Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 29 (1979)

Artikel: Limit analysis of reinforced concrete shells of revolution

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DOI: https://doi.org/10.5169/seals-23557

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Limit Analysis of Reinforced Concrete Shells of Revolution

Charge ultime de surfaces de révolution en béton armé

Traglast von rotationssymmetrischen Stahlbetonschalen

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SUMMARY
The paper deals with a static and kinematic formulation of the collapse load problem of reinforced concrete shells of revolution. A numerical method of finding the collapse load intensity, the collapse mode and the stress resultants within the plastified zones is represented.

RESUME
L'article traite de la formulation statique et cinématique de la charge ultime de surfaces de révolution en béton armé. L'article présente une méthode numérique permettant de déterminer la charge ultime, le mode de rupture et les contraintes résultantes dans les zones plastiques.

ZUSAMMENFASSUNG
Eine statische und kinematische Formulierung des Traglastproblems von rotations-symmetrischen Stahlbetonschalen wird gegeben. Eine numerische Methode zur Bestimmung der Kollapslast, des Bruchmechanismus und der Spannungsresultierenden innerhalb der plastifizierten Bereiche wird angegeben.

1. INTRODUCTION

Tensile failure of concrete is the primary ingredient of the non-linear behaviour of concrete structures. Crack-initiation and slow crack-propagation up to failure form a very complex mechanicsm which is still not fully understood. Thus predictive statements must be treated with considerable caution in view of the limitations of the underlying fracture model, the high sensitivity of the constitutive parameters and the numerical distortion of the cracking process.

In two previous publications by the authors [1, 2] , the smeared finite element approach to cracking was explored in detail. In particular the high sensitivity of the ultimate load of a thick-wall tube was noted with regard to small variations of the tensile strength as well as the finite element mesh. In view of the limited experimental evidence a careful study was proposed for crack-initiation and crack-propagation in plain concrete in the presence of large stress gradients. The primary objective was to assess the redistribution capacity of such a tensile specimen during the cracking process. In particular, the question of slow crack growth versus instable fracturing was of main concern. From the standpoint of numerical prediction, the study revolved around the fundamental postulates of smeared and discrete crack analysis methods.

As an example for the analysis of cracking we consider a thick-walled concrete ring subjected to internal pressure. This problem was selected to study the strength and deformation behaviour of a structural component with a non-uniform tensile stress field. The experimental study was carried out at the IBIW III of the Technical University Munich, the results of which were reported recently in reference [3] . Parallel, numerical investigations were carried out at the ISD of University of Stuttgart, of which some results are reported below [2] .

2. SMEARED CRACK APPROACH

It is this approach which is normally adopted for the ultimate load analysis of structures. In compression, this method is fully acceptable for modelling degrading material behaviour e.g. because of progressing micro-cracking in the form of nonlinear elastic, elastoplastic and endochronic constitutive models. In tension, the limited strenght is of primary importance, rather than the nonlinear deformation behaviour. In particular the discontinuous reduction of strenght to "no tension" behaviour is of interest in the case of brittle fracture. The ensueing stress redistribution is normally accomplished by initial load iteration which imposes this constritutive restraint by iterative correction of the linear elastic response. The smeared approach distributes the effect of localized cracking over a contributory area, for example a finite element, in which the strenght degrades continuously according to the concepts of a continuum. Thus, the finite element method with its weak equilibrium statements is particularly suited for the smearing or better the averaging of constitutive statements in finite neighbourhoods.

At this stage there is no need for a detailed account of the finite element initial load strategy, the basic concepts are well established and are summarized for the analysis of concrete fracture for example in ref. [1] . In our example the pressurized ring is subjected to biaxial tension-compression, whereby the circumferential tension is responsible for radial cracking leading ultimately to fracture. Thus, the material behaviour is virtually linear elastic up to the tension cut-off, from which on ideally brittle and ideally plastic models bound the actual softening behaviour of the heterogeneous concrete material.

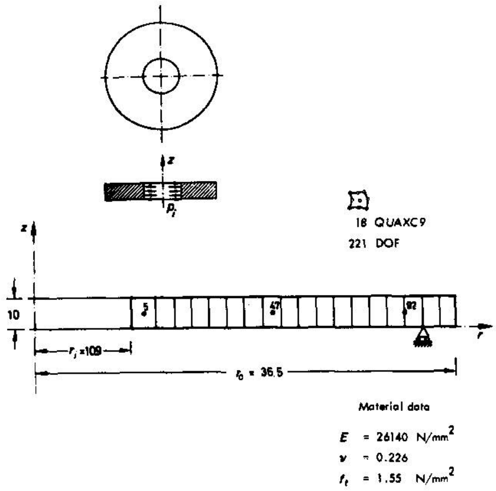

FIG. 1 Configuration and axissymmetric idealisation

Fig. 1 shows the axissymmetric idealisation of the concrete ring with 18 QUAXC9 elements and the underlying material data. Assuming that there is no axial effect due to friction at the supports or due to non-uniform shrinkage, we can restrict our analysis to a single element layer, in which cracking takes place in the radial direction only. Clearly, the axissymmetric idealization implies that the radial cracks are distributed uniformly over the circumference and that the non tension zones reduce the thickness of the ring accordingly. The material parameters are those of the original test data before they were adjusted for the age of the actual experiment, whereby the splitting tensile strenght is used directly as strength parameter $f_t = \beta_{sz} = 1.55 \text{ N/mm}^2$ without further manipulation.

In the smeared crack analysis of brittle and ductile failure behaviour the excess tensile tangential stress is redistributed whence the tensile strength f_t is reached. In this way the actual softening behaviour is bounded by the two limiting cases of discontinuous strength reduction for no-tension behaviour and non strength reduction for ideally plastic behaviour. Clearly, plain concrete exhibits primarily brittle behaviour in tension, however, the localization of discrete cracks is always accompanied by extensive microcracking over a finite neighbourhood (analogous to the tributory area of the smeared approach), such that a continuous softening model would be most realistic. However, we should be aware that this softening branch is actually not a material property, it rather depends on the boundary conditions, the stress gradients or rather the redistribution capacity of the structure and last not least on the inhomogeneous composition of concrete. In this spirit, the softening model is a computational tool rather than a constitutive property for describing the reserve strength of the structure beyond that of an isolated material point.

The results of the smeared crack analysis are compared in Fig. 2 with the experimental results at sector 8, when the primary crack leads ultimately to rupture. For the proper assessment of the axissymmetric prediction the tangential strain data of the eight sectors is also averaged along the inner and outer surface of the ring and along the midsurface. For completion, Fig. 3 shows the overall load-deformation behaviour at the three surfaces, in which the circumferential variations of Fig. 2 are averaged. All figures illustrate clearly the low strength prediction of the brittle postulate and the high strength value of the ductile model. Similar, the rupture strain more than doubles when we go from the brittle model to the ductile one. The local behaviour in Fig. 2 illustrates the apparent ductility at the sector 8 with the primary crack, where the deformation increases continuously up to ε_r = 0.54 % at rupture (this includes the crack opening). On the other hand the average values of the eight sectors show strain levels which are of the order of those normally put forward by concrete technologists for the maximum level of tensile strain ε_{ave} = 0.15 % .

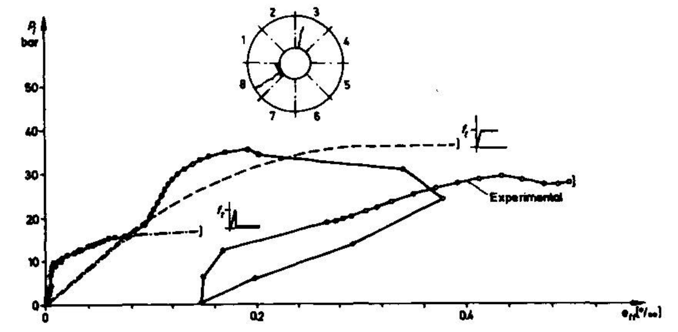

FIG. 2 Comparison of numerical and experimental results at the inside of sector 8

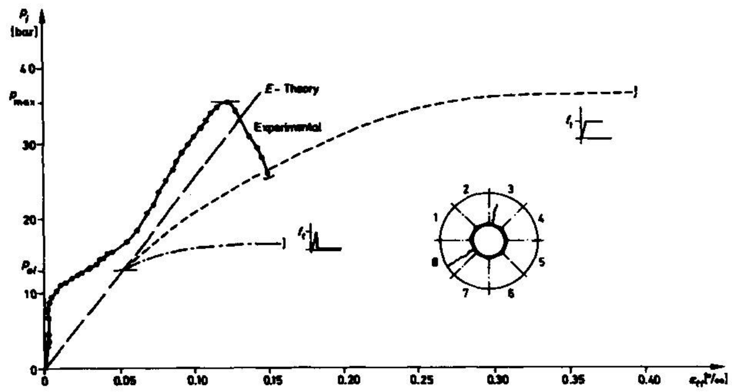

FIG. 3 Average response behaviour at inside

The tensile strength value of f_t = 1.55 N/mm^2 restricts the elastic regime to pressure levels below p^{el}_{max} = 13.0 bar . Table 1 summarizes these numerical and experimental results together with the average strain levels at the inner surface. We observe that the brittle failure model increases the maximum pressure only slightly from the elastic limit p^{el}_{max} = 13.0 bar to p^{br}_{max} = 16.62 bar , while the ductile failure model mobilizes considerable strength reserves in the structure, p^{du}_{max} = 36.50 bar . Note that these numerical strength values were obtained by an incremental iteration algorithm (initial load method with constant stiffness), where the ultimate load capacity was determined by successive refinement of the load steps. The actual failure load was localized by reducing the load step down to Δp_{min} = 0.2 bar near collapse and iterative corrections in order to assure that the cracking process reached the outside ring surface at ultimate pressure.

APPROACH	p_{max} [bar]	$\varepsilon_{tt}\big\vert_{r=r_i}$ [‰]	$\sigma_{tt} = \sigma_{tt}(r)$ $[N/mm^2]$
Experiment	35,48	0,15	-
Elastic	13,00	0,06	f_t
Brittle	16,62	0,16	f_t $f_t = 1.55$ 0,486 r_o
Ductile	36,50	0,40	f_t

TABLE 1 Results of axissymmetric crack approach

The average response behaviour in Fig. 3 indicates clearly that there is some mechanism which delays the actual response as compared to the elastic prediction. If we neglect this initial stiffening regime, which is most likely caused by friction effects at the supports, then the subsequent response regime is nearly linear elastic. The gradual softening due to internal microcracking is most pronounced at the inner surface loading to a gradual deviation from the initial linear elastic behaviour before failure. Note that a tensile bending strength of $f_t = 4.05\ N/mm^2$ the most simple elastic failure postulate would lead to a very accurate strength prediction of the ring $p^{el}_{max} = 33.86$ bar . However, it is doubtful if the high tensile bending strength can be mobilised and applied to the ring where the circumferential stresses are entirely in tension (without crack arresters due to a compression zone).

Since the brittle and ductile failure models account for gradient effects by the stress redistribution capacity of the structure, they should resort to the centric strength values. Therefore, some gradual softening must be taking place in order to increase the ultimate load capacity of the brittle failure model (the ultimate strength of the brittle model is increased by approximately 20%). On the deformation side the ductility or rather the tangential rupture strain reaches twice the value at the elastic limit, see Table 1.

Altogether, the experimental data would compare quite well with a continuous softening model whose prediction of strength and ductility falls somewhere between the results the ideally brittle and ductile failure approach. The brittle postulate is certainly too low even if we would increase the strength of the value of the tensile bending test $f_t = 1.97\ N/mm^2$ since radial cracking does not lead to a uniform reduction of the load carrying area. In contrast, the ideally plastic model infers unlimited ductility, a material property which cannot be mobilized in plain concrete, not even in compression.

3. DISCRETE CRACK APPROACH

The experimental evidence clearly indicates that the initial micro-cracking leads ultimately to the localization of a single discrete crack at failure. Therefore, the question arises if a discrete crack analysis provides additional insight into the actual fracture mechanism. To this end, we examine first the concepts of linear fracture mechanics where we essentially start from an existing crack configuration and study its stability. Thereafter, we explore the possibility of predicting slow (stable) crack growth in the tensile ring specimen.

For the discrete crack analysis we consider a quarter of the ring configuration which implies symmetry along the x- and y-direction. With other words, we assume that the primary crack along the positive direction of the x-axis is accompanied by a secondary crack in the negative direction. Recall that the formation of two opposite cracks was actually observed in the experiment near the ultimate pressure.

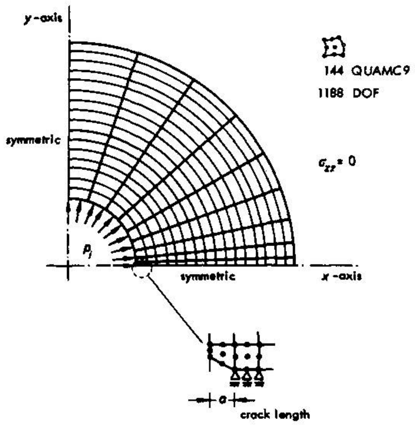

FIG. 4 Plane stress idealisation of discrete crack

Fig. 4 illustrates the finite element mesh with 144 QUAMC9 membrane elements. A discrete crack is introduced simply by releasing the kinematic nodal constraints at the x-axis of symmetry (for example in slow crack propagation the supressed degrees of freedom are released node by node according to the crack criterion).

Traditionally, the principal question of fracture mechanics circles around the stability of a given crack configuration. In essence we start already from a predetermined discrete crack and study stability (stationary crack versus unbounded crack growth) by comparing for example the stress intensity K_I at the crack tip with the critical value K_{IC} from material testing. The basic postulate rests on the assumption that crack propagation is controlled exclusively by the local stress intensity at the crack tip. Thus, the linear theory of fracture mechanics is a priori not concerned with the initiation of cracks in intact specimens without flaws and even less with the slow extension of a given crack. The crucial stability problem is solved by testing of so-called fracture mechanics specimen which yield the critical value of stress intensity or equivalent quantities such as the crack opening displacement or the crack extension force.

In what follows, we apply these concepts which were originally established for the brittle fracture of metals. To this end a discrete crack is introduced along the x-axis of symmetry by releasing the nodal degrees of freedom in this zone. The stress intensity factor is computed for different crack lengths via the compliance method where K is a measure of the energy release rate as the crack length a is increased by Δa . Clearly, there are several other methods for the finite element analysis of K_I , such as the direct calculation from the resulting stress or displacement field, via contour integrals or singularity elements and nonlinear mappings. However, in conjunction with the finite element displacement method the energy approach is the most appropriate global technique.

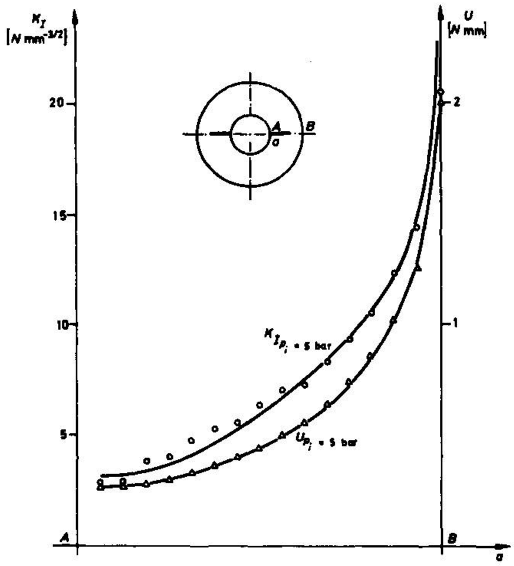

FIG. 5 Stress intensity K_I and strain energy versus crack length a

Fig. 5 illustrates the variation of the stress intensity value K_I in dependence of the crack length a together with the elastic strain energy U for an internal pressure $p = 5$ bar . Note that both results yield a continuous increase of stress intensity and strain energy with increasing crack length.

Traditionally the discrete crack analysis was the first approach which was adopted for tracing slow crack growth. Originally, this method was implemented by releasing the nodal compatibility whence the principal tensile stress was reached. At this stage the stress concentration or rather the stress singularity at the crack root was removed by a coarse finite element mesh (numerical damping) such that a simple strength criterion could be applied to propagate cracking. In subsequent proposals less sensitive variables such as the crack-opening displacement or the strain energy itself were utilized as criterion for crack extension. In this context we recall that the critical value of stress intensity or energy release rate predicts only catastrophic fracture and not slow crack extension. Thus, the appropriate criterion for crack initiation and slow crack propagation would have to be determined from a set of additional test data (calibration experiments) with a rather restrictive regime of application.

Independently of the choice of crack criterion it is intriguing that the discrete crack analysis is never able to predict slow crack growth in the tensile ring specimen. Fig. 5 clearly shows that the energy (and correspondingly all local crack criteria) increases with the crack length; thus the crack extension cannot be arrested without introducing additional dissipation mechanisms, such as damping of the numerical solution, due to the spatial discretisation or continuous softening and gradual damage accumulation, respectively. It is surprising that in the case of a tensile stress field the discrete crack approach differs fundamentally from the smeared approach. While the former model predicts catastrophic failure immediately upon local crack initiation without further redistribution, the latter method accomodates slow crack extension with an associated stress redistribution. Thus spatial smearing of cracks implies in reality numerical damping because of the spatial distribution of cracks over a tributory area. In this way it corresponds to an intuitive concept of damage accumulation over a finite region, especially in conjunction with a gradual softening postulate, which is equivalent to a continuous degradation of strength until the tributory area is fully damaged.

3. CONCLUDING REMARKS

Clearly, the physical performance of the pressurized ring exhibits three response regimes, (i) the linear elastic region without damage, (ii) the hardening region with continuous damage accumulation due to progressive micro-cracking, and (iii) the localization of discrete cracks near the ultimate load accompanied by excessive damage accumulation up to rupture (continuous increase of strains at the expense of degrading strength).

None of the two numerical models is able to describe the entire process of cracking. Obviously, the smeared crack approach is most appropriate for the second response regime, which is however small in the present example, while the discrete crack method is better suited after localisation of a discrete crack takes place, although no stable crack growth can be predicted at this stage.

In our view point "smearing" and "continuous softening" are two computer-oriented strategies which introduce dissipation for stabilizing unbounded crack growth in tensile specimens. Clearly, other phenomena play a very important role for tensile fracture, here we only mention the stochastic nature of the cracking process which starts with a random distribution of strength and initial imperfections (flaws) and which continues with the probability of damage and damage accumulation with redistribution. In ref. [4] these stochastic concepts have been used successfully for interpreting size and gradient effects in tensile test specimens. However, at this stage the probabilistic fracture analysis of real concrete components is still unrealistic because of the large number of nonlinear analyses necessary for calculating the statistical distribution of strength.

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