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Autor: Anderheggen, E. / Theiler, J.

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Computer Aided Optimum Design of Concrete Slabs

Minimalisation de l'armature des dalles à l'aide de l'ordinateur

Computerunterstützte optimale Bemessung der Armierung von Platten

E. ANDERHEGGEN
Professor of Applied Computer Science
Swiss Federal Institute of Technology
Zurich, Switzerland

J. THEILER
Research Associate
Swiss Federal Institute of Technology
Zurich, Switzerland

SUMMARY
A computer based optimum design procedure satisfying various practical design constraints is presented for finding the minimum weight reinforcement distribution for concrete slabs. The procedure uses finite element analysis and is derived from the shake-down theorem of the theory of plasticity. The approach used represents a combination of automatic optimum design and interactive computer aided design methods.

RESUME
Le rapport présente une méthode servant à minimaliser le volume d'armature des dalles en béton armé tout en tenant compte de certaines conditions dictées par la pratique. Le procédé utilise la méthode des éléments finis et est basé sur le théorème fondamental de la théorie de la plasticité. Le programme de dimensionnement offre à l'utilisateur la possibilité de modifier les données pendant le déroulement du programme.

ZUSAMMENFASSUNG
Es wird eine den praktischen Gegebenheiten angepasste Methode zur optimalen Bemessung der Armierung von Platten beschrieben. Das Verfahren benützt die Methode der Finiten Elemente und basiert auf dem Einspielsatz der Plastizitätstheorie.
Es ist ein computerunterstütztes interaktives Bemessungsverfahren mit automatischen Optimierungsalgorithmen.

1. INTRODUCTION

This paper reports on a research project presently in progress. While the main ideas on which the project is based seem today to be well understood, the development of the complex computer program needed for practical applications has not yet reached the stage where numerical results can be obtained. Therefore general conclusions concerning the applicability of the suggested design method can not be drawn yet. Experience also shows that the final system might look considerably different as it is planed today.

The aim of the project is the development of a computer based design procedure for finding the minimum weight reinforcement for concrete slabs of given geometry subjected to any kind of dead- and life-loads taking into account different kinds of practical design restrictions. Only the plate-bending action of the slabs shall be considered.

Most practically oriented civil engineers are rather sceptical towards automatic optimum design procedures. They feel - with good reasons - that the process of designing real-life structures involves too much personal experience, feeling and imagination to be left to a computer program developed by some stranger. Today much more attention is paid to interactive computer aided design methods, where the computer only checks given designs while the task of finding an "optimum", whatever that means, is left to the designer. The main problem of this approach is to make man - machine - communication so easy and to have the computer answering so quickly that some kind of a dialog between the designer sitting in front of a terminal and the computer becomes possible.

The procedure discussed here represents a combination of both approaches: while the designer is still expected (at least in the final stages of the design process) to interact with the computer sitting in front of a terminal, the computer, whenever requested, will have to perform optimality search by linear programming methods and show his results quickly and clearly. In fact, the chances that such an approach will prove useful for real-life problems rely on the facts that today's computers (a DEC-10 is used for this project) are powerful enough for performing complex calculations without keeping the user waiting too long, that man - machine communication, specially due to computer graphics, has become easy and also that the problem considered while of considerable practical significance, is one of the best suited for optimum design procedures based on the theory of plasticity.

2. THEORETICAL BACKGROUND AND OVERVIEW OF THE DESIGN METHOD

The design method suggested here is based on the shake-down theorem of the plasticity theory and was first used for framed structures by one of the co-authors in 1965 (see [1,2]). When applied to plate-bending problems the shake-down theorem says that if it is possible to find any distribution of residual moments (i.e. any homogeneous stress state) which, combined with the ideal-elastic moment distribution for every possible loading case, nowhere violates the plasticity conditions, then the structure will eventually stabilize or "shake-down" for any possible loading cycle.

With the usual assumption that reinforcement has no influence on the elastic behaviour of concrete structures, the moment distributions due to external loads can be obtained by linear-elastic finite element analysis. By prescribing as additional unit load cases different initial curvature distributions any number of residual homogeneous moment distributions can also be obtained by finite element analysis. The design problem can then be stated as follows:

A linear combination of these unit load cases leading to an optimum residual homogeneous moment distribution has to be found and added to the linear-elastic moment envelopes due to the external loads. Optimality is achieved when the weight of the reinforcement needed for the combined moment envelopes is minimal. According to the shake-down theorem, this procedure will result in the design of a structure where plastic deformations may only occur during the first load cycles, which is certainly an appropriate design criterion for reinforced concrete slabs.

Assuming that the plastic resistances needed to satisfy the plasticity conditions for the combined moment envelopes throughout the slab are linear functions of the amount of reinforcement in a number of chosen "check-points", the optimum design problem stated above can be formulated as a linear program for minimizing the total steel weight. The unknown parameters to be determined are the cross-sectional areas A_1 to A_{NG} of NG predefined groups of steel bars as well as the amplitude-factors X_1 to X_{NH} multiplying each of the NH homogeneous load cases considered.

The user of this computer-based design procedure will have to specify length, position and direction of different groups of steel bars (possibly of several alternative groups among which the linear program algorithm will look for a minimum weight solution) as well as a number of homogeneous unit load cases to be used for optimization. These will be specified by introducing a constant unit initial curvature in a given direction in one or more elements of the linear-elastic finite element model.

All static calculations, both for the external loads and for the initial curvature loads are performed by linear-elastic finite element analysis. The hybrid model with triangular and quadrilateral linear-moment plate-bending elements described in [3] and [4] is used for this purpose. Shear deformations are not be taken into account. Column-supports and elastic foundations are treated by means of "elastically" supported elements.

3. DERIVATION OF THE LINEAR PROGRAM

The total steel weight or the total steel volume V of NG predefined groups of steel bars is to be minimized. Each of these groups covers a rectangular or a parallelogram-shaped portion of the slab of length L_g (g = 1 to NG) and is positioned near the top or the bottom surface of the slab for providing negative or positive bending resistance. If A_g denotes the total cross-sectional area of all bars of given length L_g belonging to the g-th bar group, the design optimality criterion can be expressed in scalar or matrix notation as follows:

$$V = \sum_{g=1}^{NG} L_g \cdot A_g = \{L\}^T\{A\} \quad \rightarrow \quad \text{Minimum} \tag{1}$$

The plasticity conditions will be checked in a sufficiently large number NC of "check-points" chosen in such a way that no violation will occur elsewhere in the slab. This is done by using the following well-known plasticity conditions valid for relatively low degrees of reinforcement:

$$-n_\varphi \leq m_\varphi \leq p_\varphi \tag{2}$$

where n_φ, p_φ and m_φ represent the negative and positive bending resistances and the bending moment in any direction φ. As suggested by Wolfensberger [5] the angle φ can be eliminated and the non-linear conditions Eq. (2) can be linearized by introducing eight linear inequality constraints for the bending and twisting moments m_x, m_y, and m_{xy} in two orthogonal coordinate directions x and y. In matrix notation these eight inequalities for a check-point c (c = 1 to NC) are given by:

$$\begin{aligned} \{m_c\}_{max} &\leq \{p_c\} \\ -\{m_c\}_{max} &\leq \{n_c\} \end{aligned} \qquad (3)$$

where the moment-envelope vectors $\{m_c\}_{max}$ and $\{m_c\}_{min}$ and the positive and negative resistance vectors $\{p_c\}$ and $\{n_c\}$ are defined by:

$$\{p_c\} = \begin{Bmatrix} P_x + P_{xy} \\ P_x - P_{xy} \\ P_y + P_{xy} \\ P_y - P_{xy} \end{Bmatrix}_{\text{at } c} \qquad \{n_c\} = \begin{Bmatrix} N_x + N_{xy} \\ N_x - N_{xy} \\ N_y + N_{xy} \\ N_y - N_{xy} \end{Bmatrix}_{\text{at } c} \qquad \{m_c\} = \begin{Bmatrix} m_x + m_{xy} \\ m_x - m_{xy} \\ m_y + m_{xy} \\ m_y - m_{xy} \end{Bmatrix}_{\text{at } c} \qquad (4)$$

A detailed derivation of the positive and negative plastic bending resistance coefficients P_x, P_y, P_{xy}, N_x, N_y and N_{xy} for orthogonal and non-orthogonal reinforcement can be found in [5].

If the reinforcement is relatively low it is reasonable to assume, at least for practical design purposes, that the plastic resistance vary linearly with the reinforcement, implying that this has no influence on the lever-arm of the internal forces. The resistance coefficients of the vectors $\{p_c\}$ and $\{n_c\}$ can then be expressed by linear functions of the reinforcement areas A_1 to A_{NG} as follows:

$$\begin{aligned} \{p_c\} &= [P_c]\{A\} \\ \{n_c\} &= [N_c]\{A\} \end{aligned} \qquad (5)$$

where the coefficients of the 4xNG matrices $[P_c]$ and $[N_c]$ represent the resistance contributions due to a unit reinforcement area $A_g = 1$ provided that the c-th check-point lies within the surface of the slab covered by the g-th steel bar group.

The maximum and minimum moment-envelope vectors $\{m_c\}_{max}$ and $\{m_c\}_{min}$ introduced in Eq. (3) result, as explained earlier, from the superposition of the linear-elastic moment-envelopes $\{m_c^{ext}\}_{max}$ and $\{m_c^{ext}\}_{min}$ due to the external loads and the corresponding vector $\{m_c^{hom}\}$ due to NH homogeneous load cases of unknown amplitudes X_1 to X_{NH}. This leads to:

$$\begin{aligned} \{m_c\}_{max} &= \{m_c^{ext}\}_{max} + \{m_c^{hom}\} = \{m_c^{ext}\}_{max} + [H_c]\{X\} \\ \{m_c\}_{min} &= \{m_c^{ext}\}_{min} + \{m_c^{hom}\} = \{m_c^{ext}\}_{min} + [H_c]\{X\} \end{aligned} \qquad (6)$$

the coefficients of the 4xNH matrix $[H_c]$ (as well as those of the vectors $\{m_c^{ext}\}_{max}$ and $\{m_c^{ext}\}_{min}$) being found by linear-elastic finite element analysis.

Introducing Eqs. (5) and (6) in Eq. (3) the eight plasticity conditions at a check-point c can be written as follows:

$$\begin{aligned} \{m_c^{ext}\}_{max} + [H_c]\{X\} &\leq [P_c]\{A\} \\ -\{m_c^{ext}\}_{min} - [H_c]\{X\} &\leq [N_c]\{A\} \end{aligned} \qquad (7)$$

Design constraints formulated as maximum or minimum allowable reinforcement areas can also be introduced:

$$\{A_{min}\} \leq \{A\} \leq \{A_{max}\} \qquad (8)$$

From Eqs. (1), (7) and (8) the following linear program for the unknowns X_1 to X_{NH} and A_1 to A_{NG} (see also Fig. 1) is obtained:

$$\left.\begin{array}{lr} V = \{L\}^T\{A\} \quad \rightarrow \quad \text{Minimum} & \\ 0 \leq -\{m_c^{ext}\}_{max} - [H_c]\{X\} + [P_c]\{A\} & (c = 1 \text{ to } N_c) \\ 0 \leq \{m_c^{ext}\}_{min} + [H_c]\{X\} + [N_c]\{A\} & (c = 1 \text{ to } N_c) \\ 0 \leq -\{A_{min}\} + \{A\} & \\ 0 \leq \{A_{max}\} - \{A\} & \end{array}\right\} \qquad (9)$$

This linear program can be considerably simplified. The minimum reinforcement inequalities can be immediately eliminated by introducing as design variables, instead of the A_g's, non-negative $\bar{A}_g$ parameters defined by:

$$\{\bar{A}\} = \{A\} - \{A_{min}\} \geq 0 \qquad (10)$$

Also, it will certainly not be necessary to formulate all eight linear plasticity conditions in all check-points. The values of the moment envelopes due to the external loads will show that many plasticity checks are most probably not necessary (e.g. positive moment checks over a column support) thus allowing a great reduction in the number of inequalities to be considered. Maximum reinforcement inequalities will also, in many cases, not be introduced for all bar groups.

The linear program (9), simplified as explained, will be solved in core by the simplex algorithm. It should also be noted, that the designer, as explained later, will be able during the design process to introduce or to delete any A- or X-variable and any linear inequality constraint he wishes. The computer will then have to solve each time the modified linear program starting from the previous solution.

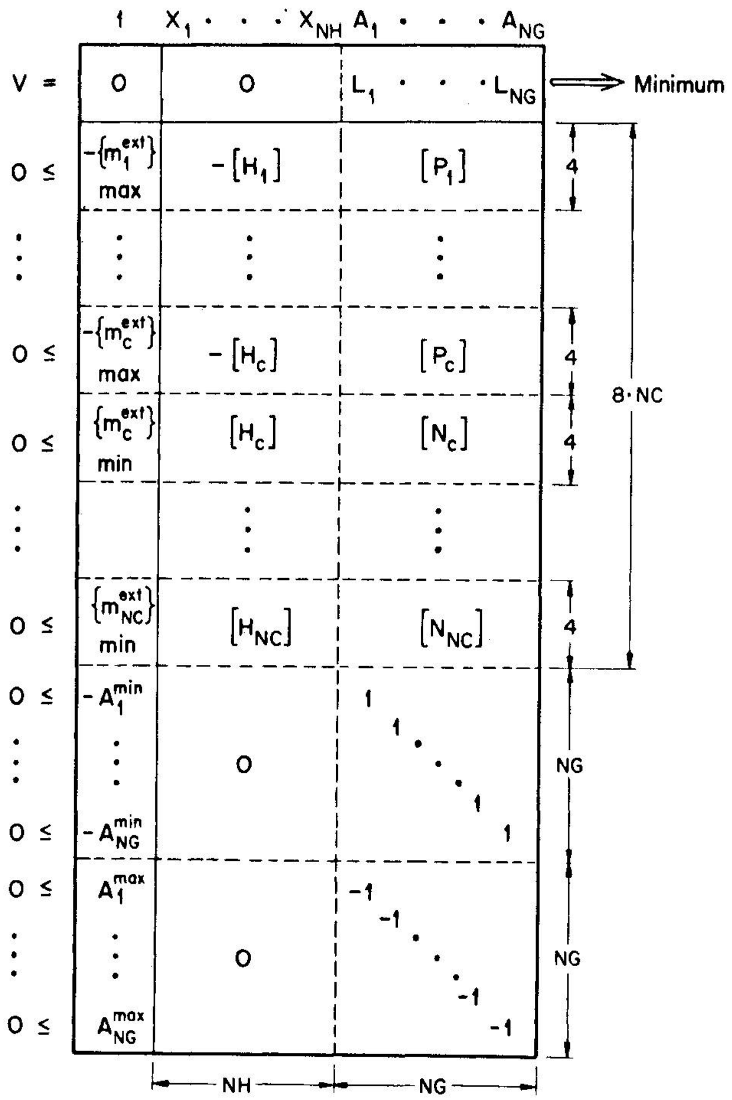

Fig. 1: Tableau form of the linear program (9)

4. INTERACTIVE DESIGN PROCEDURE

In a first step the designer has to specify, as usual in finite element analysis, all structural and load data necessary to determine the moment-envelopes in all possible check-points, i.e. in all joints and in the center of all elements. These values as well as the data needed for analysing the additional homogeneous load cases to be specified later (local load vectors for three unit initial curvatures within each element, triangular half-inverse of the global stiffness matrix, etc.) are then saved on secondary storage.

In a second step the following design data have to be specified or, whenever possible, automatically determined by the program:

a) Lengths,positions and directions of all groups of reinforcement steel bars covering rectangular or parallelogram-shaped portions of the slab. Net reinforcements with steel bars in two orthogonal directions as well as bar groups of identical cross-sectional area but covering two or more distinct portions of the slab can also be specified.
b) Minimum and maximum allowable reinforcement for any bar group.
c) Criteria for determining which of the eight possible linear plasticity conditions have to be considered in any check-point. In most cases the program will be able to determine these automatically by examining the values of the moment-envelopes due to the external loads.
d) Homogeneous load case informations concerning the direction of unit initial curvatures in one or more elements. If no such load is specified, the slab will be designed assuming no plastic moment redistribution.

With these data the program will be able to determine the linear-elastic moment distributions for the homogeneous load cases, set up the coefficient matrix of the linear program and solve this in core by the simplex algorithm. At the end of this step (as well as at the end of all subsequent steps) the program will check all linear plasticity conditions originally ignored. If any of these is found to be violated, the corresponding inequalities are introduced into the linear program and a modified optimum solution is found.

As these two steps will generally require a considerable amount of computing time, the corresponding program sections will not allow direct interaction with the designer. However, input preparation is made easy by the use of a simple problem-oriented input language described by few easily understandable syntax diagrams (see also [3] and [4]).

Full line by line interaction based on a command language also described by syntax diagrams will be possible in the subsequent design steps. Within each of these the designer will be able to request anyone of the following actions:

a) introduce a new reinforcement bar group or delete an existing one
b) change, add or delete a minimum or maximum reinforcement constraint
c) require a reinforcement area to assume a given value. This may be desirable when the designer choses to use a certain number of steel bars of standard diameter corresponding to a total cross-sectional area not identical to the optimum value found by the linear program.
d) introduce new homogeneous load cases hoping that these will help to further reduce the total steel weight.

Each of these steps implies the addition or the deletion of some variables or some inequality constraints requiring the previous solution to be modified, which, in general, can be done with a relatively little computational effort. The designer can then be informed on the effects of the action he took (change in total steel weight, changes in single reinforcements, plasticity conditions becoming active, i.e. exactly fulfilled, or inactive, etc.) in order to be able to plane his next design step.

In fact, if the program has to interact in real time with a designer changing at each step his design specifications until a satisfactory and in all respects practicable reinforcement distribution is found, the problem of man - machine communication becomes of crucial importance. Certainly extensive graphical output and possibly some graphical input capabilities have to be incorporated into the program. The designer should be able to see at a glance which parts of the slab are more heavily stressed and which are not. This will help him finding the most favourable position and shape of each reinforcement bar group and also tell him how to assume the homogeneous load cases. These will probably lead to the most favourable moment redistribution when initial curvatures are introduced in the directions and in the elements where moments are large.

It is too early to discuss these points in detail. It should only be mentioned that the graphical capabilities of a storage-tube Tektronix 4014 terminal connected with a DEC-10 computer appear to be adequate for this project.

5. OUTLOOK

Optimum design and interactive computer aided design procedures have attracted and continue to attract much attention and much research work. It is a fact, however, that at least in civil engineering such procedures are today very seldom used for practical purposes. In awareness of this it would be illusory to expect that procedures similar to the one described here will very soon become standard tools of practicing structural engineers. The main scope of our project, which is nevertheless quite an ambitious one, is therefore to assess as clearly as possible for a well defined and actually relatively simple practical optimum design problem the feasibility of the approach.

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