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## Collapse of Reinforced Concrete Voided Slabs

La ruine de dalles en béton armé avec des ouvertures

Versagen von Stahlbetonhohlplatten

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**SUMMARY**
The paper derives an upper bound to the collapse load of a circular voided slab bridge simply supported along two opposite edges and loaded symmetrically. The critical mechanism involves flexural yield lines in combination with lines of shear failure.

**RESUME**
Une valeur supérieure est obtenue pour la charge ultime d'un pont-dalle avec des trous circulaires, appuyé simplement le long de deux bords opposés et chargé symétriquement. Le mécanisme critique implique des lignes de rupture causées par la flexion combinées avec des lignes de rupture causées par la flexion combinées avec des lignes de rupture causées par le cisaillement.

**ZUSAMMENFASSUNG**
Ein oberer Grenzwert für die Traglast einer symmetrisch belasteten, entlang zweier gegenüberliegender Seiten frei drehbar gelagerten Hohlplattenbrücke mit kreiszylindrischen Aussparungen wird hergeleitet. Der massgebende Mechanismus ist durch eine Kombination von Fliessgelenklinien und Schubbruchlinien gekennzeichnet.

## 1. INTRODUCTION

A reinforced concrete voided slab bridge tested at the Cement and Concrete Association [1] collapsed by the formation of a mechanism which involved lines of shear failure in addition to conventional flexural yield lines. The slab had a depth of void ratio of 0.786 and was loaded to collapse by means of a 16-wheel vehicle positioned centrally. The maximum load attained was 455 kN but this fell instantly to 414 kN. This load was held until longitudinal shear cracks formed near to the outer wheels of the vehicle when the load fell to 373 kN. On attempting to apply further load, longitudinal top and bottom flexural yield lines developed together with transverse hogging yield lines near to the supports. The central strip of slab, bounded by the longitudinal shear/flexural yield lines, then continued to rotate about the transverse hogging yield lines with distortion of two voids occurring. A theoretical analysis of such a collapse mechanism is considered in this paper.

## 2. UPPER BOUND ANALYSIS

### 2.1 Assumptions

It is assumed that the concrete is rigid-perfectly plastic, has a modified Coulomb yield criterion, zero tensile strength, compressive strength given by $f_e = \nu f_c$ where $\nu$ is an effectiveness factor and $f_c$ is the cylinder strength which is assumed to be 80% of the cube strength, and the normality rule of plastic flow obtains. The reinforcement is assumed to be rigid-perfectly plastic and to carry only axial stresses.

### 2.2 Initial collapse

A general circular voided slab loaded symmetrically with respect to its centre is considered. The proposed initial collapse mechanism is shown in Fig. 1. The displacement rate ($\dot{\delta}$) is taken to be normal to the plane of the slab since the restraint of the rigid material each side of the shear failure lines is likely to prevent any outward movement of this material relative to the central portion of the slab.

If q is the ultimate shear per unit length measured in the span direction, m and $\alpha m$ are the sagging and hogging longitudinal moments of resistance respectively, P is the total applied load and w is the self weight of the slab per unit area, then the work equation is

$$P = \frac{2}{(2\ell-c-2d)} \left\{ 4m\,(b_b + \alpha b_t) + \left[2q - w\,(b_t + b_b)\right] (\ell^2 - d^2) \right\} \tag{1}$$

The minimum value of P is obtained when

$$\ell = \frac{c+2d}{2} + \sqrt{\frac{c^2+4cd}{4} + \frac{4m\,(b_b + \alpha b_t)}{2q - w\,(b_t + b_t)}} \tag{2}$$

### 2.3 Value of q.

The value of q is obtained by considering the dissipation rate per unit length, measured in the span direction, of the concrete and of any vertical stirrups crossed by the shear failure line. The dissipation rate in the concrete is given by [2]

$$\dot{D}_c = 0.5 f_e \ell_e (1-\cos\theta)\dot{\delta}_x$$

where $\ell_e$ is the length of concrete in the failure line and $\dot{\delta}_x$ is the displacement rate at a particular section x.

The dissipation rate in the stirrups is given by

$$\dot{D}_s = n\, A_{ss}\, f_{ys}\, \dot{\delta}_x$$

Where n is the number of rows of stirrups crossed by the failure line, $A_{ss}$ is the area per unit length measured in the span direction of the stirrups and $f_{ys}$ is the yield stress of the stirrup reinforcement.

Hence $q = 0.5 f_e \ell_e\,(1-\cos\theta) + n\, A_{ss}\, f_{ys}$ (3)

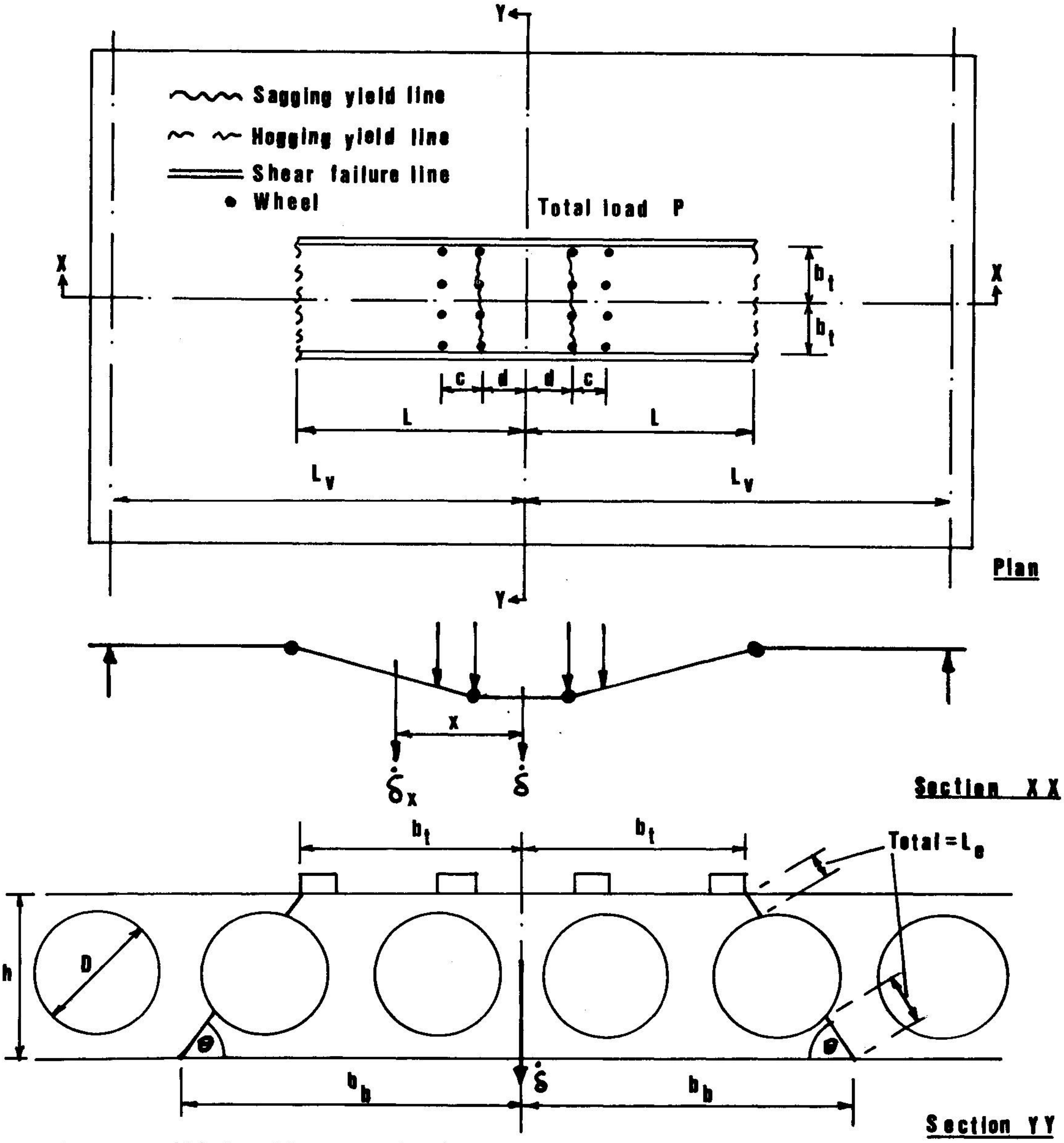


Fig. 1 Initial collapse mechanism

## 2.4 Value of $\nu$

The effectiveness factor ($\nu$) reflects the ductility of the concrete in compression and depends upon the concrete strength and the conditions under which the the concrete is stressed. However very little experimental evidence is available for voided slabs although ASTER [3] has tested a transverse strip of a slab with a depth of void ratio of 0.75 and having no shear reinforcement. An analysis indicates that $\nu = 0.13$ which is small because of the flexibility of the cross-section of a voided slab and of the discontinuous failure surface.

## 2.5 Subsequent mechanism

It is proposed that after initial failure in accordance with the above mechanism, a subsequent mechanism involving distortion of the failed cells takes place as shown in Fig. 2.

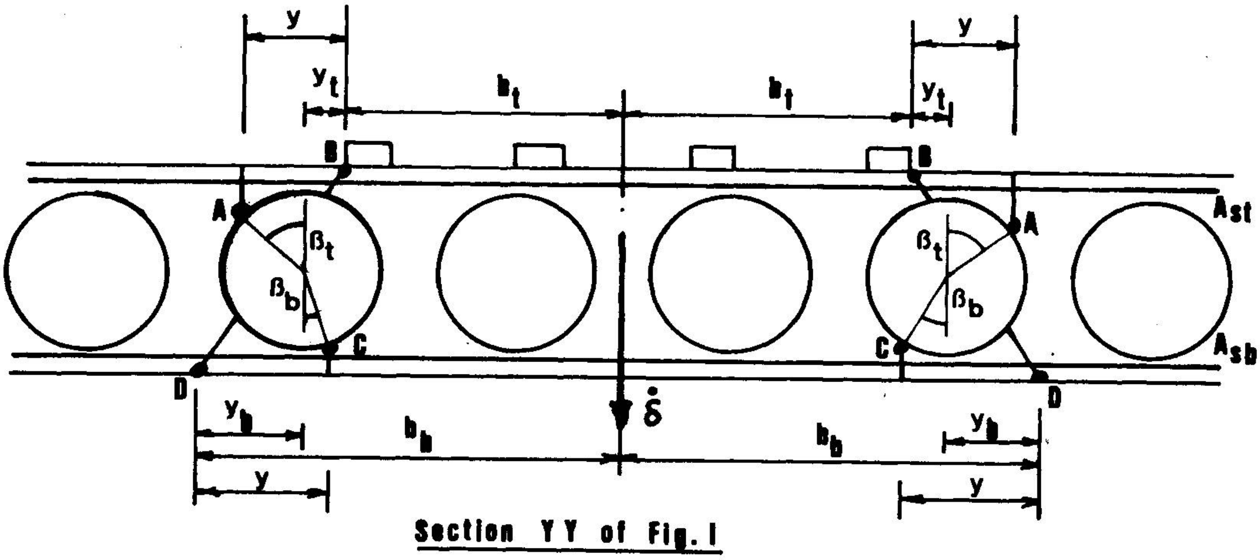


Fig. 2 Subsequent collapse mechanism

The positions of the centres of rotation B and D in Fig. 2 are determined by the initial shear failure and, of A and C, by minimising the load with the constraint that

$$y_t + \frac{D}{2} \sin \beta_t = y_b + \frac{D}{2} \sin \beta_b = y \quad (4)$$

Assuming that, compared with the initial mechanism, the reduction in the rate of work of the self weight of the bottom flange is compensated by the increase of that of the top flange then the total rate of external work is unchanged. The dissipation rate in the mechanism is given by

$$\dot{D} = \dot{\delta} \left\{ \frac{2m \left[(2b_b - y) + \alpha (2b_t + y)\right]}{\ell - d} + \frac{\zeta f_{yt} \left[A_{st} (h - D \cos \beta_t) + A_{sb} (h - D \cos \beta_b)\right] (\ell + D)}{y} \right\} \quad (5)$$

where $\zeta$ is a membrane enhancement factor and $f_{yt}$ is the yield stress of the transverse reinforcement in the flanges.

In general an analytical solution for a minimum P is unobtainable. However, if $y\,(1-\alpha)$ is small compared with $2\,(b_b + \alpha b_t)$ and $y_t \simeq y_b = \bar{y}$ so that $\beta_t \simeq \beta_b = \bar{\beta}$ then a minimum P is found for

$$\ell = \frac{c+2d}{2} + \sqrt{\frac{c^2+4cd}{4} + \frac{4m(b_b+\alpha b_t)}{\zeta\eta - w(b_t+b_b)}} \qquad (6)$$

where
$$\eta = \frac{f_{yt}\,(A_{st}+A_{sb})(h-D\cos\bar{\beta})}{\bar{y}+0.5\,D\sin\bar{\beta}} \qquad (7)$$

and
$$\tan\bar{\beta}/2 = \frac{h-D}{2y+\sqrt{4\bar{y}^2+h^2-D^2}} \qquad (8)$$

## 3. COMPARISON WITH TEST DATA

### 3.1 Initial failure

Dimensions relating to the failure line were observed to be $\theta = 48.4^{\circ}$ $\ell_e$ = 151 mm, $y_t$ = 122 mm, $y_b$ = 126 mm, $2b_t$ = 596 mm, $2b_b$ = 1092 mm and n = 0.

The concrete cube strength was 52.3 N/mm$^2$ and if $\nu$ is taken to be the value derived from the analysis of Aster's strip then $f_e$ = 5.44 N/mm$^2$. Equation 4 then gives, with n = o, q = 0.138 kN/mm

The sagging and hogging longitudinal moments of resistance are respectively 137 and 21.5 kNm/m; thus $\alpha$ = 0.157. The other relevant data are $\ell_v$ = 2650 mm, c = 450 mm, d = 225 mm and w = 3.46 kN/m$^2$.

From equation 2, $\ell$ = 1608 mm whereas the observed value was 2370 mm; and, from equation 1, P = 878 kN which is much greater than the peak load of 455 kN attained or the load of 414 kN at which the slab 'yielded' in shear.

An explanation of this gross overestimate of the collapse load could be that when the slab first fails in shear only the concrete in the immediate vicinity of the load is deformed sufficiently to 'yield' in shear and that once the slab commences to 'yield' in shear the deformations in the vicinity of the load are too large for aggregate interlock to occur across the shear crack and the dissipation rate in the vicinity of the load falls to zero. It might thus be more appropriate to ignore the dissipation rate in those parts of the lines of shear failure which extend beyond the loading vehicle when calculating the peak load, and to ignore the dissipation rate in those parts of the lines of shear failure within the length of loading vehicle when calculating the lower load at which 'yield' of the slab in shear occurs.

### 3.2 Estimate of peak load

Neglecting the dissipation rate in those parts of the shear failure lines beyond the loading vehicle, the peak load is given by

$$P = \frac{2}{2\ell-c-2d}\left\{4m(b_b+\alpha b_t) + 2q\,\left[2(c+d)(\ell-d)-c^2\right] - w(b_t+b_b)(\ell^2-d^2)\right\} \qquad (9)$$

For a minimum P, it is found that $\ell = \ell_v$ and thus the peak load is estimated to be 523 kN which should be compared with the observed peak load of 455 kN.

### 3.3 Estimate of shear yield load.

Neglecting the dissipation rate in those parts of the shear failure lines within the loading vehicle, the yield load is given by

$$P = \frac{2}{2\ell-c-2d}\left[4m(b_b+\alpha b_t) + 2q(\ell-c-d)^2 - w(b_t+b_b)(\ell^2-d^2)\right] \quad (10)$$

For a minimum P, it is found that

$$\ell = \frac{c+2d}{2} + \sqrt{\frac{c^2+4cd}{4} + \frac{4m\ (b_b+\alpha b_t)-2qcd}{2q - w(b_b+b_t)}} \quad (11)$$

from which $\ell$ = 1563 mm and the yield load, from equation 10, is 481 kN which should be compared with the observed yield load of 414 kN.

### 3.4 Subsequent mechanism

$\zeta$ is taken to be 2. It is reasonable to take $\bar{y}$ as the mean of $y_t$ and $y_b$ and thus $\bar{y}$ = 124 mm. Then from equation 9, $\bar{\beta} = 12.4^o$; from equation 8, $\eta = 0.0499$; from equation 7, $\ell$ = 2322 mm; and P = 425 kN. The observed values of $\bar{\beta}$, $\ell$ and P were $14^o$, 2370 mm and 373 kN. The calculated load exceeds the observed value by 14%.

## 4. CONCLUSIONS

Upper bounds to the collapse load of a circular voided reinforced concrete slab bridge loaded symmetrically have been presented. The analysis overestimates the peak, shear and distortional yield loads by 15%, 16% and 14% respectively. However, a number of simplifications and, in some cases, somewhat arbitrary assumptions have been made in the analysis.

## REFERENCES

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## 5. APPENDIX

### 5.1 Membrane enhancement factor

The mechanism shown in Fig. 2 neglects any membrane action in the flanges although such action must take place. In order to allow for membrane action, the enhancement factor ($\zeta$) is introduced in equation 5.

If full lateral restraint is assumed, the enhancement factor can be assessed by considering the transverse section of a flange as a beam.

Since full restraint will not occur, and in the absence of a complete analysis of the membrane effects, $\zeta$ is estimated in this paper to be 50% of the full restraint value.