Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 29 (1979)

Artikel: Punching shear failure of hollow concrete spheres

Autor: Morley, C.T.

DOI: https://doi.org/10.5169/seals-23549

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 23.02.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

III

Punching Shear Failure of Hollow Concrete Spheres

Poinçonnement d'une coque sphérique en béton

Durchstanzversagen von Kugelschalen aus Beton

C.T. MORLEY
Lecturer
University of Cambridge
Cambridge, England

SUMMARY
The paper extends the plastic theory of punching shear failure to treat hollow concrete spheres. Graphs showing the theoretical predictions are presented, and some comparisons are made with experimental results for cylinders under concentrated radial loads.

RESUME
L'analyse plastique du poinçonnement est appliquée à une coque sphérique. Les résultats théoriques sont présentés graphiquement et quelques comparaisons sont faites avec des résultats expérimentaux obtenus pour des cylindres soumis à une force concentrée radiale.

ZUSAMMENFASSUNG
Die plastische Berechnung des Durchstanzversagens wird auf den Fall von Kugelschalen aus Beton ausgedehnt. Die theoretischen Voraussagen werden in graphischer Form dargestellt. Einige Vergleiche werden gemacht mit Ergebnissen von Versuchen an Zylindern, die durch in radialer Richtung wirkende Einzellasten belastet wurden.

1. INTRODUCTION

The purpose of this brief note is to extend to hollow concrete spheres the plastic theory of punching shear failure presented by Braestrup [1] for flat slabs. In slabs with zero tensile strength the optimum failure surface extends right out to the support, giving low failure loads, and it is necessary to introduce a small non-zero tensile strength in order to confine the failure surface and produce reasonable results. In a spherical shell under a radial point load the curvature of the shell will tend to confine the failure surface in punching shear, and the plastic theory should predict reasonable failure loads even if the concrete is assigned zero tensile strength. In what follows the extended theory is presented, using Braestrup's notation as far as possible, and some experimental results on cylinders are reported.

2. BASIC ASSUMPTIONS

A concrete spherical shell or dome of thickness h and internal radius R is assumed to be loaded by an inward radial force P applied to a rigid disc of diameter d_o, as shown in Fig.1. The shell is supported well away from the region of interest. In a punching shear failure a rigid axisymmetric plug of concrete defined by a 'failure surface' with generatrix AB is assumed to move inwards along the axis of P relative to the rest of the shell. The concrete is taken to be a rigid perfectly plastic material whose yield condition is the modified Coulomb failure criterion with angle of internal friction ϕ and zero tensile strength. Deformations are governed by the associated flow rule of plasticity theory (the normality condition).

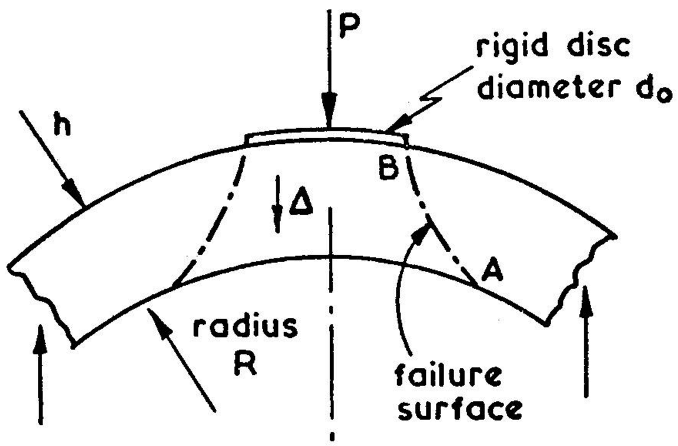

Fig. 1 A punching shear failure.

Following Braestrup, an upper bound on the failure value of P is found by writing the work equation for failure on an assumed surface, and the optimum failure surface giving the least upper bound is found by the calculus of variations. Since displacement is along the P axis hoop strains are zero everywhere and the concrete is in plane strain in planes containing the P axis: the appropriate yield locus is Fig.5(b) of Braestrup's paper, with $f_t = 0$. All deformation is assumed to occur in a narrow zone at the failure surface, which is a surface of revolution defined by the generatrix $r = r(x)$, Fig.2.

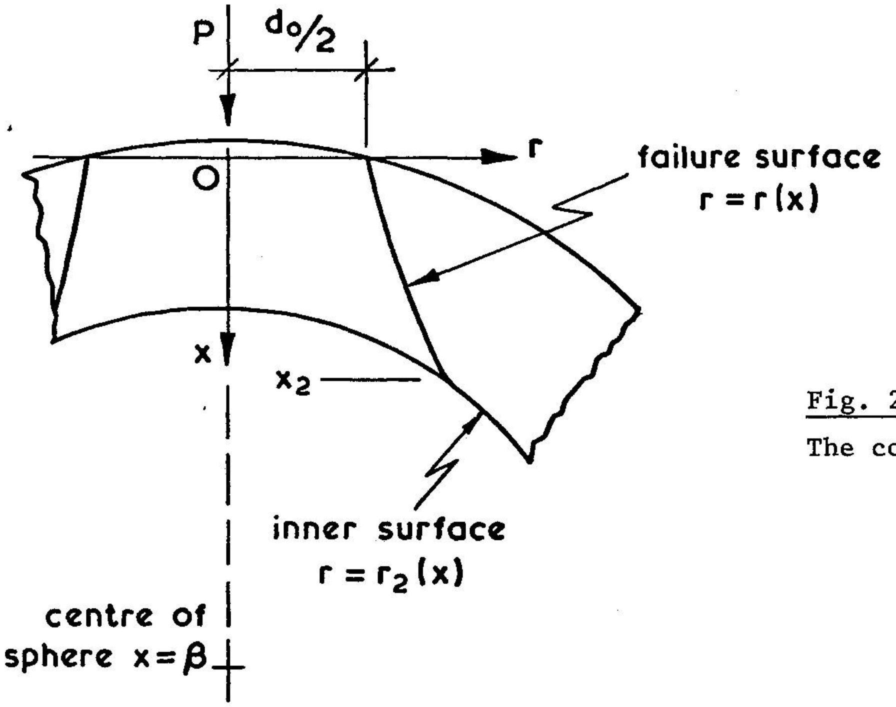

Fig. 2

The co-ordinate system.

On writing the work equation, the upper bound on the failure load is given by the equivalent of Braestrup's equation (9),

$$P = \pi f_c \int_o^{x_2} r(\sqrt{1 + (r')^2} - r')\, dx \qquad (1)$$

where a dash indicates differentiation with respect to x. In contrast to the situation in flat slabs, reinforcement running parallel to the curved shell surfaces will be compressed in such a failure, and therefore contribute to the energy dissipation. Here we ignore the contribution of such reinforcement, so that equation (1) only gives the failure load for an unreinforced shell.

3. THE OPTIMUM FAILURE SURFACE

The problem now is to find the function r(x) which minimises the load P in equation (1), subject to the condition dictated by the plane-strain yield locus that

$$r' \geqslant \tan\phi \qquad (2)$$

The additional difficulty in the case of spheres is that the upper limit of integration x_2 is itself variable because of the curvature of the inner shell surface.

We consider first the case when the minimising curve always has a slope greater than $\tan\phi$. The minimising curve presumably has r continuous, but discontinuities in slope r' would seem to be permissible on physical grounds. However, the Weierstrass-Erdmann corner conditions (ref. 2. p.33) show that the minimising curve for (1) will have continuous slope. According to the calculus of variations (see eg Pars [2] or Irving and Mullineux [3], the minimising curve r = r(x) will then satisfy the appropriate Euler equation, which for a functional of the form $\int F(x,r,r')dx$ can be written

$$\frac{\partial F}{\partial r} - \frac{d}{dx}\left(\frac{\partial F}{\partial r'}\right) = 0 \tag{3}$$

On substituting from (1) this reduces to

$$1 + (r')^2 - r.r'' = 0 \tag{4}$$

whose solution may be written

$$r = a \cosh\left(\frac{x}{a} + b\right) \tag{5}$$

where a and b are constants.

The upper limit of integration x_2 is variable but the failure surface must end on the inner shell surface $r = r_2(x)$. In these circumstances the optimising function $r(x)$ must satisfy the so-called 'transversality condition'.

$$F + (r_2' - r')\frac{\partial F}{\partial r'} = 0 \tag{6}$$

at the upper limit $x = x_2$ (ref.2 p.96, ref. 3 p.362). From (1) and (6)

$$r_2' = r' + \sqrt{1 + (r')^2} \tag{7}$$

which reduces, using (5), to

$$r_2' = \exp\left(\frac{x}{a} + b\right) \tag{8}$$

at $x = x_2$.

It turns out that in many cases the catenary curve satisfying (5) and (8) and passing through the edge $(0, d_o/2)$ of the loaded area violates condition (2) near $x = 0$. The portion of the optimising generatrix near $x = 0$ will then be a straight line of slope $\tan\phi$, so that part of the failure surface is conical. Consideration of a series of catenaries satisfying (5) and (8) and passing through different points on this straight line then shows that the optimising generatrix is tangent at some point $x = x_1$ to this line. This may be confirmed by considering an analogy with a heavy string, and a numerical investigation shows that the stationary value found for the integral (1) is indeed a minimum.

We then obtain, independently of R/h, if a is positive,

$$x_1 = a \operatorname{cosec}\phi - \frac{d_o}{2}\cot\phi \tag{9}$$

and

$$b = \sinh^{-1}(\tan\phi) - x_1/a \tag{10}$$

The equation $r = r_2(x)$ for the inner circle may be written

$$r_2^{\,2} = R^2 - (\beta - x_2)^2 \tag{11}$$

where β is a known constant. This may be combined with (5) and (8) to give

$$a = 2(\beta - x_2)\,[1 - (\beta - x_2)^2/R^2] \tag{12}$$

Equations (9), (10) and (12) give the important parameters x_1, b and a in terms of the upper limit x_2, for which an equation can be found by combining these and the transversality condition (8) to give

$$\frac{\beta - x_2}{\sqrt{R^2 - (\beta - x_2)^2}} = \exp\{\sinh^{-1}(\tan\phi) - \operatorname{cosec}\phi + \frac{2x_2 + d_o\cot\phi}{4(\beta - x_2)[1 - (\beta - x_2)^2/R^2]}\} \quad (13)$$

This equation for x_2 may be solved by iteration or by a graphical method. The possibility of obtaining numerical solutions by this approach was pointed out to me by P. R. Hunter.

Once the optimal failure surface has been found the corresponding failure load is obtained from equation (1) which becomes

$$\frac{P}{\pi fc} = (\sec\phi - \tan\phi)\frac{x_1}{2}(d_o + x_1\tan\phi) + \frac{a}{2}(x_2 - x_1)$$

$$+ \frac{a^2}{4}\left[\exp\left(-2\left(\frac{x_1}{a} + b\right)\right) - \exp\left(-2\left(\frac{x_2}{a} + b\right)\right)\right] \quad (14)$$

4. SOME TYPICAL SOLUTIONS

The optimising curves r(x) for the case R = 6h and various punch diameters are plotted in Fig. 3, for the same angle of friction as used by Braestrup, $\tan\phi = 3/4$. Notice that an appreciable proportion of the failure surface is conical even for $d_o = 0$, and that this proportion increases as the punch diameter increases until the optimal failure surface becomes entirely conical for d_o/h greater than about 3.3.

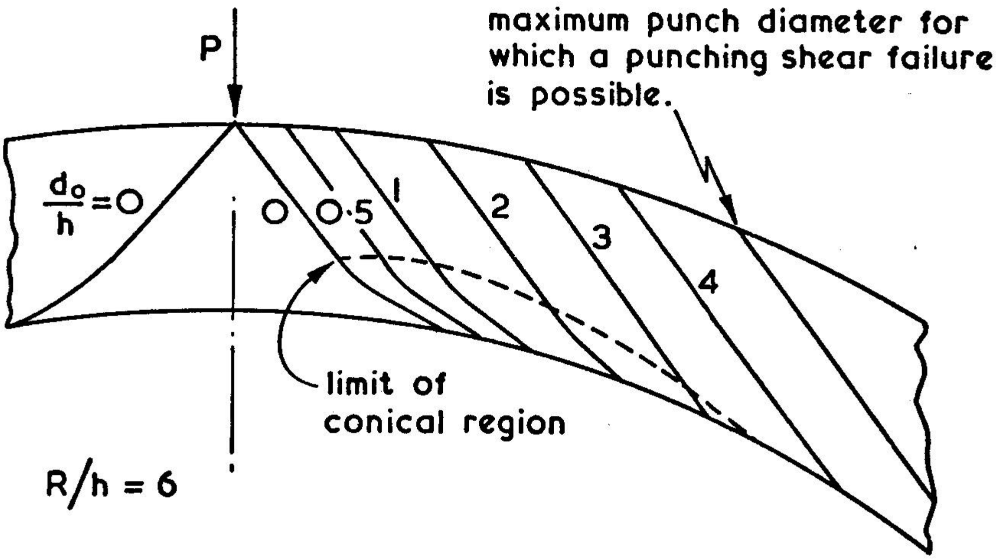

Fig. 3 Optimising curves for various punch diameters.

If the loaded area is large enough a punching shear failure is impossible because the cone with slope $\tan\phi = 3/4$ never intersects the inner surface of the shell. In these circumstances some other failure mode must intervene, presumably some form of bending failure. For punch diameters of practical interest, if the shell curvature is less marked, say R/h greater than about 10, the proportion of the optimal failure surface which is conical reduces as d_o/h increases, as in Braestrup's Fig. 8.

Some calculated failure loads are plotted in Fig. 4, which shows the dimensionless measure of failure load $P/\pi f_c h(d_o + 2h)$ used by Braestrup as a function of the shell curvature R/h for different punch diameters d_o/h. The increase of punching shear strength with shell curvature is clear.

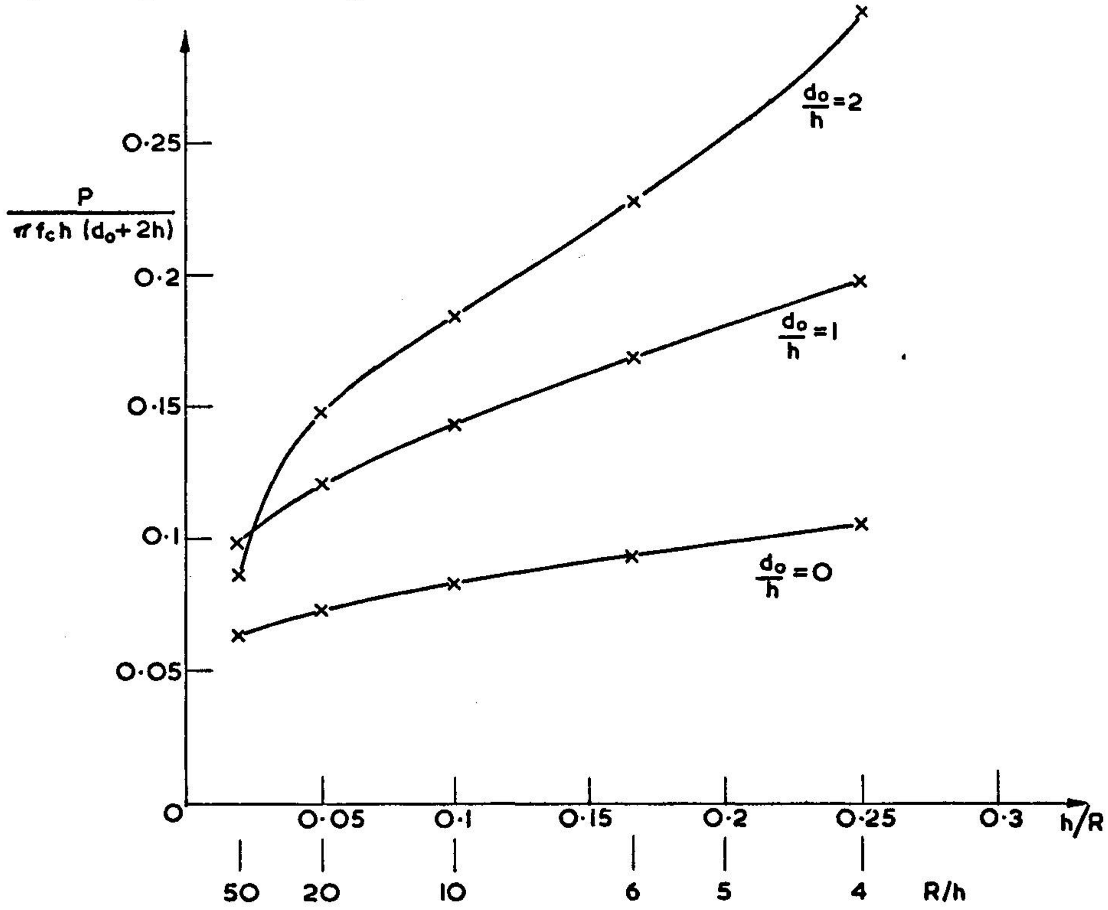

Fig.4 Theoretical failure load against curvature for a spherical shell.

5. SOME EXPERIMENTAL RESULTS ON CYLINDERS.

Some preliminary tests on concrete cylinders under concentrated radial load have recently been carried out in Cambridge by P. R. Hunter. The specimens were lengths of commercial spun-concrete sewer pipe, which were provided with diaphragms cast in situ and were supported on the laboratory floor all along a generator. The wall thickness was approximately 40 mm and the internal radius approximately 150 mm. The pipes had only nominal reinforcement, and small diameter cores drilled from them gave mean estimated cube strengths of 70.5 N/mm^2 for pipe 1, 57.5 N/mm^2 for pipe 2.

The pipes were loaded radially inwards through square steel plates cemented on to the concrete surface. Plates of various sizes were used, and in some cases pipes were retested with larger plates placed over the hole left by a previous test to failure. Various bending cracks developed during the tests, but in all cases failure occurred by punching out of a plug of concrete, square at the outer surface to match the steel loading plate. In the longitudinal cylinder direction the failure pieces were elongated as shown in Braestrup's Fig. 9, with the failure surface in some cases reaching to the nearest diaphragm. The failure pieces were much shorter in the hoop direction, as the present theory predicts, often with a steep failure surface close to the loading plate, corresponding to the predicted conical part of the failure surface.

Since the tests were on bought-in specimens the results were inevitably rather scattered, but some dimensionless failure loads are plotted in Fig. 5 against equivalent punch diameter. Also shown is the curve for a flat slab ($R/h \to \infty$) of concrete with a tensile strength f_t of $f_c/400$, from Braestrup's Fig. 10. The upper theoretical curves in Fig. 5 show the prediction of the theory for spherical shells having $f_t = 0$ and $R/h = 4$ and $R/h = 6$ respectively. For all the theoretical curves the yield strength f_c in compression is taken as 0.6 times the measured cube strength f_{cu}.

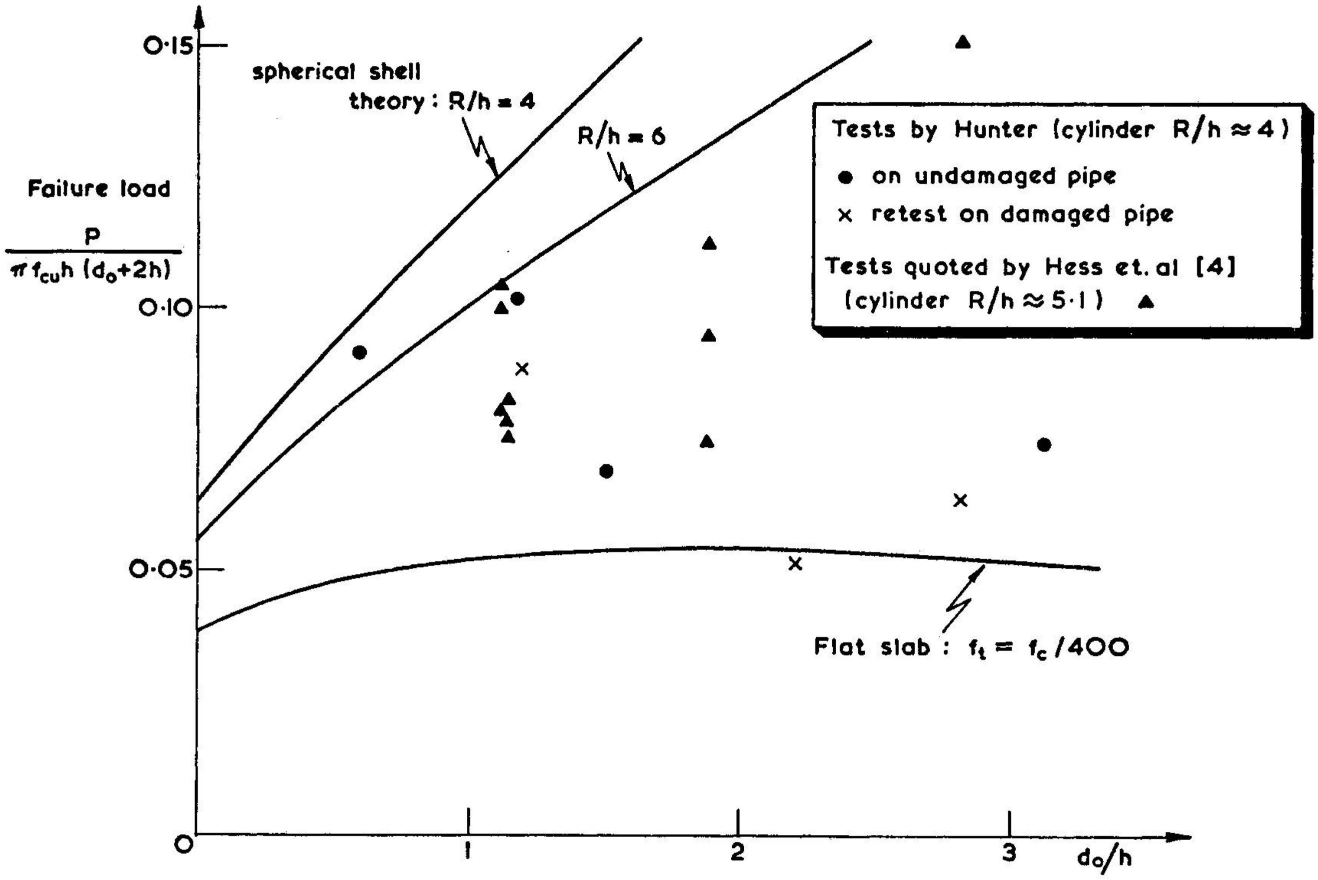

Fig. 5 Experimental results on cylinders in punching shear.

Also shown on the figure are some experimental results quoted by Hess [4] for cylinders with $R/h \approx 5.1$, taking the measured cylinder strength as 80% of the cube strength. These shells had about 1.6% of steel in the hoop direction.

One would expect the test results for cylinders to lie between the predictions for a sphere and a flat plate. This seems to be roughly true for small punch diameters, but the Cambridge results for larger punch diameters seem rather to follow the flat-plate predictions. Perhaps the large lateral compressive forces which should accompany the theoretical localised punching failure cannot easily be provided in a cylinder for large punch diameters, so that the failure mode is not pure punching but involves some bending. Clearly, more extended comparison of the theory with test results for spheres and cylinders is desirable.

6. CONCLUSIONS

A simple extension has been made to Braestrup's plastic theory of punching shear in flat slabs, to enable spherical shells with zero tensile strength to be treated. The theory predicts a substantial increase in failure load with shell curvature, but this increase is not very apparent in the results of the preliminary tests on cylindrical shells.

ACKNOWLEDGEMENT

I have had some very fruitful discussions with Dr. E. N. Fox, particularly about the application of the calculus of variations to this problem.

REFERENCES

1. Braestrup, M. W. Punching shear in concrete slabs. IABSE Colloquium on Plasticity in Reinforced Concrete, Copenhagen 1979, Introductory Report p. 115-136.

2. Pars, L. A. An introduction to the calculus of variations, Heinemann, London, 1962. 350 pp.

3. Irving, J. and Mullineux, N. Mathematics in physics and engineering. Academic Press, New York and London, 1959. 883 pp.

4. Hess, U. et. al. Gennemloking af jernbetonplader. Structural Research Laboratory, Technical University of Denmark, Report No. R 90, 1978, 63 pp.