I

Investigating the Stress-Strain Characteristics of Diagonally Cracked Concrete

Examen des relations entre contraintes et déformations dans le béton fissuré diagonalement

Untersuchung der Spannungs-Dehnungs-Beziehungen von diagonal gerissenem Beton

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SUMMARY
Experiments aimed at determining the relationships between the average principal compressive strains and the average principal compressive stresses in diagonally cracked reinforced concrete are described. It is demonstrated that these relationships are influenced by the magnitude of the maximum co-existing shear strains.

RESUME
Des essais avaient pour but d'examiner les relations entre les déformations principales moyennes de compression et les contraintes principales moyennes de compression dans le béton armé fissuré diagonalement. On montre que ces relations sont influencées par la plus grande valeur des déformations dues au cisaillement.

ZUSAMMENFASSUNG
Es werden Versuche beschrieben, die durchgeführt wurden, um die Beziehungen zwischen den mittleren Hauptdruckdehnungen und den mittleren Hauptdruckspannungen in diagonal gerissenem bewehrten Beton zu bestimmen. Es wird gezeigt, dass diese Beziehungen von der Grösse der maximal auftretenden Schiebungen beeinflusst werden.

1. INTRODUCTION

Before the load-deformation response of members subjected to shear and/or torsion can be predicted the stress-strain relationships for the reinforcement and for the diagonally cracked concrete must be known.

Because the diagonal cracks cause considerable variations in the values of local strains and stresses it has been found useful [1] to formulate the relationships in terms of average strains and average stresses. The average strains can be thought of as strains measured over a base length which is several times the crack spacing.

Shown in Fig. 1(a) is a typical length of a diagonally cracked reinforced concrete member. The average stress conditions which exist at some point in the diagonally cracked concrete (e.g. at mid-depth) can be represented by a Mohr's circle of stress such as that shown in Fig. 1(b). The average strain conditions which exist at the same point in the diagonally cracked concrete can be represented by a Mohr's circle of strains such as that shown in Fig. 1(c). With reference to Figure 1, the question that will be discussed in this paper is: "What is the relationship between the average principal compressive stress, f_d, and the average principal compressive strain, ε_d?"

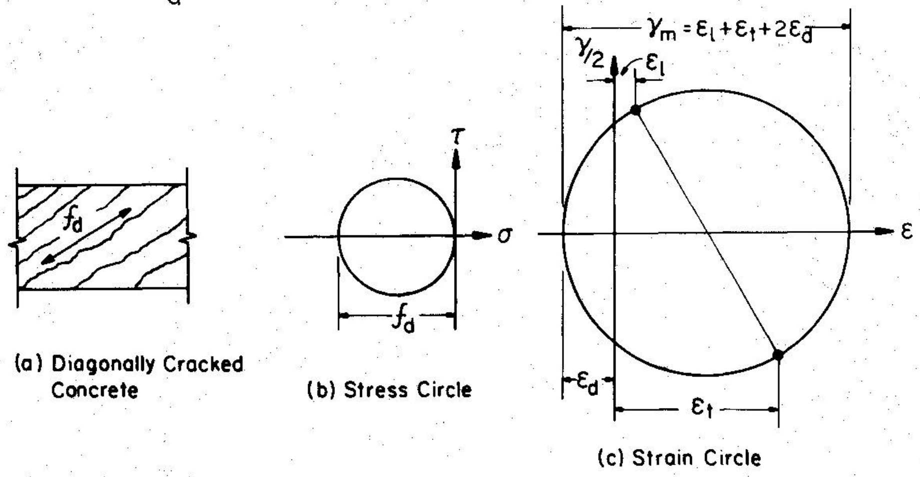

Fig. 1 Stress and Strain Conditions for Diagonally Cracked Concrete

In earlier studies ([2], [3], [4]) it was assumed that f_d could be related to ε_d by the usual stress-strain curve determined from a cylinder test on the concrete. However, the concrete strain conditions which exist in a cylinder test (see Figure 2) are substantially different from the strain conditions of the diagonally cracked concrete (Figure 1). Will these different strain conditions, which can be described in terms of the ratio of the maximum shear strain, γ_m (i.e. the diameter of the strain circle) to the principal compressive strain, ε_d, effect the stress-strain characteristics of the concrete?

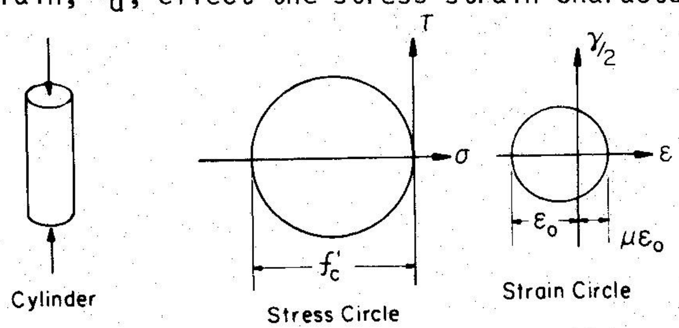

Fig. 2 Stress and Strain Conditions for a Control Cylinder

2. EXPERIMENTAL SET-UP

To investigate the effect of the strain ratio, γ_m/ε_d, on the relationship between f_d and ε_d a number of concrete panels were loaded in the manner described in Fig. 3.

Each test set-up consisted of two unreinforced concrete panels cast between three steel columns. The concrete panels, which were 915 mm high, 255 mm wide and 32 mm thick, were attached to the steel columns by being cast around steel teeth which in turn had shear studs attached (see Fig. 3).

The panels were loaded by jacking upwards the central steel column while calibrated inclined tension links held down the outer two steel columns. The lines of action of the hold down forces passed through the centre point of each panel.

The horizontal expansion of the panels was controlled by two hydraulic jacks which loaded two sets of external horizontal calibrated rods. The vertical expansion of each panel was controlled not only by the steel columns but also by eight 19 mm diameter external vertical steel rods.

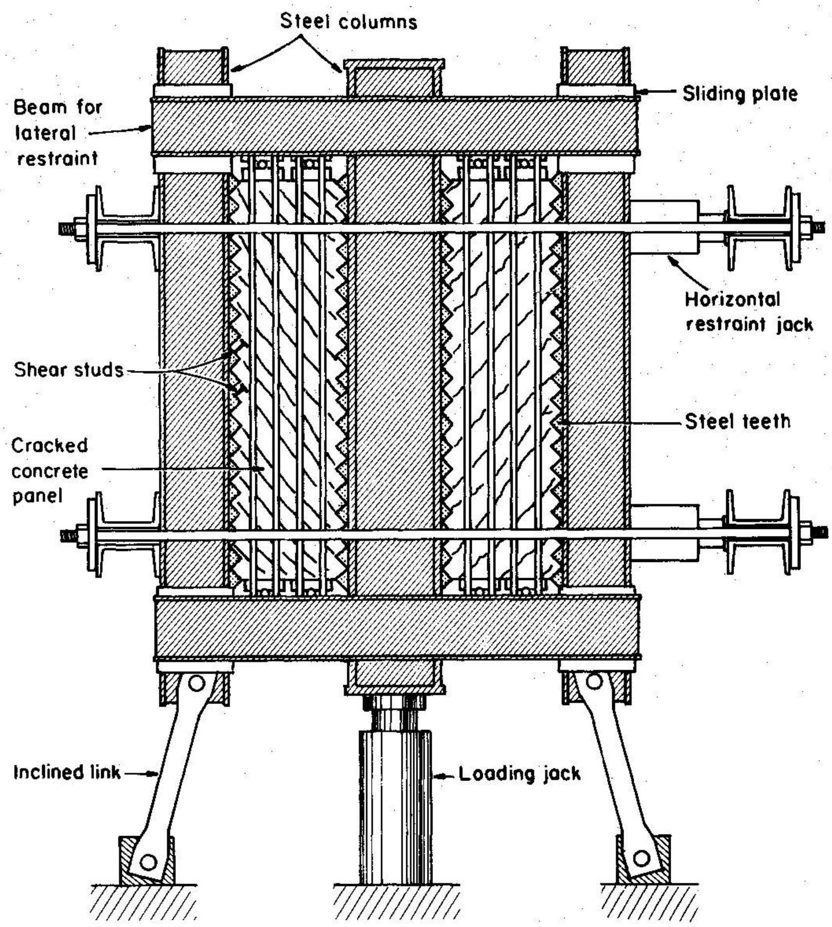

Fig. 3 Test Rig for Loading Panels in Shear

By adjusting the force applied by the horizontal jacks it was possible to change the average horizontal strain in the panel, and hence it was possible to change the ratio of shear strain to horizontal strain. This strain ratio was the prime variable between the different test panels.

The average strain in the concrete panels was determined by measuring the relative movements of the steel columns. These average strain readings were complemented by a series of local strain readings using a demountable mechanical strain gauge with targets located on steel plugs cast in the concrete.

During the test external diagonal steel braces were mounted over one of the two concrete panels. This enabled each panel to be tested separately.

3. EXPERIMENTAL RESULTS

The results for four panel tests are summarized in Table 1.

Panels 3 and 3A were cast at the same time from the same concrete mix. At the time of testing the compressive strength of this concrete, f_c', as determined from three 75 mm x 75 mm x 300 mm prisms, was 17.3 MPa and this stress was attained at a compressive strain, ε_0, of 2.0×10^{-3}. For Panels 4 and 4A, f_c' was 31.7 MPa and ε_0 was 2.4×10^{-3}.

The shear stress, v, and the horizontal compressive stress, σ_h, acting on the central vertical plane of each panel at each load stage (L.S.) are listed in Table 1. These stresses were calculated from the measured forces in the horizontal rods and the inclined tension links by means of the free body diagram shown in Fig. 4.

The average change in angle between the originally horizontal and vertical lines (i.e. shear strain γ_{vh}) and the average horizontal tensile strain, ε_h, are also listed in Table 1. These strain values were calculated from the measured deformations of the panel illustrated in Fig. 5. From the local strain readings it was found that the average vertical strain ε_v, remained essentially at zero.

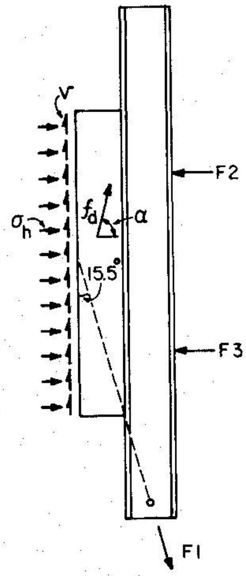

Fig. 4 Forces and Stresses

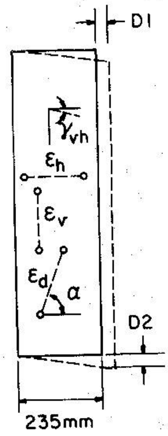

Fig. 5 Deformations and Strains

For each load stage the average value of the principal compressive strain in the concrete, ε_d, the angle of inclination, α, of this principal compressive strain, and the average value of the maximum shear strain, γ_m were all calculated from the measured values of γ_{vh} and ε_h by using the Mohr's circle of strain shown in Fig. 6.

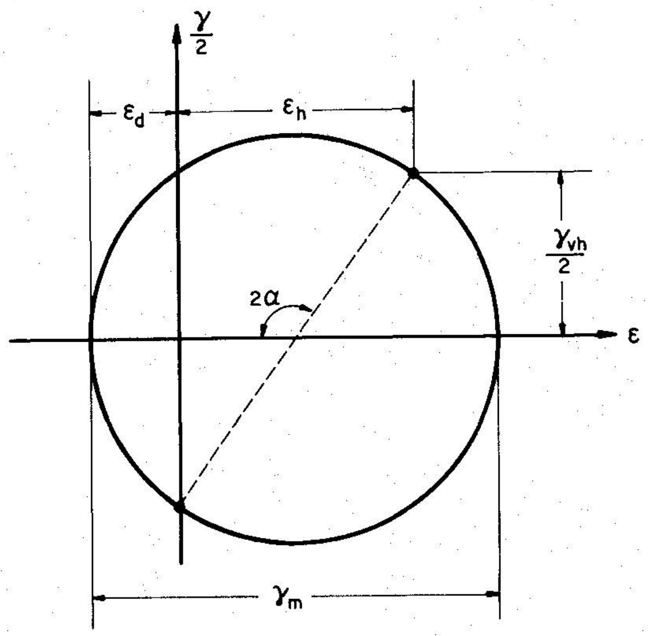

Fig. 6 Strain Relationships for Test Panels

The average value of the principal compressive stress in the concrete, f_d, listed in Table 1, was calculated from the measured values of v and α by the relationship:

$$f_d = v\left(\tan\alpha + \frac{1}{\tan\alpha}\right) \qquad \ldots (1)$$

This equation is based on the assumptions that there are no tensile stresses in the concrete and that the angle of inclination of the principal compressive stress is equal to the angle of inclination of the principal compressive strain.

If it is assumed that tensile stresses can exist in the cracked concrete then the tabulated values of σ_h, v and α can be used to calculate the principal tensile stress in the concrete, f_t, from the relationship:

$$f_t = \frac{v}{\tan\alpha} - \sigma_h \qquad \ldots (2)$$

and the principal compressive stress then becomes:

$$f_d = v\left(\tan\alpha + \frac{1}{\tan\alpha}\right) - f_t \qquad \ldots (3)$$

Obviously the value of principal compressive stress calculated from Eq. (3) will be less than the value calculated from Eq. (1). As would be expected neglecting the tensile stresses causes a more significant change for the early load stages. Thus for Panel 4 at load stage 1 the value of f_d given by Eq. (3) would be 0.76 MPa as opposed to the 1.45 MPa given by Eq. (1), whereas at load stage 9 the two estimates of f_d would be 14.26 MPa and 15.06 MPa. Because it is desired not to over-estimate any loss in stiffness due to shear strain the higher estimates of f_d (i.e. those given in Table 1) will be used in the remainder of this paper.

During each test an attempt was made to control the strain ratio γ_m/ε_d. Each test started by jacking upwards the central steel column while at the same time having only a minimum force in the horizontal restraining rods. While the panels remained uncracked they had no tendency to expand laterally and hence ε_h remained zero and the value of the ratio γ_m/ε_d remained at two. Once cracking commenced the panels tended to expand laterally as the central load was increased. This expansion was allowed to occur until the desired value of γ_m/ε_d was reached. Thereafter the magnitudes of the horizontal restraining forces were continuously adjusted in an attempt to maintain a constant value of the ratio γ_m/ε_d. However, as can be seen from Table 1, it did not prove possible to maintain γ_m/ε_d at a truly constant value.

For Panels 3, 3A and 4, the crushing failure of the concrete was relatively gradual, enabling "post-peak" readings to be taken (for 3 the peak load occurred between LS 8 and 9 at a shear stress of about 5.7 MPa). However, the highly restrained concrete of Panel 4A essentially disintegrated at the peak stress in a failure that was explosively abrupt. A view of one of the more typical failures is given in Fig. 7.

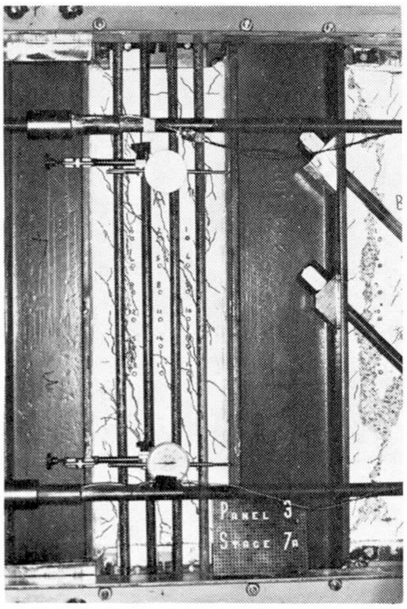

Fig. 7 Panel 3A at Failure

6. OBSERVED STRESS-STRAIN CHARACTERISTICS

The observed relationships between the principal compressive stress, f_d, and the principal compressive strain, ε_d, for Panels 3 and 3A and for a control prism cast from the same concrete are shown in Fig. 8. The numbers written above the curves in Fig. 8 are the measured values of the strain ratio γ_m/ε_d.

From Fig. 8 it would appear that the relationship between f_d and ε_d depends on the strain ratio γ_m/ε_d. For a given value of ε_d the higher the value of γ_m/ε_d the lower the value of f_d. Furthermore, the maximum value of f_d that can be attained also appears to be a function of γ_m/ε_d.

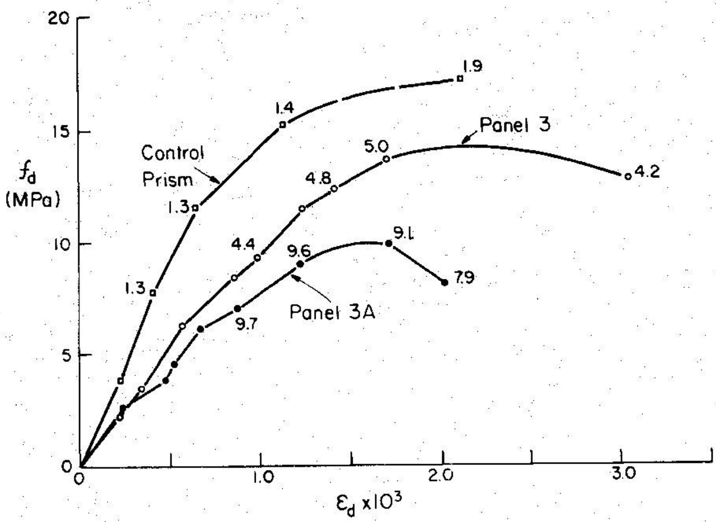

Fig. 8 Observed Stress-Strain Characteristics - Panels 3 and 3A

Fig. 9 shows the relationships between f_d and ε_d for Panels 4 and 4A and for a control prism cast from the same concrete. It is worthy of note that for Panel 4A the calculated value of f_d reached 97% of f'_c. This corresponded to an applied shear stress which was 47% of f'_c.

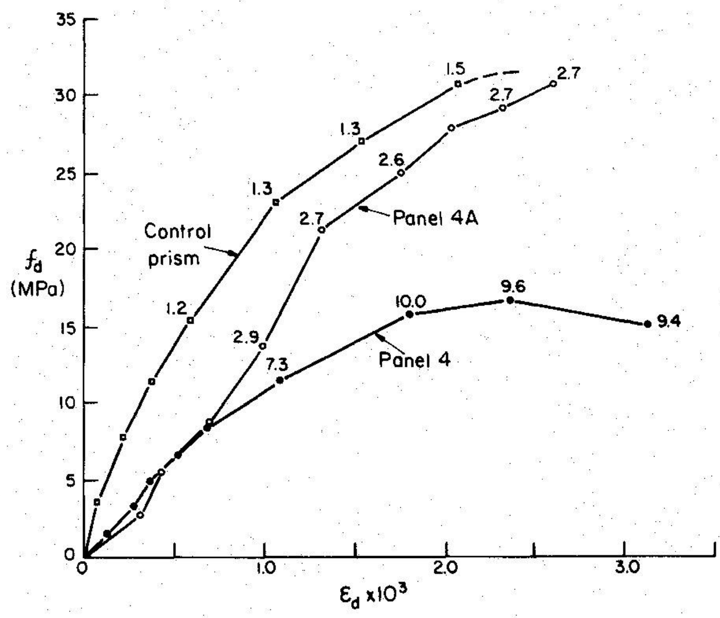

Fig. 9 Observed Stress-Strain Characteristics - Panels 4 and 4A

On the basis of the small number of tests reported in Figures 8 and 9, it appears that the value of ε_d at which f_d reaches its peak value is not greatly influenced by the value of γ_m/ε_d. Further, for any given value of γ_m/ε_d it seems that the f_d versus ε_d curve could be approximated by a parabola. Based on these observations the following relationship is tentatively proposed:

$$\frac{f_d}{f'_c} = \frac{5.5}{4 + \gamma_m/\varepsilon_d} \left[2\frac{\varepsilon_d}{\varepsilon_o} - \left(\frac{\varepsilon_d}{\varepsilon_o}\right)^2 \right] \qquad \ldots (4)$$

The predictions of Equation (4) are compared with the experimental results in Fig. 10. In this figure the predicted relationships between f_d and ε_d for four different values of γ_m/ε_d are shown. Also shown are experimental points corresponding to measured values of f_d and ε_d. The numbers written above the points are the measured values of γ_m/ε_d. It can be seen from Fig. 10 that the predictions of Equation (4) are in general conservative for these experiments.

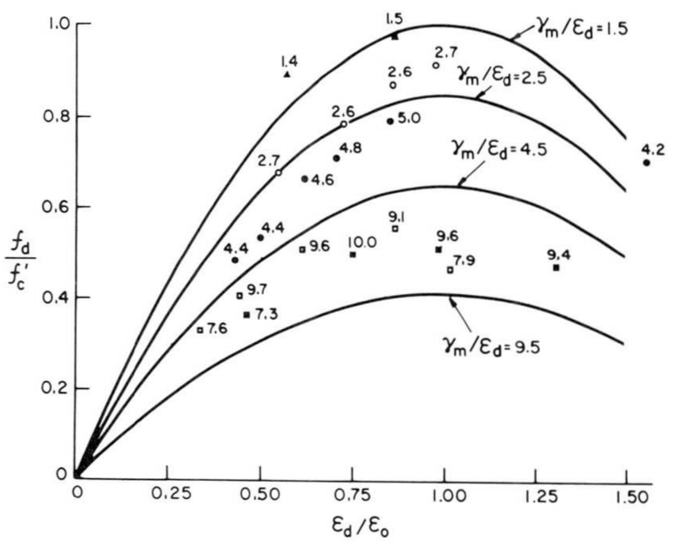

Fig. 10 Predicted and Observed Stress-Strain Relationships

7. CONCLUDING REMARKS

The experiments reported in this paper demonstrate that for a given value of principal compressive concrete strain the value of the resulting principal compressive concrete stress depends on the magnitude of the maximum co-existing shear strain. In other words, f_d is a function of both ε_d and γ_m.

The particular function suggested in the paper, Eq. (4), was based on only a very limited number of tests and hence should be regarded as tentative.

In an attempt to obtain more extensive data on the stress-strain characteristics of diagonally cracked reinforced concrete a new test rig has recently been installed at the University of Toronto. This rig, which is shown in Fig. 11, will enable 1 m x 1 m x 75 mm panels of reinforced concrete to be tested under a wide variety of stress conditions, including the condition of pure shear.

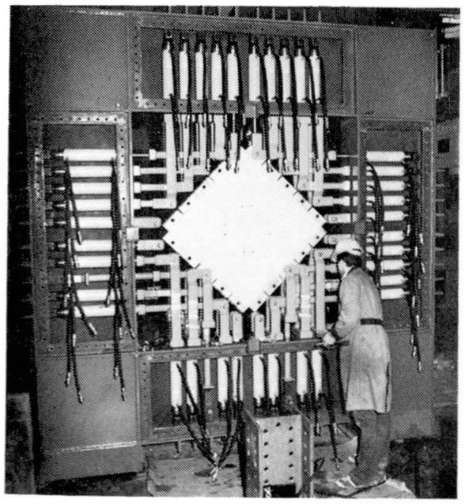

Fig. 11 New Test Rig for Shear Panels

ACKNOWLEDGEMENTS

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TABLE 1: Experimental Results for Four Panels

SPECIMEN		L.S.	v (MPa)	σ_h (MPa)	γ_{vh} x 10^3	ε_h x 10^3	α (°)	γ_m x 10^3	ε_d x 10^3	f_d (MPa)	γ_m/ε_d
$\varepsilon_0 = 2.0 \times 10^{-3}$, $f'_c = 17.3$ MPa	PANEL #3	1	1.12	0.21	0.47	0.04	47	0.47	0.22	2.24	2.2
		2	1.68	0.01	0.83	0.14	50	0.84	0.35	3.41	2.4
		3	2.77	0.41	1.93	1.00	59	2.18	0.59	6.28	3.7
		4	3.54	0.45	3.15	2.01	61	3.74	0.86	8.34	4.4
		5	3.92	0.59	3.70	2.40	61	4.41	1.00	9.24	4.4
		6	4.74	0.77	4.69	3.19	62	5.67	1.24	11.45	4.6
		7	4.94	0.88	5.51	3.98	63	6.80	1.41	12.21	4.8
		8	5.52	1.00	6.85	5.16	63	8.57	1.71	13.66	5.0
		9	5.31	1.46	11.06	6.74	61	12.95	3.11	12.52	4.2
	PANEL #3A	1	1.37	0	0.50	0	45	0.50	0.25	2.76	2.0
		2	1.74	0.63	1.47	0.64	57	1.60	0.48	3.83	3.3
		3	1.81	0.13	2.26	1.94	65	2.98	0.52	4.76	5.7
		4	2.02	0.08	3.44	3.75	69	5.09	0.67	6.03	7.6
		5	2.13	0.17	5.22	6.83	71	8.60	0.88	6.93	9.7
		6	2.74	0.27	7.19	9.32	71	11.77	1.23	8.90	9.6
		7	3.01	0.32	9.88	12.32	71	15.80	1.74	9.79	9.1
		8	2.68	0.44	10.62	11.88	69	15.93	2.03	8.00	7.9
$\varepsilon_0 = 2.4 \times 10^{-3}$, $f'_c = 31.7$ MPa	PANEL #4	1	0.72	0.04	0.34	0	45	0.34	0.17	1.45	2.0
		2	1.45	0.03	0.87	0.37	57	0.95	0.29	3.17	3.3
		3	2.12	0.07	1.33	0.83	61	1.57	0.37	5.00	4.3
		4	2.76	0.12	1.90	1.22	61	2.26	0.52	6.48	4.3
		5	3.42	0.26	2.80	2.21	64	3.57	0.68	8.69	5.3
		6	4.01	0.43	5.48	5.77	68	7.96	1.09	11.55	7.3
		7	4.62	0.57	11.00	14.70	72	18.40	1.80	15.72	10.0
		8	5.06	0.69	13.85	17.99	71	22.70	2.36	16.45	9.6
		9	4.63	0.81	18.15	23.13	71	29.40	3.14	15.06	9.4
	PANEL #4A	1	1.40	0	0.59	0	45	0.59	0.31	2.83	2.0
		2	2.85	0.05	1.06	0.13	49	1.07	0.47	5.72	2.3
		3	4.18	0.31	1.93	0.63	54	2.03	0.70	8.79	2.9
		4	6.52	2.52	2.76	0.91	54	2.90	1.00	13.72	2.9
		5	10.39	4.34	3.47	0.94	53	3.59	1.32	21.52	2.7
		6	12.04	4.86	4.45	1.06	52	4.57	1.76	24.76	2.6
		7	13.46	5.37	5.24	1.30	52	5.39	2.05	27.72	2.6
		8	14.08	5.40	6.06	1.57	52	6.26	2.34	29.10	2.7
		9	14.86	5.45	6.84	1.87	53	7.09	2.61	30.83	2.7

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