.F., V.Jensen, H.H.Christensen, M.P.Nielsen, M.W.Braestrup & Finn Bach: On the behaviour of cracked reinforced concrete beams in the elastic range, Structural Research Laboratory, Report under preparation.

[13] Sigurbjarnarson, Kristjan: Effektivitetsfaktoren på beton ved bøjning (The effectivity factor of concrete in bending) MSc-Thesis, Structural Research Laboratory, Technical University of Denmark, Lyngby 1978.

[14] CEB-FIP model code for concrete structures, Comité Euro-International de Béton, 1978.

V

Direct Design by Concrete Flow

Dimensionnement direct en considérant la transition des forces dans le béton

Direkte Bemessung durch Betrachtung des Kraftflusses im Beton

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SUMMARY
The paper presents a method of handling the equations for axial force, bending moment, shear force and torsion. The principal stress failure criterion for concrete, which is in mixed force and geometry variables, is re-written as a criterion in force variables only. This enables the longitudinal force requirements for shear and torsion to be simply obtained and incorporated in the beam axial force and flexure equations with which the designer is familiar. The method is general and adaptable to any cross-section shape or reinforcement layout and the equations are in a form suitable for direct design.

RESUME
La méthode présentée traite les équations pour une sollicitation par flexion, par une force axiale, par un effort tranchant et par un moment de torsion. Le critère de rupture pour le béton est exprimé à nouveau en termes de forces. Ainsi on peut incorporer les forces agissantes longitudinalement dues à la torsion et au cisaillement dans les équations bien connues qui décrivent l'action de la flexion et de la force normale. La méthode est générale et on peut l'adapter à une forme de poutre et une disposition de l'armature quelconque. Les équations résultantes permettent un dimensionnement pratique.

ZUSAMMENFASSUNG
Es wird eine Methode zur Handhabung der für eine Beanspruchung durch Biegung, Normalkraft, Querkraft und Torsion geltenden Beziehungen dargestellt. Die Bruchbedingung des Betons wird in Kraftgrössen ausgedrückt. Damit können die Längskräfte infolge Torsion und Querkraft in die üblichen Beziehungen für Biegung und Normalkraft einbezogen werden, mit denen der Ingenieur vertraut ist. Die Methode ist allgemein und kann für beliebige Querschnittsformen und Bewehrungsanordnungen angepasst werden. Die auftretenden Gleichungen eignen sich für eine direkte Anwendung bei der Bemessung.

1. INTRODUCTION

The safety stage comparison of design in structural concrete is, by international concensus, made at the level of member cross-section action. By this choice the design problem is decomposed into two modelling sub-problems, one above and one below the comparison level [1]. The higher level problems which will not be considered in this paper is the mapping of all loadings into the load-effect space. The lower level problem is most appropriately treated as a plasticity problem and consists of mapping the bounds of the safe domain in the same load-effect space. The most general such space is six-dimensional consisting of axial force, two bending moments, two shears and a twisting moment. A definitive stage of modelling in a two-dimensional axial force and bending space was reached some years ago [2] [3] and this led, possibly without adequate appreciation of the consequences, to the adoption of internal action as the comparison level. The most serious consequence has been a forced semi-rational empiricism when it comes to adding the shear dimension [4] and dissatisfaction by the profession with the resulting complexity of rules.

There has also been steady progress toward a consistent rational approach. The state of the art is reviewed by Thürlimann [5]. He presents the assumptions and principles and develops a rigorous approach via the space truss concept in which both upper and lower bound principles are satisfied simultaneously. The starting point is the failure criterion for concrete at a level intermediate between cross-section action and stress, namely force per unit length applied to a concrete sheet. This will be called force flow as a generalisation of the accepted term shear flow. The criterion is a very simple one, a square one in principal force flow space. However complete definition of a principal stress and hence of a principal force flow requires the specification of a geometrical variable, orientation, as well. As a result the crack orientation (or principal compression orientation) plays a major part in the section analysis. The practising designer appears, however, to find such techniques esoteric and unrelated to the models with which he is familiar. He is familiar with the equations for axial force and bending applied to the right cross-section and the purpose of this paper is to show how terms for shear and torsion may be directly included in these familiar equations.

2. THE CONCRETE FORCE FLOW CRITERION

If we define C_x as the longitudinal force per unit length when the shear flow is denoted by C_{xs} (Figure 1) Thürlimann's square criterion restricts C_1 and C_2 to the range $0 \leqslant C_{1,2} \leqslant C^n$ where $C^n = kF_c't$ t being the thickness of the sheet and k a factor modifying the cylinder strength F_c'.

Force flows transform according to the same transformation of axes laws as stresses. Hence the limit

$0 \leqslant C_{1,2}$ implies that

$$\frac{C_x + C_s}{2} - \sqrt{\left(\frac{C_x - C_s}{2}\right)^2 + C_{xs}^{\,2}} \geqslant 0$$

which simplifies to :-

$$C_{xs}^{\,2} - C_x C_s \leqslant 0 \qquad (1)$$

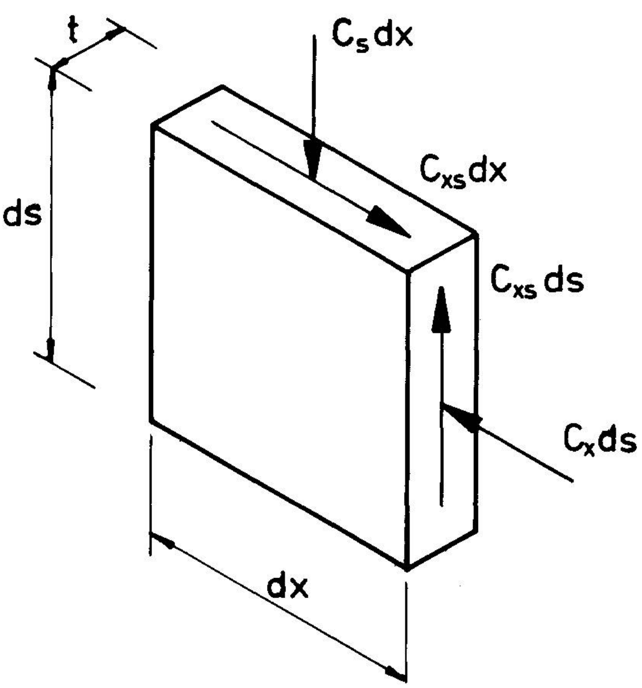


Figure 1. Concrete Sheet Element

Similarly $C_{1,2} \leqslant C^n$ implies that

$$C_{xs}^2 - (C^n - C_x)(C^n - C_s) \leqslant 0 \qquad (2)$$

Relationship (1) is the important one for designing under-reinforced beams and will be the key to most important design cases. The boundary where (1) co-incides with (2) is readily shown to be

$$C_x + C_s = C^n \qquad (3)$$

and satisfaction of (3) is the key to certain over-reinforced cases.

For design purposes it will be shown that the very simple inequality (1) is extremely powerful but to understand the physical significance the designer must be aware that failure controlled by (1) is dilatant. Any sheet failing by satisfaction of (1) is dilating in the x and s directions thus imposing on reinforcement in these directions a tensile strain and hence a tensile force. The apparent anomaly of compatibility of concrete under compression and parallel steel under tension is thus resolved.

The analytical demonstration of this dilatancy is achieved by simple application of the flow rule of plasticity theory [6] which yields

$$\begin{aligned} \dot{\varepsilon}_x &= -\lambda C_s \\ \dot{\varepsilon}_s &= -\lambda C_x \qquad (4) \\ \dot{\gamma}_{xs} &= 2\lambda C_{xs} \end{aligned} \qquad\qquad \begin{aligned} \dot{\varepsilon}_x &= \lambda (C^n - C_s) \\ \dot{\varepsilon}_s &= \lambda (C^n - C_x) \qquad (4a) \\ \dot{\gamma}_{xs} &= 2\lambda C_{xs} \end{aligned}$$

3. APPLICATION TO MODE 1 TORSION, SHEAR AND BENDING

The failure state of a properly reinforced beam subjected to torsion, shear and bending is modelled using the dilatant concrete sheet subsystem by choosing the concrete sheets in the form of a polygonal tube which is restrained from bursting by the tensile stirrup force. The limit on this restraint against dilation is set by the stirrup yield strength but the effectiveness of this limit is set by the effectiveness of the force transfer between sheet and stirrup.

The potential circumferential force flow controlled by the stirrup is given by :

$$C_s = \frac{A_w f_{wy}}{s} \qquad (5)$$

This is the limit which applies in criterion (1) so that if the required shear flow C_{xs} can be determined then the required value of C_x follows from (1) namely

$$C_x = \frac{C_{xs}^2}{C_s} = \frac{C_{xs}^2}{\frac{A_w f_{wy}}{s}} \qquad (6)$$

The statics of determining values of C_{xs} is well established in the space truss theory. If the dilatant sheets circumscribe an area A_o then the shear flow in these sheets due to twisting moment T is given by

$$C_{xs} = \frac{T}{2A_o} \qquad (7)$$

or for a rectangular section

$$C_{xs} = \frac{T}{2y_1z_1} \qquad (8)$$

For a simple rectangular beam, shear flows due to shear force V are given by :

$$C_{xs} = \frac{V}{2y_1} \qquad (9)$$

The effects of T and V are additive on one face and subtractive on the other so that the faces carry force flows as follows in the x direction :-

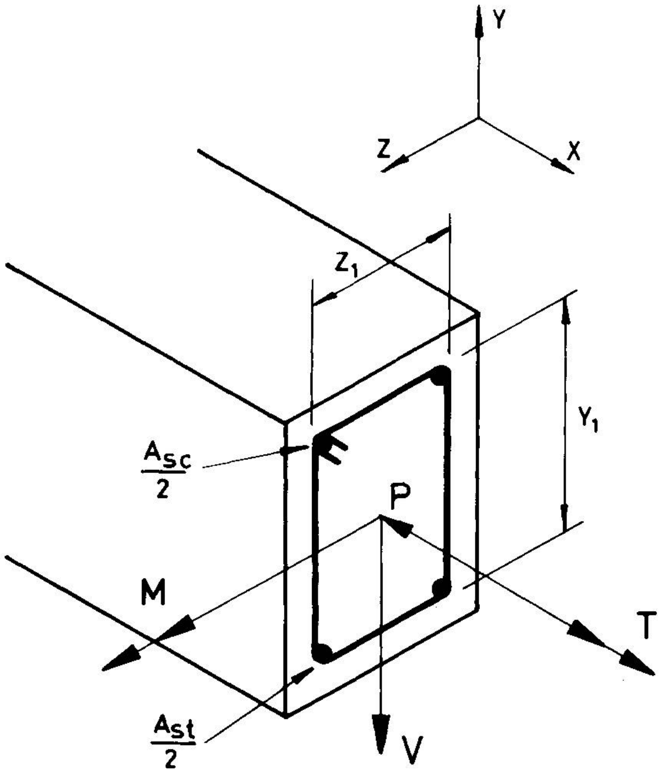
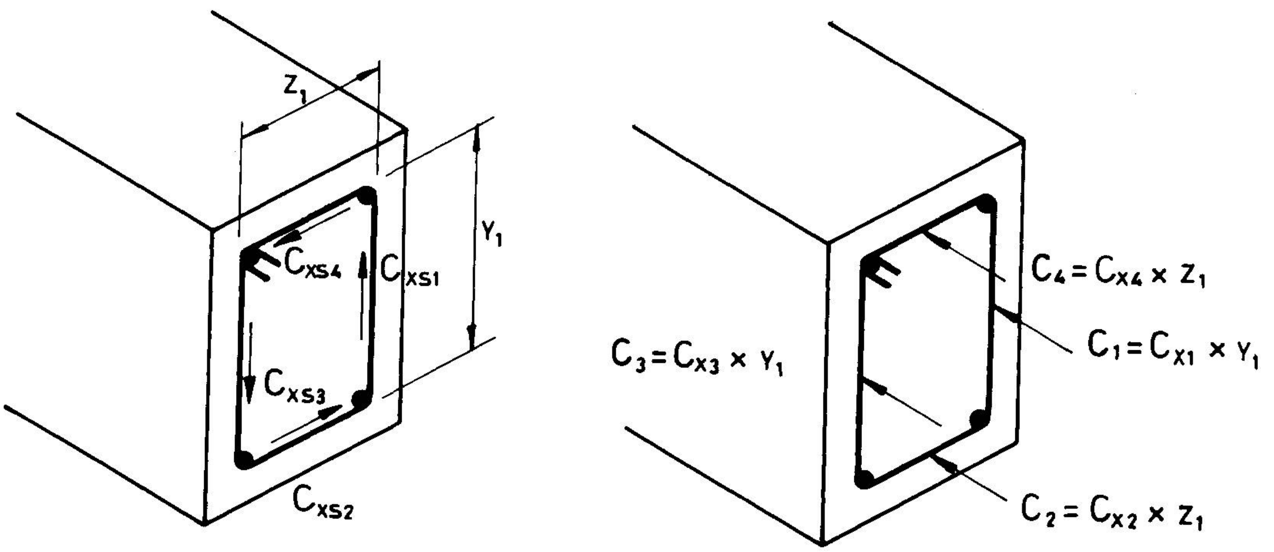


Figure 2. Rectangular Section and Loading



(a) Shear flows (b) Associated Longitudinal Forces

Figure 3. Rectangular Beam.

$$C_{x_1} = \frac{\left(\frac{T}{2y_1z_1} - \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}} \qquad C_{x_2} = C_{x_4} = \frac{\left(\frac{T}{2y_1z_1}\right)^2}{\frac{A_wf_{wy}}{s}} \qquad C_{x_3} = \frac{\left(\frac{T}{2y_1z_1} + \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}} \qquad (10)$$

The resultant force C_i on the ith face is the product of C_{xi} and the length of the ith face (Figure 3) and is located at the mid-point. These forces are readily incorporated in the equations for axial force and flexure, if an additional compressive force C_o is assumed on one of the faces as the compressive stress block due to bending. For failure mode 1 (sagging bending) it will act on the C_4 face so that the equations are :-

$$C_1 + C_2 + C_3 + C_4 + C_o - A_{st}f_{sy} = P \qquad (11)$$

$$(A_{st}f_{sy} - C_2)y - (C_1 + C_3)\frac{y_1}{2} = M \qquad (12)$$

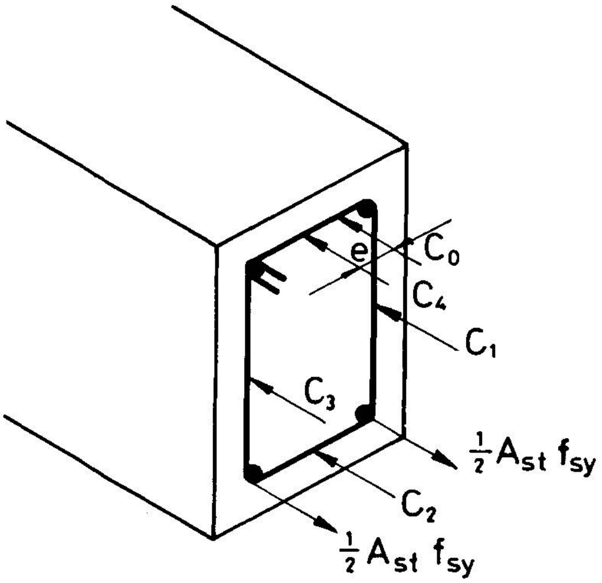


Figure 4. Forces in beam flexure equations.

Equation (11) is the standard one for determining the neutral axis location but the simplifying assumption implicit in all theories such as this is that the location of the compressive stress block resultant is known so that lever arms in [12] are independent of the solution of (11). No new theory would be required to produce more rigorous forms where the magnitude of C_o would determine the depth of the stress block and the lever arm would depend in turn on this. Substituting from (10 in (12) produces

$$M = A_{st}f_{sy}y - \frac{\left(\frac{T}{2y_1z_1}\right)^2}{\frac{A_wf_{wy}}{s}} z_1y_1 - \frac{\left(\frac{T}{2y_1z_1} - \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}} y_1\frac{y_1}{2} - \frac{\left(\frac{T}{2y_1z_1} + \frac{V}{2y_1}\right)^2}{\frac{A_wf_{wy}}{s}} y_1\frac{y_1}{2} \qquad (13)$$

In a design situation no further development is needed. A trial selection of $\frac{A_wf_{wy}}{s}$ would immediately lead to a solution for $A_{st}f_{sy}$ and vice versa since all other terms would be numerical ones at this stage. Hence the designer can obtain design parameters directly by an equation with which he is familiar. The power of the method is that exactly the same procedure may be adopted for more complex cross-sections and the tensile reinforcement need not be treated as equivalent stringers because the moments of individual bars may be included in the moment equation. The practical advantages of relating the analysis to familiar equations are obvious. As an analytical approach it has an important advantage in common with the space truss theory over the author's earlier more complex approach [7] and the skew bending theory [8] because it can analyse the Collins' [9] reductio-ad-absurdum beam with all steel external to the concrete as a standard case.

The analytical extension of (13) is trivial but confirms that the simple steps do in fact produce an interaction relationship identical with the space truss theory and rigorous skew bending theory. Re-arrangement of terms in (13) readily leads to :-

$$\frac{M}{M_o} + \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 \leq 1 \qquad \text{where } M_o = A_sf_{sy}d \qquad (14)$$

$$T_o = 2y_1z_1\sqrt{\frac{A_wf_{wy}}{s}\,\frac{A_{st}f_{sy}}{y_1 + z_1}} \qquad V_o = 2y_1\sqrt{\frac{A_wf_{wy}}{s}\,\frac{A_{st}f_{sy}}{y_1}}$$

4. ECCENTRICITY OF COMPRESSIVE FORCE

Derivation of mode 3 (top yielding) and mode 2 (side yielding) equations is equally direct, and the resulting interaction relationships are equally consistent with space truss and skew bending. Of greater significance is the new insight that the method offers into the complete identification of all longitudinal forces and their location so that the statics of the final design is thoroughly understood by the designer.

For the mode 1 case given above, the inequality of the shears in the two y legs means that C_o must be eccentric (Figure 4) for zero moment about the second axis, i.e.

$$C_o\, e = (C_3 - C_1)\, \tfrac{1}{2}z_1 \tag{15}$$

But e has maximum value $\tfrac{1}{2}z_1$ and when this is satisfied in (15) and C_o is solved from (11) using a value of $A_{st}f_{sy}$ which satisfies (12), (15) becomes :-

$$\tfrac{1}{2} \geq \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 + \frac{2TV}{T_oV_o}\sqrt{\frac{y_1}{y_1 + z_1}} \tag{16}$$

What is significant is that (16) is the mode 2, sideways bending, interaction equation with zero top steel. In fact locating C_o at $\tfrac{1}{2}z_1$ leads to simultaneous derivation of the mode 1 and mode 2 failure criteria. The equation now becomes:-

$$A_{st}f_{sy} + \tfrac{1}{2}A_{sc}f_{sy} - (C_1 + C_2 + C_3 + C_4 + C_o) = P \tag{17}$$

$$A_{st}f_{sy}\, y_1 - C_2y_1 - \tfrac{1}{2}(C_1 + C_3)\, y_1 = M \tag{12}$$

$$\tfrac{1}{2}A_{st}f_{sy}\, z_1 + \tfrac{1}{2}A_{sc}f_{sy}\, y_1 - C_3z_1 - \tfrac{1}{2}(C_2 + C_4)z_1 = 0 \tag{18}$$

Equation (12) is unchanged and leads to the mode 1 equation and equation (18) satisfies equilibrium in lateral flexure and leads directly to the mode 2 interaction relationship

$$\tfrac{1}{2}\left(1 + \frac{A_{sc}f_{sy}}{A_{st}f_{sy}}\right) = \left(\frac{T}{T_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 + \frac{2TV}{T_oV_o}\sqrt{\frac{y_1}{y_1 + z_1}} \tag{19}$$

Again (17) is not directly involved in the steel design but evaluation of C_o from it would be a step in the establishment of the requirements of the eccentric compression crushing area in a rigorous analysis since C_1 to C_4 are all in a dilatant tensile strain, state. In fact the equations may be seen to be those of a rigid plastic analysis with a skew neutral axis such that C_o acts at the corner position assumed and the rest of the section is in tensile strain state.

It is not proposed to discuss the mode 3 equations since they do not illustrate any new aspect of the method, but they would, of course, be written and utilised in a practical design situation. If they showed that A_{sc} needed an increase to cover the hogging bending situation this would not require any iteration for the rectangular beam, because, although C_o would now be at a bottom corner, equation (18) would apply to this case as well. The advantages of the method do not lie in a particular formula which it can generate but in the understanding it can provide to the designer as to why the reinforcement is required and the control it gives him in decision-making as to how much is required. Some other aspects in which it assists the designer and potential refinements will now be discussed.

5. WEB CRUSHING IN SHEAR

The minimum thickness of the concrete sheet implicit in the solution may be derived if (3) is assumed to hold at failure. In the beam design equations considered, for instance, this leads to :-

$$t = \frac{\dfrac{C_{xs}^2}{\dfrac{A_w f_{wy}}{s}} + \dfrac{A_w f_{wy}}{s}}{kF'_c} \tag{20}$$

The case of web crushing in shear is readily treated by the method. The equation for under-reinforced design in shear is (13) with T = 0. If the web width is such that t as determined by (20) is greated than $\frac{b}{2}$ then design for web crushing is required. In that case design of the stirrup to yield simultaneously with the crushing is readily achieved. Knowing C_x from (6), (3) becomes :-

$$\frac{(C_{xs})^2}{C_s} + C_s = C^n \quad \text{or} \quad \frac{\left(\dfrac{V}{2y}\right)^2}{\dfrac{A_w f_{wy}}{s}} + \frac{A_w f_{wy}}{s} = kF'_c \frac{b}{2} \tag{21}$$

Solving for $\frac{V}{2y}$, $\frac{V}{2y} = \sqrt{\frac{A_w f_{wy}}{s}\left(kF'_c \frac{b}{2} - \frac{A_w f_{wy}}{s}\right)}$ (22)

This semi-circular relationship between V and C_s is the one that was derived by Nielsen and Braestrup [10] for reinforced concrete web crushing and by Chitnuyanondh et al [11] for prestressed concrete web crushing. However, when coupled with the general theory it goes further than these methods because it then allows C_x to be obtained since C_x and C_s must sum to C^n . Hence the associated longitudinal steel equations may be written. In the resulting design the strain rates will be the sum of (4) and (4a).

6. CONCLUSIONS

A general method has been presented for the organization of the calculations which follow from the basic assumptions of plasticity theory when applied to reinforced concrete members subjected to axial force, bending, shear and torsion. The designer's problem has been transformed into writing the equilibrium equations for the resultants of longitudinal internal stresses, a problem with which he is familiar from the establishment of the interaction equations for axial force and bending. The method is independent of the shape of the cross-section or of the disposition of the reinforcement on the faces of the member. The limits of under-reinforcement can be identified and the web crushing case for beam shear may be designed directly.

NOTATION

b	web thickness
e	eccentricity of C_o
f_{st}, f_{sc}, f_{wy}	yield stress for A_{st}, A_{sc}, A_w steel
k	reduction factor for F'_c
s	(i) perimetral co-ordinate used in x,s co-ordinate system
	(ii) stirrup spacing used in $\frac{A_w f_{wy}}{s}$
t	concrete sheet thickness
x,y,z	beam co-ordinate system
A_o	area bounded by shear flow
A_{st}	steel area on bottom face
A_{sc}	steel area on top face
A_w	stirrup area
C_o	compressive force due to P,M.
C_i	compressive force due to C_{xi} on the ith sheet

C_x, C_s, C_{xs}	forces per unit length, force flows, on concrete sheet
F'_c	concrete cylinder strength
M	bending moment
P	axial force
T	twisting moment
V	shear force
M_o, P_o, T_o V_o	ultimate values of M,P,T,V when acting alone
$\dot{\varepsilon}_x$, $\dot{\varepsilon}_s$, $\dot{\gamma}_{xs}$	strain rate
λ	scalar multiplier

REFERENCES

1. CLYDE, D.H. The safety verification problem in structures. Proc. Fifth Australasian Conference on the Mechanics of Structures and Materials, Melb., 1975, pp 103-112.

2. HOGNESTAD, R. A Study of Combined Bending and Axial Load in Reinforced Concrete Members. Univ. of Illinois Bulletin, No.399. 1951.

3. GRANHOLM, H. A General Flexural Theory of Reinforced Concrete with Particular Emphasis on the Inelastic Behaviour of Concrete and Reinforcement. Stockholm, Almqvist and Wiksell. 1965. 209p.

4. ACI 428-62. Shear and Diagonal Tension. ASCE Joint Committee Report. ACI journal. Proceedings Vol.59, Jan 1962 pp 1-30, Feb pp.277-333, March 1962 pp. 353-395.

5. THÜRLIMANN, B. Plastic analysis of reinforced concrete beams. Introductory Report, IABSE Colloquium, Plasticity in Reinforced Concrete, Copenhagen.

6. PRAGER, W. Introduction to Plasticity. Reading. Addison-Wesley. 1959. 148p.

7. CLYDE, D.H. A general theory for reinforced concrete elements. Sixth Aust. Conference on the Mechanics of Structures and Materials, Christchurch,N.Z. 1977. pp 327-333.

8. ELFGREN, L., KARLSON, I., and LOSBERG, A. Torsion-bending-shear interaction for concrete beams. Journal Structural Div.Proc. ASCE V.100 No.ST8,1974, pp. 1657-1676.

9. MITCHELL, D. and COLLINS, M.P. Diagonal compression field theory - a rational model for structural concrete in torsion. Journal American Concrete Institute. V.71, 1974, pp.396-408.

10. NIELSEN, M.P. and BRAESTRUP, M.W. Plastic Shear Strength of Reinforced Concrete Beams. Bygningsstatiske Meddelelser. V.46,No.3. 1975 pp.61-69.

11. CHITNUYANONDH,L., CAMPBELL, T.I., BATCHELOR, B.del and CSAGALY,P. Shear failure due to web crushing in prestressed concrete I Beams. Proc.American Concrete Institute Symposium, Philadelphia. 1976.

V

Optimal Shear Design of Beams with CEB-FIP Model Code

Dimensionnement optimal à l'effort tranchant des poutres à l'aide du code modèle CEB-FIP

Optimale Schubbemessung von Balken nach der CEB-FIP Mustervorschrift

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SUMMARY
The contribution presents a simpler and more convenient formulation of C.E.B.-F.I.P. Model Code. The findings of an extensive numerical investigation are given for the economical choice of shear and longitudinal tensile reinforcement, based on an inclination of the compression field $\theta = 31^{\circ}$ which is shown to belong in any case to the optimal solution.

RESUME
La contribution présente une formulation plus simple et plus commode des Recommandations C.E.B.-F.I.P. relatives à l'effort tranchant. On y expose les résultats d'une vaste recherche numérique quant au choix économique des armatures d'effort tranchant et des armatures longitudinales, en ayant adopté au préalable un angle d'inclinaison du champ de compression dans l'âme $\theta = 31^{\circ}$, valeur qui appartient en tout cas à la solution optimale.

ZUSAMMENFASSUNG
Eine einfache Formulierung der CEB-FIP Empfehlungen für die Schubbemessung wird gegeben. Resultate einer ausgedehnten numerischen Untersuchung im Hinblick auf die wirtschaftliche Auslegung von Schub- und Längsbewehrung werden dargestellt. Es wird gezeigt, dass die angenommene Druckfeldneigung $\theta = 31^{\circ}$ in jedem Fall zur optimalen Lösung führt.

INTRODUCTION.

The 1978 issue of the C.E.B.-F.I.P. Model Code contains |1| a realistic proposal for the shear design of concrete beams which is still based on the truss analogy but is generalized and completed by a set of limitations and improvements drawn from a lot of theoretical and experimental results |2|. In its refined presentation, called *accurate method*, it leaves to the designer the choice of the Θ crack inclination instead of the usual value $\Theta = 45°$, according to the original concept of Ritter and Mörsch |5|. On the value of Θ, depend the amount of shear reinforcement and the increase of the longitudinal tensile reinforcement. Both types of reinforcement are varying in the opposite direction : if Θ is chosen so that the amount of shear reinforcement increases, the supplement of longitudinal reinforcement decreases. Thus, the question is : does an optimum inclination angle Θ exist, that minimizes the total amount of reinforcement required by shear loading ? The purpose of present paper is to bring an answer to such a question, when case of concentrated loads near the supports is excluded.

The notations used in present paper are similar to those of the C.E.B.-F.I.P. Model Code ; the main of them are listed at the end.

C.E.B. - F.I.P. RECOMMENDATIONS.

The shear limit state can be reached either by diagonal compression in the concrete, causing crushing of the web, either by tension in the web reinforcement which reaches its design strength . The applied design shear V_{sd} must fulfil the following conditions on the resistant shear forces V_{Rd2} and V_{Rd3} for shear reinforcement and web concrete respectively :

$$V_{sd} \leqslant V_{Rd2} \tag{1}$$

$$V_{sd} \leqslant V_{Rd3} \tag{2}$$

The truss analogy can be remarkably improved by taking account in a restricted range, in addition to the usual shear force V_{wd} carried by truss action, of a contribution V_{cd} corresponding to the shear force carried by the compression flange and other effects, so that the resistant design shear force for web concrete reads :

$$V_{Rd3} = V_{cd} + V_{wd} \tag{3}$$

with : $V_{cd} = 2{,}5\ V_{Rd}$ (if $V_{sd} \leqslant 2{,}5\ V_{Rd}$) (4a)

$= 0$ (if $V_{sd} \geqslant 7{,}5\ V_Rd$) (4b)

$$V_{Rd} = A_{sw}(0{,}9\ d/s)\ f_{ywd}(\text{cotg}\ \alpha + \text{cotg}\ \Theta)\ \sin\alpha \tag{5}$$

V_{Rd} is a codified resistant design shear.

The resistant design shear force for reinforcement, is given below, and has an upper bound :

$$V_{Rd2} = 0{,}6\ f_{cd}\ b_w\ d(\text{cotg}\ \alpha + \text{cotg}\ \Theta)\ \sin^2\Theta \leqslant 0{,}45\ f_{cd} b_w\ d\ \sin 2\ \Theta \qquad (6)$$

In any case, the shear reinforcement must comply with a specified minimum amount :

$$\rho_w = A_{sw}/sb_w\ \sin\alpha \geqslant \rho_{w\ min} \qquad (7)$$

Last, in order to control the crack width for the serviceability state, the value of the inclination Θ is bounded as follows :

$$3/5 \leqslant \text{cotg}\ \Theta \leqslant 5/3 \qquad (8)$$

The longitudinal tensile reinforcement should be increased to resist the following additional tensile force :

$$\Delta F_{t\ell} = \Delta A_{s\ell}\ f_{y\ell d} = V_{sd}^2 s\ /\ 2\ A_{sw} f_{ywd}\ d\ \sin\alpha - V_{sd}\ \text{cotg}\ \alpha \qquad (9)$$

In the authors' opinion, it is more convenient to develop the explicit form of above expressions and to substitute the shear stresses τ to the shear forces V and the characteristic values to design values by using a load faction $\gamma_c = 1{,}5$. It is shown elsewhere |4| that the factors τ_{Rd}, τ_{cd} and $\rho_{w,min}$ can be written by means of analytical expressions, which thus adequately replace tables of numerical values. Taking account of these facts, the whole set of design requirements becomes :

a) minimum shear reinforcement :

$$A_{sw}/sb_w \geqslant (0{,}01\ f_{ck} + 0{,}2)\ \sin\alpha\ /f_{ywk} \qquad (10)$$

b) web concrete strength :

$$\tau_{sd} \leqslant \text{MIN}\ \left[0{,}4\ f_{ck}(\text{cotg}\ \Theta + \text{cotg}\ \alpha)\ \sin^2\Theta\ ;\ 0{,}3\ f_{ck}\ \sin 2\Theta\right] \qquad (11)$$

c) shear reinforcement strength :

$$A_{sw}/sb_w \geqslant 1{,}278\ \left[(\tau_{sd} - \tau_{cd})/f_{ywk}\right]\ /\ (\text{cotg}\ \Theta + \text{cotg}\ \alpha)\ \sin\alpha \qquad (12)$$

with $\tau_{cd} = \text{MAX}\ (0\ ;\ 0{,}03\ f_{ck} + 0{,}375 - 0{,}5\ \tau_{sd})$ (13)

d) inclination of the diagonal concrete compression field :

$$3/5 \leqslant \text{cotg}\ \Theta \leqslant 5/3 \qquad (14)$$

The requirement on the concrete strength is governed by a relation between the applied design shear stress and the characteristic concrete strength, and is without influence on the reinforcement design. It may thus be canceled from the design equations, under the condition that it be checked independently.

The bounds of the design, drawn in a figure $(A_{sw}/sb_w) = f(\text{tg}\Theta)$, are two vertical lines for (14), and horizontal line for (10) and an hyperbola for (12). The feasible domain is hatched on figure 1 ; it may take several configurations with respect to the relative position of the hyperbola and the horizontal line (figure 2).

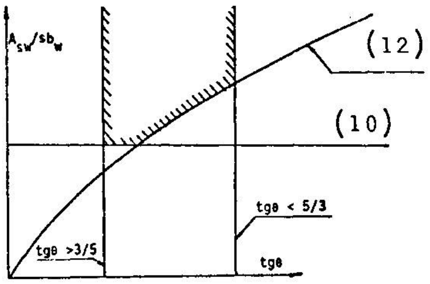
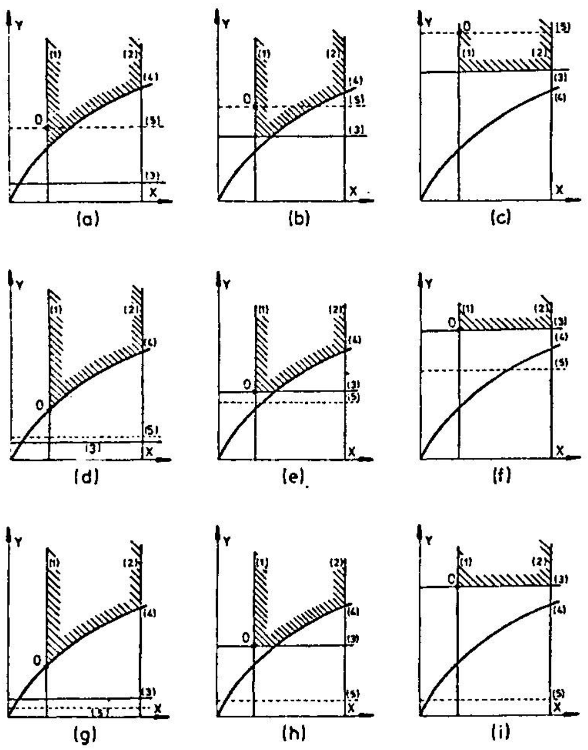


Figure 1 - Feasible domain



$X = tg\,\theta$

$Y = A_{sw}/sb_w$

(1) et (2) : bounds of θ

(3) : minimum shear reinforcement

(4) : shear reinforcement strength

(5) : unconstrained minimum.

Figure 2 - Configurations of the feasible domain.

OPTIMUM DESIGN.

The objective to optimize is the total amount of reinforcement needed by the shear design ; in fact, it would apply the cost of these reinforcements. If factor ρ is the cost ratio of shear - to tensile longitudinal reinforcement, the unit cost may be written :

$$(F/b_w d) = \rho(A_{sw}/sb_w)\ \lambda\ (1 + \varepsilon b_w/d) + (\Delta A_{s\ell}\ /b_w d) \qquad (15)$$

whilst the unit supplementary longitudinal reinforcement is :

$$(\Delta A_{s\ell} / b_w d) = \xi \tau_{sd}^2 / (A_{sw}/sb_w) \, f_{ywk} \, f_{y\ell k} - \eta \cdot \tau_{sd} / f_{y\ell k} \tag{16}$$

where λ, ε, ξ and μ are numerical coefficients.

A specific value of (A_{sw}/sb_w) minimizes the objective function and corresponds to the annulment of the first derivative of this function. It is found to be :

$$(A_{sw}/sb_w)_{min} = \tau_{sd} \sqrt{\xi/\rho\lambda \, (1 + \varepsilon b_w/d) \, f_{ywk} \, f_{y\ell k}} \tag{17}$$

This minimum is called *unconstrained* because it does not interfere with the limits of the problem, and is represented by an horizontal line in figure 2. When this latter line does not intersect the feasible domain, the optimum value of (A_{sw}/sb_w) is obtained from the nearest apex of the feasible domain ; if the contrary is true, it is derived from the unconstrained minimum and a certain variation range of $tg\Theta$ is associated with the optimum value of (A_{sw}/sb_w), (see cases e, f, h, i of figure 2) so that this latter does not correspond to a unique value of Θ. It must however be observed that the lower bound $tg\Theta = 3/5$ belongs in any case to the optimal solution. It is the reason why this value, which corresponds to $\Theta = 31°$, is selected for the further optimization process ; it is in complete agreement with experimental results obtained for beams subject to rather distributed loading |2|.

It is said above that the objective function depends on a cost factor ρ, the value of which is generally comprised between 1 and 1,5 as pointed out by THURLIMANN |3|. A numerical investigation proved that the influence of this factor is small ; as, in addition, the value of ρ is likely to vary with respect to the factory, the country and to the labor-to material cost ratio, it is reasonable to put $\rho = 1$.

Finally, the design procedure, based on the recommendations of the C.E.B.-F.I.P. Model Code, can be summarized as follows :

To check the web concrete :

$$(\tau_{sd}/f_{ck}) \leqslant \upsilon \, (\%) \tag{18}$$

To design the shear reinforcement :

by selecting for (A_{sw}/sb_w) the largest value of the three following expressions :

- minimum shear reinforcement : $\beta(0{,}01 \, f_{ck} + 0{,}2)/f_{ywk}$ (19)
- shear reinforcement strength

$$\delta \left[\tau_{sd} - \mathrm{MAX} \, (0 \; ; 0{,}03 \, f_{ck} + 0{,}375 - 0{,}5 \, \tau_{sd}) \right] / f_{ywk} \tag{20}$$

- unconstrained minimum :

$$\tau_{sd} \sqrt{\xi / (1 + \varepsilon b_w/d) \, f_{ywk} \, f_{y\ell k}} \tag{21}$$

To design the additional longitunal tensile reinforcement :

$$(\Delta A_{s\ell}/b_w d) = \xi \, \tau^2_{sd} \; / \; (A_{sw}/sb_w) \, f_{ywk} \, f_{y\ell k} - \eta \, \tau_{sd}/f_{y\ell k} \tag{22}$$

The numerical coefficients β, δ, ξ, η, ν, ε and λ depend on the inclination Θ of cracks and α of the stirrups, and on the configuration of the shear reinforcement in the cross-section of the beam. They are given in references |4| and |6| for different types of reinforcement.

Let us insist on the fact that the above formulation remains general and does not yet at all depend on the value $\Theta = 31°$. Except for the unconstrained minimum, it represents, in the author's opinion, thus a more simple convenient presentation of the C.E.B.-F.I.P. Recommendations.

To make easier the design procedure for shear, a lot of charts can be drawn, each of them being specific of the configuration of the stirrups and of the yield stress of shear - and longitudinal reinforcement respectively. A full set of charts drawn for $\Theta = 31°$ are available and can be provided by the authors.

ECONOMICAL CHOICE OF SHEAR REINFORCEMENT.

On base of the C.E.B.-F.I.P. Recommendations for shear design and of an inclination $\Theta = 31°$ for the compression field, as discussed above, a lot of numerical simulations have been performed.

Several parameters are investigated :

a) the geometrical configurations of the shear reinforcement (figure 3) : closed stirrups with inclination $\alpha = 45°$, 59° and 90°, and single and closed nets with resultant inclination $\alpha = 45°$ and 59° ;
b) the steel grades : S 220, S 400 and S 500. ;
c) the compressive concrete strength : C 20, C 30, C 40 and C 50 ;
d) the aspect ratio fo the cross-sectional dimensions : $d/b_w = 1, 2, 3, 4$ and 5.

From this extensive work |6|, it can be concluded that :

1. the economical classification of the shear reinforcement does not depend on the value of the applied design stress τ_{sd}, except for small values of τ_{sd} for which the minimum amount of shear reinforcement is governing ;
2. the type of optimal reinforcement does not depend on the compressive concrete strength ;
3. the aspect ratio d/b_w only influences the choice of the type of shear reinforcement if both shear - and longitudinal reinforcement are made of high strength steel ;

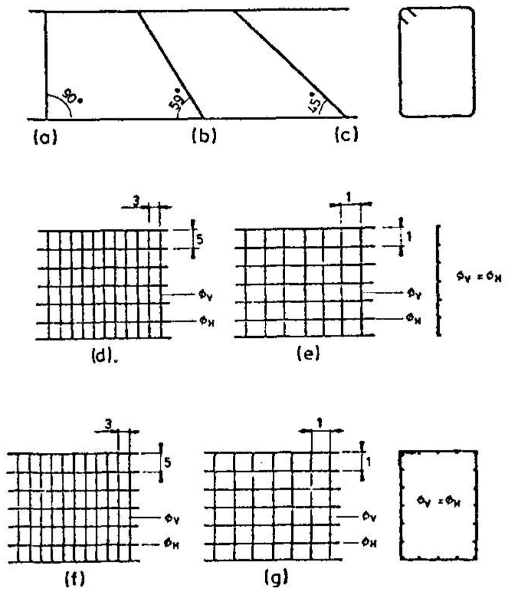


Figure 3 : Geometrical configurations of reinforcements.

A simple practical proposal for the choice of an economic shear reinforcement can be recommended as follows :

- shear reinforcement S 220 : stirrups with $\alpha = 45°$;
- shear reinforcement S 400 or S 500 :

  $f_{y\ell k} \geqslant f_{ywk}$: stirrups with $\alpha = 45°$;

  $f_{y\ell k} < f_{ywk}$: single net with resultant inclination $\alpha = 45°$.

It leads to values of the objective function which only differ of 3 to 5 % from these obtained with effective optimal configurations.

A comparison between vertical and inclined stirrups shows that the use of inclined shear reinforcement allows an economy of 20 to 25 % of the total amount of reinforcement required by shear, the reference being the solution with vertical stirrups. Both inclinations $\alpha = 45°$ and $\alpha = 59°$ lead to nearly the same economy ; for practical reasons, inclined stirrups with $\alpha = 45°$ is highly recommended.

Investigation on the value of Θ shows that the choice of $\Theta = 31°$ instead of $\Theta = 45°$ leads to a decrease of the amount of shear reinforcement and to an increase of the longitudinal tensile reinforcement, so that a global economy results, which reaches 20 to 13 %, if τ_{sd} exceeds 7,5 % of f_{ck}. If the contrary, the economy decreases until zero when the minimum shear reinforcement becomes governing.

Usually, the steel grade for longitudinal tensile reinforcement is decided prior to that of shear reinforcement ; then the maximum economy requires to use for shear reinforcement the highest steel grade and the following configuration : net and 45° stirrups when longitudinal reinforcement is of type S 220 - S 400 and S 500 respectively.

CONCLUSIONS.

The formulation of the C.E.B.-F.I.P. Recommendations for shear design is improved and used for a numerical simulation with $\theta = 31°$, which belongs in any case to the economical solution. It is shown that inclined stirrups with $\alpha = 45°$ and single net with resultant inclination $\alpha = 45°$, are the most economical configurations. Inclined stirrups are about 20 % more economical than vertical ones, whilst with the choice of $\theta = 31°$, an economy of 10 to 20 % can be expected with respect to $\theta = 45°$. Last, one shows how to choose the configuration and the steel grade of shear reinforcement, when the steel grade of longitudinal reinforcement is specified, in order to obtain the least amount of both types of reinforcements.

REFERENCES.

1. C.E.B.-F.I.P. : Code-Modèle pour les structures en béton. 3d Edition of the International Recommendations C.E.B.-F.I.P., 1978.
2. C.E.B.-F.I.P. : Bulletin d'information n°126 : Shear and Torsion, June 1978.
3. GROB, J. and THURLIMANN, B. : Ultimate strength and Design of Reinforced Concrete Beams under Bending and Shear. Proceedings of the I.A.B.S.E., 36-II, 1976, pp. 105-120.
4. MAQUOI, R. and RONDAL, J. : L'armature optimale d'effort tranchant dans les poutres en béton. To be published in Annales des Travaux Publics de Belgique.
5. MORSCH, E. : Die Schubsicherung der Eisenbetonbalken - Beton und Eisen, 1927, H.2, pp.27-35.
6. RONDAL, J. and MAQUOI, R. : Considérations économiques sur le choix d'une armature d'effort tranchant. I.A.B.S.E. Proceedings P-22/79, May 1979.

NOTATIONS.

A_{sw} : cross-sectional area of shear reinforcement ;

$\Delta A_{s\ell}$: additional longitudinal tensile reinforcement ;

F : objective function ;

$V_{cd}(\tau_{cd})$: shear force (stress) carried by compression flange ;

V_{Rd2}, V_{Rd3} : resistant shear for shear reinforcement, for web concrete ;

$V_{Rd}(\tau_{Rd})$: codified resistant design shear force (stress) ;

$V_{sd}(\tau_{sd})$: applied design shear force (stress) ;

V_{wd} : shear carried by truss action ;

b_w : web breadth ;

d : effective depth ;

$f_{cd}(f_{ck})$: design (characteristic) compressive strength of concrete ;

$f_{ywd}(f_{ywk})$, $f_{y\ell d}(f_{y\ell k})$: design (characteristic) yield strength for shear reinforcement, for longitudinal reinforcement ;

s : stirrup spacing ;

α : inclination of stirrups ;

θ : inclination of compression field ;

ρ_w : minimum geometrical percentage of shear reinforcement ;

ρ : cost factor ;

β, δ, ϵ, λ, ξ, η, ν : numerical values depending on the configuration of shear reinforcement and/or on the inclinations θ and α .

V

Design of Reinforced Concrete Beams Based on Directive 34 of the Swiss Code SIA 162

Dimensionnement des poutres en béton armé sur la base de la directive 34 des normes suisses SIA 162

Bemessung von Stahlbetonträgern nach der Richtlinie 34 der schweizerischen Norm SIA 162

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SUMMARY
In April 1976 the new Directive No 34 of the Swiss Code SIA 162 was introduced for the design of reinforced and prestressed concrete beams. The design rules for beams under bending, shear, torsion and combined actions are based on plastic solutions for strength using a truss model with variable inclination of the concrete compression struts.

RESUME
En avril 1976 la nouvelle directive 34 des normes suisses SIA 162 a été introduite pour le dimensionnement pratique des poutres en béton armé et précontraint. Les règles pour le dimensionnement des poutres soumises à la flexion, à l'effort tranchant, à la torsion et à des efforts combinés sont basées sur des solutions plastiques pour la résistance en utilisant un modèle de treillis avec inclinaison variable des bielles en béton.

ZUSAMMENFASSUNG
Im April 1976 wurde die neue Richtlinie 34 der schweizerischen Norm SIA 162 für die Bemessung von Stahlbeton- und Spannbetonträgern eingeführt. Die Bemessungsregeln für Balken unter Biegung, Querkraft, Torsion und kombinierten Beanspruchungen stützen sich auf plastische Lösungen für den Bruchwiderstand unter Verwendung eines Fachwerkmodells mit variabler Neigung der Betondruckdiagonalen.

1. INTRODUCTION

Structural design requires a suitable structural behavior under service loads and an acceptable factor of safety against failure. The Directive 34, observing these design principles given in the Swiss Code SIA 162, unifies the design of reinforced and prestressed concrete structures. Hence an adequate behavior of a structure under service loads with regard to cracking and deflections must be ensured. Since the safety of a structure depends directly on the ultimate strength the determination of the strength is of primary importance. In the Directive 34 the same truss model is assumed as failure model for all actions as bending, shear, torsion and combined.

The Directive 34 of the Swiss Code SIA 162 offers two different design procedures. First, for the design of a member cross-section, an adequate factor of safety against failure has to be applied to the internal actions (bending moments, shear forces etc.) determined elastically.

$$\frac{W_r}{s_W} \geq S'(s_{Li} \cdot L_i) \quad \text{or} \quad W_r \geq S^* = s_W \cdot S'(s_{Li} \cdot L_i) \tag{1}$$

with

L_i	Design Loads	W_r	Theoretical Resistance of Member Cross-Section
S'	Internal Actions due to $(s_{Li} \cdot L_i)$	s_{Li}	Load Factors $(0.8 \leq s_{Li} \leq 1.4)$
S^*	Internal Actions for Required Minimum Resistance	s_W	Resistance Factor $(s_W = 1.3)$

Additionally a design method is employed based on an adequate carrying capacity of a structure with respect to standard design load configurations.

2. TRUSS MODEL

To describe the static behavior of reinforced and prestressed concrete beams a truss model is used with the longitudinal bars acting as stringers, the stirrups as vertical ties and the concrete diagonals as inclined struts. In order to simplify the mathematical treatment the longitudinal reinforcement is first assumed to be concentrated into corner stringers.

If a beam element is subjected to a bending moment M^*, an axial force N^* and a shear force Q^*, the equilibrium conditions lead to the following stringer and stirrup forces:

$$B = Q^* \cdot \frac{s_B}{h_o} \cdot \tan\gamma$$

$$Z_o = -\frac{M^*}{h_o} + \frac{N^*}{2} + \frac{Q^*}{2} \cdot \cot\gamma \tag{2}$$

$$Z_u = \frac{M^*}{h_o} + \frac{N^*}{2} + \frac{Q^*}{2} \cdot \cot\gamma$$

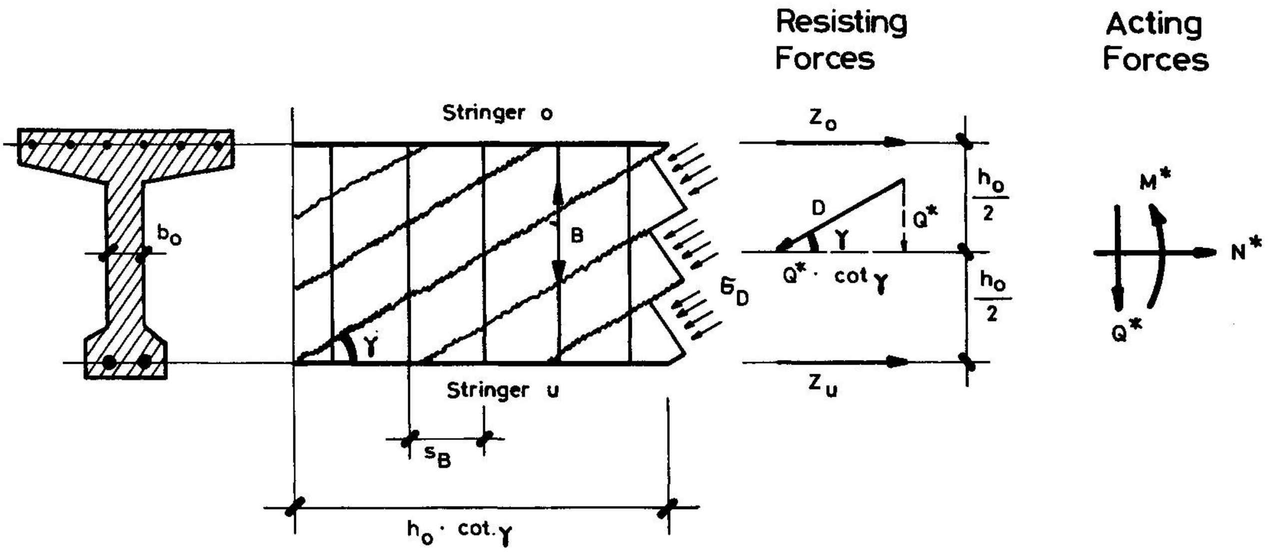


Fig. 1 Truss Model

The stresses in the reinforcement and the concrete have to be limited. The stringer and stirrup forces must not violate the yield conditions of the steel, and the concrete stresses must not produce a concrete failure. Concrete failure can be caused by crushing of the concrete compression zone or of the concrete compression struts.

If the concrete stress in the compression zone is limited, the extension of the compression zone must be considered. Thus the assumption of concentrated stringer forces in the corners does no longer hold. The distance y between compression and tension resultant gets smaller than h_o, Fig. 1. This fact can easily be taken into account by using the well-known bending theory. The tensile force due to shear is then added to the resultant forces due to bending and axial force.

$$Z_o = Z_o\ (M^*,\ N^*) + \frac{Q^*}{2} \cdot \cot\gamma$$
$$Z_u = Z_u\ (M^*,\ N^*) + \frac{Q^*}{2} \cdot \cot\gamma \qquad (3)$$

The stresses σ_D in the concrete compression struts are caused by the diagonal force D, Fig. 1.

$$\sigma_D = \frac{Q^*}{b_o \cdot h_o} \cdot \frac{1}{\cos\gamma \cdot \sin\gamma} = \tau^* \cdot \frac{1}{\cos\gamma \cdot \sin\gamma} \qquad (4)$$

From tests, Ref. [3], the following bounds have been chosen for the inclination of the concrete compression struts.

$$0.5 \leqslant |\tan\gamma| \leqslant 2.0 \qquad (5)$$

Further explanations are given in Refs. [1], [2] and [6].

3. DESIGN OF REINFORCED CONCRETE BEAMS

3.1 Bending and Axial Force

The design of beams subjected to a bending moment and an axial force is based on the well-known bending theory.

For an advantageous design procedure all material strength properties must be put on the same basis. The Directive 34 of the Swiss Code SIA 162 generally uses nominal strength values according to the 5%-fractile for both steel and concrete.

$$\begin{aligned} &\text{Concrete} : \beta_r = 0.60 \cdot \beta_w \simeq 0.75 \cdot \beta_c \\ &\text{Steel} : \sigma_f = \sigma_{2.0} \end{aligned} \tag{6}$$

with

β_r Nominal Compressive Strength of Concrete

β_w Cube Strength (16%-Factile)

β_c Cylinder Strength (16%-Fractile)

σ_f Nominal Yield Stress of Steel

$\sigma_{2.0}$ Yield Stress for 2 ‰ Permanent Strain (5%-Fractile)

If prestressing steel is present, then the initial strain in the prestressing steel is taken into account in calculating the ultimate strength. This formulation leads to an unified approach for any possible choice of prestressing between reinforced and fully prestressed concrete.

3.2 Shear

The nominal shear stress is a measure of the concrete stress in the web:

$$\tau^* = \frac{Q^*}{b_o \cdot h_o} \tag{7}$$

No shear cracks occur below the value τ_r. Therefore the shear is carried by the concrete alone and none or only a nominal shear reinforcement is required for $\tau^* \leqslant \tau_r$.

The values of τ_r given in the current Swiss Code SIA 162 and in the new Directive 34, depend on the concrete strength β_w (cube strength).

Cube Strength β_w (N/mm2)	:	20	30	40	⩾	50
Approx. Cylinder Strength β_c	:	16	24	32	⩾	40
Design Strength β_r	:	12	18	24	⩾	30
Design Shear Stress τ_r	:	0.8	1.0	1.2		1.4

(8)

On the other side the nominal shear stress τ^* should not exceed the maximum value τ_{max} which depends on the concrete strength and the maximum stirrup spacing s_B.

$$\text{Normal Spacing} : \tau_{max} = 5\,\tau_r \text{ for } s_B \leqslant \frac{h_o}{2} \text{ but } s_B \leqslant 30 \text{ cm} \tag{9}$$

$$\text{Close Spacing}: \tau_{max} = 6\,\tau_r \text{ for } s_B \leqslant \frac{h_o}{3} \text{ but } s_B \leqslant 20 \text{ cm}$$

Between the uncracked state, $\tau^* \leqslant \tau_r$, and the fully developed truss action, about $\tau^* \geqslant 3\,\tau_r$, a transition takes place. Observations on test beams and recent theoretical investigations, Refs. [6] and [7], show that the initial shear strength of the uncracked concrete is reduced progressively. For design purposes the following approximation is used:

$$\begin{aligned} &\text{For } \tau_r < \tau^* < 3\,\tau_r : Q_b = \frac{1}{2} \cdot (3\,\tau_r - \tau^*) \cdot b_o \cdot h_o \\ &\text{For } \tau^* \geqslant 3\,\tau_r \quad : Q_b = 0 \end{aligned} \tag{10}$$

This additional contribution is taken into account for the design of the stirrup reinforcement by deducting Q_b from the applied shear force Q^*. Thus the final design equations are obtained.

Stirrup Reinforcement F_B (Q):

$$\begin{aligned} &F_B\,(Q) \cdot \sigma_{fB} \geqslant (Q^* - Q_b) \cdot \frac{s_B}{h_o} \cdot \tan\gamma \\ &\text{but} \\ &F_B\,(Q) \cdot \sigma_{fB} \geqslant \frac{1}{2} \cdot \tau_r \cdot b_o \cdot s_B \text{ (min. stirrup reinf.)} \end{aligned} \tag{11}$$

Longitudinal Reinforcement F_L (Q) due to Shear:

$$F_L\,(Q) \cdot \sigma_{fL} \geqslant \frac{1}{2} \cdot Q^* \cdot \cot\gamma \tag{12}$$

In the Directive 34, closer limits than given by Eq. (5) are prescribed for the inclination γ of the concrete compression struts.

$$\frac{3}{5} \leqslant |\tan\gamma| \leqslant \frac{5}{3} \tag{13}$$

The cracking evaluation of test beams, Ref. [2], showed an adequate behavior under service loads, if the limiting values from Eq. (13) are used.

For design purposes the Eqs. (11) and (12) are put in a non-dimensional form. In Fig. 2 the corresponding design diagrams are given for the practical inclination range of the compression struts, e.g. from about 30^o to 45^o.

As shown in Fig. 2 smaller inclinations γ lead to a reduction of the stirrup reinforcement but to an increase of the longitudinal reinforcement. Usually the longitudinal reinforcement is such, that the inclination $\tan\gamma$ can be fixed to the value 3/5. Furthermore, as shown in Ref. [2], the selection of an angle γ approaching 30^o gives the most economical solution.

So far the influences of a variable depth of the beam and of inclined prestressing tendons have not been considered. They can be taken into account by substituting the applied shear force Q^* by the effective web shear force Q^*_{eff}. In order to simplify the design procedure the force Z_s, Fig. 3, in the tendon is taken equal to the prestressing force V of the tendon.

$$Q^*_{eff} = Q^* - \frac{M^*}{h_o} \cdot 2 \tan\frac{\delta}{2} - V \cdot (\sin\alpha - \frac{y_s}{h_o} \cdot \cos\alpha \cdot 2 \tan\frac{\delta}{2}) \tag{14}$$

The design convention for the forces and the angles are indicated in Fig. 3.

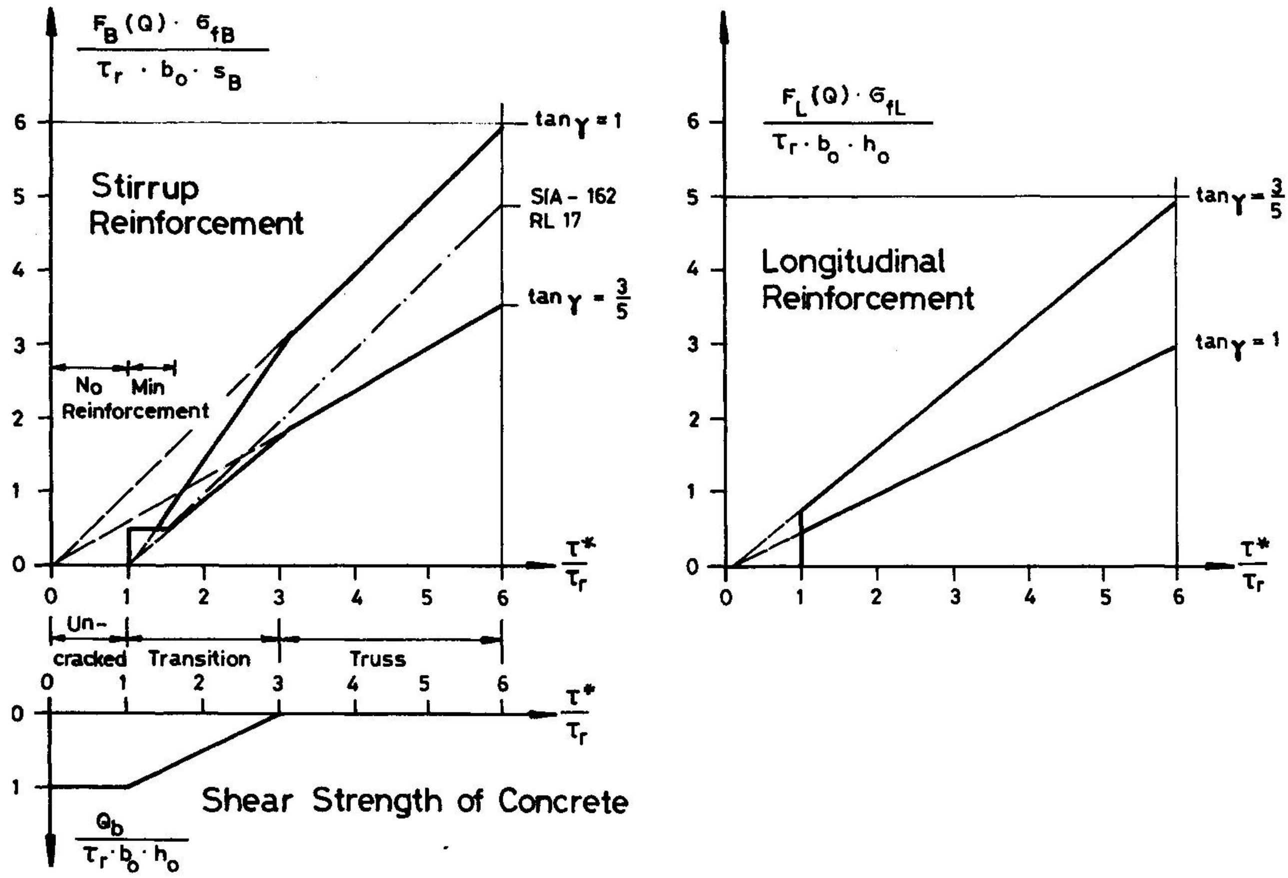


Fig. 2 Design of Shear Reinforcement

3.3 Torsion and Combined Actions

In a box beam torsion produces a constant shear flow S^* around the perimeter u_o.

$$S^* = \tau^* \cdot t_o = \frac{T^*}{2 F_o} \tag{15}$$

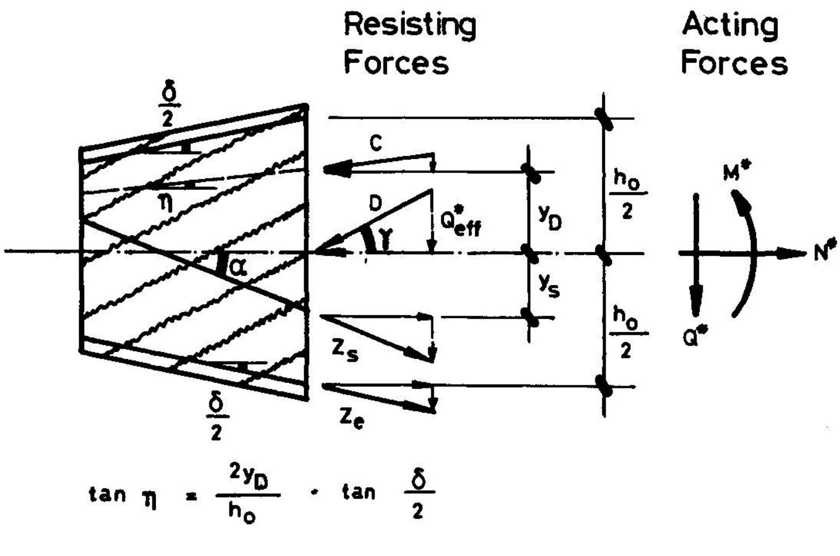


Fig. 3 Variable Depth and Inclined Tendons

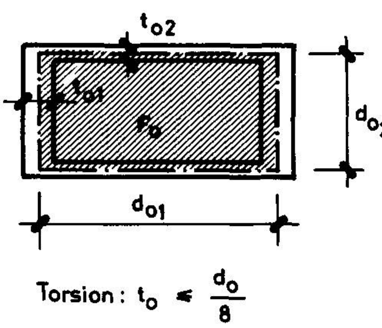


Fig. 4 Effective Wall Thickness for Torsion

Assuming the same static model for both shear and torsion, the design equations for shear (shear flow Q^*/h_o) can easily be transformed for torsion (shear flow $T^*/2\,F_o$).

$$\frac{Q^*}{h_o} \rightarrow \frac{T^*}{2F_o} \qquad \frac{Q_b}{h_o} \rightarrow \frac{T_b}{2F_o} \qquad b_o \rightarrow t_o \qquad h_o \rightarrow u_o \tag{16}$$

Torsion produces an additional effect due to the distortion of the side walls of a box beam. The concrete stresses in the compression struts caused by the truss action are superimposed by secondary bending stresses, Ref. [3]. In the Directive 34 these bending stresses are limited in a pragmatic manner by adopting a cautious value for the effective wall thickness t_o, Fig. 4.

$$t_o \leqslant \frac{d_o}{8} \tag{17}$$

If the nominal shear stress due to torsion is calculated by considering Eq. (17), the bounds for the nominal shear stress are the same for both torsion and shear.

The design of beams subjected to combined actions can be made by superposition of the stresses and reinforcements respectively due to bending, shear and torsion. However in practical design a more efficient procedure, pointed out next, is recommended.

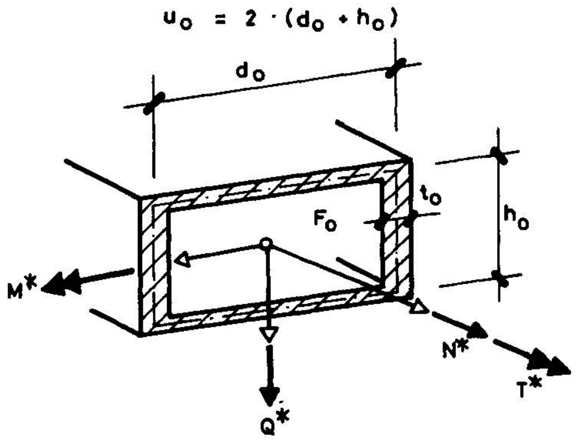
The design of the stirrup reinforcement and the control of the concrete compression struts can be based upon the calculation of the shear resultants in the side walls of a beam. The design rules given in Sec. 3.2 for shear are then applied to these shear resultants. For the example shown in Fig. 5 the following shear resultants R^* are obtained in the webs:

$$R^* = \frac{Q^*}{2} \pm \frac{T^*}{2F_o} \cdot h_o \tag{18}$$

The design of the longitudinal reinforcement and the control of the compression zone can be based upon the effective axial force Z^* due to the applied axial force, the shear force and the torque, Fig. 5.

$$Z^* = N^* + |\cot\gamma| \cdot (|Q^*| + |T^*| \cdot \frac{u_o}{2F_o}) \tag{19}$$

The tensile force due to shear acts at the centroid of the two webs, whereas the tensile force due to torsion acts at the centroid of the perimeter u_o around the area F_o.



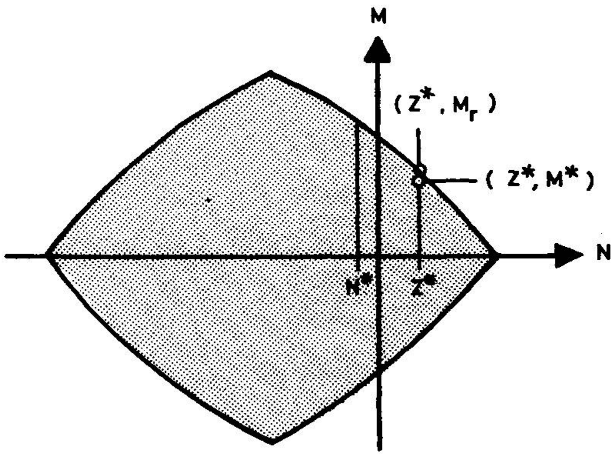


Fig. 5 Combined Actions

A sufficient resistance of the member cross-section is obtained, if the stress point (Z^*, M^*) lies inside the interaction curve, Fig. 5, determined by application of the bending theory.

CONCLUSION

As generally can be stated, design methods are better the simpler they are. A simple and unified design approach including a clear safety concept leads to safe structures, because the designer gets a sound idea of the static behavior and of the carrying capacity. It is felt that plastic analysis, especially the static method, helps to develop a simple, unified and reasonably accurate design concept.

REFERENCES

[1] THUERLIMANN, B., GROB, J., LUECHINGER, P.: "Torsion, Biegung und Schub in Stahlbetonträgern" (Torsion, Bending and Shear in Reinforced Concrete Beams), Institut für Baustatik und Konstruktion, ETH Zürich, Autographie zu Fortbildungskurs für Bauingenieure aus der Praxis, 1975.

[2] GROB, J., THUERLIMANN, B.: "Ultimate Strength and Design of Reinforced Concrete Beams under Bending and Shear", Publications, International Association for Bridge and Structural Engineering (IABSE), Vol. 36-II, p. 105, 1976.

[3] LAMPERT, P., THUERLIMANN, B.: "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending", Publications, International Association for Bridge and Structural Engineering (IABSE), Vol. 31-I, p. 107, 1971.

[4] LUECHINGER, P.: "Bruchwiderstand von Kastenträgern aus Stahlbeton unter Torsion, Biegung und Querkraft" (Ultimate Strength of Box-Girders in Reinforced Concrete under Torsion, Bending and Shear), Institut für Baustatik und Konstruktion, ETH Zürich, Bericht Nr. 69, Birkhäuser Verlag Basel und Stuttgart, 1977.

[5] MUELLER, P.: "Plastische Berechnung von Stahlbetonscheiben und -balken" (Plastic Analysis of Walls and Beams of Reinforced Concrete), Institut für Baustatik und Konstruktion, ETH Zürich, Bericht Nr. 83, Birkhäuser Verlag Basel und Stuttgart, 1978.

[6] MARTI, P., THUERLIMANN, B.: "Fliessbedingung für Stahlbeton mit Berücksichtigung der Betonzugfestigkeit" (Yield Criteria for Reinforced Concrete Including the Tensile Strength of Concrete), Beton- und Stahlbetonbau, Heft 1, 1977.

[7] NIELSEN, M.P., BRAESTRUP, M.W., BACH, F.: "Rational Analysis of Shear in Reinforced Concrete Beams", Proceedings, IABSE, P-15/78, Mai 1978.

[8] RICHTLINIE 34 ZU NORM SIA 162, "Bruchwiderstand und Bemessung von Stahlbeton- und Spannbetontragwerken" (Directive RL 34 to Structural Design Code SIA 162 (1968)), Schweizerischer Ingenieur- und Architekten-Verein, Zürich, 1976.

SUMMARY OF DISCUSSION - SESSION 5

1. Brittleness and Plastic Design

The discussion focused on some problems of major practical interest where the theory of plasticity does not seem applicable.

Both K.H. Gerstle and J. Blaauwendraad discussed the structural response of a substance of brittle nature. The latter reported on a cooperation established in the Netherlands between researchers on the mechanical behaviour of the constituent materials and on numerical modelling. This has led to the development of a finite element computer programme incorporating the effects of cracking. This so-called micro-model had been applied successfully to several structural elements on which tests had also been performed, e.g. the well-known wall-beam experiments performed by Leonhardt and Walter. A connection was drawn to session 4 on numerical methods by discussing the advantages of applying hybrid displacement methods when developing such numerical methods for structural design.

K.H. Gerstle referred to at least three problems discussed where failure is initiated by brittle tension, i.e. punching (presented by M.W. Braestrup), splitting and anchorage (presented by U. Hess), and flexural shear in beams without shear reinforcement (presented by M.P. Nielsen). He therefore asked the Danish group whether elastic or elasto-cracking analysis had been carried out to compare with the plastic solutions and the tests.

Such investigations have not been performed, but M.P. Nielsen further mentioned the scepticism of the Danish research group when they found that in the application of plastic analysis to problems apparently so brittle in nature as anchorage, splitting, corbels, punching etc., excellent results were found when comparing with tests. Until further investigations have been made, the assumption at present is that the stress-strain relationship for concrete in tension is of the same nature as in compression, including a falling branch of considerable length, and that the application of plastic analysis to these problems is a fully reasonable and rational approach. As reported by M.W. Braestrup, the theory also holds excellently for the lok-test.

2. Effectiveness Factor

J. Witteveen supplemented his discussion to Session 2 on the origin and magnitude of the effectiveness factor ν , and made the criticism that the value of ν in the different cases depends much on the intuition of the individual designer. M.P. Nielsen supported further research on the determination of ν for the individual cases considered, but at present the formula $\nu = 2/\sqrt{\sigma_c}$ (MPa) seems to be a fair guess on the safe side, as values lower than these had seldom been found. He further reminded the discussion group that the elastic stress distribtuion in certain cases can be very wrong, e.g. when stress concentrations occur.

3. Safety Levels and Codified Plastic Design

A.B. Sele had applied the beam-shear model presented by J.F. Jensen in Session 2 to survey data from the UK. Extensive computer calculations were made in order to determine the level II reliability based on the BS CP 110, the ACI

318-77, the DIN 1045 (1972) and the rules of Det Norske Veritas for off-shore structures.

The results showed good agreement between the different rules. However, the absolute level of calculated reliability depends heavily on the codified loadings adopted. The relative picture may be altered significantly for other load types.

4. Bond and Anchorage

The tensile strength of concrete is often overlooked in our models. This simplification is in several cases unsatisfactory and leads to wrong results, e.g. in punching, and in splitting in the anchorage zone, as pointed out by R. Tepfers. In the anchorage tests performed by Rathkjen and further treated by U. Hess, an upper bound solution was applied, and transverse friction at the supports was neglected though some friction must have been present in the tests. It was furthermore pointed out that in the presentation by U. Hess only the last test series contained stirrups in the anchorage zone, all other series had no stirrups.

5. Optimal Design

Based on many years experience with the development of precast, reinforced slabs, A. Sawczuk presented his recommendations on the rational reinforcing of such slabs. Economy was assured by seeking maximum load carrying capacity for minimum reinforcement. Favourable service performance was reached by allowing minimal redistribution of sectional forces. Simple and quick execution was achieved by following certain technological conditions concerning welding, producing simplified reinforcing nets.

In R. Maquoi's interesting presentation it was stated that considerable economy was achieved when applying inclined stirrups as compared to vertical stirrups. As pointed out by M.W. Braestrup, this is only the case when the shear stresses are high and the concrete strength thus determining, whereas the advantage disappears when low shear stresses are exhibited. R. Maquoi supplemented this by emphasizing that codes such as the CEB-FIP Model Code should contain supplementary information such as the information he had given, instead of just giving the two limiting values of the stirrup inclination.

It seems obvious that the various manuals of CEB for example should profit from such valuable practical information, so clearly exemplified by R. Maquoi.

S. ROSTAM

SUMMARY OF GENERAL DISCUSSION

1. Plastic Analysis in Structural Design

During the previous sessions it had been demonstrated that plastic analysis of concrete structures has achieved broad acceptance among concrete researchers.

The impression could therefore be had that all problems have been solved and that the theory was ready to be applied as a general basis, and not just as a yield line theory for slabs as it has been so successfully applied for decades. This conclusion could not be drawn from the Colloquium, nor was it the aim of the participants.

On the contrary, the final discussion showed that the theory was maturing, and that its successful application to numerous well-defined problems had given confidence to its coming role as a rational means of designing concrete structures in the ultimate limit state.

However, the discussions had also focused on assumptions and basic problems needing further consideration.

2. Properties of Plain Concrete

The general discussion concentrated to a large extent on whether concrete does in fact comply with the basic assumptions of plasticity.

K.H. Gerstle concentrated on the actual properties of plain concrete. He referred to a cooperative research project involving several US research institutes exploring multiaxial concrete behaviour. This investigation has been under way for several years and the following main results were presented by way of slides of three-dimensional models of the obtained stress-strain relationships.

- Six sets of σ-ε curves of biaxial tests with $\sigma_1/\sigma_2 = 1/3$ showed wide scatter though they were obtained from supposedly identical tests. So large was the scatter that it, according to Gerstle, appears that probabilistic methods would have to be used to reach a rational prediction.
- Similar tests, but from different laboratories and using different test methods, showed so large a scatter that it was of dubious value to assign a single curve to describe the behaviour of the concrete in the actual structure.
- Experimentally determined biaxial and triaxial strength envelopes exhibited so large a scatter that they, according to Gerstle, provide food for thought, and should be kept in mind when evaluating the results of analyses and tests.

The fact that reinforced concrete is a composite material led several participants to view its behaviour as an interaction between the constituent materials only to be fully understood when the behaviour of the individual materials is understood.

M.D. Kotsovos was in agreement with K.H. Gerstle when seeking to study the behaviour of reinforced concrete using a realistic representation of the properties of plain concrete. No successful attempt to do so has been made to date which, according to Kotsovos, was the most important reason why various proposed theories may only predict the behaviour of certain structural members under certain types of stress conditions. In this respect, he could see no link between the contents of session 1 on constitutive equations and the application of the theory of plasticity presented in the rest of the sessions.

He based this conclusion on the following comments on the various assumptions presented, describing the basic properties of concrete:

- Failure mode of concrete.
  Various types of modes of failure (fracture) have been considered, e.g. crushing under compression, cleavage under tension, shear, punching shear etc. However, it has been established by experiments that fracture is caused by crack propagation in the direction of the maximum principal compressive stress, and this is valid under any state of stress.

- Strength properties of concrete.
  A Coulomb-Mohr failure criterion has been assumed to describe adequately the strength properties of concrete under triaxial stress states. However, this representation is not correct since the criterion does not describe the effect of the intermediate stress, which has been found by experiment to be significant.Furthermore it is rather doubtful whether this criterion can realistically predict the strength under relatively high triaxial states of stress.

  Under plane stress conditions, the square failure envelope with zero tensile strength seems to be generally acceptable. However, in practice there is always some stress, though debatably minimal, in the third direction. If this stress is tensile, the proposed envelope is unsafe, even under relatively small compressive stresses on the plane of loading. In this case concrete will crack on a plane parallel to the plane of loading.

- Stress-strain relationships of concrete.
  Typical stress-strain relationships for concrete likely to be encountered in practice are shown in Figure 8 of the paper by Kotsovos in Session 1. Similar is the shape of the relationships under biaxial compression, whereas under tension the descending portion is non-existant.

  In view of this evidence, elastic (or rigid) - plastic idealization is hardly justified according to Kotsovos. It may be justified under high biaxial states of stress, as shown in Figure 6 of the above-mentioned paper, but it would be difficult, if not impossible, to define any yield line under such states of stress.

The remark by Kotsovos on the stress-strain relationships of concrete in tension, as having no falling branch, should be compared with the comments by M.P. Nielsen during the discussion of Session 2. M.P. Nielsen feels that the successful ability of plastic analysis to describe phenomena dependent on the tensile strength of concrete, indicates the presence of a falling branch in tension.

This focuses on a difference of opinion on a matter of apparently considerable importance in the understanding and further development of the rational theory of plasticity to treat reinforced concrete structures. In view of this, further well-considered tests on the stress-strain relationship of concrete in tension should be carried out, and the application of the results in the theory should be considered further.

3. Restraints

C.T. Morley presented an engineering approach to the same problems by discussing the restraints necessary to achieve a sufficient load carrying capacity. Also S.M. Uzumeri concentrated on the beneficial effect of confined concrete.

Shear strength can, as indicated by C.T. Morley, derive from inclined forces, provided sufficient restraint is available, as in a clamped beam. This is equivalent to the tension field theory valid for steel plate girders, where the restraint is provided by the flanges, and for masonry structures, where the foundation can provide the necessary restraint.

In local punching shear problems the necessary restraint is provided by the surrounding slab, and the assumed failure mode, as presented by M.W. Braestrup, is an upper bound approach.

According to Morley some difficulty does arise in practice, when ensuring that some other failure mechanism does not intervene. This was exemplified with punching tests on cylinders, where, for small punch diameters, the failure load, as expected, fell between the loads predicted by the sphere and the flat slab theories respectively. For larger punch diameters the theoretical beneficial effect of shell curvature seemed, however, not to appear. Whether this was due to lack of restraint, or whether another failure mechanism was beginning to intervene, could, according to Morley, not be determined.

Finally his opinion was that the problems involved in applying the plastic theory to reinforced concrete were not caused by the neglect of the tensile strength of concrete, but by applying it to brittle, over-reinforced elements, which has led to the frequent discussion of the introduction of the effectiveness factor during these sessions.

4. Effectiveness Factor

B. Thürlimann supplemented the discussion of Session 2 by stating the two different effects covered in a rational way by ν . These were on the one hand the local effects giving rise to local complicated stress distributions, and on the other hand the influence of both the strain history and the softening up of concrete at collapse which enters into the conditions of the kinematics. At present a different ν-factor is presented for shear, for punching, for anchorage etc. and Thürlimann hoped that the Danish research group, as the originators of this factor, would give it some reconsideration.

5. Deformation of Capacity

The deformation capacity of concrete obviously has a decisive effect on the ability of plastic analysis to model the behaviour of reinforced concrete structures at ultimate.

P. Lenkei presented the results of a recent research project aimed at determining the deformation, cracking and redistribution of internal forces in reinforced slabs for loads increasing up to the maximum value. The behaviour of slabs during this transition process is greatly influenced by the level of orthotropy and by the differences in the principal directions of the resistance of the slab and of the external moments. Lenkei showed that the same ultimate load could be reached by going through different values of deflections and different crack patterns. The differences are in some cases not negligible, and should be considered when serviceability considerations are determining.

6. Energy Absorbing Capacity

Until now only static and proportional loading has been considered, but K.J. Pittner recalled that a multitude of structures have to be designed to resist severe loading cases of a dynamic nature such as earthquakes.

In these cases the dynamic design load can be much higher than the static resistance of the structure. Such designs have to rely on the prediction of the energy absorbing capacity of the structure. This requires a means of determining the deformation capacity as the induced energy is to dissipate as much and as locally as possible.

According to Pittner, a prediction of the energy absorbing capacity requires knowledge of:

- how much reinforcement can be plastified,
- which plastic strains can be accepted, and
- how bonding between concrete and reinforcement can be avoided in order to achieve maximum elongation of the reinforcing bars and thus reach a maximum of deformation capacity.

He thus questions whether the limit state design has to achieve an extreme resistance force of the structure or an extreme capacity to dissipate energy. The latter would thus lead to a basically different definition of structural safety and to revised design principles.

Plastic analysis seems to be only part of the means of designing structures subjected to dynamic or impulsive loading. The presentation by Pittner is therefore a reminder that a supplementary plastic design criterion treating the influence of such loading should be presented.

7. Structural Safety

The interaction between plastic analysis and structural safety was discussed by J. Schneider, who presented a summary of a recommendable paper by W. Bosshard entitled "Structural Safety - a Matter of Decision and Control", published in the IABSE Periodica 2/1979, Surveys S-9/79. The final recommendation by J. Schneider was that scientific models should not be directly pushed into the codes, but should go through a proper filter, e.g. a code writing committee. This could have special interest when viewing the subjects treated at this Colloquium.

8. Secondary Effects in Shear

Possible contributions to the shear carrying capacity represented by secondary effects such as aggregate interlock or dowel effect are often discussed. In this connection T. Brøndum-Nielsen pointed out that shear force which can be transferred across a crack by dowel action is negligible, unless a stirrup happens to be located in the immediate vicinity of the crack, thus preventing the axial reinforcement from being pulled down ("zipper action"). He illustrated this with a photo of a statically indeterminate bridge slab, test loaded to failure. Immediately after a bending compression failure had occurred at mid-span, the axial reinforcement was pulled down over a length of several metres under the sole action of the shear force corresponding to dead load.

9. Notations

Finally A. Holmberg pointed out that only in rare cases were the notations in the numerous presentations equal to those given by CEB and FIP on the basis of ISO 3898.

A further unification in this respect would undoubtedly facilitate a rapid communication with the audience and with the readers of the papers.

10. Complete Design Basis for Concrete Structures

This Colloquium has demonstrated that plastic analysis is developing into an operational method of treating reinforced and prestressed concrete structures in the ultimate limit states.

In a report on the future activities of the IABSE, A.L. Bouma announced a Colloquium planned to take place in Delft in 1981. The subjects to be treated would cover serviceability conditions of concrete structures.

This promises to be a most valuable supplement to the present Colloquium, with the IABSE thus having fulfilled its commitment to cover the two integrated fields in modern concrete design, the one being the ultimate limit state conditions and the other being the serviceability limit state requirements.

S. ROSTAM