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The Bimoment Method for Hillerborg Slabs

La méthode du bimoment pour les dalles-Hillerborg

Die Bimomentsmethode für Hillerborg-Platten

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SUMMARY
This contribution introduces a new macroscopic principle of static equilibrium for segments of Hillerborg plate of large size. These elements can then be seen to be more nearly statically determinate than had been realized. Exact plastic design of Hillerborg plates is often a practical routine design-office activity.

RESUME
L'article présente un nouveau principe macroscopique d'équilibre statique pour des éléments de grande dimensions de dalles-Hillerborg. Le degré d'indétermination statique est plus petit que celui auquel on pouvait s'attendre. L'analyse plastique exacte de dalles-Hillerborg fait souvent partie de l'activité de routine d'un bureau d'ingénieurs.

ZUSAMMENFASSUNG
Ein neues makroskopisches Gleichgewichtsprinzip für Elemente von Hillerborg-Platten wird eingeführt, und es wird gezeigt, dass der Grad der statischen Unbestimmtheit dieser Elemente geringer ist als erwartet. Die plastizitätstheoretisch vollständige Bemessung von Hillerborg-Platten ist oft ein praktisches Handwerkszeug für die übliche Bemessungstätigkeit.

The papers by Morley and by Nielsen in the Introductory Report provide excellent statements of the current research situation in regard to plastic behaviour of slabs. The subject is certainly a difficult one - one has only to reflect that we are now approaching a half-century of work since Johansen began his pioneering efforts and to consider the number of eminent engineers who have contributed in that time. It is satisfying then that worthwhile progress is being made into matters of quite basic and fundamental importance.

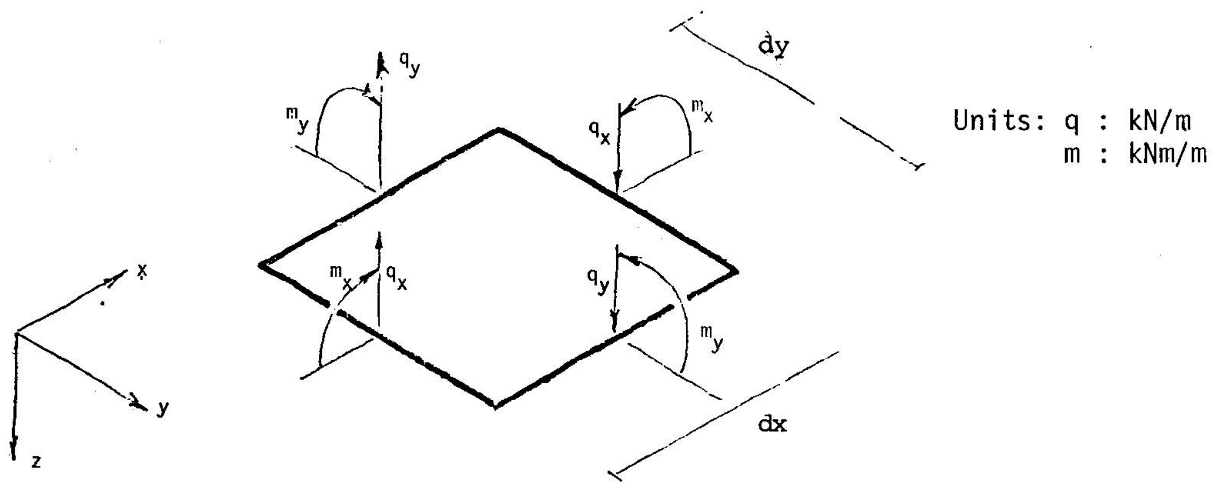
Nevertheless it does seem that it will be some time yet before the more advanced matters considered in this session are fully resolved and reduced to routine design-office procedures. The present situation in design practice is not entirely satisfactory and it seems then that designers will need to seek some interim approach of a rather more pragmatic character until these more basic issues are resolved. It cannot be assumed that all designers will have easy access to computer facilities at all times. Neither is it desirable that designers become totally reliant on such facilities. We seek then "here-and-now" design procedures which will provide for straight-forward design with no more equipment than a pocket calculator.

The purpose of this contribution is to suggest that there is a good deal of unrealised potential in Hillerborg's Simple Strip Method and to show how that Method can be improved a more satisfactory design procedure. This can be done by re-examining the equilibrium conditions for a rectangular segment of Hillerborg plate of finite (non-differential) size.

Consider then a small (differential) element of Hillerborg plate as sketched in Fig. 1. The coordinate axes and the element edges are to be taken parallel to the directions of the reinforcing mesh. We assume here that these are perpendicular although the extension to skew reinforcement would not seem to be difficult so long as there are just two reinforcement directions. Following Hillerborg it is assmed that the local twisting moments 'm_{xy}' and 'm_{yx}' are zero everywhere. It follows that there are only local distributed shear forces and local distributed bending moments on each edge. The variation of these stesses across the width 'dx' of the element involves expressions like:

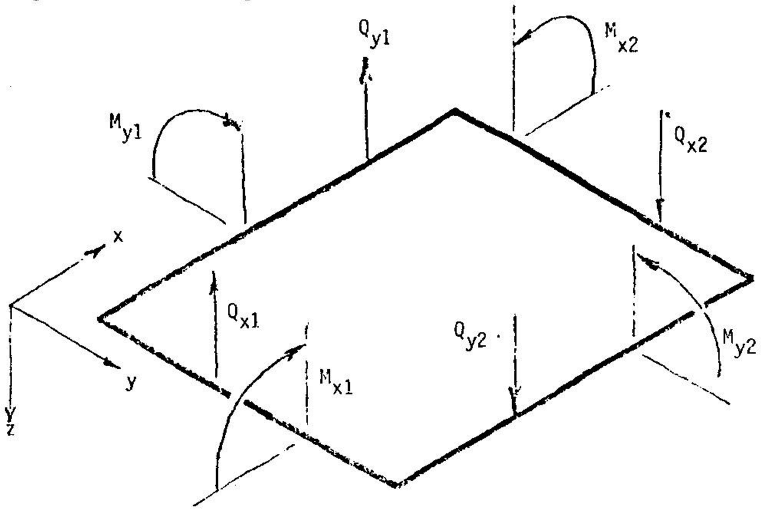
$$m_x + dm_x$$

We do not record these here because the equilibrium conditions for a differential element are already well-known and our present interest is to formulate the equilibrium conditions for a <u>large</u> element.



<u>Fig. 1</u> Local Distributed Stress-Resultants on a <u>Small</u> Element

Consider then a typical large element shown in Fig. 2. The scale of this element may be of the order of several metres on each edge and it may incorporate several Hillerborg strips in each direction. The external equilibrium conditions for this element involve the 8 independent stress-resultants shown together with a further 8 independent coordinates specifying the position of action of each of these stress-resultants measured along the relevant edge. Thus there are, in total, 16 indpendent stress-resultant variables involved in the overall equilibrium of the segment. The conventional state-of-the-art would suggest that these are subject to 3 independent overall equilibrium conditions so that, the large element is 13 times hyperstatic externally. The proposition of this contribution is that there are, in fact, 4 independent overall equilibrium conditions so that the element is only 12 times hyperstatic externally. And, of course, this proposition applies to any and every sub-element resulting from sub-division of the element. It turns out that, while the degree of reduction in hyperstasy appears slight it is, combined with the usual yield conditions, often sufficient to permit exact plastic design as a matter of practical routine in many common design-office situations.



Units: Q : kN
M : kNm

Fig.2 Stress-Resultants on a Large Element

We consider now a virtual displacement in the form of a small unit hyperbolic paraboloid (Fig. 3):

$$z = xy$$

The position of the origin '0' is arbitrary except only that it is in the plane of the element. Under this displacement, the generators of the hypar remain straight so there is no curvature in the directions of the reinforcing mesh and so no virtual work is done by the bending moments except on the perimeter. In this sense then the above is a virtual rigid body displacement.

All of the quantities involved in the virtual work equation will have dimensions of force times two distinct leverarms e.g. kNm^2. It may be somewhat misleading to reuse Vlasov's term "bimoment" in this context but it does have a certain logical inevitability.

We define then the "Restoring Bimoment" of the above plate about origin '0' as the virtual work done by the perimeter moments under the above virtual displacement. Similarly we define the "Overturning Bimoment" about origin '0' as the virtual work done by the loads. It is usually more convenient to include the virtual work done by the perimeter shears in the overturning bimoment as if they were perimeter line-loads. On this basis the virtual work equation becomes, very simply:

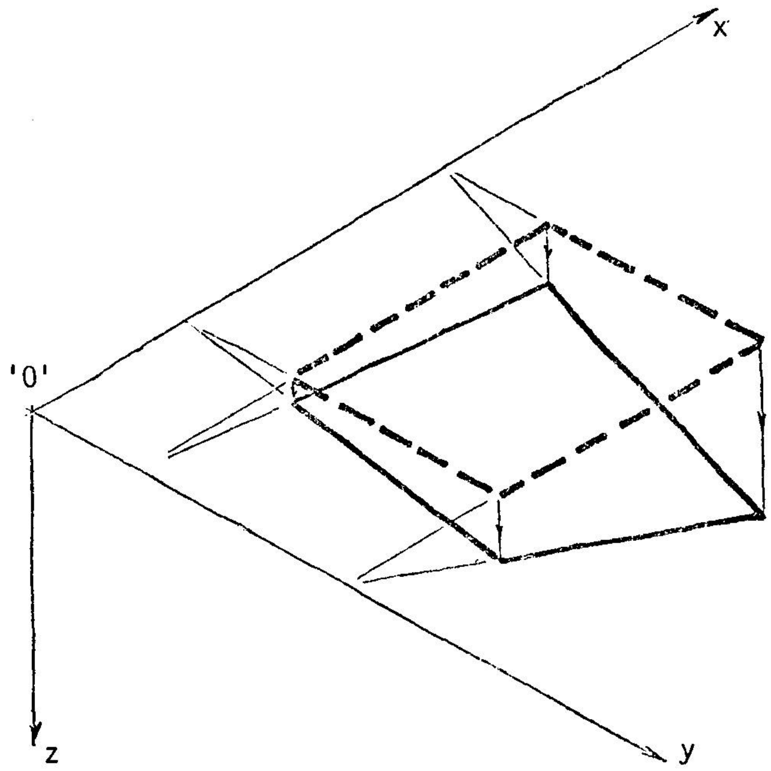


Fig. 3

Overturning Bimoment = Restoring Bimoment

This result can be expressed in mathematical terms using double integrals. Indeed it can be proved by integrating the local (differential) equilibrium equation with the aid of Green's theorem (two-dimensional integration by parts). To do so would obscure the simplicity of the result. In a routine design calculation the quantities concerned can, almost invariably be evaluated from direct, simple physical considerations. It is not necessary either to become pre-occupied with matters of sign convention. The correct sign is usually quite obvious and can, in any case, always be resolved by sketching out the virtual displacement.

For any particular segment it is always possible to find four independent bimoment equilibrium equations. These include the known three equilibrium equations e.g. moments about each of two axes and equilibrium of total load with perimeter shears. Indeed these known conditions can be regarded as bimoment conditions in which the origin has been pushed to infinity in one or other or both directions. In any case it is clear that we now have available a good deal more equilibrium information than we had expected. This is surely significant.

The author has developed the above approach and used it in many actual routine design calculations since mid 1977. The principal advantage is that it provides simple direct relationships between moment-fields and total loads. It is possible then to avoid the initial arbitrary assignment of strip-widths and strip-loads as suggested in the original Hillerborg proposals. This makes it possible to produce "practical optimum" designs every time and to do so within the constraints of Code minimum reinforcement content, reasonable simplicity of construction etc.

However the author does conclude all designs with a Hillerborg solution showing the strip-loads calculated from the assigned moment-fields. Whether or not such a solution is theoretically essential, it provides an independent check against gross errors of calculation and this is always desirable.

Is the Bimoment Method a Lower Bound approach? It uses a macroscopic form of the equilibrium conditions but, theoretically at least, it can be applied to progressively smaller elements and, in the limit, this process amounts to differentiation and necessarily re-establishes the local (differential) equilibrium conditions. In practice the author only uses bimoment methods to assign sufficient of the moment-field to determine reinforcement (including, sometimes, cut-off and curtailment positions) and then uses Hillerborg methods to complete and check the statical solution. This combination seems to work very well.

In many cases it is possible to do exact plastic design of slabs regarded as Hillerborg plates. It would seem that such designs are necessarily very efficient lower bound designs for slabs regarded as Johansen plates. In practice the steel in quite extensive areas of slabs is determined by Code rules on minimum reinforcement content. It is usually possible to ensure that this steel is fully utilised at yield strength under design load.

In other cases, particularly those involving a re-entrant or near-point load or support acting integrally with the slab, it is not possible to find an exact plastic solution even when the slab is regarded as a Hillerborg plate. These situations seem synonymous with those in which the, usually, "secondary" effects of shear and strain-hardening have substantial significant. It is not reasonable to expect rigid plastic thin plate theory to provide "exact" solutions in such complex 3-dimensional situations. Design in such situations is a linear programming "game". Success in such situations does depend on the judgement and intuition of the designer but, then, these are the skills possessed by experienced designers and the bimoment approach does provide equilibrium information in a form most easily assimilated and used. Safety is not an issue, because all designs can be checked by Hillerborg procedures, but economy and suitability will still depend on the individual approach. In this matter then Engineering remains an art as well as a science.

A longer paper [Ref. 1] expected soon attempts to cover many of the points omitted because of the limited length of this contribution.

REFERENCES

1. GURLEY, C.R., "Bimoment Equilibrium of Finite Segements of Hillerborg Plate", Publication pending, Magazine of Concrete Research, London

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