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Autor: Pedersen, C.

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Shear in Beams with Bent-Up Bars

L'effort tranchant dans les poutres avec des barres relevées

Schub in Balken mit aufgebogener Bewehrung

C. PEDERSEN
Ph. D. Student
Technical University of Denmark
Copenhagen Lyngby, Denmark

SUMMARY
The shear strength of reinforced concrete beams with bent-up bars as shear reinforcement is analysed by means of the upper bound theorem of the theory of plasticity. The upper bound solutions are compared with the results from a number of tests and good agreement is found.

RESUME
La résistance au cisaillement de poutres en béton armé avec des barres relevées est examinée par la méthode cinématique de la théorie de la plasticité. Les valeurs extrêmes de la charge ultime sont comparées avec les résultats d'essais et une bonne concordance peut être constatée.

ZUSAMMENFASSUNG
Der Schubwiderstand von Stahlbetonbalken mit aufgebogenen Stäben als Schubbewehrung wird mit der kinematischen Methode der Plastizitätstheorie untersucht. Die oberen Grenzwerte für die Traglasten werden mit Ergebnissen einer Anzahl von Versuchen verglichen, und eine gute Übereinstimmung wird festgestellt.

1. INTRODUCTION

The purpose of this investigation was to show the usability of the theory of plasticity on this type of problem. In previous examinations by K.W. Johansen and H.C. Sørensen it has been claimed that the contribution of bent-up bars to the shear strength is independent of the inclination of the bar at the bending point. (c.f. [57.1] and [72.1]). Furthermore, in these examinations (in which a model with fixed concrete strut inclination is used) it is claimed that the contribution of the bent-up bars to the shear strength of the beam is usually over-estimated by 41%. However, no failures due to this have been found. It is thus reasonable to ask if the plastic model for shear in reinforced concrete (c.f. [78.2] and [75.1]) gives a more precise description of the problem. This paper is a short version of a similar paper in Danish, [78.3], which gives a more profound description of the analysis.

2. BASIC ASSUMPTIONS

The reinforcing steel and the concrete are both assumed to be rigid, perfectly plastic. The yield locus of the concrete in plane stress is shown in Fig. 1. The associated flow rule is assumed to be valid. The state of stresses in the beam is assumed to be plane. f_c^* denotes the effective concrete strength and is related to the uniaxial compression strength by the equation

$$f_c^* = \nu f_c \qquad (1)$$

ν is a positive figure less than 1 which takes into account that the basic assumptions are never fulfilled in practice, that is, the concrete has not an unlimited capacity of deformation under constant stress and the state of stress is never quite plane.

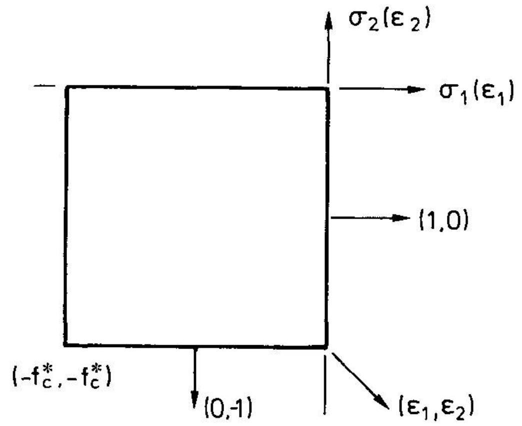

Fig. 1 Yield Locus for Concrete in Plane Stress.

3. THEORETICAL UPPER BOUND SOLUTIONS

The beam in Fig. 2 with only one bent-up bar in the shear span is considered.

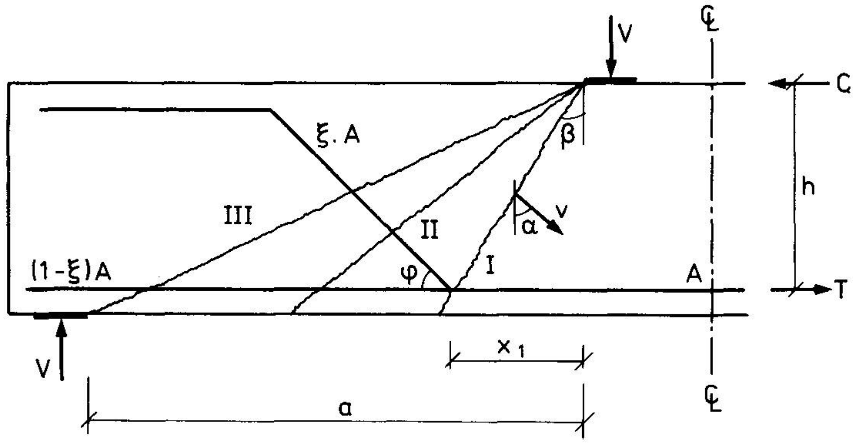

Fig. 2 Failure Mechanisms for Beams with only one Bent-up Bar.

For the failure mechanism corresponding to yield line II it is easy to calculate the external work and the internal work (dissipation) using the basic assumptions (c.f. [78.2]). The following upper-bound solution is obtained by putting the external work equal to the internal work.

$$\frac{\tau}{f_c} = \tfrac{1}{2}\nu\left(\sqrt{1+\mathrm{tg}^2\alpha}\cdot\sqrt{1+\mathrm{tg}^2\beta} - \mathrm{tg}\,\alpha - \mathrm{tg}\,\beta\right) + \phi(1-\xi+\xi\cos\varphi) + \xi\,\phi\sin\varphi \qquad (1)$$

The angles α and β are found by minimizing (1). This yields the following values

$$\mathrm{tg}\,\beta = a/h \qquad (2)$$

$$\sin\alpha_o = \begin{cases} \dfrac{\tfrac{1}{2}\nu - \Gamma}{\tfrac{1}{2}\nu\sqrt{1+\mathrm{tg}^2\beta}} & \text{for } \Gamma \leq \dfrac{\nu}{2} \\ 0 & \text{for } \Gamma > \dfrac{\nu}{2} \end{cases} \qquad (3)$$

The reduced degree of longitudinal reinforcement is given by

$$\Gamma = \phi(1-\xi+\xi\cos\varphi) \qquad (4)$$

For yield line I similar results can be obtained. In this case $\Gamma = \phi$ and $\mathrm{tg}\,\beta = x_1/h$.

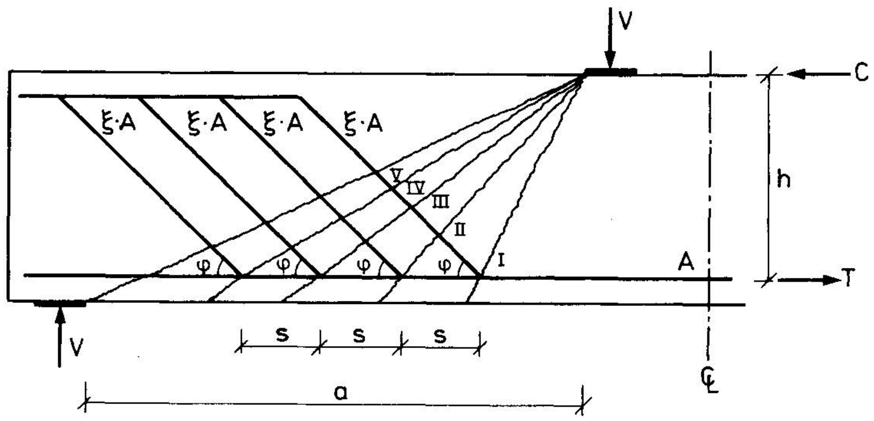

Fig. 3 Failure Mechanisms for a Beam with several Bent-up Bars.

Fig. 3 shows a beam with several bent-up bars in the shear span. This beam has to be investigated for each yield line I, II, III, IV and V. The investigations are carried out using (1) to (4) with varying values of $\mathrm{tg}\,\beta$, ξ and Γ.

Alternatively, this beam is analysed by means of the usual expression for the shear strength of a beam with inclined stirrups as shear reinforcement. The shear strength is then given by

$$\frac{\tau}{f_c} = \begin{cases} \dfrac{\nu}{2}\left(\sqrt{1+\lambda^2} - \lambda\right) + \psi\sin^2\varphi(\lambda+\cot\varphi) & , \quad 0 \leq \psi \leq \psi_o \\ \sqrt{\psi\sin^2\varphi(\nu-\psi\sin^2\varphi)} + \psi\cos\varphi\sin\varphi & , \quad \psi_o \leq \psi \leq \dfrac{\nu}{2}\dfrac{1+\cos\varphi}{\sin^2\varphi} \\ \dfrac{\nu}{2}\cot\dfrac{\varphi}{2} & , \quad \psi \geq \dfrac{\nu}{2}\dfrac{1+\cos\varphi}{\sin^2\varphi} \end{cases} \qquad (5)$$

where

$$\psi = \frac{\xi A f_y}{b\,s\,f_c\sin\varphi} \quad \text{and} \quad \psi_o = \frac{\nu}{2}\,\frac{\sqrt{1+\lambda^2}-\lambda}{\sin^2\varphi\sqrt{1+\lambda^2}} \qquad (6)$$

Finally, a beam with combined stirrups and bent-up bars as shear reinforcement is considered.

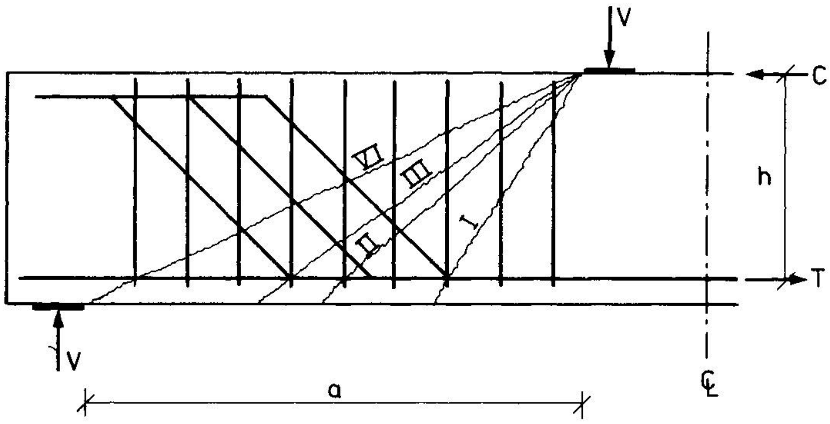

Fig. 4 Reinforced Beam with Stirrups and Bent-up Bars.

For the failure mechanism corresponding to yield line I the following expression for the shear force at failure is found

$$V = n A_s f_{ys} + A f_y \operatorname{tg} \alpha + \tfrac{1}{2} \nu f_c \frac{1-\sin(\alpha+\beta)}{\cos\alpha \cos\beta} \tag{7}$$

From (7) it is seen that the contribution of the stirrups to the shear force is independent of α. This means that α_o has to be determined as previously.

4. COMPARISON BETWEEN CALCULATED RESULTS AND RESULTS OBTAINED BY TESTS

In this section, a comparison between results calculated by the preceeding upper-bound solutions and results obtained by tests is drawn.

The results from the tests have all been found in the literature. The values of ν have all been determined so that accordance between theory and test was obtained. Finally, these values of ν are compared with the values recommended in [78.2].

4.1 Tests by K. Özden and K.W. Johansen

In [67.1], 4 tests with beams with bent-up bars and 2 tests on beams with combined stirrups and bent-up bars are described. The beam T-11 has been omitted from the present analysis because this beam showed typical brittle failure.

BEAM	h	b	f_c	ϕ	α_o	tg β	τ/f_c	$(\tau/f_c)_E$
	m	m	MPa				ν=0.60	
T5	0.2902	0.110	34.0	0.523	0^o	1.29	0.103	0.133
T12	0.2744	0.110	34.0	0.549	0^o	0.820	0.142	0.151
T16	0.2672	0.160	34.0	0.416	0^o	0.468	0.191	0.113

Table 4.1.1 Calculated and Measured Results (ν = 0.60)

For the beams T-13 and T-14, the following results are found.

BEAM	h	b	f_c	ϕ	α_o	tg β	τ/f_c	$(\tau/f_c)_E$
	m	m	MPa				ν=0.70	
T13	0.2718	0.110	31.1	0.610	0^o	0.828	0.177	0.165
T14	0.2714	0.110	31.1	0.627	0^o	2.045	0.120	0.129

Table 4.1.2 Calculated and Measured Results (ν = 0.70)

4.2 Tests by F. Leonhardt and R. Walter

The results from 3 tests carried out by F. Leonhardt and R. Walter are described in [63.1]. One of the beams showed flexural failure and is omitted in this analysis.

Strain measurements on the bent-up bars in the tests in [67.1] and [63.1] indicate that no yielding has occured in these before failure. It is therefore reasonable to assume that the yield line runs from the edge of the loading plate to the first bending point of the longitudinal reinforcement.

BEAM	h	b	f_c	ϕ	α_o	$tg\,\beta$	τ/f_c	$(\tau/f_c)_E$
	m	m	MPa				$\nu=0.75$	
TA5	0.335	0.160	19.0	1.433	0^o	0.716	0.235	0.227
TA17	0.345	0.160	25.2	0.777	0^o	0.406	0.253	0.248

Table 4.2.1 Calculated and Measured Results (ν = 0.75)

4.3 Tests by K. Leksukhum and R. Smith

K. Leksuhkum and R. Smith [71.1] have carried out a number of tests on beams with different types of web reinforcement. Here, the tests with bent-up bars are analysed by means of (11) and the following results are found.

BEAM	λ	ψ_o	f_c	$\rho\, f_y$	ψ	τ/f_c	$(\tau/f_c)_E$	ν
		$\nu=0.70$	MPa	MPa		$\nu=0.70$		
BI-1	2.60	$4.67\cdot10^{-2}$	22.8	1.12	$4.91\cdot10^{-2}$	0.1533	0.150	0.666
BI-2	2.63	$4.57\cdot10^{-2}$	23.6	1.12	$4.75\cdot10^{-2}$	0.1505	0.136	0.554
BI-3	2.60	$4.67\cdot10^{-2}$	24.8	1.12	$4.52\cdot10^{-2}$	0.1463	0.141	0.639
BI-4	2.70	$4.36\cdot10^{-2}$	23.6	1.12	$4.75\cdot10^{-2}$	0.1505	0.146	0.653
BI-5	2.88	$3.87\cdot10^{-2}$	20.9	1.12	$5.36\cdot10^{-2}$	0.1611	0.163	0.719
BI-6	2.91	$3.80\cdot10^{-2}$	24.7	1.12	$4.53\cdot10^{-2}$	0.1465	0.150	0.739
BI-7	2.34	$5.63\cdot10^{-2}$	24.4	1.40	$5.74\cdot10^{-2}$	0.1675	0.181	0.837
BI-8	2.34	$5.63\cdot10^{-2}$	22.6	1.40	$6.19\cdot10^{-2}$	0.1748	0.212	1.090
BI-9	3.00	$3.59\cdot10^{-2}$	23.6	0.70	$2.97\cdot10^{-2}$	0.1162	0.114	0.675
BII-11	2.72	$4.30\cdot10^{-2}$	23.4	0.86	$3.68\cdot10^{-2}$	0.1307	0.146	0.871

Table 4.3.1 Calculated and Measured Results

4.4 Tests by P. Regan and M.H. Khan

In [71.2], two series of tests by P. Regan and M.H. Khan are described. Series A had both stirrups and bent-up bars as web reinforcement, while series K had only bent-up bars as web reinforcement. The results are shown in Table 4.4.1 which is shown on the next page.

4.5 Calculated values of ν compared with the estimated values

In the preceeding sections, the values of ν have been estimated so that agreement between the measured results and the calculated results could be obtained. In this section, these values of ν are compared with the values calculated from the formulae given in [78.1]. The values of ν from the preceeding sections are denoted US and the calculated values are denoted CA.

According to [78.1], the following expression can be used to calculate ν when the beam has stirrups as web reinforcement. The unit of f_c is MPa.

$$\nu = 0.8 - \frac{f_c}{200} \qquad (8)$$

As can be seen from table 4.5.1, the values of ν from the tests by P. Regan and M.H. Khan are somewhat smaller than the values calculated. This may be due to the fact that the bending points in these tests are not placed symmetrically about the vertical plane through the beam axis.

BEAM	f_c	f_y	f_{ys}	d	φ	ν	V	V_u	NOTES
	MPa	MPa	MPa	mm	grd.		kN	kN	
A2	34.54	276	275	12.7	45^o	0.55	298	280	(1)
A3	31.85	276	275	12.7	45^o	0.55	277	260	(2)
A4	32.31	269	275	9.5	45^o	0.55	284	280	(2)
A5	35.38	276	275	12.7	30^o	0.55	289	264	(3)
A6	30.54	276	275	12.7	60^o	0.55	298	284	(4)
A8	27.85	276	275	12.7	45^o	0.60	286	280	(2)
A10	34.54	276	275	12.7	45^o	0.55	313	282	(3)
A13	30.08	346	275	12.7	45^o	0.55	308	300	(2)
A14	32.62	346	275	12.7	45^o	0.55	287	280	(2)
A15	31.77	434	275	12.7	45^o	0.55	336	308	(2)
A17	24.85	640	275	12.0	45^o	0.65	343	338	(2)
A18	26.85	620	275	8.0	45^o	0.70	391	398	(5)
K1	24.15	427	-	15.9	26.5^o	0.70	265	300	(6)
K2	27.62	276	-	12.7	45^o	0.65	302	280	(2)
K4	31.54	276	-	12.7	45^o	0.55	198	178	(3)

Notes: The critical yield line runs from the edge of the loading plate to

1. the third bending point.
2. the second bending point
3. the third stirrup.
4. the second stirrup.
5. the fourth stirrup.
6. the edge of the bearing plate.

Table 4.4.1 Analysis of 15 Tests by Regan and Khan

BEAM	US	CA	±	BEAM	US	CA	±
T5	0.60	0.63	-	A2	0.55	0.63	+
T12	0.60	0.63	-	A3	0.55	0.64	+
T16	0.60	0.63	-	A4	0.55	0.64	+
T13	0.70	0.64	+	A5	0.55	0.62	+
T14	0.70	0.64	+	A6	0.55	0.65	+
TA5	0.75	0.71	-	A8	0.60	0.66	+
TA17	0.75	0.67	+	A10	0.55	0.63	+
BI-1	0.66	0.69	-	A13	0.55	0.65	+
BI-2	0.55	0.68	-	A14	0.55	0.64	+
BI-3	0.64	0.68	-	A15	0.55	0.64	+
BI-4	0.65	0.68	-	A17	0.65	0.68	+
BI-5	0.72	0.70	-	A18	0.70	0.67	+
BI-6	0.74	0.68	-	K1	0.70	0.68	-
BI-7	0.84	0.68	-	K2	0.65	0.66	-
BI-9	0.68	0.68	-	K3	0.55	0.64	-
BII-11	0.87	0.68	-				

\+ Stirrups and bent-up bars
\- Only bent-up bars

Table 4.5.1 Estimated and Calculated Values of ν

5. CONCLUSION

It appears that the theory of plasticity provides a model which gives a reasonably good discription of the shear-strength of a reinforced concrete beam with bent-up bars as shear-reinforcement. The calculated values of the shear-strength are in good agreement with the measured ones.

The use of only bent-up bars is not advisable because of the risk of the concrete being crushed at the bending point, but a combination of stirrups and bent-up bars may be used with good results.

6. NOTATIONS

a	Length of the shear span
A	Area of the longitudinal reinforcement
A_s	Area of one cross-section of the stirrups
b	Web width
C	Compression stringer force
f_c	Uniaxial compression strength of the concrete
f_c^*	Effective strength of the longitudinal reinforcement
f_y	Yield stress of the longitudinal reinforcement
f_{ys}	Yield stress of the stirrups
h	Effective shear depth
n	Number of stirrup cross-sections cut by a yield line
s	Distance between bending points
T	Tensile stringer force
v	Relative displacement in a yield line
V	Calculated failure load
V_u	Measured failure load
x_1	Distance between the edge of the loading plate and first bending point
α	Angle between vertical and displacement vector
α_o	Optimal value of α
β	Angle between vertical and the yield line
Γ	Reduced degree of longitudinal reinforcement
ξ	Ratio between the area of the bent-up bars cut by a yield line and A
λ	a/h
ν	The web effectiveness factor
τ	Uniformly distributed shear stress
$(\frac{\tau}{f_c})_E$	Experimental shear strength
φ	Inclination of a bent-up bar
ψ	Mechanical degree of shear reinforcement
ψ_o	The value of ψ for which the yield line runs between the edges of the loading plate and the bearing plate
ϕ	Mechanical degree of longitudinal reinforcement

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