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Finite Element Approach to Optimization of Slab Reinforcement

Optimalisation de l'armature des dalles au moyen de la méthode des éléments finis

Optimierung der Plattenbewehrung mittels finiter Elemente

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SUMMARY
A numerical procedure is presented which enables an optimization of reinforcement to be carried out in the preliminary design of concrete slabs. The method is based upon discretization of the slab by means of triangular finite elements. A rigid-plastic behaviour of the slab is assumed. The reinforcement volume is minimized by linear programming taking into account technological constraints where necessary.

RESUME
La méthode exposée permet d'optimaliser l'armature des dalles en béton dans une phase préliminaire du projet. La méthode s'appuie sur la discrétisation des dalles en éléments finis triangulaires et suppose que le comportement des dalles est rigide-plastique. Le volume de l'armature est minimisé par programmation linéaire en tenant compte, si nécessaire, des contraintes d'exécution pour les variables du projet.

ZUSAMMENFASSUNG
Es wird ein numerisches Verfahren beschrieben, durch welches es möglich ist, die Armierung von Betonplatten an einem vorläufigen Entwurf zu optimieren. Die Methode stützt sich auf die Diskretisierung der Betonplatte mit Hilfe von dreieckigen finiten Elementen unter der Annahme von starr-plastischem Verhalten. Das Volumen der Armierung wird durch lineare Programmierung minimiert, wobei herstellungsbedingte Schranken für die Entwurfsvariablen, soweit nötig, in Betracht gezogen werden.

1. INTRODUCTION

The yield-line theory [1] belongs to the most widely used tools of the plastic design. Despite its purely kinematical nature, this method provides, when properly used, a conservative estimate of the reinforcement of concrete slabs. This is due to such effects neglected in the yield-line theory as the steel hardening, arching and membrane action. Each of them is favourable to the safety of design.

It is rather simple to find an adequate collapse mode for a conventionally shaped and loaded plate of an uniform reinforcement. However the primary task of an engineer is rather to look for the most efficient reinforcement pattern than to analyse a given slab. This can be accomplished by means of the finite element method and linear programming as shown in a paper [2]. The aim of the present article is to recall the main features of such approach. As far as discretization is concerned the present method is similar to those proposed by Anderheggen, Knöpfel [3] and Kawai [4].

2. OPTIMUM PLASTIC DESIGN AS LINEAR PROGRAMMING PROBLEM

It is well known that linear programming (LP) is far more numerically efficient than any other method of constrained optimization. Therefore it is natural to try to convert an engineering optimization problem into the shape of the LP-problem. Considering design of reinforced concrete slabs on the ground of the ultimate load theory one has to introduce two main assumptions in order to achieve this goal:

1) the minimized volume of reinforcement should be a linear function of the principal yield moments,

2) the yield surface should be piecewise-linear.

The first assumption means that dependence of the arm of stress couple acting in the yielded cross-section upon the area of reinforcement is neglected. The second one can be regarded as numerical approximation of the true convex yield surface. Of course the consequences of such an approximation for the collapse mechanism must be taken into account.

Let an arbitrary structure be discretized in such a way that its mechanical behaviour is represented by the following vectors: a stress $\underline{s} \in E^m$, strain $\underline{q} \in E^m$, load $\underline{p} \in E^n$, displacement $\underline{w} \in E^n$ and plastic modulus $\underline{c} \in E^k$. Taking into account two basic assumptions listed above one can formulate the optimum design problem as follows:

a) a static approach -

minimize $V = \underline{1}^t \underline{c}$,

subject to:

$$\underline{C}^t \underline{s} = \underline{p}, \qquad (1)$$

$$\underline{G}^t \underline{c} - \underline{N}^t \underline{s} \geqslant \underline{0},$$

$$\underline{c} \geqslant \underline{0};$$

b) a kinematic approach -

maximize $\dot{W} = \underline{p}^t \underline{\dot{w}}$,

subject to:

$$\underline{C}\,\underline{\dot{w}} - \underline{N}\,\underline{\dot{\lambda}} = \underline{0}, \qquad (2)$$

$$\underline{G}\,\underline{\dot{\lambda}} \leqslant \underline{1},$$

$$\underline{\dot{\lambda}} \geqslant \underline{0}.$$

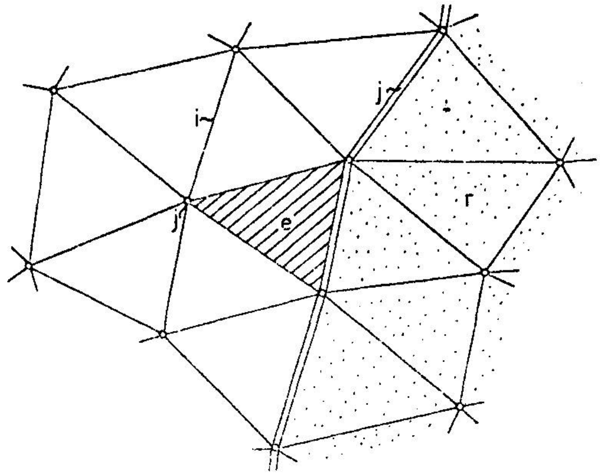

Fig. 1 Finite-element Mesh

A cost function of the primal problem (1) expresses the minimized volume of reinforcement. Here $\underline{1} \in E^k$ is a constant vector of the cost factors. The first constraint in the static approach is the equilibrium equation that relates the stress $\underline{s}$ to the given ultimate load $\underline{p}$. The second one describes a convex polyhedron of admissible stresses. The dual problem (2) reads as a search of the maximum external power over a set of the collapse mechanisms kinematically compatible with the strain rate $\underline{\dot{q}}$ that follows from the associated flow rule:

$$\underline{\dot{q}} = \underline{N}\,\underline{\dot{\lambda}}\,. \qquad (3)$$

The second constraint in (2) is the optimality condition relating the plastic multipliers $\underline{\dot{\lambda}}$ to the cost factors $\underline{1}$. For practical purposes it is advisable to replace the last constraint in (1) by

$$\underline{c}^- \leqslant \underline{c} \leqslant \underline{c}^+ \qquad (4)$$

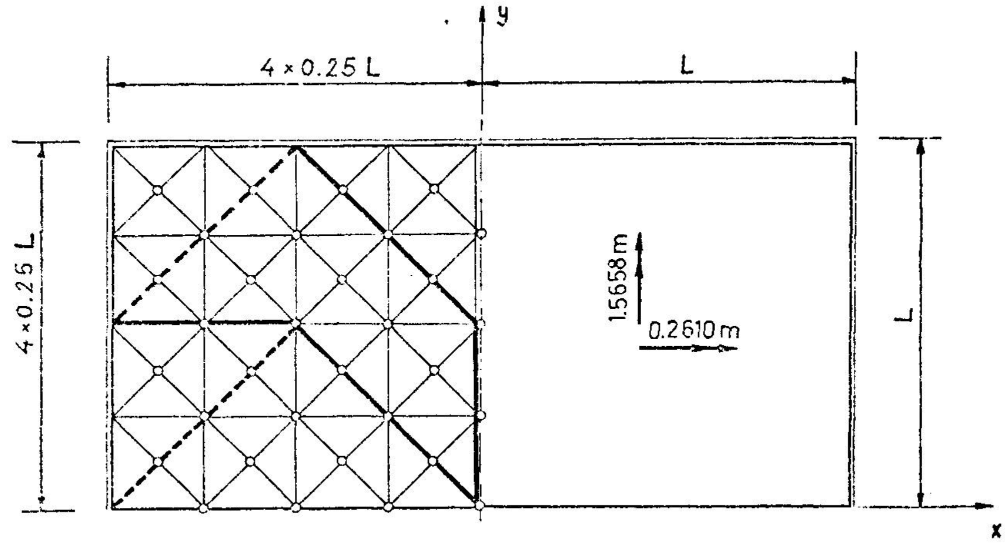

Fig. 2 Rectangular Slab with Free Edge - Optimal Solution for Single Bottom Reinforcement Grid

where $\underline{c}^-$, $\underline{c}^+$ are fixed bounds for the design variable $\underline{c}$. Since the dual problems (1)-(2) are equivalent one can use any one of them as an input for the simplex routine which provides the solution $\underline{c}^*$, $\underline{s}^*$ and $\underline{\dot{w}}^*$, $\underline{\dot{\lambda}}^*$.

3. DISCRETE MODEL OF REINFORCED SLAB

A mesh of triangular finite elements as shown in Figure 1 was chosen for discretization. It was assumed that $\dot{w}$ is linear while the moments M_x, M_y, M_{xy} are constant over an element. The slope discontinuities $\dot{\theta}_i$ along the edges of each triangle are collected into the strain rate vector $\underline{\dot{q}}$. The nodal deflection rates $\dot{w}_j$ enter the vector $\underline{\dot{w}}$. Rational technology requires the reinforcement to be composed from a small number of grids, each of them having a constant mesh and constant diameter of steel rods. Therefore prior to optimization the area of the slab should be divided into a small number of regions of constant principal yield moments. Theoretically a region can include single element but usually there are many elements in it. The adjacent regions are connected via the narrow strips that can rotate independently about their longitudinal axes. The rates of such rotations $\dot{\psi}_j$ are included into $\underline{\dot{w}}$.

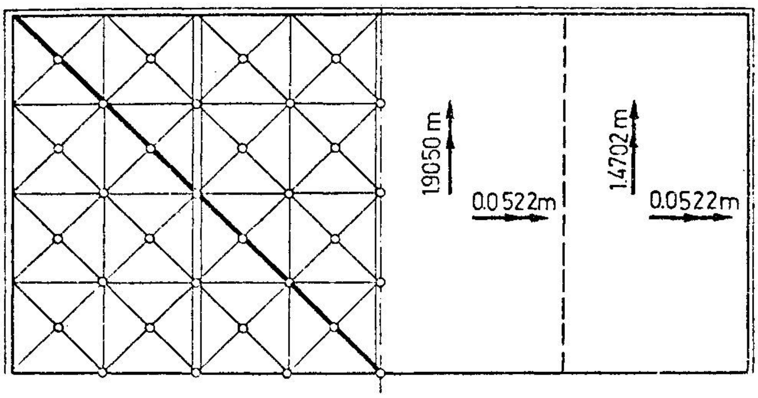

Fig. 3 Rectangular Slab with Free Edge - Optimal Solution for Two Bottom Reinforcing Grids

These connectors provide a continuous slope of the slab along the lines of discontinuity of the yield moment.

In the static description of the slab the entries of $\underline{s}$ are the bending moments M_i acting normal to the edges of triangles. The components of $\underline{p}$ are the nodal forces and the external moments attached to the connecting strips. The vector of design variables $\underline{c}$ collects the principal yield moments for each region. Denoting by x and y the orthogonal directions of reinforcing bars, common for the entire slab, one has four design parameters for a region: the yield moments m_x, m_y for positive bending (bottom reinforcement) and the yield moments m_x', m_y' for negative bending (top reinforcement). The yield criterion for this discrete model reads: i-th line of the mesh is at the yield when (a) the positive bending moment M_i reaches the ultimate value

$$m_i = m_x \sin^2 \alpha_i + m_y \cos^2 \alpha_i, \qquad (5)$$

or (b) the negative moment reaches the value

$$m_i' = m_x' \sin^2 \alpha_i + m_y' \cos^2 \alpha_i. \qquad (6)$$

Here α_i denotes the angle between i-th line and the x-axis.

The cost factors l_k result from the expression of the reinforcement volume as the linear function of the principal yield moments. Usually these factors are taken proportional to the areas of regions with constant reinforcement. A detailed derivation can be found in [2] as well as the modifications of the model (1)-(2) for the cases of prescribed orthotropy and/or asymmetry of reinforcement.

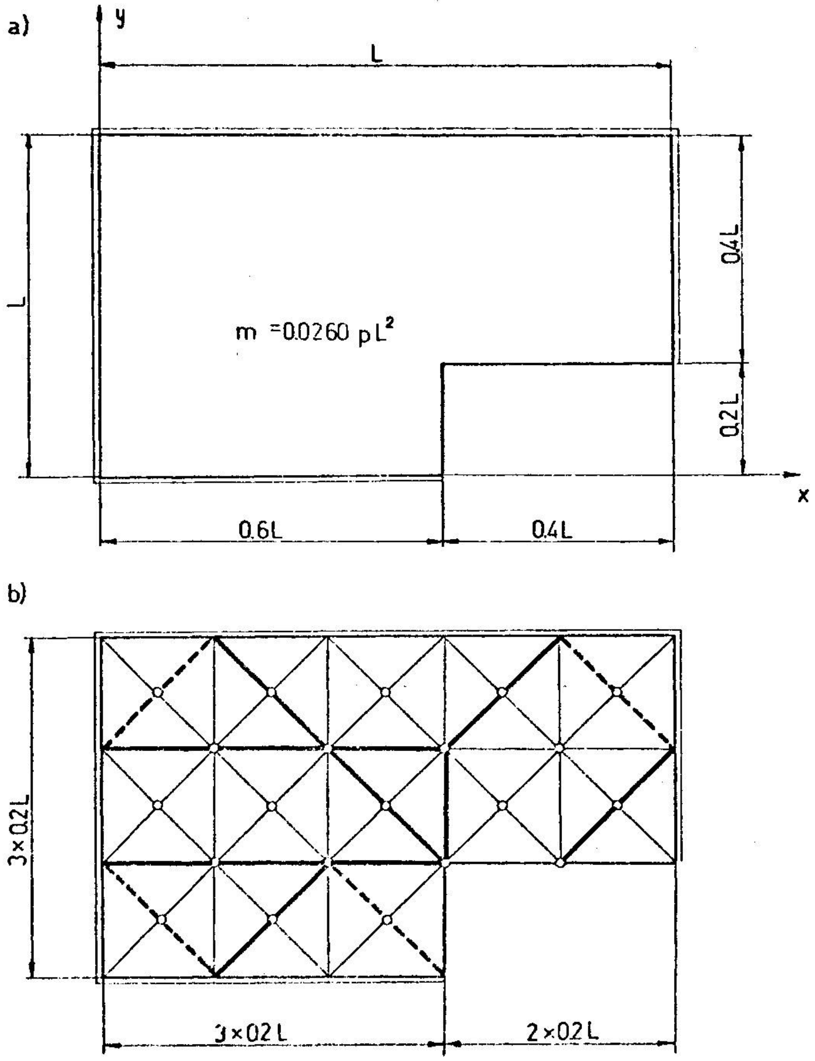

Fig. 4 Simply Supported Slab with Cut-off - Isotropic Reinforcement: a) Dimensions and Yield Moment, b) Discretization Mesh and Collapse Mechanism.

4. NUMERICAL EXAMPLES

The first example (Figure 2) concerns the rectangular slab with three edges simply supported and the fourth edge free. The following optimum values of the principal yield moments were obtained for a single bottom reinforcement grid:

$$m_x^* = 0.30\ pL^2 \qquad m_y^* = 0.05\ pL^2 \tag{7}$$

Here p denotes the transversal pressure and L is the length of the shorter edge of the slab. The optimum orthotropy factor is

$$\mu^* = m_y^*/m_x^* = 0.1667 \tag{8}$$

The volume of reinforcement for this design is 8 % less as compared to the isotropic plate.

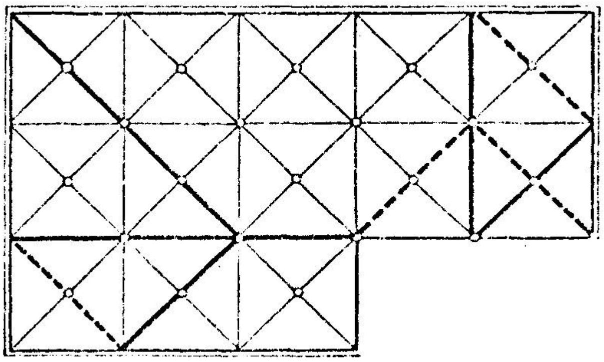

Fig. 5 Simply Supported Slab with Cut-off - Discretization Mesh and Collapse Mechanism for Optimum Orthotropy

The second example (Figure 3) shows the optimum solution for the same slab but having two reinforcement grids. It was assumed additionally that for technological reasons the yield moment should not be less than $0.01\ pL^2$. The optimum values of the principal yield moments are:

for the central region: $m_x^* = 0.365\ pL^2$, $m_y^* = 0.01\ pL^2$ (9)

for the outer region: $m_x^* = 0.282\ pL^2$, $m_y^* = 0.01\ pL^2$ (10)

This solution reduces the reinforcement volume by 13 % in comparison to the isotropic case. Finally Figures 4 and 5 show the results for a slab with cut-off. The solution for isotropic case is depicted in Figure 4 while the optimum values

$$m_x^* = 0.667 \times 10^{-2}\ pL^2, \quad m_y^* = 3.85 \times 10^{-2}\ pL^2 \tag{11}$$

correspond to the collapse mechanism shown in Figure 5.

5. CONCLUSIONS

The computer based version of the yield-line method offers a cheap tool for preliminary design of slabs. Computational effectiveness of the algorithm makes it possible to run several trial optimizations with differently chosen reinforcement patterns. After a final choice has been made on the ground of the rigid-plastic approach, one has to check whether other requirements, such as a sufficient stiffness and crack resistance, are met. The final design can be recalculated by the present method in order to establish its safety factor against plastic collapse.

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