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On the Effectiveness Factor in Plastic Analysis of Concrete

Le facteur d'efficacité et l'analyse plastique du béton

Zur wirksamen Festigkeit und der plastischen Berechnung von Beton

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SUMMARY

When an uncorrected classic theory of plasticity is used to calculate the carrying capacity of a concrete structure, we may obtain a value that is on the unsafe side. In practical applications, therefore, we reduce the compression strength by an empirically determined effectiveness factor ν before using it in a plastic analysis. The article describes how we can, on the basis of our knowledge of the stress-strain curve for concrete in uniaxial compression, employ theoretical means to arrive at a reliable ν-value. The theoretical ν-value is compared with the empirical ν-value for various concrete strengths.

RESUME

La charge ultime d'une structure en béton armé peut être surestimée si on utilise la théorie de la plasticité classique sans modification. C'est pourquoi on introduit un facteur d'efficacité ν pour diminuer la résistance du béton. Habituellement les facteurs ν sont déterminés expérimentalement. L'article décrit comment on peut calculer une valeur ν sur la base d'une relation connue entre les contraintes et les déformations du béton obtenue à partir d'un essai simple de compression. Des valeurs ν théoriques sont comparées avec des valeurs empiriques pour des qualités différentes de béton.

ZUSAMMENFASSUNG

Die Traglast von Betontragwerken kann überschätzt werden, wenn die klassische Plastizitätstheorie ohne Modifikation angewendet wird. Deshalb wird für die praktische Anwendung die Betondruckfestigkeit um einen empirisch ermittelten Faktor ν abgemindert. Der Beitrag zeigt, wie ein Wert ν auf Grund theoretischer Überlegungen ausgehend vom Spannungs-Dehnungs-Diagramm des Betons unter einachsigem Druck bestimmt werden kann. Für verschiedene Betonfestigkeiten werden theoretische Werte ν mit empirischen verglichen.

1. THE USE OF THE THEORY OF PLASTICITY FOR CONCRETE

The use of the theory of plasticity for the analysis of the carrying capacity of a structure has several advantages.

It gives simpler calculations than an analysis in which the entire behaviour is followed from a purely elastic start to an elastic-plastic stage at which the carrying capacity is not increased by further deformations. The calculations can even sometimes be done manually.

As only the ultimate state is considered, no knowledge of the elastic properties is required, i.e., the carrying capacity can be determined on the basis of less information than is required for a complete elastic-plastic analysis.

However, the theory of plasticity also has a number of disadvantages.

For example, it is based on the very drastic assumption that, for strains of arbitrary magnitude, the material retains its maximum stress, in other words, that the strength is not lost again.

Furthermore, the normality criterion is assumed to apply.

From the lower-bound theorem it follows that these assumptions are the most optimistic assumptions regarding the carrying capacity that can be imagined from a knowledge of the failure criterion. It is therefore obvious that direct use of the theory of plasticity may give results that are on the unsafe side.

2. INTRODUCTION OF THE EFFECTIVENESS FACTOR ν

In order to take account of the fact that the assumptions are not always satisfied, a correction is introduced - the effectiveness factor ν less than or equal to 1, see for instance [78.1], by which the characteristic strength values of the material are multiplied before they are used for calculation of the carrying capacity of a structure.

We can find this correction for a specific type of structure by comparing theoretically calculated capacities for the structure with the capacities measured in tests.

There is, however, another method by means of which we can, on certain assumptions, calculate an effectiveness factor on the safe side, on the basis of our knowledge of the material's stress-strain relationship. This method is demonstrated in the following.

3. THEOREM FOR DETERMINATION OF SAFE STRESS-STRAIN CURVES

3.1 Formulation

Let us imagine two structures 1 and 2 with identical geometry. The structures are subjected to the same, proportionally increasing load with the load parameter P, and it is assumed that the strain increments at each point have the same signs during the entire development of the load.

With these assumptions we can relate a strain-energy function W to each point of each structure, whereby the stresses σ_{ij} can be obtained from the strains ε_{ij} by derivation:

$$\sigma_{ij} = \frac{\partial W(\varepsilon_{ij})}{\partial \varepsilon_{ij}}$$

It is now assumed that as long as the strains lie within certain bounds $f(\varepsilon_{ij}) \leq 0$ or, written symbolically, $\varepsilon \leq \varepsilon_g$ we have:

$$W_1 \geq W_2$$

The carrying capacities P_1 and P_2 of the structures are each defined as the maximum value P can reach during loading.

We now wish to prove that

$$P_1 \geq P_2$$

provided that the strain in structure 1 does not exceed ε_g at any point for $P \leq P_1$.

3.2 Proof

For an arbitrary displacement field that satisfies the geometrical boundary conditions, we get the potential energy:

$$\Pi_{p1} = \int_V (W_1 + \varphi)dV + \int_{A_T} \psi \, dA$$

$$\Pi_{p2} = \int_V (W_2 + \varphi)dV + \int_{A_T} \psi \, dA$$

where φ and ψ are the conservative volume and surface-load potentials, V is the volume of the structure, and A_T is the part of the boundary with given surface loads.

For displacements lying within certain bounds determined by $\varepsilon \leq \varepsilon_g$ throughout, written symbolically $u \leq u_g$, the following applies:

$$W_1 \geq W_2$$

and thus

$$\Pi_{p1} \geq \Pi_{p2}$$

If $P < P_2$, then Π_{p2} has a minimum. From this it follows that Π_{p1}, whose possible minimum lies within $u \leq u_g$, has a lower bound. This implies that $P \leq P_1$, for if P were bigger than P_1, Π_{p1} could reach arbitrarily low values.

It should be noted that, theoretically, the minimum for Π_{p1} can occur for unbounded values of u, corresponding to 1 having become a mechanism, but this means that $P = P_1$, so we still have $P \leq P_1$.

It is thus demonstrated that if $P < P_2$ then $P \leq P_1$, and thus

$$P_1 \geq P_2$$

4. THE REAL STRESS-STRAIN DIAGRAM

The measurement of stress-strain curves for concrete in uniaxial compression when the maximum stress has been exceeded and the stress decreases with increasing strains presents problems since the field will be unstable unless the testing machine or other parts of the test arrangement are sufficiently stiff to prevent the elastic energy in the arrangement from being released in a sudden failure. The literature therefore contains very little information on such tests.

At the University of Illinois, Wang, Shah and Naaman have carried out some compression tests on concrete cylinders, in which the stresses were measured for strains up to 6‰ [78.2]. The stiffness of the test arrangement was increased by placing a steel pipe around the concrete cylinder, whereby the pipe was subjected to the same deformations as the concrete.

The authors of [78.2] propose an analytical expression for the stress-strain curve of the form

$$Y = \frac{AX + BX^2}{1 + CX + DX^2}$$

where Y is the stress in relation to the maximum stress, σ/σ_c and X is the strain in relation to the strain for maximum stress, $\varepsilon/\varepsilon_0$. A, B, C and D are constants. For each concrete strength, two sets of constants are determined, one for the rising branch $\varepsilon \leq \varepsilon_0$ and one for the falling branch $\varepsilon \geq \varepsilon_0$. The constants are determined on the basis of certain selected information, viz. that the slope for $\varepsilon = 0$ and the secant slope for $\sigma = 0.45\ \sigma_c$ both must be the secant slope to $\sigma = 0.45\ \sigma_c$ measured in the test. Further, that $\sigma = \sigma_c$ and $d\sigma/d\varepsilon = 0$ for $\varepsilon = \varepsilon_0$, and that the falling branch must pass through two specific points: the point at which the test curve has the inflexion $(\varepsilon_i, \sigma_i)$ and the point at which $\varepsilon = \varepsilon_{2i} = 2\varepsilon_i - \varepsilon_0$.

On the basis of the tests, the necessary information is found by statistical methods solely as a function of the concrete strength σ_c. A curve is thereby obtained for each value of σ_c (in the interval 21 to 77 MPa covered by the tests).

The authors themselves think that the curves can be extended someway past the 6‰, and they show an example of this in which $\sigma_c = 41$ MPa. However, this extension is only possible for values of σ_c in a narrow zone around $\sigma_c = 40$ MPa, since the limit value of σ for $\varepsilon \rightarrow \infty$, viz., $\sigma_c B/D$ is approximately zero here, whereas it is positive for smaller strength values and negative for higher strength values.

For example, we obtain:

$\sigma \rightarrow 13.4$ MPa for $\varepsilon \rightarrow \infty$ and $\sigma_c = 20$ MPa

$\sigma \rightarrow -10.0$ MPa for $\varepsilon \rightarrow \infty$ and $\sigma_c = 70$ MPa

A curve that is to be used for relatively large strains must satisfy the reasonable requirement that it has the asymptotic value zero and that negative values must not occur. Here, therefore, the calculations are not based directly on the curve shown in [78.2] drawn in full line in fig. 1, but on a curve obtained by substituting the requirement that the curve shall pass through the point $(\varepsilon_i, \sigma_i)$ by the requirement that it shall have the asymptotic value zero, the dashed curve in fig. 1. In the interval $0 < \varepsilon < 6‰$, the deviations between the two curves are insignificant compared with the deviations between the analytical curves and the measured curves, see fig. 1.

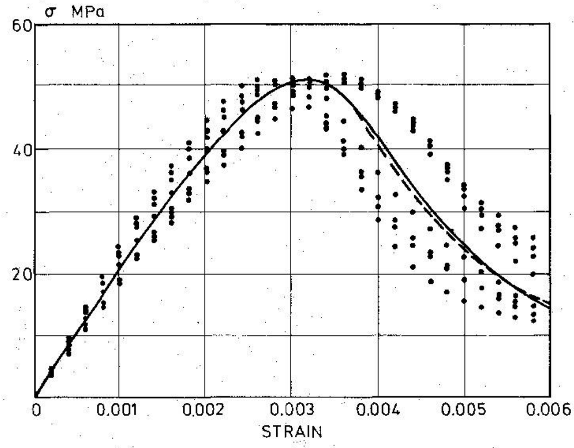

Fig. 1. Measured stress-strain curves and
_______ : the analytical curve from [78.1].
------- : the modified curve with the asymptotic value zero.

5. THEORETICAL DETERMINATION OF ν

We now wish to use the theorem for the case of uniaxial compression of concrete. From the curve found in section 4, the area below it, i.e. the strain energy for $\varepsilon = \varepsilon_g$, $W(\varepsilon_g)$ is determined for different values of the boundary strain ε_g. For a curve consisting of two straight lines, a linear-elastic branch with the secant modulus mentioned earlier as slope and a plastic branch with constant stress, we then calculate the plastic stress which gives the same value of $W(\varepsilon_g)$, see fig.2.

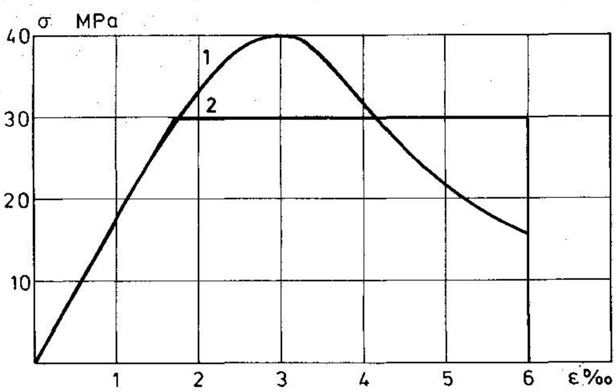

Fig. 2. 1:The analytical stress-strain curve.
2:The corresponding linear-elastic, perfectly plastic stress-strain curve.

With one insignificant exception around the transition between the elastic and the plastic branch, we find that

$$W_1\ (\varepsilon) \geq W_2\ (\varepsilon) \qquad \text{for} \qquad \varepsilon \leq \varepsilon_g$$

where the index 1 refers to the curve found in the tests, and the index 2 refers to the linear-elastic, perfectly plastic curve.

The ratio between the maximum stress on curve 2 and that at curve 1 is ν.

With the assumptions of our theorem, and provided the elastic strains are sufficiently small in relation to ε_g, we now know that a value for the carrying capacity of a structure calculated in accordance with the theory of plasticity with the compression strength $\nu\,\sigma_c$ will give a safe value for the actual structure. We have thus arrived at a method of calculating the effectiveness factor directly on the basis of a concrete compression test.

The calculation of ν as a function of ε_g and σ_c gives the result shown in fig. 3. The curves fall with increasing value of σ_c, since it has been found that the stress-strain curves for a strong concrete fall relatively more rapidly after passing the peak than in the case of weak concrete.

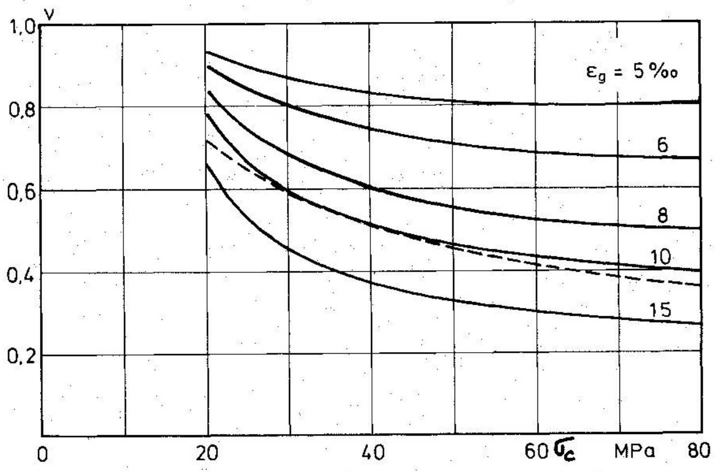

Fig. 3. ν as a function of ε_g and σ_c. The dashed curve is the curve $\nu = 3.2/\sqrt{\sigma_c}$, σ_c in MPa from fig. 4.

6. COMPARISON WITH TESTS

The question is now: which limit strain is the right one? This question requires detailed investigation and cannot be finally answered here. In the investigations in [67.1], measurements are taken of the strains in the web of concrete beams loaded to shear failure. In some cases the measurements showed a compressive strain of up to 8‰, and these measurements were interrupted before failure.

A comparison with empirically determined ν-values can be carried out by means of the results in [79.1]. Here, ν is determined for the shear strength of beams without shear reinforcement. In one of the test series [62.2], the only varying parameter is σ_c.

The test results group themselves as shown in fig. 4, in which the curve $\nu = 3.2/\sqrt{\sigma_c}$, σ_c in MPa is included.

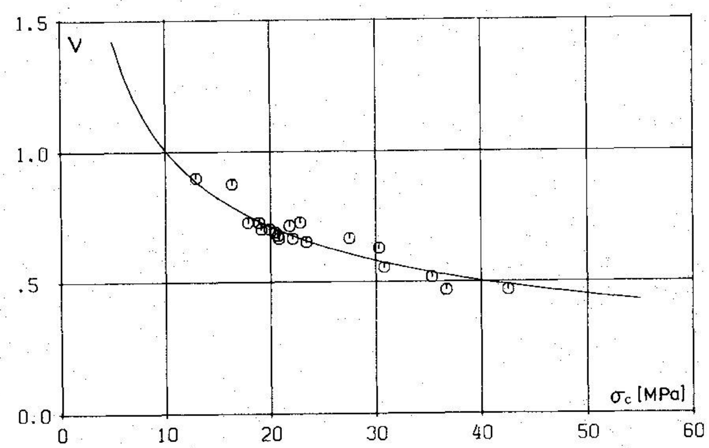

Fig. 4. Empirically determined ν-curve: $\nu = 3.2/\sqrt{\sigma_c}$, σ_c in MPa and test results from [79.1]

This curve is also shown dashed in fig. 3. It will be seen that these test results correspond well with the ν-curves for $\varepsilon_g \sim 10\,^{\circ}/oo$.

7. REMAINING THEORETICAL ASSUMPTIONS FOR USE OF THEORY OF PLASTICITY

The theory of plasticity assumes strains of arbitrary magnitude, i.e. of arbitrary magnitude in relation to the elastic strains.

In the stress-strain diagram found, there is a finite relationship between the plastic and the elastic strains. This limited yield capacity naturally results in a lower ultimate strength than calculated using the theory of plasticity, but with reasonably selected limit strains, this reduction will be of little significance. Nevertheless, in principle, it must be taken into account in another way.

As the validity of the theorem is not limited by the use of different limit strains at different points of the structure, a higher limit strain and thus a lower effective stress can be used at places where particular big strains can be expected, for instance near corners. If the highest strain here was used as the limit strain for the whole construction, the effective stress would be unnecessarily low.

8. EFFECT OF REINFORCEMENT ON THE ν-VALUE

In a compression test on a concrete cylinder reinforced against strain in the direction perpendicular to the load, this reinforcement will somewhat increase the stiffness on the rising branch of the stress-strain curve. On the falling branch, where the concrete has a tendency during crushing to get very big strains in directions perpendicular to the direction of the compression, the reinforcement will get big stresses and will thus to a great degree limit the reduction of the concrete stress.

With the method of calculation introduced here, this will result in an increase in ν. In other words, it is possible to determine the relationship between the effectiveness factor and the degree of reinforcement on the basis of tests on reinforced concrete cylinders.

9. CONCLUSION

It may seem very risky to use the optimistic assumptions of the theory of plasticity for calculating the carrying capacity of a structure made of a material like concrete. In practice, however, the applicability of this method of analysis has been demonstrated in many cases in [78.1]. This article shows that its use can also be justified theoretically provided a suitable effectiveness factor is introduced.

It is shown that the reason why the full compression strength of the concrete cannot be utilized in the theory of plasticity is to be found in the falling stress-strain curve of the concrete.

The empirically found decrease of the effectiveness factor with the strength of the concrete is directly explained on the basis of compression tests on concrete cylinders.

A method is given for calculating the effectiveness factor on the basis of the maximum strain. However, here we need to know more about the size of the strains that occur in a structure when the ultimate strength is reached. We also need measurements of the stress-strain curves for concrete strains of 10-15‰.

However, on account of other assumptions made in the analysis of certain types of structure, we cannot expect the introduction of a ν-factor, determined as described here, to remove all systematic discrepancies between test results and theoretical results.

10. REFERENCES

[62.1] Van den Berg, F.J.: Shear Strength of Reinforced Concrete Beams without Web Reinforcement, part 2 - Factors Affecting Load at Diagonal Tension Cracking. Journal of the ACI - Proceedings, Vol. 59, pp 1587, Nov. 1962.

[67.1] Ozden, K.: An Experimental Investigation on the Shear Strength of Reinforced Concrete Beams. Faculty of Civil Engineering, Technical University of Istanbul, 1967.

[78.1] Nielsen, M.P., Bræstrup, M.W., Jensen, B.C. and Bach, F.: Concrete Plasticity. Danish Society for Structural Science and Engineering, 1978.

[78.2] Wang, P.T., Shah, S.P. & Naaman, A.E.: Stress-Strain Curves of Normal and Lightweight Concrete in Compression. Journal of the American Concrete Institute, November 1978, No. 11, Proceedings V.75, pp 603-611.

[79.1] Roikjaer, M., Pedersen,C., Bræstrup, M.W., Nielsen, M.P. and Bach, F.: Bestemmelse af ikke-forskydningsarmerede bjælkers forskydningsbæreevne. Structural Research Laboratory, Technical University of Denmark, 1979.