IV

Comparison of Plastic Prediction with STANIL/1 Analysis

Comparaison de l'analyse plastique avec le programme STANIL/1

Vergleich plastischer Berechnungen mit Berechnungen nach STANIL/1

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SUMMARY
The Danish group led by M.P. Nielsen published in 1978 a plastic analysis for the prediction of the ultimate shear failure load in beams. This method holds where unlimited ductility of steel and concrete can be assumed. In The Netherlands a nonlinear program, STANIL/1 is available to determine in which cases the plastic approach is admissible. The program uses concrete beam elements with main bending reinforcement and vertical web reinforcement. Results of some performed comparisons will be shown.

RESUME
Le groupe danois de Nielsen a publié en 1978 une méthode plastique pour calculer les charges ultimes de poutres soumises au cisaillement. Cette méthode est valable avec l'hypothèse d'une ductilité illimitée du béton et de l'acier. On a développé, aux Pays-Bas le programme non linéaire STANIL/1 à l'aide duquel on peut examiner si l'analyse plastique est applicable. Le programme utilise des éléments de poutre de béton avec une armature principale de flexion et une armature de cisaillement. Quelques résultats sont comparés.

ZUSAMMENFASSUNG
Nielsens Dänische Gruppe publizierte 1978 eine plastische Methode zur Berechnung der Schubbruchlast von Balken. Diese Methode ist anwendbar, wenn ein unbeschränktes Verformungsvermögen von Stahl und Beton angenommen werden darf. In den Niederlanden wurde das nichtlineare Rechenprogramm STANIL/1 entwickelt, mit dessen Hilfe beurteilt werden kann, in welchen Fällen die plastische Berechnung zulässig ist. Das Programm verwendet Beton-Balkenelemente mit Biegelängsbewehrung und vertikaler Schubbewehrung. Die Ergebnisse einiger durchgerechneter Vergleiche werden dargestellt.

1. INTRODUCTION AND SCOPE

During the IASS-symposium on Nonlinear behaviour of reinforced spatial structures at Darmstadt, 1978, a presentation has been given of the researchproject 'Beton-mechanica' in The Netherlands. A number of subprojects is on its way for experimental studies of a crackzone and a bondzone and also a subproject for numerical models. One of these models is called in the framework of the total project the Macro-model for framed structures. This Macro-model is a computerprogram Stanil/1 which enables us to analyse the nonlinear load displacement characteristics of beams, columns and frames. The program can be used to confirm the results of an existing ultimate load prediction via a plastic analysis, but above that additional information is provided on deformation restrictions and on the needed strain capacity of the reinforcement steel and the concrete.

The program Stanil/1 is an extension of an existing program which has been published by BLAAUWENDRAAD in 1972 [1]. That program had been based on the concept of a so called 'layered' beam-element as has been used parallelly by other investigators [2], [3] . The element has proven to give very good results for load combinations of pure bending and axial forces. However, the influence of shear forces could not be simulated adequately. This problem has been solved in the now presented new program Stanil/1 which uses a beam-element taking shear deformations and the action of vertical stirrups into account as well. The element--model will be briefly described in chapter 2.

NIELSEN, BRAESTRUP and BACH [4] presented a plastic analysis for the prediction of the ultimate shear failure load in beams. This method, which is in line with previous studies of THUERLIMANN et al [5], is used for the comparison with the Stanil/1 results. The plastic analysis is based on a theory of plasticity using an equilibrium method, providing a lower bound solution and a mechanism analysis, providing an upper bound solution. The method holds if unlimited ductility of steel and concrete may be assumed. Tuning of the method with experimental results showed that it was necessary to introduce a web effectiveness factor. In [4] this effectiveness factor was explained as to account for the limited ductility of the concrete. In case of complete accordance of the theoretical plastic model and the experimental results the web effectiviness factor should have the value 1.0. In practice the factor varies between 0.7 and 0.9.

Comparing the program Stanil/1 and the plastic analysis, it can be said that Stanil/1 is more general. The ultimate load prediction of the plastic analysis is a special case in the framework of Stanil/1. This program also is capable to calculate the ultimate load, but does not need the introduction of a web effectiveness factor. But more important, Stanil/1 provides information on the stiffness under work load conditions and on the amount of cracking. Stanil/1 also shows in which cases the strain capacity is insufficient to reach the plastic prediction for the ultimate load.

In cases in which the plastic analysis is valid, at failure both the nonlinear analysis of Stanil/1 and the plastic analysis of NIELSEN et al. should give the same results. To check this, in this paper two comparisons are presented. The first comparison regards the ideal plastic model in which the web effectiveness factor has the unit value. This situation can be simulated with Stanil/1 by making the axial concrete strains in the beam zero. This is the case for extremely high percentage of main reinforcement in the tensile region and for a compression flange which has an infinite rigidity. This comparison is shown in chapter 3. The second comparison in chapter 4 regards a situation for which the web effectiveness factor is less than unity. We use for this purpose experimental results for real beams of LEONHARDT and WALTHER [6].

2. THE MACRO-MODEL (STANIL/1)

2.1 General remarks about the beam-element

The beam-element has been based on an assumed field of displacements. Main bending reinforcement is schematized to two thin layers of steel; vertical stirrups are 'smeared out' to distributed vertical strings; cracks are smeared out on the beam. Nonlinearities are accounted for as follows: Each beam is divided over its height into imaginary concrete layers and steel layers (longitudinal reinforcement). Each layer may have different material properties corresponding to its stress or strain state and these properties can be different along one layer in the several cross-sections. The steel properties are defined for uniaxial states only but the concrete properties are defined for two-dimensional plane stress states. The behaviour of a beam-element is derived from the behaviour of a number of cross-sections of the beam-element , and the behaviour of the cross-section can be derived by totalizing the material properties of all layers in the cross-section in an appropriate way. Cracking and crushing of concrete are accounted for by modifying the material properties.

2.2 Possible deformations in the beam-element.

The assumed field of displacements allows for axial strains, bending and shear deformations and is capable of simulating bond slip of the main bending reinforcement and failure of the anchoring zone of this reinforcement. Above that vertical strains are allowed to occur, so that each admissible two-dimensional strain state can be simulated in the concrete, but also the stirrups can be activated. In this way one may expect to simulate truss action in the beam, needing in that case inclined concrete diagonals and vertical hangers.

Axial strains and bending deformations.

The chosen field of displacements allows a linear variation along the axis of the beam of both the axial strain e_{xx} and the curvature κ_{xx}, needing a total of 7 degrees of freedom $(u_1, u_2, u_3$ and $w_1, w_2, \phi_1, \phi_2)$, see fig. 1.
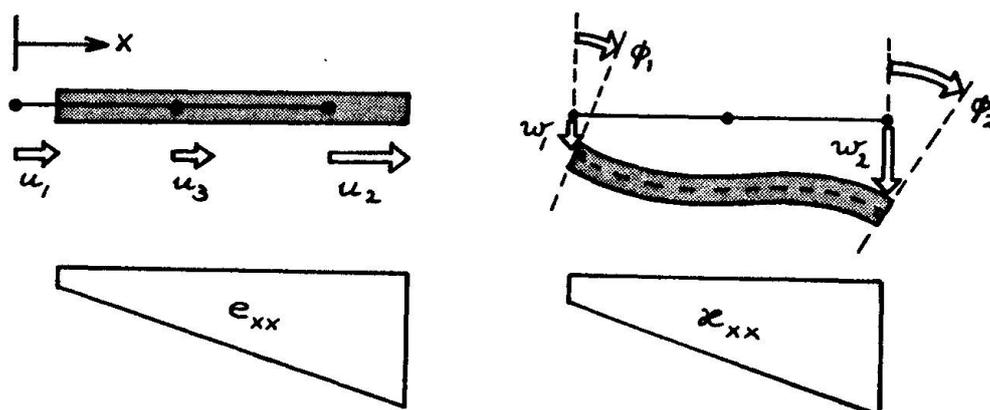

Fig. 1   Degrees of freedom and deformations for axial strain e_{xx} and curvature κ_{xx}

Shear deformation and tensile strain in stirrups.

More over the chosen field of displacements allows a linear variation along the axis of the beam of both the shear deformation γ_{xy} and the vertical strain e_s, needing another 4 degrees of freedom (γ_1, γ_2 and Δh_1, Δh_2), see fig. 2. This implies that the shear deformation and the strain in the stirrups is constant over the height of the beam. A perfect bond is assumed between the concrete and the stirrups.
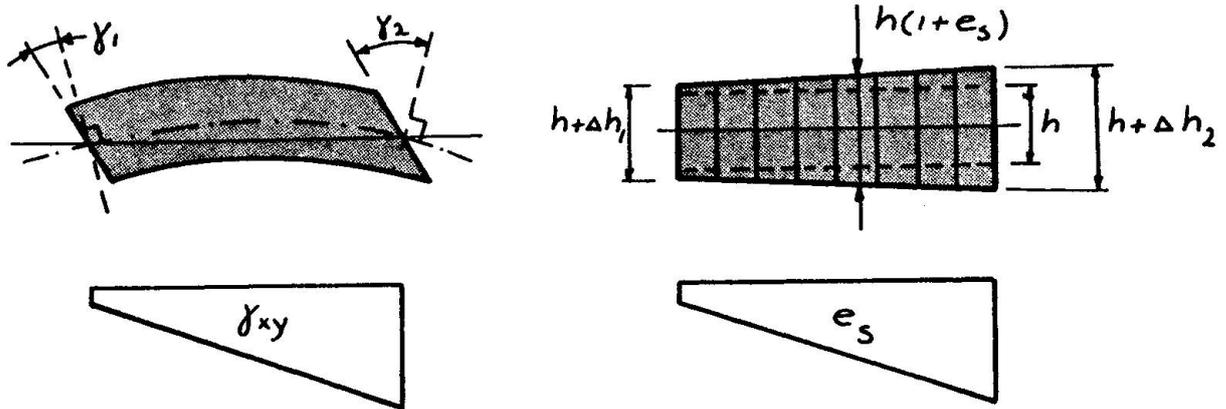

Fig. 2   Extra degrees of freedom and deformations for shear
γ_{xy} and vertical strain e_s

Steel-concrete interaction.

In order to accomplish a stiffness-interaction between longitudinal reinforcement and concrete, a possibility is created for relative movement between steel and concrete, called bond slip. This is achieved by imagining a tubular bond--spring around the bars of reinforcement. The interaction takes place as follows. Besides the already chosen field of axial displacements (u_1, u_2, u_3) for concrete, a separate field of axial displacements is chosen for steel (interpolation of the same degree as for concrete). The relative movement (bond slip) is found as the difference between the displacements of steel and concrete, resulting in three additional degrees of freedom (Δu_1, Δu_2, Δu_3). Using these parabolic interpolations for bottom and top reinforcement 6 additional degrees of freedom are necessary.
The anchoring of the main reinforcement is in fact a complex threedimensional state of strains and stresses. This is schematized with an extra point-spring between each end of the main reinforcement and the concrete in that position. Each spring results in an additional degree of freedom, being a relative axial displacement Δu.

2.3 Material properties.

The material properties of steel, concrete and bond can be inputted into STANIL/1 in multi-linear stress-strain relations c.q. multi-linear bond stress-slip relation. The failure surface for concrete is derived from the relevant relation, see fig. 3.
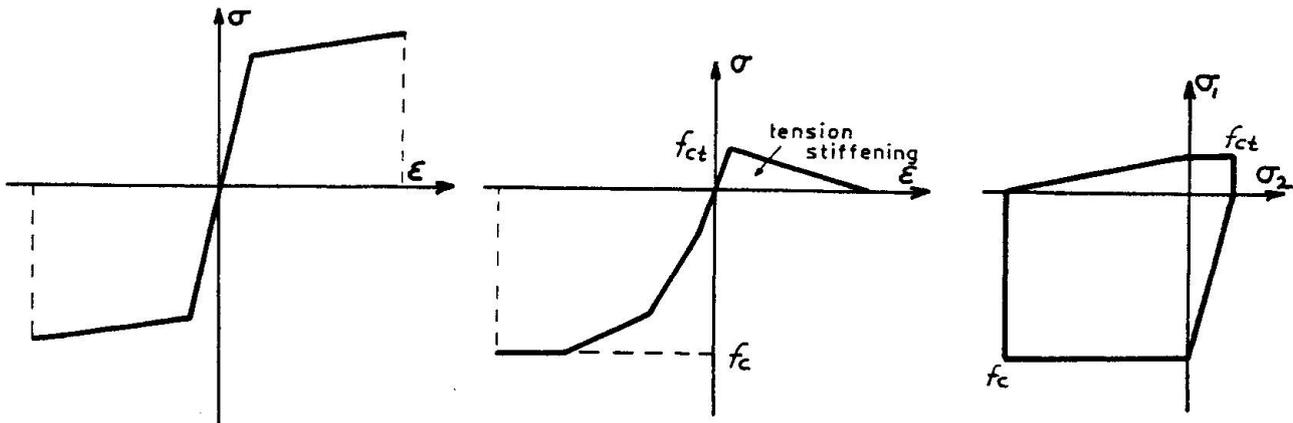
Fig. 3   Possible stress-strain   Possible stress-strain relation
             relation for steel   for concrete together with assumed
                                             failure surface

The stress-strain relations that are used for concrete in biaxial stress-states, are also derived from the uni-axial stress-strain relations. At present the relations that are used can be expressed as:

uncracked region:
$$\begin{bmatrix} d\ \sigma_{xx} \\ d\ \sigma_{yy} \\ d\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \tfrac{1}{2}E \end{bmatrix} \begin{bmatrix} d\ \varepsilon_{xx} \\ d\ \varepsilon_{yy} \\ d2\varepsilon_{xy} \end{bmatrix}$$

If in one of the principal directions, say direction 1, the tensile strength is exceeded the relation used is:

cracked region:
$$\begin{bmatrix} d\ \sigma_{11} \\ d\ \sigma_{22} \\ d\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \alpha\tfrac{1}{2}E \end{bmatrix} \begin{bmatrix} d\ \varepsilon_{11} \\ d\ \varepsilon_{22} \\ d2\varepsilon_{12} \end{bmatrix}$$

in which α is a constant to simulate the effect of aggregate interlock. If in future the other subprojects of 'Betonmechanica' on bond and cracking will be finished, it is expected to improve the three by three stiffnessmatrix and make it more dependent of the strains ε_{11}, ε_{22} and ε_{12}

Within the failure surface the stress-strain relation is regarded to be elastic. A similar assumption is made for steel and bond, the failure criteria (one-dimensional) being constituted by the extreme strains respectively extreme slip values given in the relevant relation.

3. COMPARISON FOR THE IDEAL MODEL (UNIT WEB EFFECTIVENESS FACTOR).

NIELSEN et al. [4] found for beams with vertical stirrups a relation between the nominal ultimate shearstress τ_u and a coefficient ω which is the mechanical degree for the amount of stirrups.

Fig. 4 displays this relation. The nominal shear stress τ_u is found by dividing the shear force V through the web cross-section area bh. The coefficient ω is defined by the quantities ρ, f_y and f_c, of which ρ is the degree of stirrup reinforcement, f_y the yield strength of steel and f_c the yield strength of concrete. The web effectiveness is indicated with the character υ.
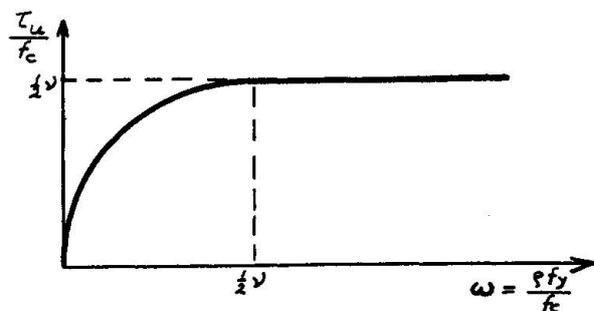

Fig. 4   Relation between τ_u, ω and υ
             according Nielsen et al.

As has been said in chapter 1, the unit web effectiveness factor corresponds with a Stanil/1 calculation for a beam with infinite rigid tensile and compression stringers. Fig. 5 shows which beam has been chosen and which material properties have been used.
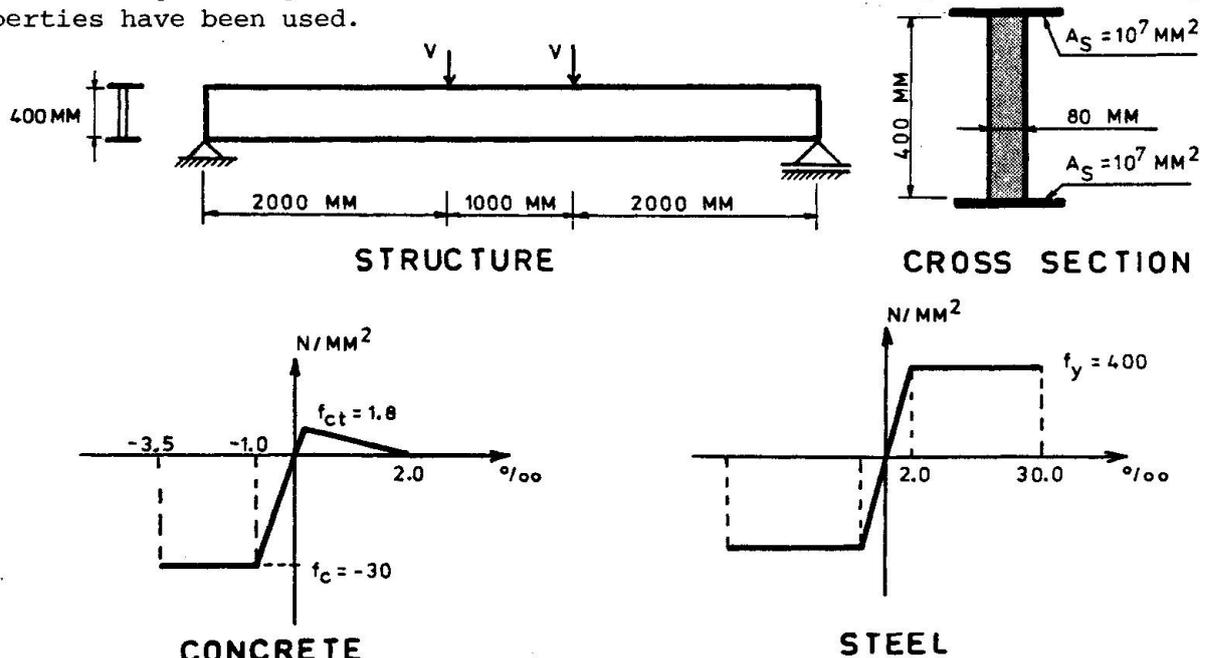

STRUCTURE   **CROSS SECTION**

CONCRETE   **STEEL**

Fig. 5   Survey of the structure that was investigated and the stress-strain relations used for a unit web effectiveness factor.

The calculation with Stanil/1 has been executed for several amounts of web reinforcement, corresponding with ω-values 0.025, 0.05, 0.075, 0.1, 0.2, 0.35, 0.5 0.6 and 0.8. In fig. 6 the results are plotted in the diagram for $\upsilon = 1$, showing perfect agreement. In all cases sufficient concrete ductility seems to be ensured to allow a plastic approach. It may therefore be concluded that in the plastic shear capacity prediction the web effectiveness factor is not needed because of the limited ductility of concrete but because of the fact that the axial strains caused by bending cannot be neglected in practical structures. The ductility of the structure as a whole is then limited due to the additional strains in the compressed concrete zone. This will be the subject of chapter 4.
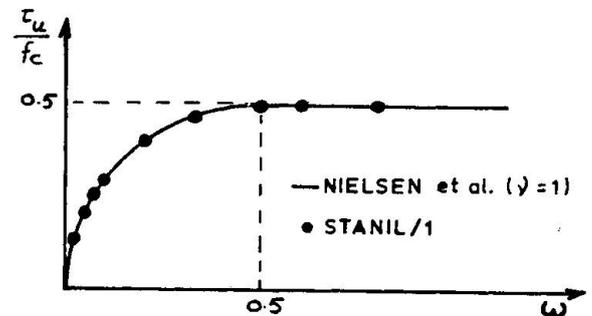

Fig. 6 Full agreement between Stanil/1 results and plastic analysis for $\upsilon = 1$

As has been said in chapter 1, the program Stanil/1 provides also additional information. In fig. 7 the load-deformation curves for the beams are shown for a number of ω-values. An extensive discussion cannot be given in this short paper, but the most important phenomena will be summarized.

- The stiffness after cracking decreases with decreasing amount of stirrups. The curves are less smooth in case of a low percentage web reinforcement. For these cases the stress-strain relation for concrete has to be refined, especially the tension stiffening.
- The initial crack inclination is $45°$ but it changes with increasing load
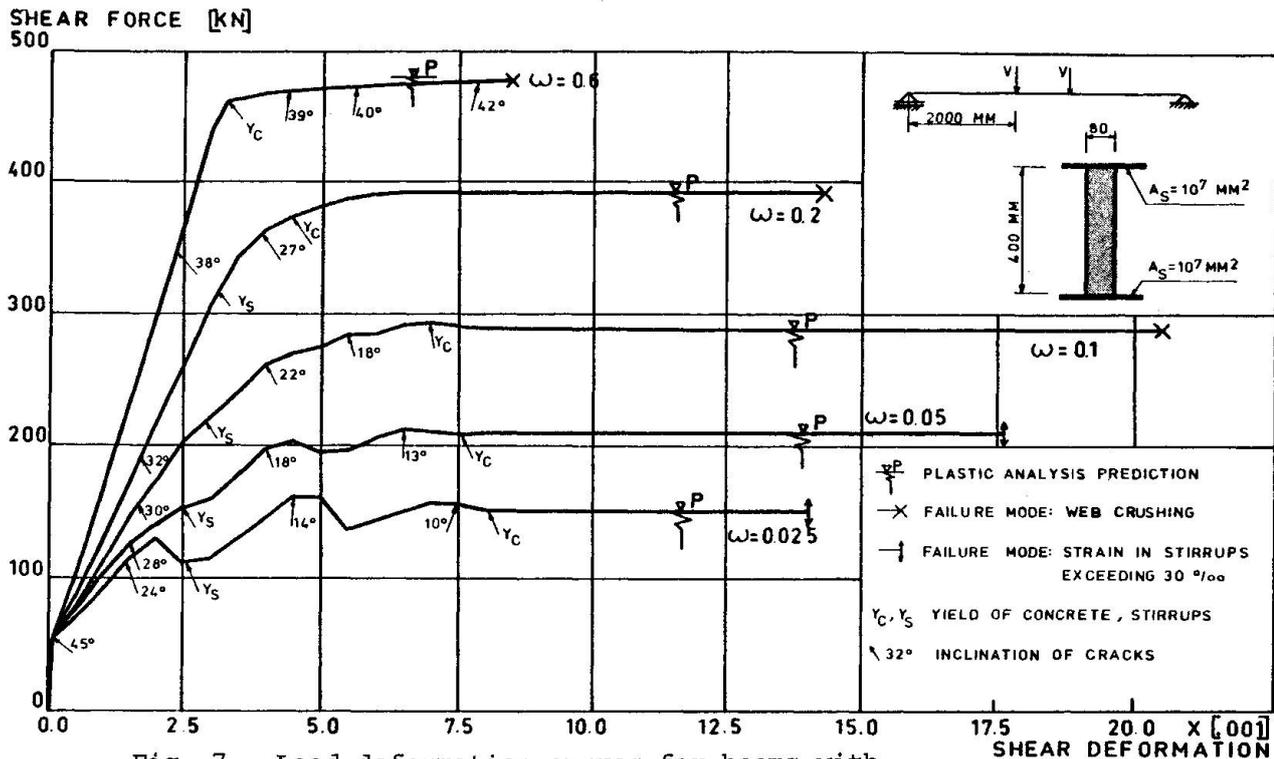
SHEAR FORCE [kN]

Fig. 7   Load deformation curves for beams with
          infinitely strong main bending reinforcement

● - At failure of the beam the web concrete yields for every value of ω, but the web reinforcement not always does. Depending on the value of ω one can notice three regimes with different failure phenomena:

- For values of ω greater than 0.5 the failure mode is web crushing; the stirrups do not yield at failure.

- For values of ω between 0.1 and 0.5 the failure mode is also web crushing, but now the stirrups do yield.

- For values of ω smaller than 0.1 the yielding web concrete does not crush. Now the (average) strains in the yielding stirrups get very large and exceed $30^{\circ}/oo$. In practice this will probably mean that stirrups crossing dominant cracks will break. However, at this failure the shear deformation $2\varepsilon_{xy}$ has allready reached a big value, which means that sufficient ductility can be ensured.

4. COMPARISON FOR PRACTICAL CASE (WEB EFFECTIVENESS FACTOR SMALLER THAN UNITY)

It has been explained in chapter 1 that experimental results only correspond with the plastic model of NIELSEN et al. when a web effectiveness factor smaller than unity is introduced in the plastic model. In [4] it has been shown that test results of LEONHARDT and WALTHER agree with a plastic analysis for $\nu = 0.86$. Two of these tested beams (TA1 and TA4) have been analysed with STANIL/1. The load system is the same as applied in fig. 5. The distance between the support and the transverse load V was divided into three elements. The T-shaped beams have been modelled for this purpose into beams with by reinforcement steel in the compression zone with the same stiffness. This is allowable if the failure mode is not controlled by the flange. The geometrical data and material properties were taken from [6]. From experience gained so far we have learnt that the use of the prism strength in Stanil/1 shows a good agreement with tests. The results of the analysis are shown in fig.8. It can be seen to which extent they agree with the plastic analysis for $\nu = 0.86$ of NIELSEN et al. and with the test results of LEONHARDT and WALTHER. We may conclude that Stanil/1 is capable to predict the ultimate nominal shear load for such cases fairly well.
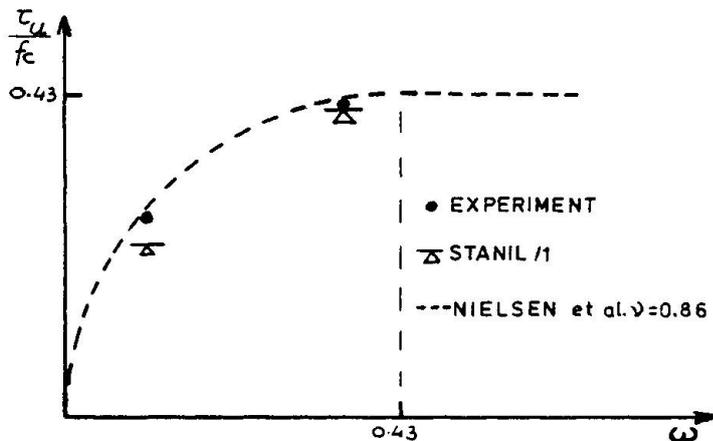
Fig. 8 Comparison of the ultimate
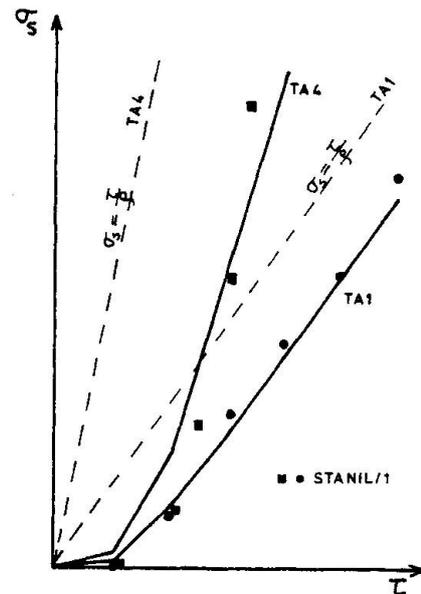strength from Stanil/1 and
experiment.

Fig. 9 Comparison of
stirrup stresses
σ_s from Stanil/1
and experiment

• From the tests it is known in which way the steel stress σ_s in the vertical web
reinforcement develops when the shear load (and thus the nominal shear stress τ)
increases. This experimental result is reproduced in fig. 9, together with the
dashed lines which would apply if the truss-analogy would hold (with inclined
bars under 45 degrees). The shown curves were found by averaging the value of
four stirrups in a certain position along the beam. The Stanil/1 results in
fig. 9 are averaged values for the corresponding points. These results fit in
a satisfactory manner with the experimental data, which means that the program
Stanil/1 seems capable to simulate the beam phenomena under realistic conditions.

6.   REFERENCES

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