Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 29 (1979)

Artikel: Plasticity and endochronic inelasticity in finite element analysis of reinforced concrete

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DOI: https://doi.org/10.5169/seals-23534

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Plasticity and Endochronic Inelasticity in Finite Element Analysis of Reinforced Concrete

Plasticité et inélasticité endochronique dans l'analyse du béton armé par éléments finis

Plastizität und endochronische Inelastizität bei der Berechnung von Stahlbeton mittels finiter Elemente

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SUMMARY
Plane reinforced concrete members subjected to monotonic and cyclic loading are analyzed by the finite element method, using different material models for concrete under compression; plasticity and endochronic inelasticity. Numerical results are compared with test results. For monotonic loading the observed behaviour is approximated equally well with both models, while the endochronic model seems to give a more realistic representation of the cyclic behaviour.

RESUME
Des éléments plans en béton armé sollicités par des charges monotoniques et cycliques sont analysés à l'aide de la méthode par éléments finis. Deux modèles différents sont utilisés pour le béton: plasticité et inélasticité endochronique. Des résultats numériques sont comparés avec des résultats expérimentaux. Tandis que le comportement sous une charge monotonique est bien décrit avec les deux modèles, le modèle endochronique semble donner une représentation plus réaliste du comportement sous une charge cyclique.

ZUSAMMENFASSUNG
Ebene Stahlbetonträger unter monoton wachsender und zyklischer Belastung werden mittels finiter Elemente untersucht. Für den Beton werden einerseits plastische und andererseits endochronisch inelastische Modelle verwendet. Numerische Resultate werden mit Versuchsergebnissen verglichen. Beide Modelle ergeben eine etwa gleich gute Näherung für das Verhalten unter monoton wachsender Belastung. Für zyklische Belastung führt das endochronische Modell zu einer realistischeren Wiedergabe des Verhaltens.

1. INTRODUCTION

In nonlinear analysis of reinforced concrete structures, the real behaviour of concrete in compression can be approximated by several theories. Commonly used are nonlinear elasticity and flow theory of plasticity. Several refinements have been introduced in these theories, enabling them to give a better representation of different effects in the nonlinear concrete behaviour [1-6].
A new approach, which is an important step in the direction of developing a more unified and comprehensive material model for concrete was proposed by Bazant and Bhat [7,8]. This formulation, termed endochronic inelasticity, is based on an extensive set of functions which fit most of the experimental observed effects in nonlinear concrete behaviour.

This paper deals with finite element analyses of plane reinforced concrete members, where the concrete behaviour in compression is approximated by a simple plasticity approach and endochronic inelasticity. Several numerical examples are presented and discussed.

2. MATHEMATICAL MODELLING OF REINFORCED CONCRETE

2.1 Concrete in compression

2.1.1 Plasticity

The stress-strain behaviour is approximated by an elastic-strain hardening plastic approach, see Fig. 1a. The stress-strain response is assumed to be linearly elastic below the stability limit, while linear strain-hardening is assumed after the initial yielding.

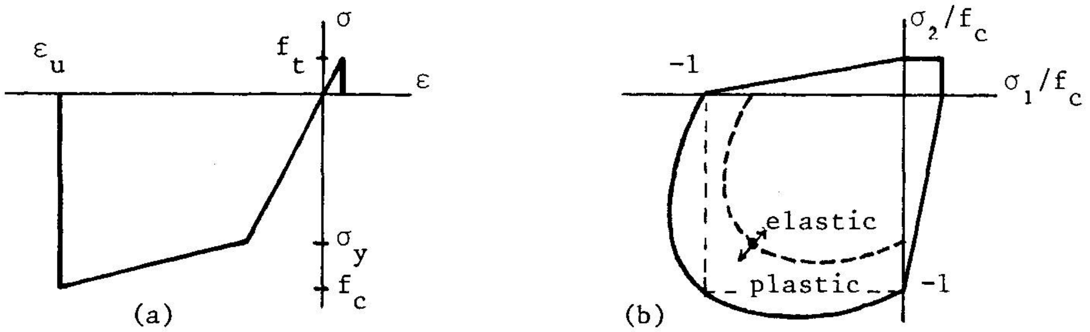


Fig.1 Plasticity model

Since only plane states of stress are considered the biaxial compressive strength is approximated by a failure envelope according to von Mises, see Fig. 1b. This is a very simple approach, and no increase of strength for equal compression in two directions is achieved. Compression failure (crushing) is assumed to occur when the compressive strength is reached.

2.1.2 Endochronic Inelasticity

The endochronic theory for concrete, with material parameters as given in Ref. [7] is used in the present study. Good agreement between theory and experiments has been demonstrated; it appears that effects like nonlinear stress-strain response, inelastic dilatancy, cyclic behaviour and multiaxial strength can be represented by the endochronic model. Fits of experimental stress-strain curve and biaxial strength are shown in Fig. 2. A detailed derivation of the theory, with fits of numerous experimental data can be found in Ref. [7].

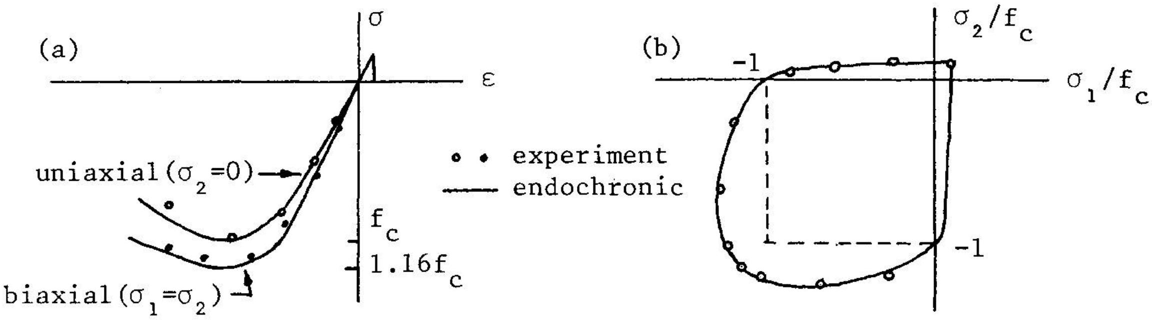


Fig.2 Endochronic model

2.2 Concrete in Tension

In the plasticity approach the tension cracking process is controlled by a maximum tensile stress criterion, while a combined stress and strain criterion is used in the endochronic approach. Cracks are assumed to open perpendicular to the highest principal stress or strain direction when the failure envelopes in tension regions in Figs. 1b and 2b are reached. At this point there are of course no shear stresses to be transferred across the crack. By further straining, however, shear strains may occur parallel to the crack. This raises the question of whether aggregate interlocking is capable of transferring shear stress over the crack. Shear transfer is taken into account by assuming that a "cracked" shear modulus is retained through a factor $0 \leq \alpha \leq 1$ times the elastic shear modulus. In the plasticity approach this factor is made dependent upon average crack widths computed in the program, while a constant value after cracking is used in the endochronic approach, see Fig. 3. In the endochronic approach, criteria for closing and reopening of cracks are introduced as demonstrated by Fig. 4.

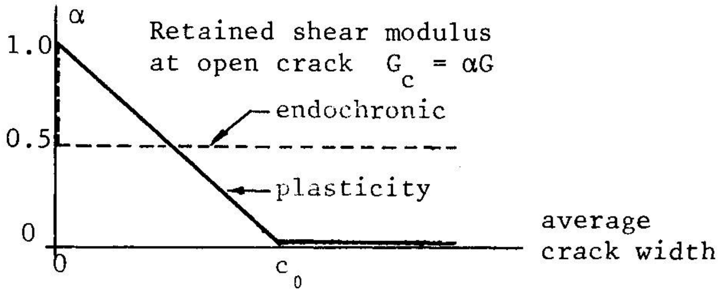
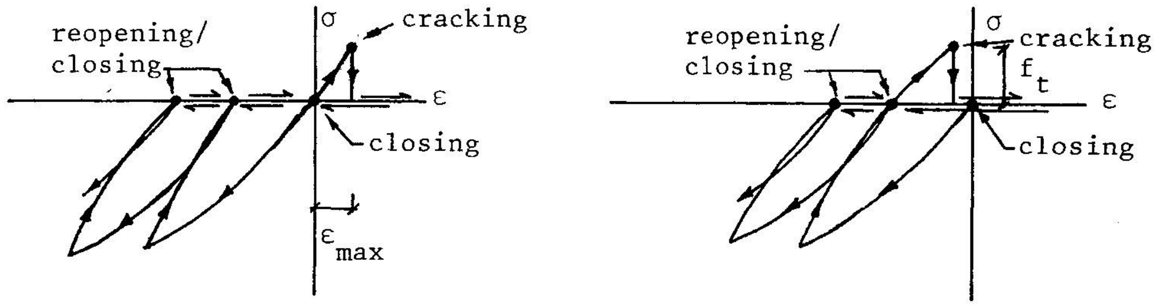


Fig.3 Shear retention factor



(a) First loading in tension (b) First loading in compression

Fig.4 Criteria for opening, closing and reopening of cracks

2.3 Reinforcement Steel Behaviour

The reinforcement steel behaviour is approximated by a uniaxial stress-strain relationship. A plasticity formulation is used, assuming linear, isotropic strain hardening after initial yielding. The stress-strain curve is assumed to be the same in tension and compression.

3. NUMERICAL SOLUTION TECHNIQUES

3.1 Finite Element Approximations

The concrete is modelled by quadrilateral, isoparametric finite elements, based on the assumption of linear interpolation functions in terms of displacements. The quadrilateral has four corner nodes with two translational degrees of freedom each. In the endochronic approach regular 2x2 Gaussian integration is used for computation of the strain energy, while the concept of selective integration according to Doherty et al [9] is used in the plasticity approach; i.e the shear strain is sampled at the centroid, but used in accumulation of strain history at the four Gaussian points. This approach improves the bending performance of the element. The reinforcement bars are modelled by simple two-noded bar elements with linear displacement interpolation. Compatibility between concrete and reinforcement is assured at common nodal points.

3.2 Solution Procedures

Incremental solution procedures are used in both approaches. In the plasticity program, an incremental (tangential) procedure is combined with a Newton-Raphson type iteration process. In reality, a modified version of the original Newton-Raphson process is used, since it is made possible to choose at what iteration steps the stiffness matrix is updated or kept constant.

In the computer program constructed around the endochronic model, a similar type of solution procedure is used. Errors due to incremental linearization are eliminated by carrying out equilibrium iterations in this case too. However, it should be noted that the endochronic formulation is not a tangential one. Special care must be taken in the equilibrium iteration process, since the parameters which account for inelastic effects are path dependent, and a wrong path may be followed during the equilibrium iterations. Such problems can be avoided by using a solution procedure as proposed in Ref.[10]. The iteration is terminated when the displacement corrections become sufficiently small, measured in terms of a modified displacement norm, see Bergan and Clough [11], or when a prescribed minimum number of cycles has been reached.

4. NUMERICAL APPLICATIONS

4.1 Monotonically Loaded Members

4.1.1 Bending Failure

A simply supported beam was tested by Burns and Siess [12] . The behaviour under monotontically increasing load is analyzed by the plasticity and endochronic models. The test specimen failed by yielding of reinforcement at 156 kN. Figure 5 shows finite element idealization, midspan load deflection curves and computed crack patterns. It appears that both models approximate degredation of stiffness due to cracking, and ultimate load with fairly good accuracy.

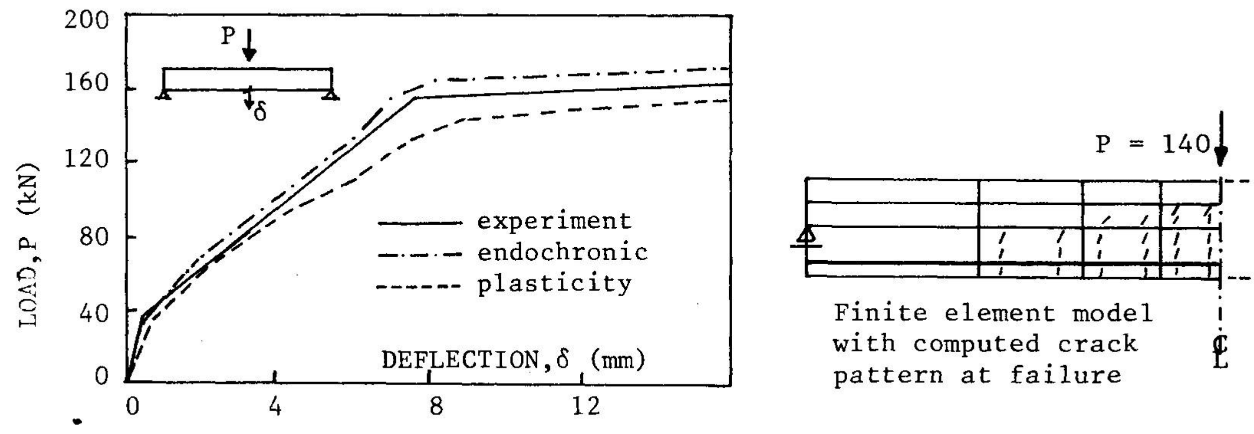


Fig.5 Results from analyses, Burns-Siess beam

4.1.2 Diagonal Tension Failure

A beam, tested by Bresler and Scordelis [13], is also analyzed using both material models. The test specimen failed by a rapid diagonal tension failure mechanism at a load level of 258,1 kN. Figure 6 shows experimental and numerical load-deflection curves. Both models show good agreement with the test with respect to cracked stiffness. As regards ultimate load at failure, no such load can be seen from the endochronic approach. The plasticity approach, however, indicated a failure load of approximately 230 kN, which is about 10 percent below the experimental value.

The different results can be explained by the different assumptions of shear retention after cracking, and the differences in the numerical integrations which are used. This is discussed in Ref. [14].

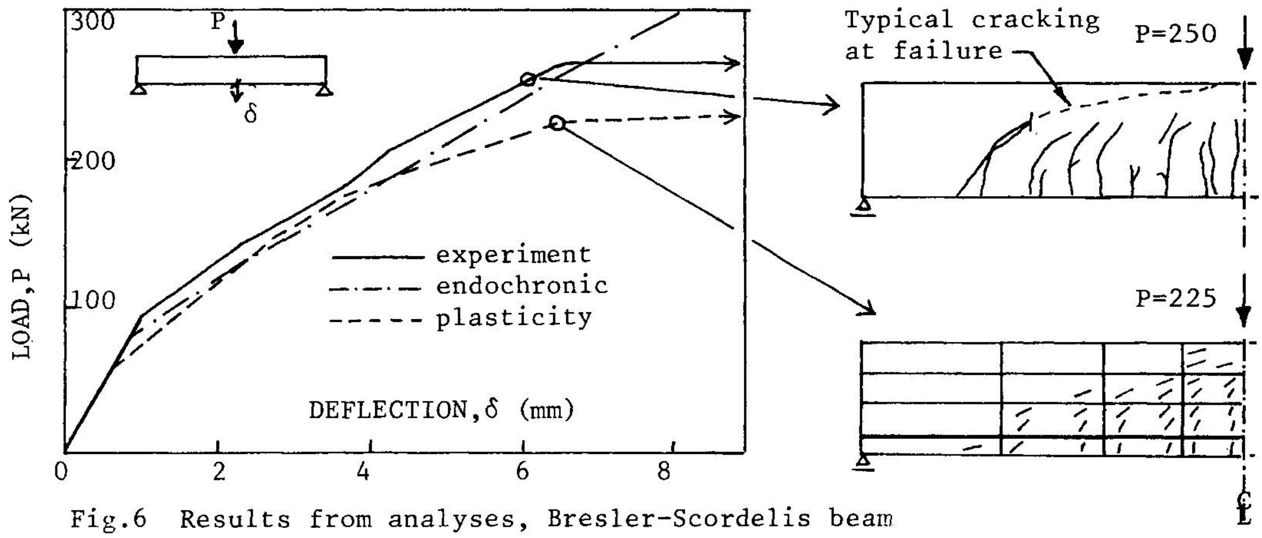


Fig.6 Results from analyses, Bresler-Scordelis beam

4.2 Cyclically loaded Members

A shear panel subjected to cyclic loading was tested by Cervenka [15]. The specimen was first subjected to two load cycles with amplitude $P=\pm 53,4$ kN, which is approximately 46 percent of an analytically predicted monotonic failure load. Tension cracking is the dominant nonlinear effect within this load range, which may be termed "elastic cycling". Further, the specimen was subjected to load cycles with amplitude $P = \pm$ 102,3 kN, which is about 88% of the predicted failure load. Yielding of reinforcement and inelasticity of concrete in compression appeared in these cycles, which then may be termed "plastic cycling".

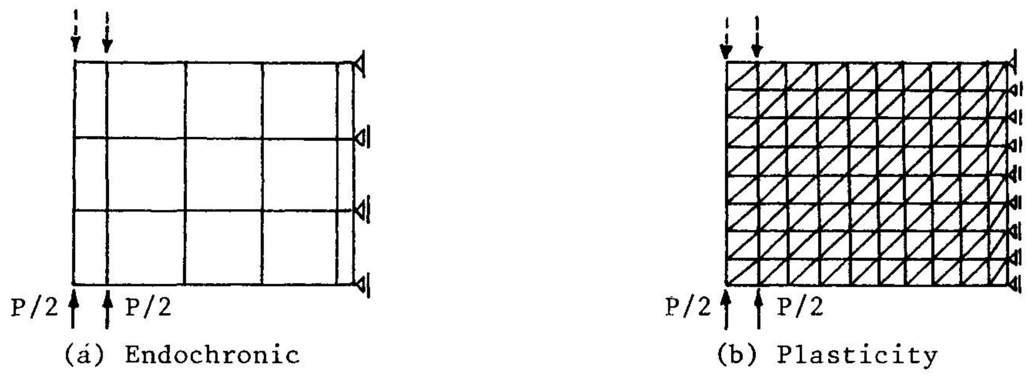


Fig.7 Finite element idealizations of Cervenka's shear panel

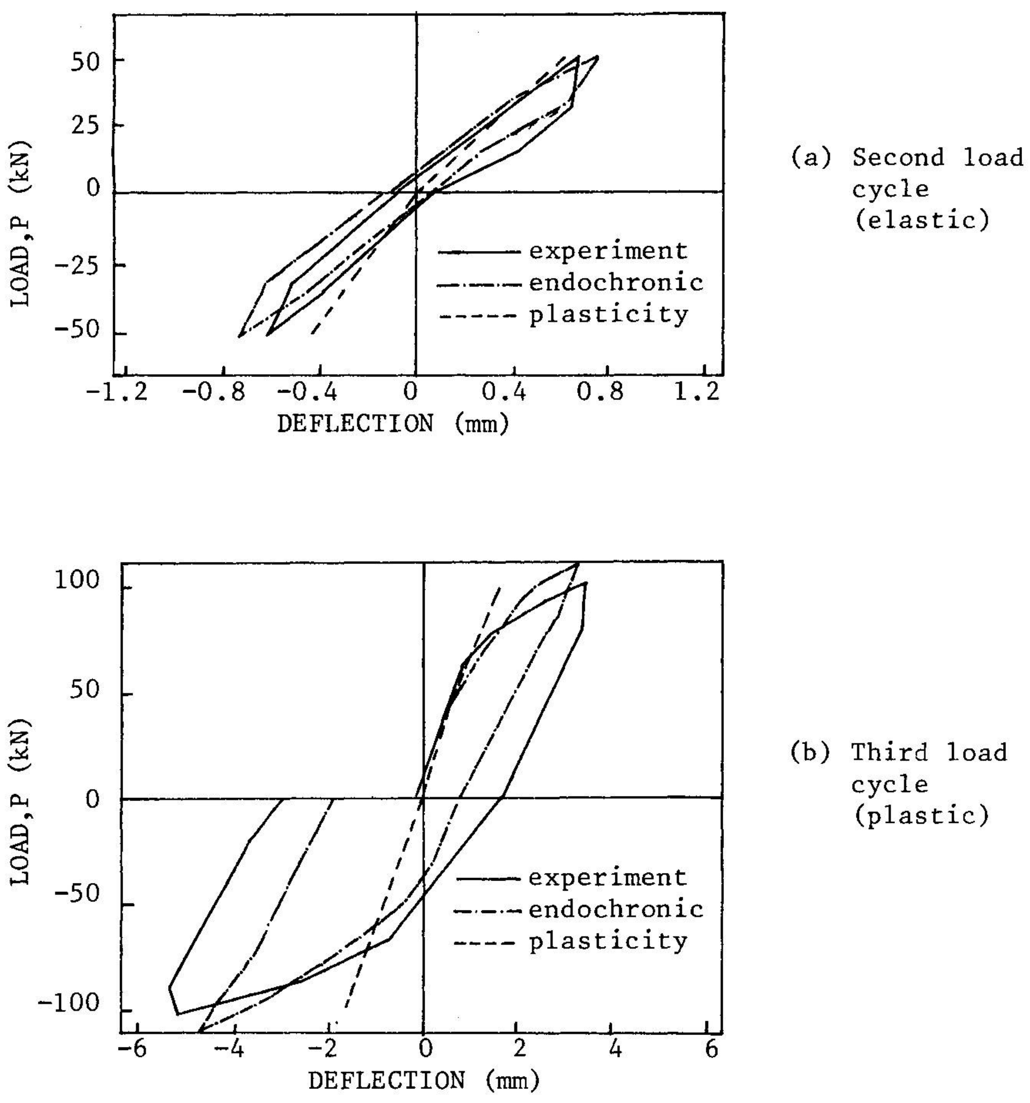


Fig.8 Results from analyses, Cervenka's shear panel

The endochronic model is used to analyze the behaviour during the three first cycles (two elastic and one plastic). A detailed description is given in Ref. [10]. The finite element model is shown in Fig. 7a. For comparison, numerical results obtained by Cervenka, using a plasticity model similar to the one in the present study are presented [15, 16, 17]. Fig. 7b shows the finite element model (triangular elements) used in the plasticity approach.

Fig. 8 a-b shows experimental and numerical load-deflection curves. It appears that the results obtained by the endochronic model compare favourably with the experiments. This is probably due to a better representation of cyclic stress-strain behaviour in the endochronic model, but also, as pointed out by Cervenka and Gerstle [17], because a crack mode with two cracks opened simultaneously at a point must be included. This possibility is not taken into account in Cervenka's analysis, while it is included in the endochronic model.

5. CONCLUSIONS

On the basis of the limited amount of results presented, the following conclusions may be drawn:

- As could be expected, the simple plasticity model and the endochronic model are both capable of approximating the behaviour of monotonically loaded members in a plane stress situation with fairly good accuracy.
- As regards cyclically loaded members, the endochronic model seems to give better results than the plasticity model used in this study. The possibility of having two cracks open simultaneously at a point should be included when cyclic behaviour is considered.

If triaxial states of stress were considered, the endochronic model must be expected to be superior to the simple plasticity model in compression, where the sensitivity to hydrostatic pressure is neglected. However, more refined plasticity models as proposed in Refs.[4, 5, 6] have to be compared numerically to the endochronic model before any general conclusions can be drawn.

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