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Yield Polyhedron of R.C. Shear Walls under Combined Forces

Polyèdre d'écoulement pour des voiles en béton armé sollicités par des actions combinées

Fliesspolyeder für Schubwände aus Stahlbeton unter kombinierter Beanspruchung

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SUMMARY
N-M-Q Yield Polyhedron of reinforced concrete unit shear walls subjected to the combined action of axial force, bending moment and shear force, is clarified analytically through the idealization of shear wall into a truss model. Calculated moment-shear force interaction is compared with test results. It becomes clear that there are two types of fracture modes of reinforced concrete shear walls, i.e. flexural yield and shear failure modes. On this yield polyhedron of shear wall it becomes possible to analyse the ultimate states of reinforced concrete structures with cantilever-type shear walls.

RESUME
En remplaçant un voile en béton armé par un treillis, on obtient un polyèdre d'écoulement soumis à l'action combinée d'un moment de flexion, d'un force axiale et d'une force de cisaillement. Des résultats d'essais sont comparés avec l'interaction calculée entre le moment de flexion et la force de cisaillement. Il y a deux types de rupture distincts: la rupture par flexion et la rupture par cisaillement. Le polyèdre d'écoulement peut former la base de l'examen de la charge ultime des structures en béton armé avec des voiles en porte-à-faux.

ZUSAMMENFASSUNG
Mit Betrachtungen an einem Ersatzfachwerk wird ein Fliesspolyeder für Schubwände aus Stahlbeton ermittelt, welche durch kombiniert wirkende Biegung, Normalkraft und Querkraft beansprucht werden. Der berechneten Interaktion zwischen Biegemoment und Querkraft werden Versuchsergebnisse gegenübergestellt. Zwei Versagensarten sind zu unterscheiden, nämlich Biegebrüche und Schubbrüche. Das Fliesspolyeder kann der Untersuchung der Grenztragfähigkeit von Stahlbetontragwerken mit kragträgerartigen Schubwänden zu Grunde gelegt werden.

1. INTRODUCTION

It has been long recognized that cantilever shear walls in multistory reinforced concrete structures are effective resisting elements against such lateral loads as earthquakes and wind loads. Many researchers [1][2][3][4][5][6] have been carried out their researches to make clear the elasto-plastic deformation and fracture behaviors of shear walls experimentally as well as analytically. As a base of deformation and fracture analysis of such structures as cantilever shear wall structures, it is desirable to develop the yield and plastification conditions of unit shear walls, which are one span and one story in cantilever shear walls.

In order to present such a base of the ultimate state analysis of structures, the senior author [7] had presented the N-M-Q yield polyhedron of rectangular and wide flange sections of elasto-plastic materials. In this paper, the yield polyhedron of unit shear walls subjected to axial force, bending moment and shear force at the upper and lower boundaries of them is developed through the idealizations of unit shear walls into a truss system. Then, moment-shear force interaction calculated for shear walls under constant axial force is compared with experimental results.

2. IDEALIZATIONS OF SHEAR WALLS

2.1 Truss Model of Shear Walls

Shear walls in multistory cantilever shear wall structures are subjected to axial force, bending moment and shear force at the upper and lower boundaries of them. These shear walls may be idealized into a truss model [8], which is composed of two vertical column elements and two diagonal brace elements hinged at the upper and lower boundaries in consequence of the observation of cracking patterns and fracture modes of tested shear walls, as shown in Fig. 1.

2.2 Equilibrium Equation of Truss Model

Axial force (N), bending moment (M) and shear force (Q), which act on the upper boundary of a truss model are in equilibrium with axial forces ($N_{c_1}, N_{c_2}, N_{b_1}, N_{b_2}$) in all elements of the truss model, as follows:

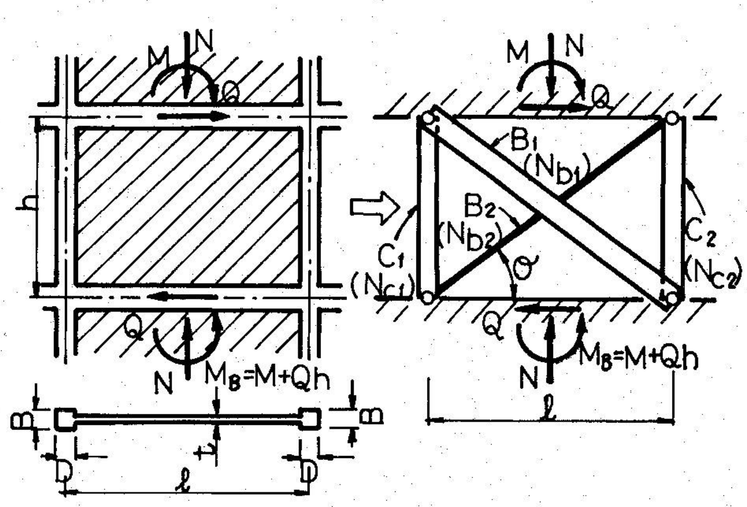

Fig. 1 Truss Model

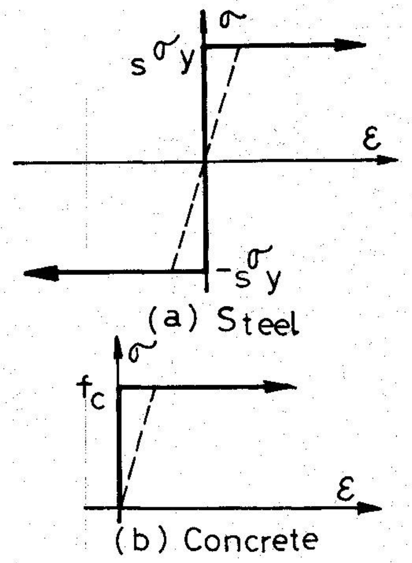

Fig. 2 σ-ε Relation

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} 1 & 1 & \sin\theta & \sin\theta \\ -\ell/2 & \ell/2 & -(\sin\theta)/2 & \ell(\sin\theta)/2 \\ 0 & 0 & \cos\theta & -\cos\theta \end{bmatrix} \begin{Bmatrix} N_{c_1} \\ N_{c_2} \\ N_{b_1} \\ N_{b_2} \end{Bmatrix} \tag{1}$$

2.3 Assumptions of Characteristics of Materials

Stress-strain relationships of concrete and reinforcing steel are assumed such as shown in Fig. 2, that is, tensile resistance of concrete is neglected and concrete shows somewhat flow after yielding.

2.4 Yield Axial Forces of Each Element in Truss Model

2.4.1 Column Elements

Side columns of shear walls are idealized into column elements. The tensile and compressive yield axial forces (N_{cy}^-, N_{cy}^+) of these elements are:

$$\begin{cases} N_{cy}^- = -\, a_c\, {}_s\sigma_y \\ N_{cy}^+ = a_c\, {}_s\sigma_y + f_c\, A_c \end{cases} \tag{2}$$

where f_c : compressive strength of concrete,
A_c : gross area of column element (= B D),
a_c : cross sectional area of longitudinal reinforcements in column,
${}_s\sigma_y$: yield stress of reinforcing steel.

2.4.2 Brace Elements

Infill concrete panels with wall reinforcements are idealized into two diagonal brace elements in the following two cases:

(a) In case I, one of brace elements carry tension and the other compression. So, the infill concrete panels are idealized into a diagonal brace element which carries only compression with a definite effective width and the wall reinforcements are idealized into a diagonal brace element which carries only tension. The cross sectional area (a_d) of this latter element is assumed:

$$a_d = p_{wh}\, h\, t \cos\theta + p_{wv}\, \ell\, t \sin\theta \tag{3}$$

where p_{wh}: horizontal reinforcement ratio in wall,
p_{wv}: vertical reinforcement ratio in wall.

Therefore, the tensile and compressive yield axial forces ($N_{by_1}^-, N_{by_1}^+$) are:

$$\begin{cases} N_{by_1}^- = -\, a_d\, {}_s\sigma_y \\ N_{by_1}^+ = B_e\, t\, f_c \end{cases} \tag{4}$$

where B_e : effective width of concrete brace element.

(b) In case II, both of brace elements carry either tension or compression alternatively. In the former, then, the infill concrete panels are neglected. In the latter, however, they are idealized into two diagonal brace elements which carry only compression with a definite effective width defined in the case I. The wall reinforcements are idealized into two diagonal brace elements which carry either tension or compression. It is assumed that the cross sectional areas of these elements are equal to $a_d/2$. Therefore, the tensile and compressive yield axial forces ($N^-_{by_2}, N^+_{by_2}$) are:

$$\begin{cases} N^-_{by_2} = -\dfrac{a_d}{2}\, {}_s\sigma_y \\ N^+_{by_2} = \dfrac{a_d}{2}\, {}_s\sigma_y + B_e\, t\, f_c \end{cases} \tag{5}$$

3. N-M-Q INTERACTION OF SHEAR WALLS

Truss model shows four modes of collapse mechanisms such as shown in Fig. 3. In these collapse mechanisms, interactions constructed for external loads applying the truss model are expressed as planes in N-M-Q space based upon the principle of virtual work, as follows.

$$\begin{cases} \text{Mode I} \quad : -N\dfrac{\ell}{2} + M + Q\,h = -\left(N^-_{cy}\,\ell + N^-_{by_1}\, h\cos\theta \right), \text{ for } B_1 : \text{comp.} \\ \text{Mode II} \quad : Q = \left(N^+_{by_1} - N^-_{by_1} \right) h\cos\theta \\ \text{Mode III} : M + \dfrac{Q}{2}h = \left(N^+_{cy} - N^-_{cy} \right)\dfrac{\ell}{2} \text{ for } B_1 : \text{comp. and } B_2 : \text{tens.} \\ \text{Mode IV} \quad : N\dfrac{\ell}{2} + M + Q\,h = N^-_{cy}\ell + N^+_{cy_1}\, h\cos\theta, \text{ for } B_2 : \text{tens.} \end{cases} \tag{6}$$

Then, the N-M-Q polyhedral interaction are expressed as the inner surface of foregoing planes corresponding to four modes of collapse mechanisms as shown in Fig. 4, that is, this N-M-Q interaction is obtained through the upper bound theorem. On the other hand, in Fig. 4, all the combinations of yield forces of all elements are shown on the surfaces, consequently, the lower bound theorem is satisfied.

In actual calculations of N-M-Q interaction, this is composed of points, which are obtained by means of substituting the stress states described in Table 1 into Eq. (1). Because infill concrete panels with reinforcements are idealized into brace elements with two different areas a_d mentioned in the section 2.4.2, two lines of $N_{b_1} = 0$, two lines of $N_{b_2} = 0$ and two lines of $Q = 0$ on the N-M-Q interaction are obtained as boundaries. Then, 3-8-13-4, 3-8-15, 2-15-16 and 1-2-16-19 planes correspond to the case II.

4. M-Q INTERACTION OF SHEAR WALLS UNDER CONSTANT AXIAL FORCES

In the design of actual structures, axial forces acting on shear walls are restricted to be lower than the balanced axial force above which a compressive column element begins to yield. An axial load is kept to be constant and lateral loads are applied incrementally in the experiments of shear walls. In order to compare experimental results with analytical values, N-M interaction under constant axial force is illustrated in Fig. 5(a) from N-M-Q interaction. The characteristic points on this M-Q interaction are obtained from Eq. (1), as:

$$\begin{cases} M_a = \left(\dfrac{N}{2} - N^-_{cy} - N^-_{by_1}\sin\theta \right)\ell & M_c = \left(\dfrac{N}{2} - N^-_{cy} \right)\ell \\ M'_a = \left(\dfrac{N}{2} - N^-_{cy} - N^-_{by_2}\sin\theta \right)\ell & Q_c = -N^-_{by_1}\cos\theta \\ Q_y = \left(N^+_{by_1} - N^-_{by_1} \right)\cos\theta & \end{cases} \tag{7}$$

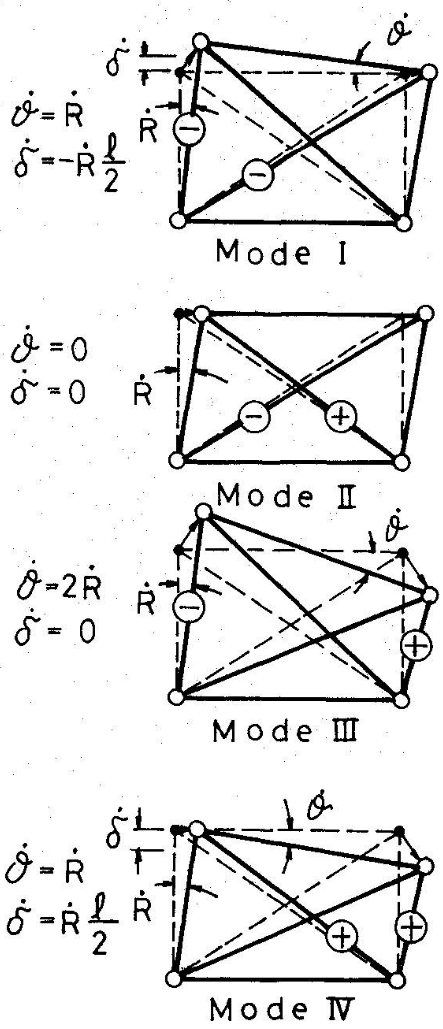

Fig. 3 Collapse Mechanisms

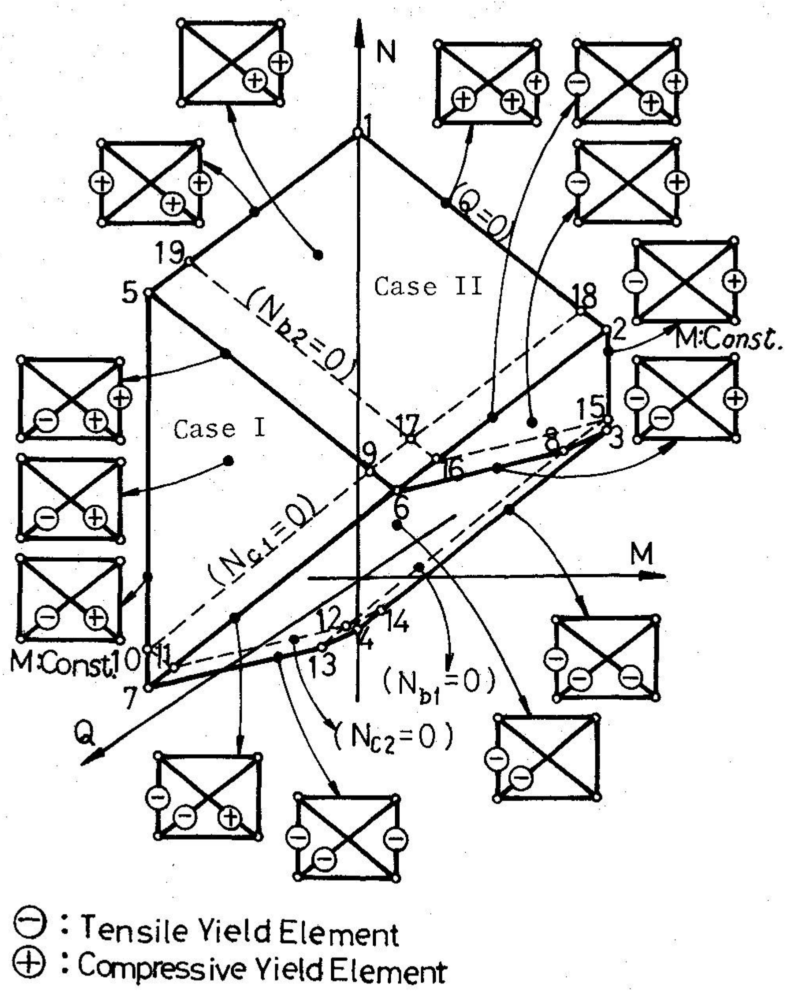

Fig. 4 N-M-Q Interaction

Table 1 Points of Intersection on N-M-Q Interaction

Points of Intersection		Stress States of All Elements of Truss Model			
		Column Elements		Brace Elements	
		N_{c1}	N_{c2}	N_{b1}	N_{b2}
All Elements Yield.	1	N_{cy}^{-}	N_{cy}^{+}	N_{by2}^{+}	N_{by2}^{+}
	2	N_{cy}^{-}	N_{cy}^{+}	N_{by2}^{+}	N_{by2}^{+}
	3	N_{cy}^{-}	N_{cy}^{+}	N_{by2}^{-}	N_{by2}^{-}
	4	N_{cy}^{-}	N_{cy}^{-}	N_{by2}^{-}	N_{by2}^{-}
	5	N_{cy}^{+}	N_{cy}^{+}	N_{by1}^{+}	N_{by1}^{-}
	6	N_{cy}^{-}	N_{cy}^{+}	N_{by1}^{+}	N_{by1}^{-}
	7	N_{cy}^{-}	N_{cy}^{-}	N_{by1}^{+}	N_{by1}^{-}
One Element is "0", Another Elements Yield.	8	N_{cy}^{-}	N_{cy}^{+}	0	N_{by1}^{-}
	9	0	N_{cy}^{+}	N_{by1}^{+}	N_{by1}^{-}
	10	0	0	N_{by1}^{+}	N_{by1}^{-}
	11	N_{cy}^{-}	0	N_{by1}^{+}	N_{by1}^{-}
	12	N_{cy}^{-}	0	0	N_{by1}^{-}
	13	N_{cy}^{-}	N_{cy}^{-}	0	N_{by1}^{-}
	14	N_{cy}^{-}	0	N_{by2}^{-}	N_{by2}^{-}
	15	N_{cy}^{-}	N_{cy}^{+}	0	0
	16	N_{cy}^{-}	N_{cy}^{+}	N_{by1}^{+}	0
	17	0	N_{cy}^{+}	N_{by1}^{+}	0
	18	0	N_{cy}^{+}	N_{by2}^{+}	N_{by2}^{+}
	19	N_{cy}^{+}	N_{cy}^{+}	N_{by1}^{+}	0

Then, a relation between bending moment of the upper boundary of unit shear walls and that of the lower is $M_B = M + Q\,h$, as shown in Fig.1. $M_B/M_a - Q/Q_y$ interaction in which M_B and Q are the maximum bending moment and the ultimate shear capacity of individual shear walls is shown in Fig.5(b). When the case in which two diagonal brace elements carry only tension is neglected, this interaction is represented with two solid lines. The thick broken line illustrated in Fig.5(b) classifies shear walls into two fracture modes, i.e. flexural yield mode and shear failure mode.

5. EFFECTIVE WIDTH OF CONCRETE BRACE ELEMENTS IN TRUSS MODEL

The effective width of concrete brace elements is determined on the basis of experimental results [2][3] which are obtained with the loading of shearing type (diagonal loading) excluding bending moment from external loads at the upper and lower boundaries of shear walls. Then, the shear capacity of concrete brace elements (Q_w) is expressed:

$$Q_w = Q_u - Q_s \tag{8}$$

where Q_u : ultimate shear capacity of tested shear walls,
Q_s : shear capacity of wall reinforcements ($= a_d\ {}_s\sigma_y \cos\theta$).

The effective width of concrete brace elements is expressed as:

$$B_e = \frac{Q_w}{f_c\ t\cos\theta} = \frac{\tau_w}{f_c}\ \frac{\ell}{\cos\theta} \tag{9}$$

where τ_w : nominal shear stress ($= Q_w/\ell\ t$).

Tested results [2][3] are plotted in $\tau_c/f_c - \beta_s p_w$ relation, as shown in Fig.6, where $\beta_s p_w = (\ {}_s\sigma_y/f_c\ )\ p_w$: reinforcing index, $p_w = p_{wh} = p_{wv}$.

It is able to recognized that τ_w/f_c is nearly equal to 0.2 independently of reinforcing index in Fig.6. Therefore, it is assumed that the effective width of concrete brace elements is equivalent to $0.2\ \ell / \cos\theta$.

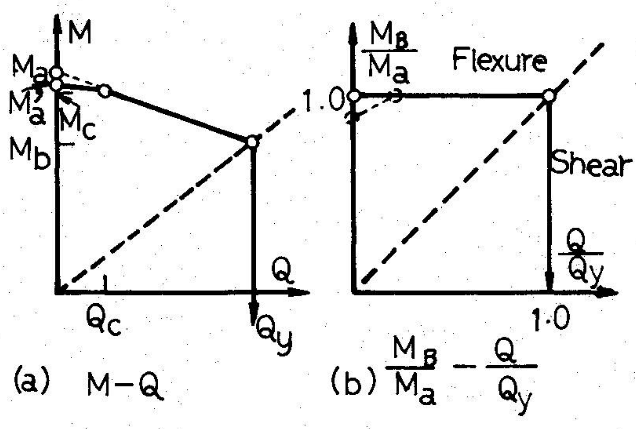

Fig. 5 Moment-Shear Force Interaction

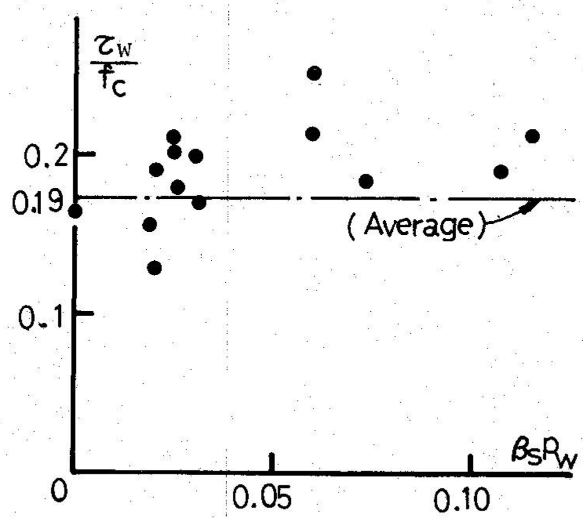

Fig. 6 $\frac{\tau_w}{f_c} - \beta_s p_w$ Relation

6. COMPARISON OF EXPERIMENTAL RESULTS WITH CALCULATED VALUES

In the analysis of foregoing chapters, although the tensile ultimate strength of reinforcing steel is neglected, calculations for reference are carried out in consideration of them. However, the compressive ultimate strength of steel is neglected.

Tested results [4][5][6] of flexural type loading of shear walls are plotted on M-Q planes, as shown in Fig.7. Since the ultimate strength of reinforcements is not described in the original document [5], it is assumed that that of column reinforcements equal to 1.5 $_s\sigma_y$ and that of wall reinforcements equal to 1.1 $_s\sigma_y$.

It is considered that the broken line in Fig.7(a) classifies flexural yield mode and shear failure mode of shear walls. Similarly, the broken line in Fig.7(b) represents the boundary of following two failure modes of shear walls. In the first mode, shear walls may fail in shear after the yielding of reinforcements of a tensile column. In the other mode, these may not fail in shear.

The tested values of ultimate bending moment are somewhat higher than calculated yield bending moment, but coincide well with calculated ultimate bending moment for the shear walls of flexural yielding type.

The ratio of ultimate shear capacities between tested and calculated shear failure type scatter widely than the flexural yielding type. This scattering may be caused by ignoring of the confining effects [3] of side columns on infill wall panels. So, if the effective width B_e of a diagonal brace element is assumed adequately, these scattering may become smaller.

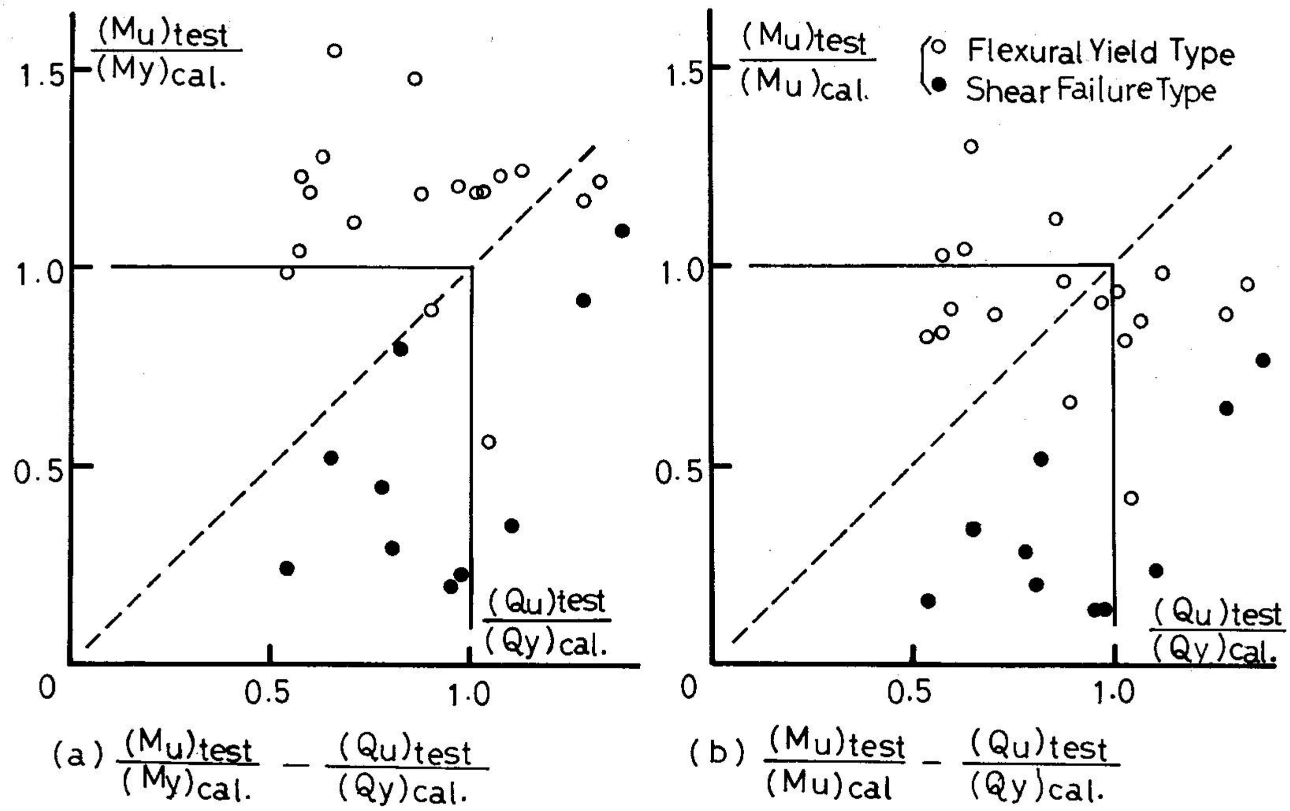

Fig. 7 Comparison of Experimental Results with Calculated Values

$(M_u)_{test} = h_0 (Q_u)_{test}$

h_0 = shear span

7. CONCLUDING REMARKS

Reinforced concrete shear walls subjected to the combined action of axial force, bending moment and shear force are idealized into truss model, such as shown in Fig.1. Then, the yield polyhedron of such shear walls is clarified analytically. In the analysis, the effective width of concrete brace elements is assumed to be $0.2\,\ell/\cos\theta$, on the basis of the experimental results of shear walls which are obtained by shear loading tests. The calculated moment-shear force interactions are compared with the experimental results of shear walls which are obtained by lateral loading tests under constant axial loads. From this analysis, therefore, possible to predict flexural yield mode and shear failure mode of shear walls and to estimate the ultimate shear resistance and bending moment of them.

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