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Nonlinear Behaviour of Reinforced Concrete Beams

Comportement non linéaire de poutres en béton armé

Nichtlineares Verhalten von Stahlbetonbalken

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SUMMARY
The computer-oriented method for the analysis of reinforced concrete beams in the nonlinear range is based upon the finite-element approach. The method is suitable for the investigation of the structural behaviour taking nonlinear material properties into consideration, and allows the simulation of crack formation and crack propagation. The structures are analyzed as two-dimensional problems.

RESUME
La méthode, basée sur les éléments finis et sur le traitement électronique des données, est utilisée pour l'analyse des poutres en béton armé dans le domaine non linéaire. Elle permet l'étude du comportement des poutres, tenant compte des caractéristiques non-linéaires des matériaux, ainsi que la simulation de la formation et de la propagation de fissures dans le béton. La méthode de résolution se limite à l'étude des problèmes plans.

ZUSAMMENFASSUNG
Die hier beschriebene EDV-orientierte Methode zur Analyse von Stahlbetonbalken im nicht linearen Bereich beruht auf dem Finite-Element-Ansatz. Sie gestattet die Untersuchung des Trägerverhaltens unter Berücksichtigung nichtlinearer Materialeigenschaften sowie die Simulation der Bildung und Fortpflanzung von Rissen im Beton. Die Lösungsmethode beschränkt sich auf ebene Probleme.

1. INTRODUCTION

In this paper a method based on the finite element approach to simulate the behaviour of reinforced concrete beams is presented. The solution method is a combined iterative and step-by-step procedure based upon the matrix displacement method. The beam-structures are analyzed as plane stress problems. For each load increment, repeated elastic solutions are performed until the displacements meet a specific tolerance.

The mathematical model consists of an assemblage of triangular concrete plate elements, steel bar elements and bond links. The displacement fields are assumed to be linear for all three parts. The elastic constants which are needed in the derivation of the element stiffness matrices are extrapolated from the pertinent uniaxial stress-strain curves. For all elements, these functions are approximated by piecewise linear polygons. The appropriate values of the material constants are found by entering the stress-strain diagram at the correspondent values of principal strains. The random change in structural configuration due to cracking in the concrete is treated by cutting the concrete triangular element in the direction perpendicular to the principal tensile stress. Bond between concrete and steel reinforcements is simulated by discrete spring-like bond links. The influence of time-dependent effects such as creep or relaxation is neglected.

The method has proven to be an effective tool for the study of crack propagation.

2. FINITE ELEMENT PROPERTIES

The finite element idealization relevant to this investigation is displayed in Figure 1.

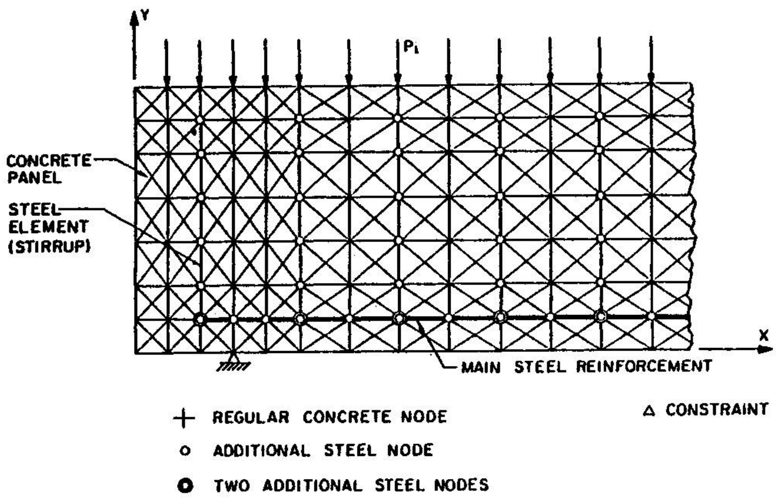


Figure 1. Finite element assemblage of a singly reinforced beam

Three kinematically and geometrically dissimilar elements as shown in Figure 2 have been chosen as basic components of the model.

For the concrete elements the constant-strain triangular panel has been adopted. This mainly because yielding takes place throughout the whole element. Thus, no problems arise in determing the state of stress. The displacement functions are:

$$u_x = c_1 x + c_2 y + c_3 \qquad 1)$$

$$u_y = c_4 x + c_5 y + c_6 \qquad 2)$$

The reinforcement occupies a relatively small volume compared to that of the concrete. It is therefore justifiable to idealize the steel tendons by simple two-force members.

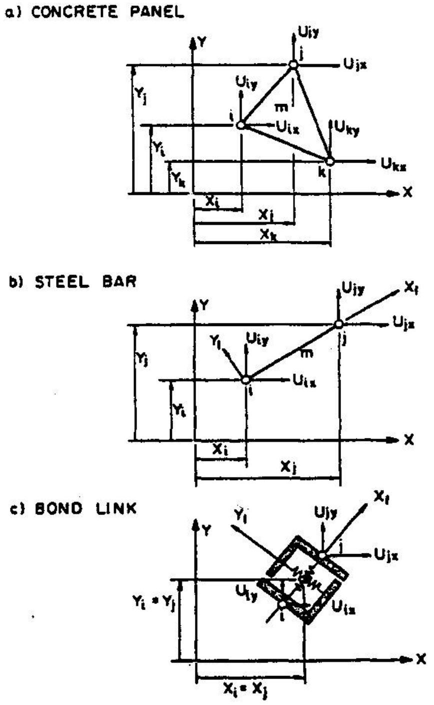


Figure 2.

Components of finite element model

To account for bond slip, the steel must be attached to the concrete by a special connection mechanism. The bond link used here is designed to allow for relative displacements between the steel bars and the concrete panels. The links are dimensionless. Nevertheless, additional nodes must be provided to permit relative displacements between adjacent concrete and steel nodes.
For steel and bond elements the displacement function used is:

$$u_1 = c_7 x + c_8 \qquad 3)$$

In the assemblage of Figure 3 the nodes of the steel bars and the connecting springs originally occupy the same geometrical position as their corresponding concrete joints.

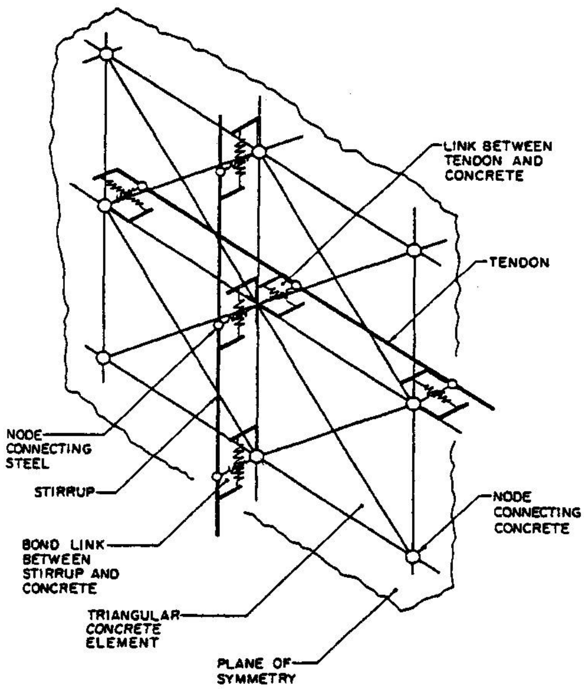


Figure 3.

Configuration of bond links

3. CONSTITUTIVE EQUATIONS

As constitutive equations for the concrete panels the following expressions in terms of principal values were used:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \begin{bmatrix} 1 & \nu_{21} & 0 \\ \nu_{12} & n & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

For steel and bond elements the constitutive equations are of the following form:

$$\begin{bmatrix} \sigma_{x1} \\ \sigma_{x2} \end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{x1} \\ \varepsilon_{x2} \end{bmatrix}$$

Where ε_{x1} and ε_{x2} are the relative displacements between the steel and the corresponding concrete node.

The influence of a crack on a continuous triangular concrete element is treated by introducing a cut in the direction perpendicular to the calculated principal tensile stress σ_1.
In this new state the element no longer has any stiffness normal to the crack surface (Figure 4). Consequentely, the concrete may be considered as a uniaxial stress condition parallel to the second principal axis. This assumption results in the following stress-strain relationship:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

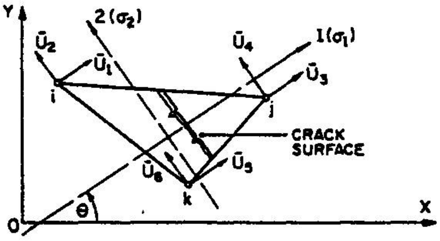


Figure 4. Cracked Element

4. STIFFNESS MATRICES

The stiffness matrices of the three types of elements may be derived by the standard equations

$$[K] = \iiint [b]^t [D] [b] \, dV$$

where $[b]$ is the strain-displacement transformation matrix and $[D]$ the matrix of elastic constants.

For the bond link the stiffness matrix is similar to the well-known matrix for a two-force member. The matrix for the cracked concrete element reduces to

$$[kn] = \begin{bmatrix} 0 & & & & & \\ 0 & 0 & & \text{sym} & & \\ 0 & x_{32}^2 & 0 & & & \\ 0 & -x_{32}x_{31} & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & x_{21}x_{32} & 0 & -x_{21}x_{31} & 0 & x_{21}^2 \end{bmatrix}$$

5. ITERATIVE PROCEDURE

The method presented here is based upon an iterative, incremental load approach. For each load increment, the whole structure is repeatedly solved as an elastic problem until closure. Consider an arbitrary concrete element during load increment i. Assume that at the end of the previous step the principal strains $\{\varepsilon_{p,i-1}\}$ and stresses $\{\sigma_{p,i-1}\}$ have been established. Based upon these values the element may be in any one of the following conditions:

a. Type 1: Elastic, isotropic

b. Type 2: Elastic, anisotropic

c. Type 3: Inelastic, anisotropic

d. Type 4: Cracked

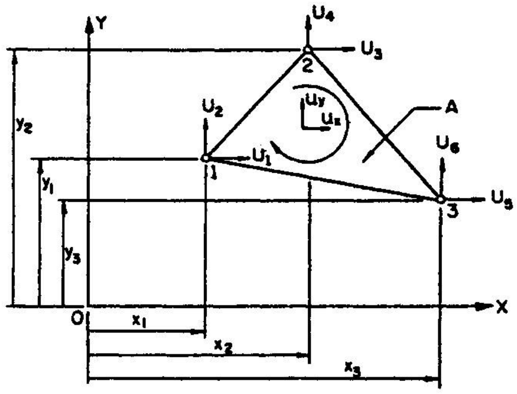


Figure 5.

Arbitrary triangular concrete element

The four cases may be visualized diagrammatically in Figures 6 and 7.

In the present computer program, principal strains $\{\varepsilon_{i-1}\}$ are used to determine the relevant material constants. After the proper modulus of elasticity E_i, and Poisson's ratio, ν_1, have been found for each element, the $[D]$ matrices are generated.

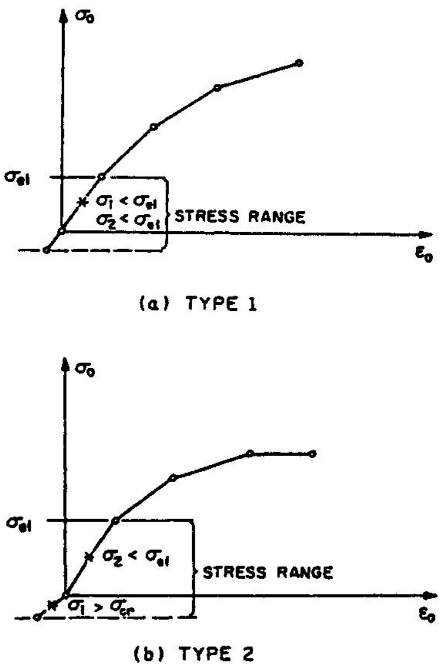


Figure 6.
Classification of concrete elements in the elastic range

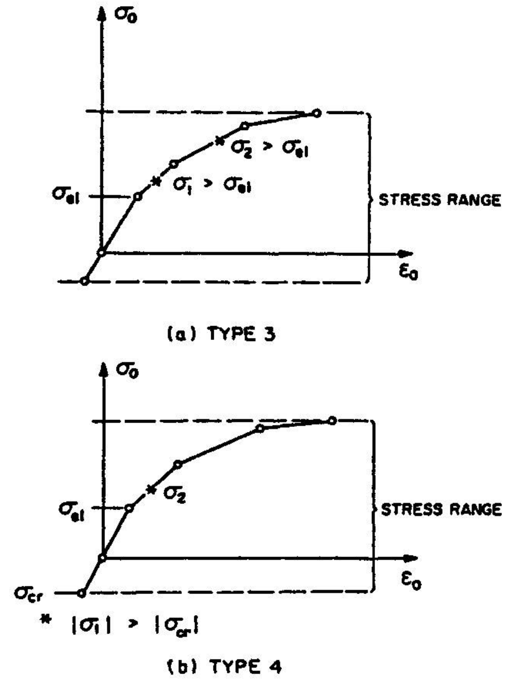


Figure 7.
Classification of concrete elements in the inelastic range

The elemental stiffness matrices $[k_i]$ follow immediately

$$[k_i] = [b]^t [D] [b]$$

or, for cases 2, 3 and 4

$$[k_i] = [R]^t [b]^t [D_i] [b] [R]$$

Next, the total stiffness matrix is assembled and solved for the incremental displacements. The incremental strains are now evaluated as

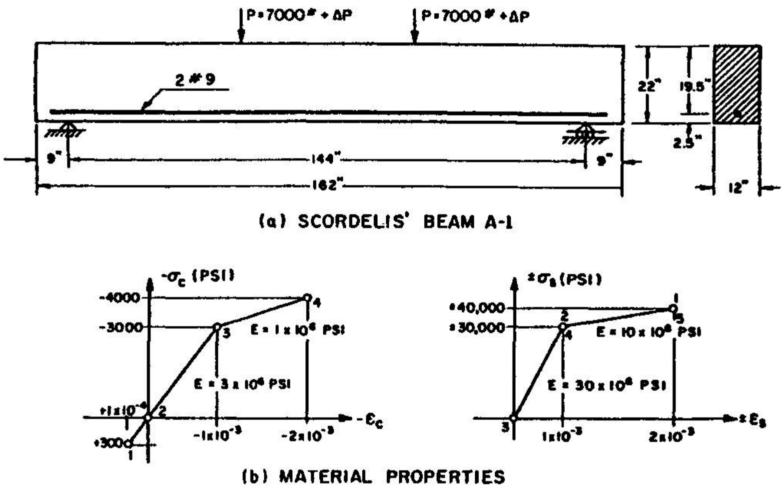
$$\{\Delta e_i\} = [b_i] \{\Delta U_i\}$$

and added to the total strains e_{i-1} of the preceding step to give the new total strains

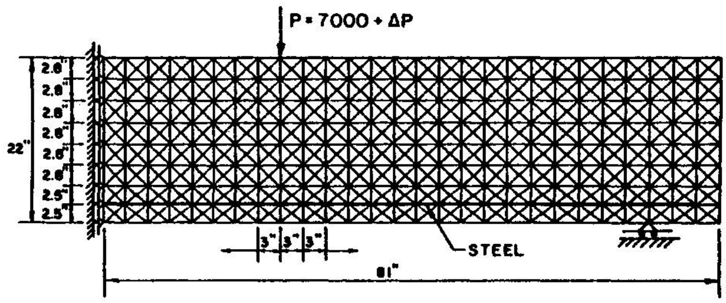
$$\{e_i^1\} = \{e_{i-1}\} + \{\Delta e_i\}$$

These values constitute a new strain situation with a corresponding new set of principal strains $\{\varepsilon_{p,i}\}$. The material properties of the following iteration cycle are again extrapolated from the stress-strain curve. The iteration is stopped after a specified tolerance is reached. Before proceeding to the next increment, all total stress and strain values are updated and stored. Similar treatment is imposed upon the reinforcements and bond links. However, the procedure here is much less involved since the matric $[D]$ reduces to a single term E_1.

Example Problem 1:

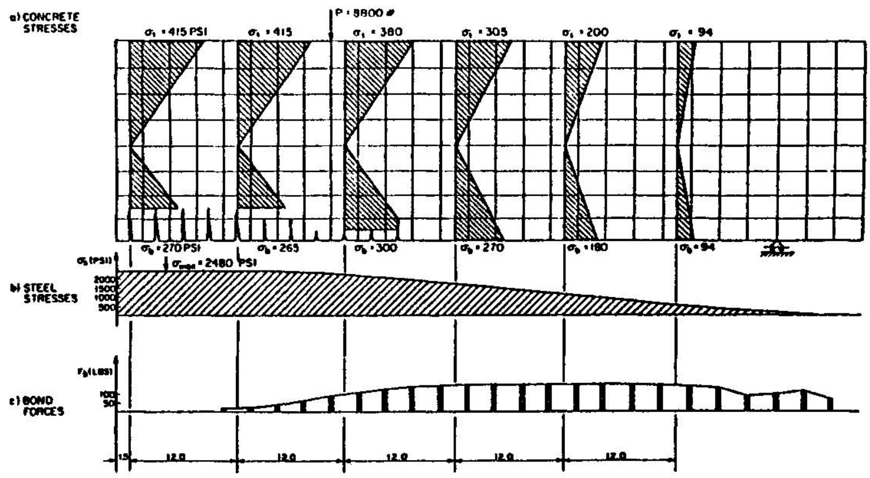


Example Problem 1: Scordelis' Beam A-1



NUMBER OF TRIANG. ELEMENTS	NTEL	=	864
NUMBER OF NODES (INCL. STEEL)	NUMNOD	=	495
NUMBER OF LOADS	NLOAD	=	1
NUMBER OF BOUND. COND.	NBOUND	=	10

Mathematical Model of Beam A-1



Example Problem 1: Stresses at P = 8800 Lbs.

Example Problem 2:

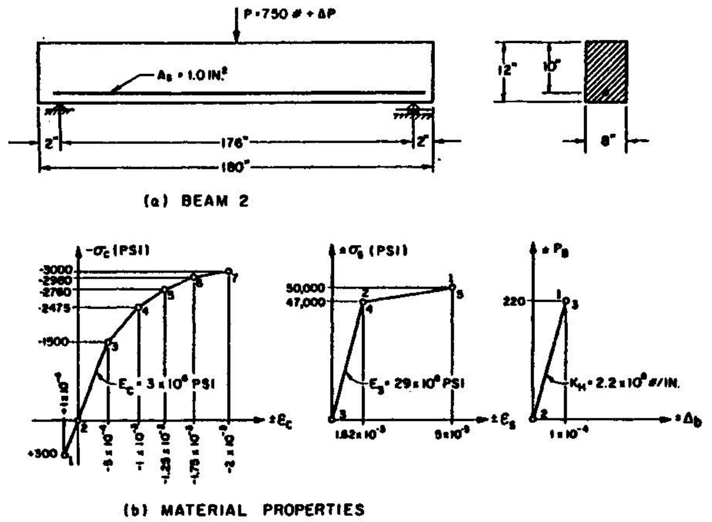


Figure 27. Example Problem 2: Simple Beam Loaded at Midspan

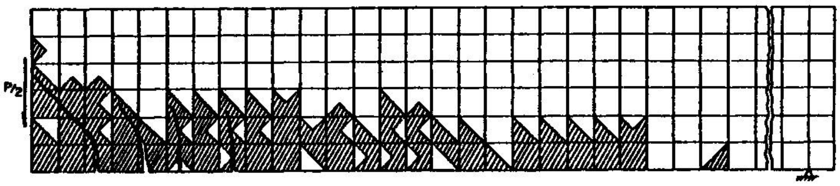
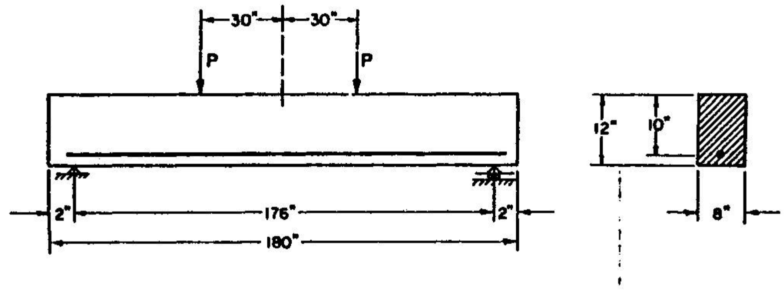


Figure 29. Example Problem 2: Crack Pattern at P = 4000 Lbs.

Example Problem 3:



FOR MATERIAL PROPERTIES, SEE FIGURE 27.

Figure 30. Example Problem 3: Simple Beam Loaded Symmetrically by Two Concentrated Loads

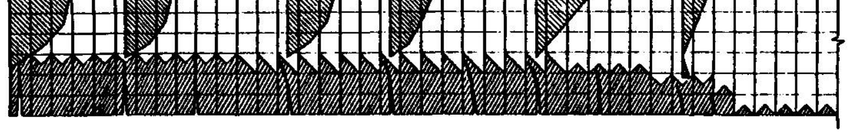


Figure 31. Example Problem 3: Crack Pattern and Stress Distribution at P = 10,000 Lbs.