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Reinforced Concrete Members in Torsion and Shear

Eléments en béton armé soumis à la torsion et au cisaillement

Stahlbetonelemente in Torsion und Schub

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SUMMARY
Progress in developing a rational model (the diagonal compression field theory) capable of predicting the behaviour of reinforced concrete members in torsion and shear is reported. The differences between the diagonal compression field theory and the procedures based on plastic analysis are highlighted.

RESUME
Cet article rend compte du progrès dans le développement d'un modèle rationnel (théorie du champ de compression diagonale) capable de prédire le comportement d'éléments en béton armé soumis à la torsion et au cisaillement. Les différences entre la théorie du champ de compression diagonale et les méthodes de l'analyse plastique sont mises en évidence.

ZUSAMMENFASSUNG
Es wird über den Fortschritt bei der Entwicklung eines rationalen Modells (Theorie des diagonalen Druckfeldes) berichtet, mit dem das Verhalten eines Stahlbetonelementes bei Torsion und Schub vorausgesagt werden kann. Die Unterschiede zwischen der Theorie des diagonalen Druckfeldes und den Verfahren, die auf plastischen Berechnungen beruhen, werden herausgestellt.

1. INTRODUCTION

During the past 10 years research aimed at developing behavioural theories for reinforced concrete in torsion and shear comparable in rationality and generality to the well known theory for flexure and axial load has been conducted at the University of Toronto.

The unsatisfactory nature of the shear and torsion "theories" currently used in North American design practice is evident if the ACI [1] chapter on shear and torsion is compared with the ACI chapter on flexure and axial load. In the flexure and axial load chapter a rational, simple, general method is explained in a few paragraphs of text. On the other hand, the shear and torsion chapter consists of a collection of complex, restricted, empirical equations which, while leading to safe designs if properly used, lack any understandable central philosophy. This lack, in the opinion of the author, is the source of many of the complaints which arise from the profession about modern design codes becoming unworkably complicated.

In this paper first the well known theory for flexure and axial load will be briefly reviewed. Then the development of comparable theories for pure torsion, torsion and bending, and shear and bending will be summarized. The paper will conclude by contrasting the theories developed at Toronto with those developed in Zurich and Copenhagen.

2. PLANE SECTIONS THEORY FOR FLEXURE AND AXIAL LOAD

Although the "plane sections" theory which is capable of predicting the behaviour of reinforced concrete beams loaded in flexure and axial load is fully described in many textbooks (e.g. [2]) it will be briefly illustrated here so that the capabilities of the theory, and the assumptions on which it is based can be more readily appreciated.

Assume that it is desired to find the moment-curvature relationship of a rectangular reinforced concrete beam from the known cross-sectional dimensions, Fig. 1(a), and the known stress-strain characteristics of the concrete, Fig. 1(b), and the steel, Fig. 1(c). As it is assumed that "plane sections remain plane" only two variables (say the concrete strain at the top, ε_{ct}, and the depth to the neutral axis, kd) are required to define the concrete longitudinal strain distribution, Fig. 1(d). For a chosen value of ε_{ct} a trial value of kd can be selected and the concrete strain distribution will then be fixed. The longitudinal concrete stresses, Fig. 1(e), can then be found from the concrete strains by using the assumed concrete stress-strain characteristics. Usually it is assumed that in compression the stress-strain curve obtained from a test cylinder, Fig. 1(b), can be used and that in tension the concrete is not capable of resisting stress. To determine the steel stress it is assumed that the strain in the steel is equal to the strain in the surrounding concrete, Fig. 1(d), and that the stress-strain characteristics obtained from a tension test of a reinforcing bar, Fig. 1(c), can be used. Knowing the stresses acting on the cross-section the resulting compression force in the concrete, C, and tension force in the steel, S, can be computed, Fig. 1(f). In the case of zero axial load, equilibrium requires that C equals S and so if this is not the case the trial value of kd must be adjusted and the calculations repeated. When the correct value of kd has been found the moment, M, corresponding to the chosen value of ε_{ct} can then be calculated,

Fig. 1(g). This moment along with the curvature calculated from the strain distribution, Fig. 1(d), will give one point on the moment-curvature plot. Repeating the calculations for different values of ε_{ct} will produce the complete moment-curvature relationship, Fig. 1(h).

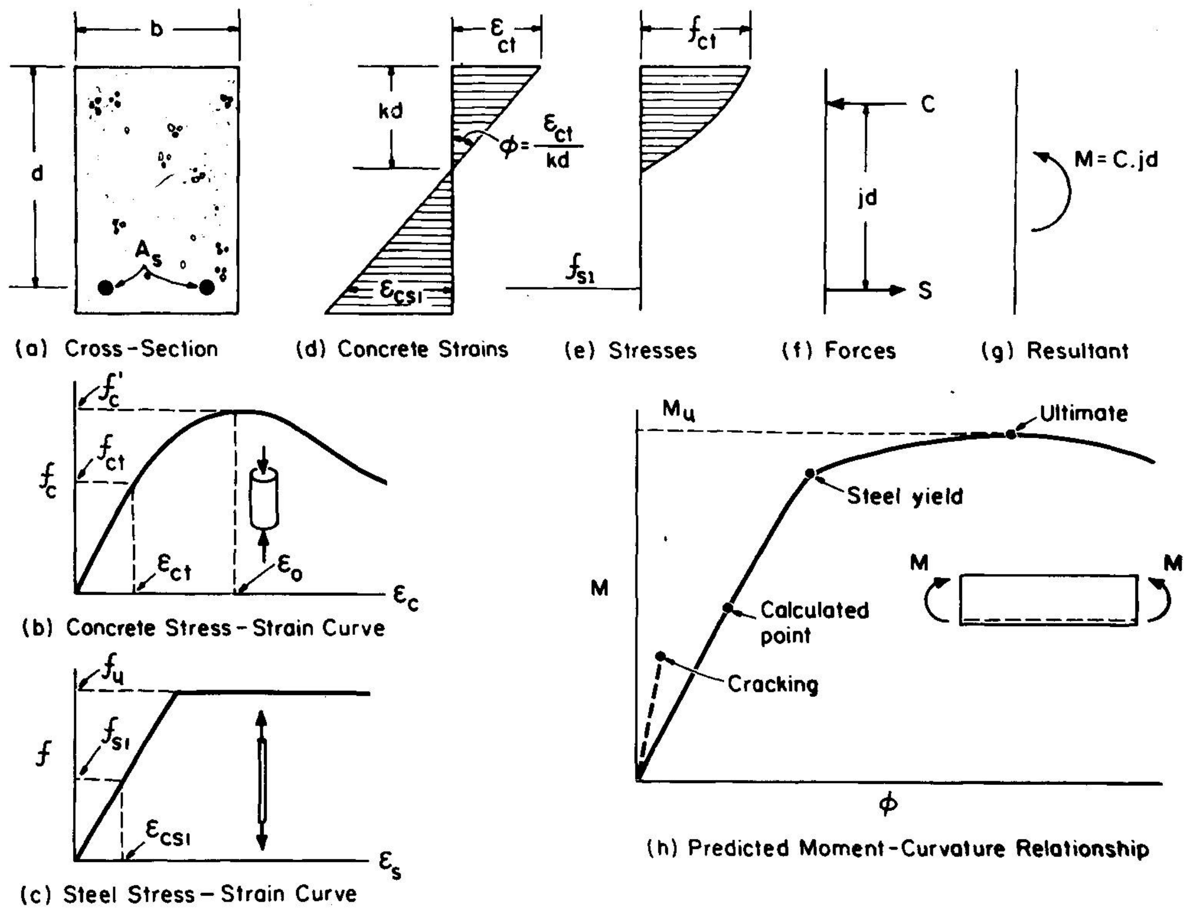

Fig. 1 Plane Sections Theory for Flexure

It should be noted that the concrete strains used in the above calculations are "average" strains rather than actual local strains. Thus the tensile strain at the level of the steel, ε_{cs1}, will be the average of high local values that will occur at crack locations and the lower values that will occur between the cracks. In a similar fashion the calculated steel stress, f_{s1}, should be representative of the average stress in the steel.

In determining the magnitude and position of the resultant compression in the concrete, C, Fig. 1(f), it is sometimes convenient to replace the actual stress distribution with an equivalent uniform stress distribution. Thus the distribution shown in Fig. 1(e) could be replaced by a uniform stress of $\alpha_1 f_c'$ acting over a depth β_1 kd where the stress block factors α_1 and β_1 have been chosen so that the magnitude and position of C do not change. For a constant width of beam, b, the values of α_1 and β_1 will depend only on the shape of the stress-strain curve, Fig. 1(b), and the value of the highest concrete strain, ε_{ct}.

The simple "plane sections remain plane" theory illustrated in Fig. 1 can be applied to quite complex problems. For example, Fig. 2 compares the predicted [2] moment-curvature response of a reinforced concrete beam subjected to reversed, cyclic loading with the experimentally determined response. Apart from the plane sections theory, only the stress-strain characteristics of the concrete and the steel under reversed, cyclic loading were required to make this prediction.

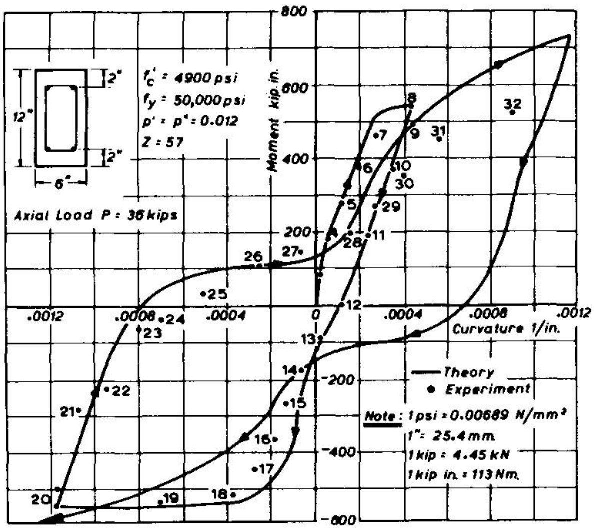

Fig. 2 Predicted Moment-Curvature Response Under Reversed Cyclic Loading [2]

3. DIAGONAL COMPRESSION FIELD THEORY FOR PURE TORSION

The diagonal compression field theory for pure torsion, which has been presented in more detail elsewhere [3], will be illustrated here by examining the problem of predicting the post-cracking torque-twist response of symmetrically reinforced concrete beams.

The theory assumes that after cracking the concrete can carry no tension and that the torsion is resisted by diagonal concrete compressive stresses which spiral around the beam at a constant angle α, Fig. 3(a). The outward thrust of these diagonal compressive stresses tends to push the corners of the beam apart which produces tension in the transverse hoops. The longitudinal components of the diagonal compressive stresses tends to push apart the ends of the beam which produces tension in the longitudinal steel.

Not all of the concrete is effective in providing diagonal compressive stresses to resist the torsion. The concrete cover outside of the hoop centreline is assumed to be ineffective because at higher loads this cover will spall off [3]. If the deformed shape of the twisted beam, Fig. 3(b), is examined it can be observed that the walls of the beam do not remain plane surfaces. Because of the curvature of the walls, ϕ_d, the diagonal compressive strains will have their maximum values at the surface, ε_{ds}, and will decrease linearly with the distance from the surface becoming tensile for depths below a certain distance, t_d. Thus in torsion as in flexure we have a depth of compression below which we may assume that the concrete, being in tension, is ineffective. The outside concrete spalls off and the inside concrete goes into tension, hence we are left with a tube of effective concrete t_d thick which lies just inside the hoop centreline.

The diagonal concrete stresses will vary in magnitude over the thickness of the effective concrete tube from zero at the inside to a value f_{ds} corresponding to

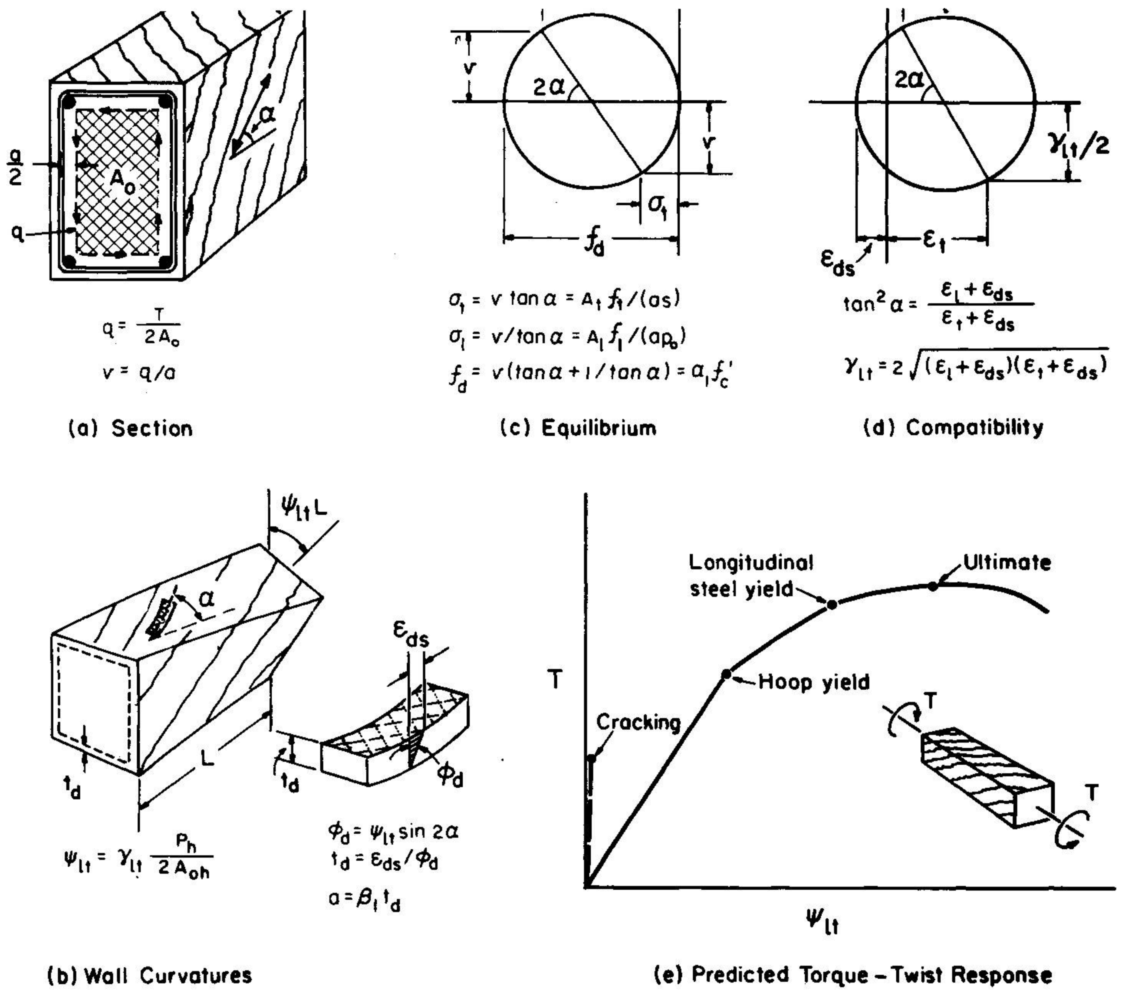

Fig. 3 The Diagonal Compression Field Theory for Pure Torsion

the strain ε_{ds} at the effective outside surface. As in flexure we can replace this actual stress distribution by a uniform stress of $\alpha_1 f_c'$ acting over a depth of $\beta_1 t_d = a$ where the stress block factors α_1 and β_1 depend on the shape of the concrete stress-strain curve and the value of ε_{ds}. The depth of this uniformly stressed concrete, a, will define the path of the shear flow, q, Fig. 3(a) and hence the terms A_o (the area enclosed by the shear flow) and p_o (the perimeter of the shear flow path).

To illustrate how a solution can be obtained let us imagine that we wish to find the response of a given beam to a given torsional load, T. We could start by estimating the equivalent depth of compression, a. From a and the known hoop geometry we could find A_o and p_o, Fig. 3(a) and then from T, A_o and a, the uniform shear stress, v, could be found. After a trial value for the angle of inclination of the principal compressive stress, α, has been chosen we can use the equilibrium equations, Fig. 3(c), to find the stresses in the transverse hoop steel, f_t, the longitudinal steel, f_ℓ, and the equivalent uniform diagonal stress in the concrete, f_d. These stresses and the appropriate stress-strain curves for the concrete and the steel, e.g. Fig. 1(b) and 1(c), can then be used to determine the tensile strains in the hoop steel, ε_t, and the longitudinal steel, ε_ℓ, and the compressive diagonal surface strain of the concrete, ε_{ds}. These strain values enable the direction of the principal compressive strain to be computed, Fig. 3(d), and hence allow the trial value of α to be checked. It is assumed that the direction of principal compressive stress coincides with the direction of principal compressive strain. After a consistent value of α has been found the strain values can be used to compute the twist of the beam, $\Psi_{\ell t}$, Fig. 3(b), (A_{oh} and p_h are the area enclosed by the centreline of the hoop and the perimeter of this area, respectively.) Once the twist is known the curvature of the walls, ϕ_d, can be calculated, Fig. 3(b). This curvature and the surface strains, ε_{ds}, define the equivalent depth of compression, a, Fig. 3(b). If the calculated value of a does not agree with the assumed value then a new estimate of a must be made and the calculations repeated. When the correct value of a has been determined then the response of the beam (i.e. the twist and the strains) at this given value of torque will have been found.

While the calculations described above perhaps sound rather formidable, it is possible to reformulate the expressions so that only one variable has to be found by trial and error [3]. With the aid of a programmable pocket calculator the complete torque-twist curve of a beam, Fig. 3(e), can then be found in about the same time as it takes to find the moment-curvature curve, Fig. 1(h).

4. COMBINED TORSION, FLEXURE AND AXIAL LOAD

The recently developed [4] compression field theory for combined torsion, flexure and axial load is essentially a combination of the plane sections theory for flexure and the diagonal compression field theory for torsion. The theory will be illustred here by discussing the manner in which the response of a beam loaded in combined torsion and flexure can be predicted.

Say that we wish to find the response of a given beam, Fig. 4(a), to a given torsional load, T, and a given flexural load, M. As in flexure the longitudinal strain distribution will be defined by two variables, Fig. 4(b) which this time we will choose as the top strain, ε_{ct} and the bottom strain, ε_b. The calculations commence by estimating ε_{ct} and ε_b. From the estimated strain distribution and the stress-strain curve of the steel the magnitude and position of the resultant tension force in the longitudinal steel, S, can be calculated, Fig. 4(e).

For any element of concrete the longitudinal stress depends not only on its longitudinal strain and stress-strain curve but also on the magnitude of the co-existing shear stress. If tension is to be avoided then when shear stresses are present there must also be longitudinal compressive stresses even when there are longitudinal tensile strains, Fig. 3(c) and Fig. 3(d). Given the shear stress, the longitudinal strain, the amount of transverse steel and the stress-strain curves, the longitudinal concrete stress can be calculated. For convenience the beam section can be divided into elements so that within each element the longitudinal strain can be taken as approximately constant, Fig. 4(a). As in the pure torsion calculations there will be a tube of effective concrete lying just inside the hoop centreline but now the thickness of the tube will vary around the cross-section. For each element an estimate is made of this thickness (i.e. the equivalent depth of compression) and from these estimates A_o and hence the shear stresses, v, in each element are calculated, Fig. 4(a). For each element an estimate is then made of the angle of inclination of the principal stress, α. Knowing α and v the transverse hoop strain ε_t and the diagonal concrete strain ε_{ds} can be calculated from the equilibrium equations of Fig. 3(c) and the stress-strain curves of Fig. 1(b) and Fig. 1(c). The estimate of α can then be checked from the basic geometric equation of Fig. 3(d), namely:

$$\tan^2\alpha = \frac{\varepsilon_\ell + \varepsilon_{ds}}{\varepsilon_t + \varepsilon_{ds}} \qquad \text{...... (1)}$$

When α values satisfying Eq.(1) have been found then the longitudinal concrete compressive stresses, σ_ℓ, can be calculated for each element, Fig. 4(d), and from these the position and magnitude of the resultant concrete compressive force, C, can be determined, Fig. 4(e).

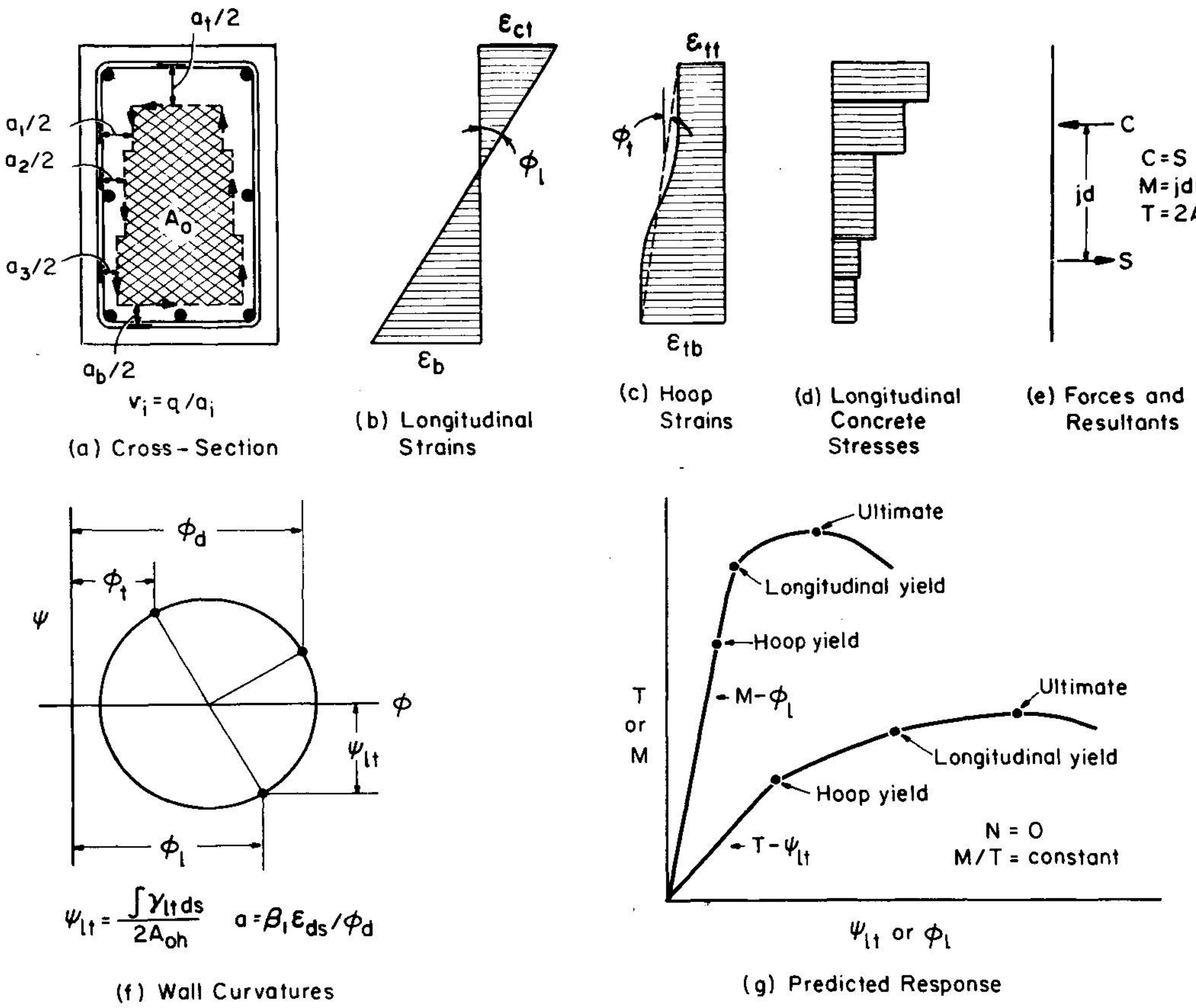

Fig. 4 The Diagonal Compression Field Theory for Torsion and Flexure

As well as giving the force C the calculations described above will have produced the distributions of transverse hoop strains, Fig. 4(c), and diagonal concrete strains around the section. These will enable the shear strains, Fig. 3(d), and hence the twist of the beam to be evaluated. From the twist, $\psi_{\ell t}$, the longitudinal curvature, ϕ_ℓ, and the transverse curvature, ϕ_t (the top and bottom faces of the beam will be curved transversely) the diagonal curvature, ϕ_d, can be calculated, Fig. 4(f). For each element ϕ_d and ε_{ds} enable the thickness a to be calculated, Fig. 4(f). If the calculated values of a do not agree with the assumed values then new estimates must be made and the calculations repeated. When the correct values of a have been found then the axial force, N, and moment, M, corresponding to the assumed longitudinal strain profile can be determined from the magnitudes and positions of the forces C and S, Fig. 4(e). If N and M do not have the desired values (in our case N should equal zero and M should equal the given value) then a new longitudinal strain profile is chosen and the whole process is repeated.

While the trial and error procedure described above sounds very laborious it luckily converges very rapidly and hence with the aid of a programmable calculator or a small computer the moment-curvature and torque-twist curves, Fig. 4(g) can be obtained relatively easily.

5. COMBINED SHEAR, FLEXURE AND AXIAL LOAD

The compression field theory has been applied to the loading cases of shear [5] and shear combined with flexure and axial load [6]. As might be expected the procedures are very similar to those for torsion and torsion and flexure. These procedures will be illustrated here by discussing the manner in which the response of a beam loaded in combined shear and flexure can be predicted.

Say that for a given beam, Fig. 5(a), we wish to find the deformations and strains associated with a given shear load, V, and a given flexural load, M. Once again the calculations commence by estimating the longitudinal strains ε_{ct} and ε_b, Fig. 5(b), from which the magnitude and position of the resultant tension force, S, in the longitudinal steel can be calculated, Fig. 5(e).

The beam is again divided into elements (this time full width strips) so that within each element the longitudinal strain can be taken as constant, Fig. 5(b). In torsion the effective width, a, was initially unknown but once we had assumed a the shear stress could be directly calculated, Fig. 4(a). In shear the width of the strips, b, is known (the side cover is again assumed to be ineffective) but the relative magnitudes of the shear stresses in the various strips can not be directly calculated. We need to make an initial estimate of the shear stress distribution, Fig. 5(c), which should of course satisfy the basic equilibrium requirement that the integral of the shear stresses over the total area must equal the shear force, V. As in the case of torsion and flexure once we know the longitudinal strain, ε_ℓ, and the shear stress, v, in a given element we can calculate the longitudinal concrete compression, σ_ℓ. From the values of σ_ℓ, Fig. 5(d), the magnitude and position of the resultant compression in the concrete, C, can be calculated, Fig. 5(e) and hence the axial load, N, and moment, M, can be determined, Fig. 5(e), and compared with the desired values.

How do we check the assumed shear stress distribution? To do this we examine a section of the beam a small distance, Δx, away from the original section, Fig. 5(f). The longitudinal strain distribution and the steel concrete stresses corresponding with the loads at this new section must be found. With the longitudinal stress distributions for these two sections known the shear stress at any depth can be calculated, Fig. 5(g). If the calculated shear stress distribution does not agree with the assumed distribution then the whole process is repeated.

Once again the trial and error procedure described above converges very rapidly and hence the predicted response (e.g. the relationship between the applied shear and the maximum hoop strain, Fig. 5(h)) can be obtained relatively easily.

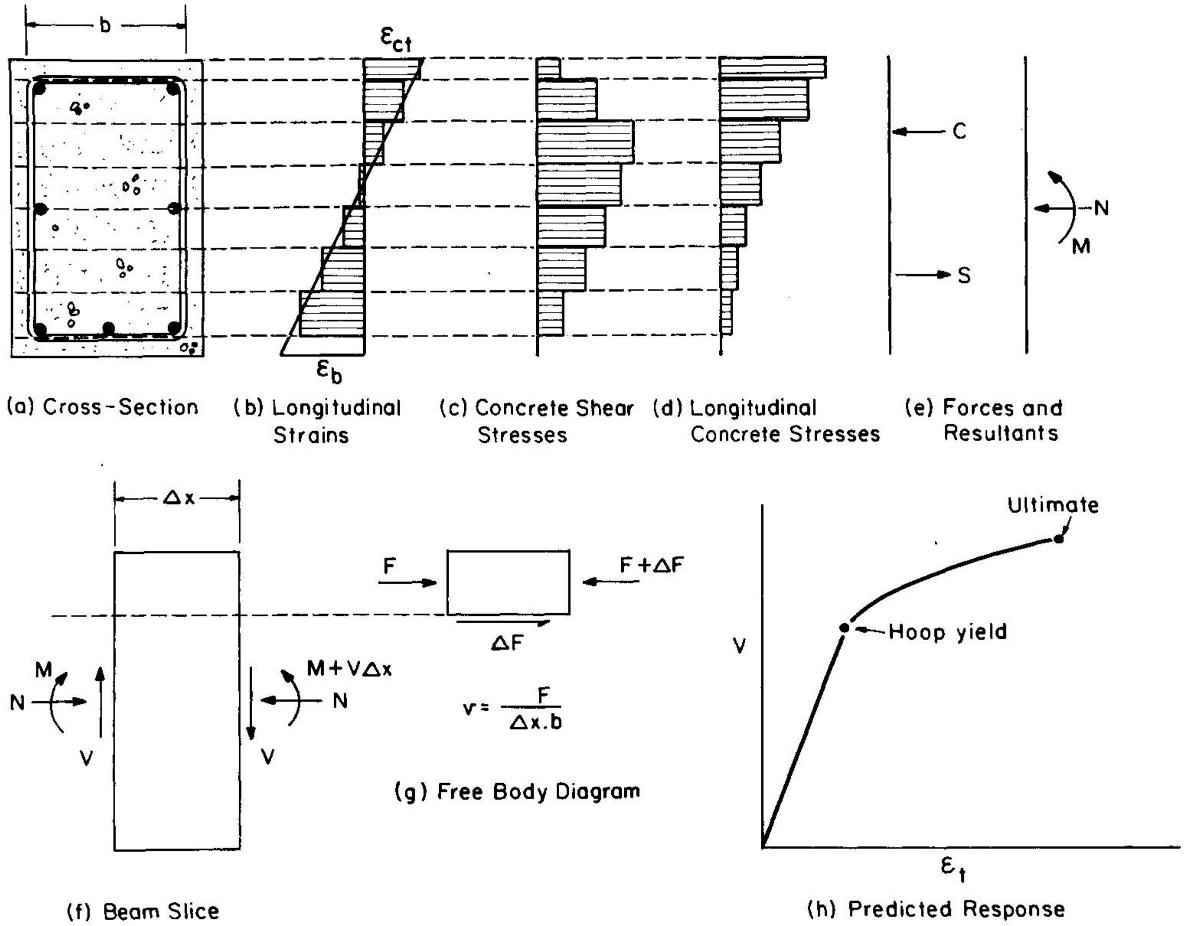

Fig. 5 The Compression Field Theory for Shear and Flexure

However, if the stress-strain curve for the diagonally cracked concrete is assumed to be the same as that obtained from a cylinder test, Fig. 1(b), the failure load of the specimen will be overestimated. While the more promising approach appears to be the modification of the stress-strain curve to allow for shear strains [7] the present procedure is to empirically limit the magnitude of the principal diagonal compressive stress, f_d, to a value f_{du} given by:

$$f_{du} = \frac{3.6\ f_c'}{1 + 2\ \gamma_m/\varepsilon_o} \qquad \text{...... (2)}$$

where γ_m is the maximum shear strain, $\varepsilon_\ell + \varepsilon_t + 2\ \varepsilon_{ds}$, and ε_o is the cylinder peak stress strain, Fig. 1(b).

6. THE DIAGONAL COMPRESSION FIELD THEORY AND PLASTIC ANALYSIS

The plastic analysis procedures for reinforced concrete beams in torsion and shear developed in Zurich [8] and Compenhagen [9] are concerned with predicting the failure loads whereas the compression field theory summarized above attempts to predict the complete load deformation response of the beams. Even if concern is restricted to only failure load predictions there are a number of singificant differences between the two approaches. Some of these differences will be illustrated below.

Shown in Fig. 6 is the observed relationship between failure torque and amount of reinforcement for 6 beams (M1 - M6) tested by Hsu [10]. For all these beams the concrete strength (28 MPa) and the steel strength (330 MPa) remained essentially constant and the volume of longitudinal steel was 1.5 times the volume of hoop steel. For small amounts of steel (Beams M1 and M2) both the hoop steel and the longitudinal steel yielded at failure. For larger amounts of steel (Beams M3, M4 and M5) only the hoops yielded at failure while for very large amount of steel (Beam M6) the beam failed before any steel yielded. As can be seen from Fig. 6 the failure torques and manner of failure for these beams are predicted well by the compression field theory.

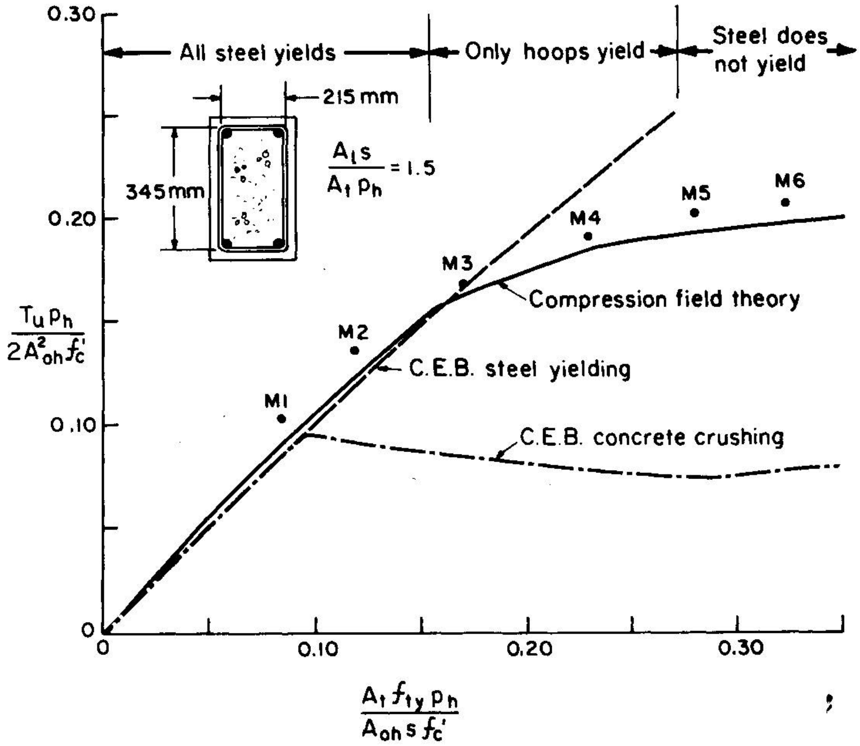

Fig. 6 Torsional Capacity versus Amount of Reinforcement

Also shown in Fig. 6 are the failure torques predicted for these beams by the provisions of the new CEB Code [11]. These code equations are based on the Zurich plastic analysis procedures. The CEB equations which assume that all the steel yields at failure of course become unconservative when the steel does not yield at failure. However, the empirical equation which is intended to predict failures in which the concrete crushes before the steel yields is very conservative for these beams. What is more, it predicts that as the amount of reinforcing steel is increased the torsional capacity will be decreased. This happens because for these beams the amount of steel was increased (for M1 - M5) by increasing the size of the reinforcing bars which had the effect of decreasing the distance between the centres of the corner longitudinal bars and for this method A_0 (the area enclosed by the shear flow) is defined as the area enclosed by lines joining the centres of the corner longitudinal bars.

To summarize, the compression field theory can predict the strains at failure, the area enclosed by the shear flow, A_o, the effective wall thickness of the wall, a, the angle of principal compression, α, and the failure torque, T_u. The plastic analysis procedures must assume the area A_0, and the wall thickness, a, and can only accurately predict α and T_u if all of the steel is yielding.

As a final point, Fig. 7 illustrates the effect of prestress on shear strength. The results of four beams (SP0 - SP3) tested by Sadler [12] all of which had the same reinforcement are shown. The main variable between these four beams was the magnitude of the uniform precompression, σ, that was applied by the unbonded central Dywidag bars.

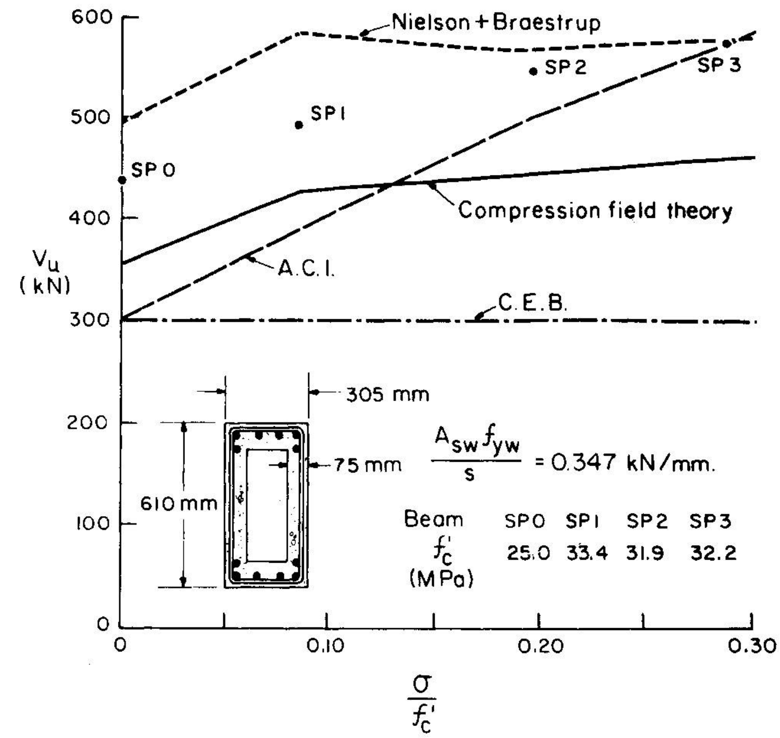

Fig. 7 Shear Capacity versus Level of Prestress

The plastic analysis procedures developed by Neilson and Braestrup [9] predict that prestressing should not influence the shear capacity and that Beam SP1 having the highest concrete strength, Fig. 7, should have the highest shear capacity. The CEB predictions [11] for these beams as well as not being influenced by the magnitude of the prestress are not influenced by the concrete strength hence all four beams are predicted to have the same strength.

In direct contradiction to the plastic analysis procedures the ACI Code [1] predicts that prestressing would very substantially increase the shear capacity of the beams. Beam SP3 is predicted to be 91% stronger than Beam SP0. The compression field theory predicts a more moderate gain in shear strength with prestress with Beam SP3 being predicted to be 29% stronger than SP0.

The experimental results showed that prestressing indeed increased the shear capacity, Fig. 7, so that Beam SP3 was 31% stronger than Beam SP0. While the trend of the experimental results was accurately predicted by the compression field theory the actual shear strengths were considerably in excess of the predictions. This was partly due to the conservative nature of the empirical stress limit, Eq.(2), and may also have been caused by end restraint of the test specimens [12].

7. CONCLUDING REMARKS

The research programme summarized in this paper has not yet resulted in a unified beam theory capable of predicting the behaviour of any reinforced concrete cross-section under any combination of loading. It is, however, believed that significant progress has been made in achieving this ultimate objective.

At present predictions for members subjected to complex loading (say all six stress resultants simultaneously) can be made with the aid of truss analogies [13], and automatic design programmes based on such analogies [14] are in use. These models, however, involve empirical assumptions as to what are the effective areas of the various components of the truss and hence they are not comparable to the "plane sections" theory for flexure which remains the "standard" against which we wish to judge all other theories.

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