Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 29 (1979)

Artikel: The stringer method applied to discs with holes

Autor: Kaern, J.C.

DOI: https://doi.org/10.5169/seals-23538

II

The Stringer Method Applied to Discs with Holes

La méthode des „stringers" appliquée aux parois avec des ouvertures

Anwendung der Stringermethode auf gelochte Scheiben

J.C. KAERN
PhD Student
Technical University of Denmark
Lyngby, Denmark

SUMMARY
In this paper a simple method — the stringer method — is discussed for constructing lower-bound solutions for reinforced concrete discs with holes. In the method normal stresses are concentrated in lines, the stringers, forming an orthogonal net. The rectangular areas between the stringers are assumed to carry pure shear only. The use of the method is described by simple examples. Further, the problem of optimizing the statical and geometrical variables is dealt with.

RESUME
Des valeurs inférieures pour la charge ultime de parois avec des ouvertures sont obtenues à l'aide d'une méthode simple. La paroi est divisée par un réseau orthogonal de „stringers". L'état de contrainte dans les éléments rectangulaires limités par les „stringers" est supposé un état de cisaillement pur. Les contraintes normales par rapport aux axes du réseau orthogonal sont concentrées dans les „stringers". L'application de la méthode est décrite à l'aide d'exemples simples. Le problème du choix optimal des variables statïques et géométriques est discuté.

ZUSAMMENFASSUNG
Untere Grenzwerte für die Traglast von gelochten Scheiben werden mit einer einfachen Methode ermittelt. Die Scheibe wird mit einem rechtwinkligen Netz von Stringern in rechteckige Elemente unterteilt. Es wird angenommen, in den rechteckigen Elementen herrsche ein Zustand reinen Schubes, und die Normalspannungen bezüglich der Netzrichtungen werden als in den Stringern konzentriert wirkende Kräfte zusammengefasst. Die Anwendung der Methode wird mit einfachen Beispielen erläutert, und das Problem der optimalen Wahl der statischen und geometrischen Variablen wird erörtert.

1. INTRODUCTION

In aeroelasticity it has for a long time been customary to apply the stringer method for the calculation of thin sheets reinforced with stringers in the elastic region. Within reinforced concrete design in the plastic region, the stringer method was applied to cylindrical shells by Lundgren [1] and to discs by Nielsen [2] [3].

In this paper it is demonstrated how the method provides a useful tool for treating discs with holes. In addition, the method is concisely described in general terms since it still seems to be rather unknown within the concrete field.

2. CALCULATION OF DISCS

Reinforced concrete discs are still often designed by calculating the stresses according to the elastic theory. However, the reinforcement is generally designed according to one or another plastic theory. Such a procedure has often great disadvantages since the elastic stress field may have large stress peaks for which a correct reinforcement is unpractical. Therefore, even when elastic design is used, the stresses are often only covered by reinforcement in some average manner.

Some simple and often-met design problems are relatively well dealt with by experiments so that empirical formulas can be developed. For discs with holes, such formulas have been developed by Kong [4].

A more rational method of design is furnished by the theory of plasticity.

If the lower bound method is applied, one needs a statically admissible stress field. The necessary reinforcement can then be determined by standard formulas. A number of statically admissible solutions for discs without holes have been given in [2] and [3]. Most of them are constructed by combining homogeneous stress fields in triangular elements. This type of stress field could also be used in discs with holes, but solutions for practical purposes have not yet been developed.

In the following it is shown how the stringer method can be applied to discs with holes.

3. THE STRINGER METHOD

The basic idea of the stringer method is that the normal stresses are imagined carried by stringers, i.e. lines along which concentrated tensile or compressive forces are located. The stringers are supposed to form an orthogonal net. The rectangular elements between the stringers are supposed to be subjected to constant pure shear. Therefore, the stringer forces vary linearly between the net points.

If necessary, the external forces of course have to be replaced by an equivalent set of concentrated forces in the net points and constant shear stresses along the boundaries of the rectangular elements.

The idealized system will normally be statically indeterminate. If the shear stresses in the rectangular elements are considered to be the unknowns, it means that a number of shear stresses can be chosen arbitrarily. When the statically indeterminate shear stresses have been chosen, the other ones can be determined by equilibrium equations. Knowing the shear stresses, the stringer forces can finally be determined.

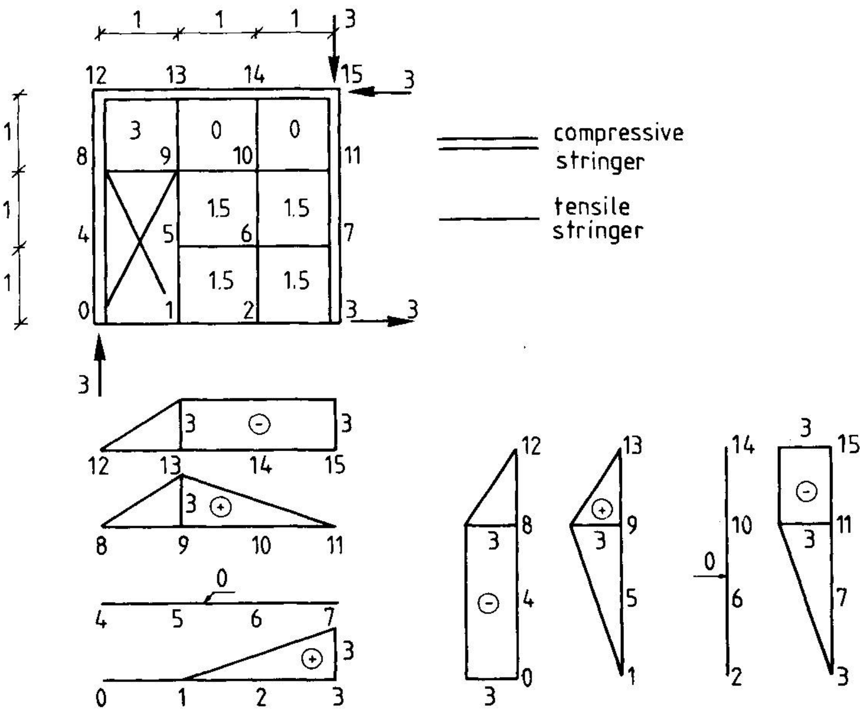

Fig. 1.

A simple example as shown in Figure 1 illustrates this procedure. This figure shows a disc with a hole 0,1,9,8. The disc may be imagined to be half of a deep beam with the depth equal to 3 units and the span equal to 6 units. The thickness is assumed to be 1 unit. In the middle section, the moment is supposed to be carried by concentrated forces at the top and bottom. The external load is a concentrated force of 3 units in the middle point at the top side. The disc has been divided into stringers as shown in the figure. Vertical projection shows that the shear stress in the element 8,9,12,13 is 3. Choosing the shear stress in 9,10,13, 14 to be zero and the shear stress in 5,6,9,10 to be 1,5, vertical projection shows that the shear stress in 1,2,5,6 is 1,5. Horizontal projection shows that the shear stress in 10,11,14,15 is 0, that the shear stress is 1,5 in 6,7,10,11 and that the shear stress is 1,5 in 2,3,6,7. Vertical projection through the 3 elements to the right is automatically satisfied. It is seen that in this case, there are 2 statically indeterminate shear stresses. The stringer forces can now easily be determined. The result is shown in the figure.

The stringer method can be explained in general terms by using Airy's stress function. The function for the force system in the stringer theory is a number of hyperbolic paraboloids, one for each rectangular element.

The stresses in rectangular coordinates x,y is generally

$$\sigma_x = \frac{\partial^2\Psi}{\partial y^2} \tag{1}$$

$$\sigma_y = \frac{\partial^2\Psi}{\partial x^2} \tag{2}$$

$$\tau_{xy} = -\frac{\partial^2\Psi}{\partial x\partial y} \tag{3}$$

σ_x and σ_y being normal stresses (positive as tensile stresses), τ_{xy} the shear stress and Ψ the Airy stress function, it is seen that the hyperbolic paraboloids give constant shear stresses in the rectangular elements since $\partial^2\Psi/\partial x\partial y$ will be constant. The jumps in the derivatives $\partial\Psi/\partial x$ and $\partial\Psi/\partial y$ in the net points determine the concentrated stringer forces.

From the general theory of plane stress, it is known that the values of Ψ and of the derivatives $\partial\Psi/\partial x$ and $\partial\Psi/\partial y$ are determined along the boundary. Using the properties of the stress function, it is easily shown that the number, N , of statically indeterminate shear stresses is

$$N = m - h - (2s - y) + (r - 3) \tag{4}$$

where

- m is the number of net points incl. the number of net points in holes.
- h is the number of rectangular elements where the shear stress is zero.
- s is the number of stringers running from boundary to boundary (incl. boundaries along holes).
- y is the number of external stringers, i.e. stringers having a boundary on one side of the stringer along the whole stringer.
- r is the number of reactions.

In the case shown in Figure 1, we have m = 16, h = 2, s = 8, y = 4 and r = 3, giving N = 2, as already found.

In the more complicated example shown in Figure 2, we have m = 86, h = 9, s = 23, y = 4 and r = 4, giving N = 36.

The validity of formula (4) is demonstrated by the following arguments: The stress function Ψ is determined by m parameters. N is found as m minus the number of requirements to the shear stress in holes and the number of requirements to the stringer forces at the boundaries. For an internal stringer, there are two boundary conditions since the stringer force is determined as differences between first derivatives of the stress function. For an external stringer with a boundary on one side along the whole stringer, the stringer force is not determined in an analogous way, but a projection equation can be formulated giving one requirement. The last term in (4) is the number of external statical indeterminate parameters.

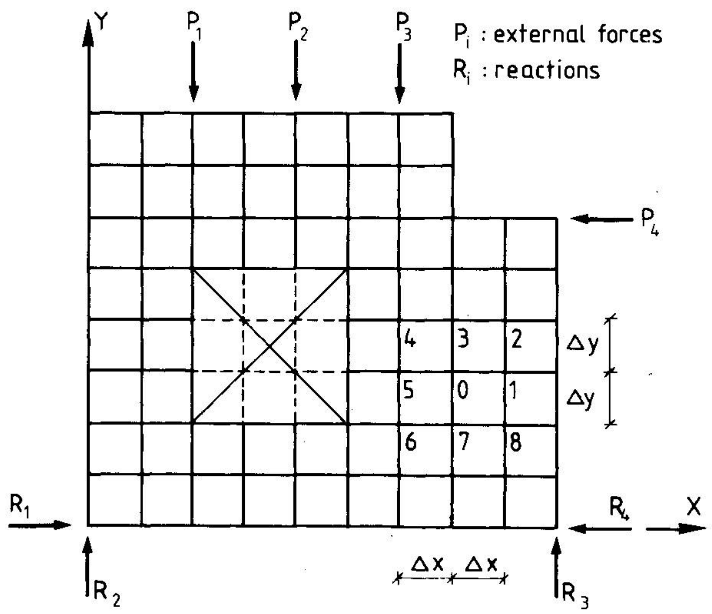

Fig. 2.

If the number of net points is large, the force system in the stringer method can be used as an approximation of a continuous stress field. If the stringer forces are distributed uniformly along Δx and Δy, Δx and Δy being the distance between the net points in the x-direction and the y-direction resp., we find at the point 0 in Figure 2:

$$\sigma_x = \frac{\Psi_3 - 2\Psi_0 + \Psi_7}{(\Delta y)^2} \qquad (5)$$

$$\sigma_y = \frac{\Psi_1 - 2\Psi_0 + \Psi_5}{(\Delta x)^2} \qquad (6)$$

$$\tau_{xy} = \frac{(\Psi_4 + \Psi_8) - (\Psi_2 + \Psi_6)}{4\Delta x \ \Delta y} \qquad (7)$$

Note that these expressions are identical to the usually applied difference approximations.

In order to get a situation at the boundary equivalent to the situation at interiour points, the boundary stringers may be placed at a distance $\Delta x/2$ and $\Delta y/2$ resp. from the real boundary.

4. OPTIMIZED STRINGER SYSTEMS

The stringer is a practical tool which can offer a quick answer to the question of how to reinforce a disc for given external loads. Therefore, the method is especially well suited for hand calculations giving simple solutions to a reinforcement problem.

In more refined calculations, one may wish to find the optimum value of the statically indeterminate quantities, i.e. the values giving the smallest amount of reinforcement. One may even wish to find the optimum lay-out of a stringer system.

As an example, consider a rectangular hole in a zone with pure shear. The question how to reinforce the disc near the hole can be answered by considering a stringer system as shown in Figure 3.

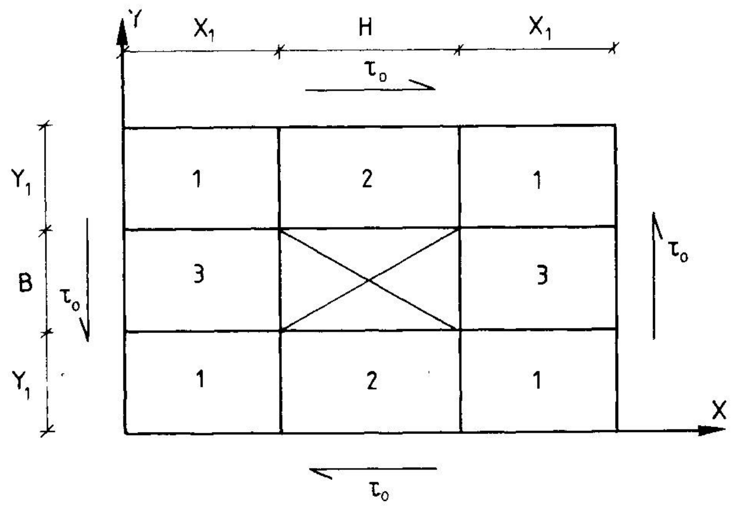

Fig. 3.

If the shear stress outside the stringer system is τ_0 and if the geometry of the stringer system is, for instance, laid down as shown in the figure, the shear stresses and the stringer forces can be calculated as explained above. Here, the reinforcement is chosen as a homogeous mesh and a constant reinforcement area along the whole length of the stringer, but different in vertical and horizontal directions. The necessary reinforcement may then be determined by the formulas given in [2] and [3]. Because of symmetry, the shear stresses in the rectangular element having the same number in the figure are equal. The shear stresses are then statically determinate if x_1 and y_1 are known. The total reinforcement volume can then be determined as a function of x_1 and y_1 and then minimized with respect to these quantities.

The result of such an optimization is shown in Figure 4.

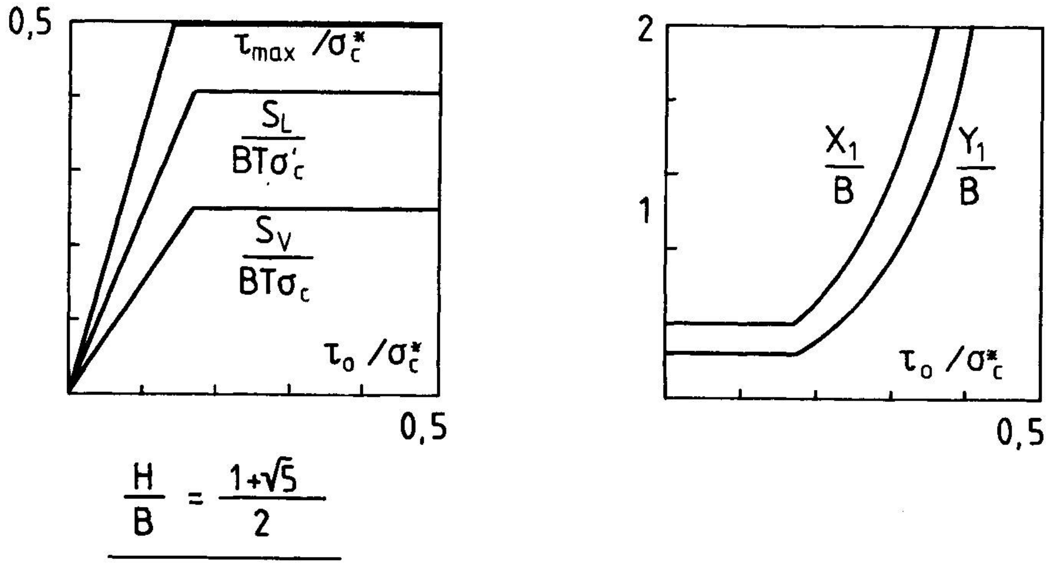

Fig. 4.

In the figure, σ_c^* is the effective concrete compression strength of the concrete, S_L is the largest value of the stringer force in vertical stringers and S_V the largest stringer force in horizontal stringers. The optimal value of the geometrical parameters x_1 and y_1 tends to infinity if τ_0 approaches $\sigma_c^*/2$, which is the largest shear stress which can be carried by concrete in plane stress conditions.

Similar optimizations can be carried out in other cases. Some results have been described in [6].

If the stringer system has such a fine mesh that a continuous stress field can be modelled, the optimization can be formulated as a linear programming problem, provided that the yield condition for the reinforced material is linearized. Such a procedure has been developed in [5], using, however, a difference approximation for the stress components as a function of the net point values of the Airy stress function. Therefore, these net point values can be used as optimization variables. However, an optimization could just as well take place by means of the stringer method, using the shear stresses as variables, whereby the boundary conditions could perhaps be handled in an easier way than in the difference method.
This, however, has yet to be demonstrated.

REFERENCES

1. LUNDGREN, H.: Cylindrical Shells. The Danish Technical Press. The Institute of Civil Engineers, Copenhagen 1949.

2. NIELSEN, M.P.: Om jernbetonskivers styrke. Polyteknisk forlag, København 1969.

3. NIELSEN, M.P.: On the Strength of Reinforced Concrete Discs. Acta Polytechnica Scandinavica, Copenhagen 1971.

4. KONG, F.K.: Structural Idealization for Deep Beams with Web Openings Further Evidence. Magazine of Concrete Research, Vol. 30, No. 103, pp. 89 - 94, June 1978.

5. BRYDER, K.: Optimeringsmetoder for 2-dimensionale legemer af ideal plastiske materialer. (Optimization Methods for 2-Dimensional Bodies of Perfectly Plastic Materials). Ph.D. thesis, Structural Research Laboratory, Technical University of Denmark, Lyngby, September 1978.

6. KÆRN, J.C.: Optimering af armering omkring huller i skiver. (Optimization of Reinforcement around holes), Structural Research Laboratory, Technical University of Denmark, Internal Report (in press), Lyngby 1979.