Yield Criteria for Elements of Reinforced Concrete Slabs

Critères d'écoulement plastique d'éléments de dalles en béton armé

Fliessbedingungen für Elemente von Stahlbetonplatten

C.T. MORLEY
Lecturer
University of Cambridge
Cambridge, England

SUMMARY
The paper reviews the present state of knowledge of the criteria for plastic flow in elements of reinforced concrete slabs. Three main topics are considered: the rate of energy dissipation in yield lines, the development of yield criteria in terms of stress resultants, and the question of how to optimise the reinforcement in an element designed for strength.

RESUME
L'étude décrit l'état actuel des connaissances des critères d'écoulement pour des plaques en béton armé. Trois sujets principaux sont traités: la vitesse de dissipation d'énergie en lignes de rupture, les critères de ruine pour un élément soumis à des moments de flexion et à des efforts de membrane, et l'armature optimale d'éléments dimensionnés à la rupture.

ZUSAMMENFASSUNG
Die heutigen Kenntnisse über Fliessbedingungen für Elemente von Stahlbetonplatten werden besprochen. Drei Hauptthemen werden erörtert: Energiedissipation in Fliessgelenklinien, Entwicklung von Fliessbedingungen für kombinierte Beanspruchung durch Biegemomente und Membrankräfte, und die Frage nach der optimalen Bewehrung von auf Bruch bemessenen Elementen.

1.   INTRODUCTION

There have been two main strands in the development of plasticity theory for
reinforced concrete structures. In the first, for instance in the truss analogy
for beams in shear or torsion, the steel and concrete are considered separately
throughout the analysis. In the second strand, for thin structures such as
slabs or shells, the structural analysis is in terms of stress resultants acting
on composite plate elements, the properties of the composite plate being derived
before the structural analysis from the properties of its steel and concrete
constituents. This survey concentrates on the second approach to thin struct-
ures in reinforced concrete, considering only flat plate structures in detail
in the expectation that the results will be applicable to shell elements with
radii of curvature large compared to their thickness.

The eight important stress resultants (per unit slab width) on a typical con-
crete plate element are indicated in Fig. 1, where the orthogonal axes x, y are
in the median plane. The element is
reinforced by layers of steel rein-
forcing bars parallel to the median
plane, each layer consisting of
parallel straight bars with uniform
area and spacing. In practice the
reinforcing bars often lie in two
orthogonal directions, which can be
taken as the x and y directions.
For cases where the steel lies in
two skew directions it may be advant-
ageous to choose axes r, s along
the bisectors of the angles
between the bars (Fig. 2).

Consideration of the strength and
yield criterion of slab elements
was pioneered in the early work of
Ingerslev [1] and Johansen [2]
on collapse of reinforced concrete
slabs, but further progress has
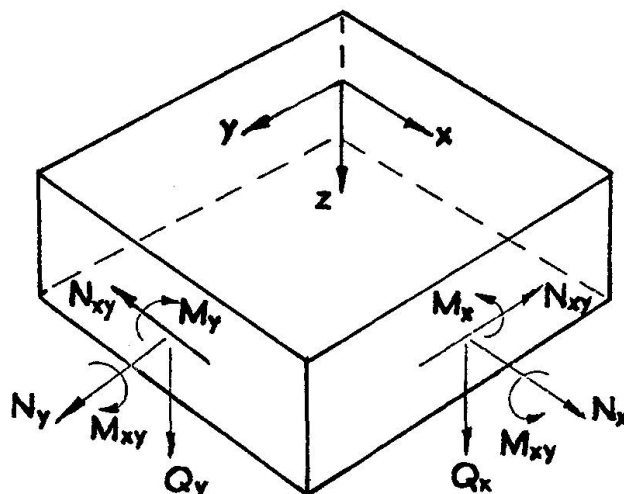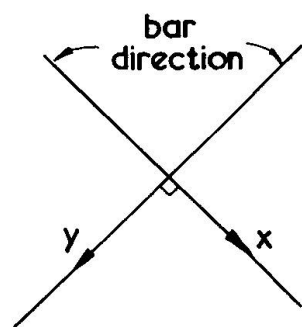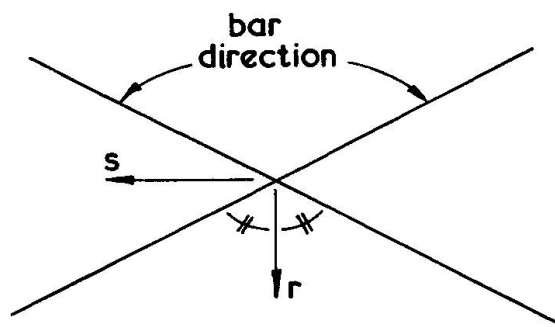
Fig. 1 Sign convention for stress-
resultants on a slab element

(a) orthogonal steel

(b) skew steel

Fig. 2 Reinforcement directions

been made since, especially in the application of plasticity theory to slab
elements carrying combinations of moment and membrane force. This paper surveys
the various methods now available for deriving the strength properties of a

composite element from the properties of its constituents, concentrating on three main topics:

- the rate of plastic energy dissipation in yield lines, which play such an important role in the kinematic analysis of plate structures,

- the development of yield criteria for plate elements in terms of stress resultants, for use perhaps in statical analysis of structures, and

- the question of how best to reinforce a slab element against the stress-resultants acting on it, perhaps to minimise the amount of steel required.

First, however, we consider the various assumptions which must be made before plasticity theory can conveniently be used to investigate the strength of a composite plate element.

2.   BASIC ASSUMPTIONS

2.1   'Smeared' Steel Layers

The stress-resultants are assumed to vary rather slowly over the slab surface, so that the diameters and spacings of the individual bars are not important. Each steel layer can then be characterised by its cross-sectional area per unit slab width normal to the bars, the layer being effectively idealised as a thin 'smeared' sheet at the level of the bar centres.

2.2   Perfect Bond

Cracks in a brittle inhomogeneous material such as concrete will of course occur at random positions, introducing strain fluctuations and local bond failure near each crack. However, it is usual to consider 'mean' strains, measured over gauge lengths crossing several cracks, and to assume that for these mean strains there is effectively perfect bond between the steel and the concrete, so that a convenient theory of the composite plate element can be developed. In practice, transferring forces from concrete to steel at bar ends may be rather difficult - but it is assumed in the theory that the necessary anchorages and bond lengths are well away from the element under consideration. Both 2.1 and 2.2 may be somewhat in error if deformation is heavily concentrated at a yield line.

2.3   Neglect of Shear Forces and Transverse Stresses

It is also usual to assume in bending problems that the transverse shear forces Q_x and Q_y have no effect on the element's strength: in the terminology of plasticity theory these forces are treated as 'generalised reactions'. Stresses normal to the slab plane, due perhaps to applied distributed loading, are also neglected. The bending strength of the element may then be evaluated for $Q_x = Q_y = 0$, in which case each layer of material will be in plane stress parallel to the median plane, and yield criteria for the constituent steel and concrete in plane stress will be needed.

Also, straight fibres normal to the median plane will remain straight and normal during plastic deformation, so that the strain-increments $\dot{e}$ at any level z can easily be determined from the middle-surface strain-increments $\dot{\varepsilon}$ and the curvature-increments κ, using formulae of the type

$$\dot{e}_x = \dot{\varepsilon}_x + z\,\dot{\kappa}_x \tag{1}$$

2.4   Properties of the steel

The steel reinforcement is taken to be elastic, perfectly-plastic, yielding at stress $\pm f_y$. The bars are assumed to carry stress in their original directions only, so that the yield-criterion for a single layer of steel reinforcement in plane stress is pencil-shaped, as shown in Fig. 3. In practice with work-hardening reinforcement difficulties can arise in allotting a value to f_y between the elastic limit and ultimate strength of the bars.

2.5   Properties of the concrete

The concrete is idealised as an isotropic elastic, perfectly-plastic material with a known yield criterion in plane stress, obeying the associated 'normality' flow-rule of plasticity theory. It has recently become common [3-8] to adopt a square yield condition in principal stress space, as shown in Fig. 4. In practice the precise shape of the criterion in the biaxial compression region is of small importance. More significant is the complete neglect of the tensile strength of concrete in bending problems, although a small tensile strength may be needed to enable the plate to carry transverse shear [9].

Fig. 3   Yield criterion for a steel layer with bars in the x-direction

Neglect of the tensile strength would seem to be unavoidable if plasticity theory with the associated flow rule is to be used with confidence for a material which is brittle in tension. Even in compression concrete is by no means a perfectly-plastic material, and the use of plast-

Fig. 4   Assumed yield criterion for concrete in plane stress

icity theory can only be justified in two circumstances, (i) if the properties of the slab element are dominated by those of the steel reinforcement, or (ii) if the concrete in compression is sufficiently ductile to allow adequate stress redistribution before any layer of material fails. In an attempt to allow for the limited strain-capacity of concrete in compression, the strength f_c used in plasticity theory is often [10,11] taken to be an 'effectiveness factor' ν times the true uniaxial strength measured say on cylinders. The value of ν will vary from one class of problem to another, and is to be determined experimentally: its value for typical slab bending problems is discussed later.

3.   PLASTICITY THEORY FOR A SLAB ELEMENT

According to the above assumptions, the steel and concrete are both taken to be perfectly-plastic materials, and we therefore expect all the theorems of plast-
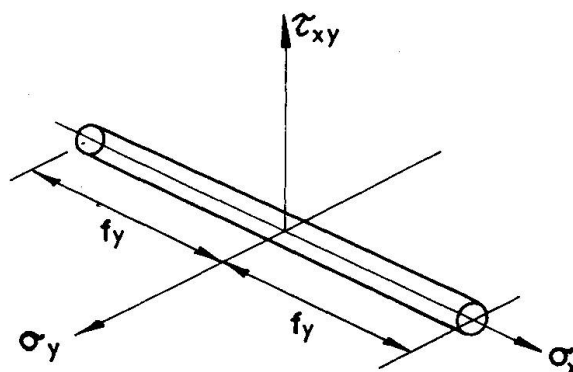
icity theory to be applicable for determining the properties of the composite plate element from those of its constituents. The composite will have a convex yield criterion in stress-resultant space, and the 'normality' flow rule will be obeyed. In principle, the yield criterion can be determined by methods of the type used by e.g. Onat and Prager [12] for homogeneous plates, and the theorem of Drucker and Shield [13] can be applied to optimise the reinforcement. However, it is not easy to present a yield criterion in six-dimensional space, and in what follows we shall mainly discuss special cases.

4.   THE GENERALISED YIELD LINE

In collapse analysis of concrete plate structures, plastic deformation is often assumed to be concentrated into narrow 'yield-lines' between rigid slab portions. The strain-increments are taken to be uniform but infinitely large in a narrow zone of infinitesimal width Δ (Fig. 5), leaving finite displacement increments

slab portion
A

'yield line'

displacement δ

rotation θ

x   small width Δ

ϕ

t

slab portion B

n

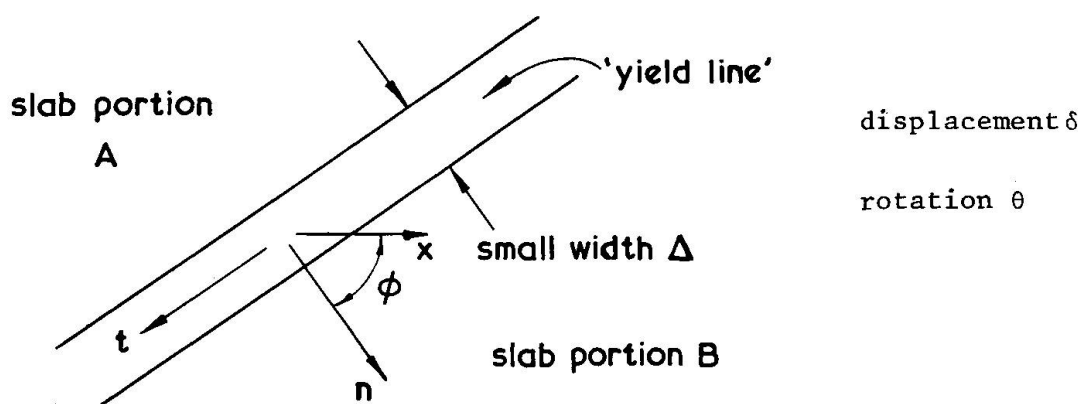Fig. 5   A yield line between two rigid slab portions

across the zone. In a flat plate, only three non-zero displacement increments, namely δ_n, δ_{nt} and θ_n, are possible at such a discontinuity [14].

Strictly speaking, a yield criterion in terms of plate stress resultants is not required in a kinematic analysis: the energy dissipation at any level z can be calculated from the yield criterion for the material at that level, and the total dissipation is then determined by integration. However, the generalised yield line plays such a large part in the theory as to justify further study.

At every level z in the yield line the longitudinal strain-increment $\dot{e}_t$ will be zero. Except in the special case $\dot{\delta}_{nt} = 0$, the principal plastic strain-increments in the concrete will then be of opposite sign, and the stress point will be at the corners B or D of the concrete yield criterion. For assumed values of δ_n, $\dot{\delta}_{nt}$ and θ_n, integration gives the contribution of the concrete to N_n, N_{nt} and M_n, and hence to the plastic energy dissipation. Expressions for the concrete stress-resultants on the yield line are in effect obtained in parametric form: the two parameters could be taken as the ratios $\dot{\delta}_{nt}/\dot{\delta}_n$ and $\dot{\delta}_n/\dot{\theta}n$. Cookson [15] gives explicit parametric formulae and comparisons with some proposed simplified approximations. The contribution of the steel can be determined similarly and added to that of the concrete: this corresponds to translating the surface for concrete only in (N_n, N_{nt}, M_n) space in various directions depending on the steel properties, to sweep out the full yield criterion for the reinforced plate. Some examples involving yield-lines with shear displacement $\dot{\delta}_{nt}$ have been solved [16, 17].

The stress-resultants N_t, M_t and M_{nt} on the yield line do not contribute to the
energy dissipation. They need not be evaluated in an upper-bound calculation,
and might be regarded as generalised reactions. They can however be determined
by the same integration process and the six stress-resultants on the n, t axes
(Fig. 5) can then be transformed to other co-ordinates, e.g. the x, y axes in
line with the steel, to give points on the general yield criterion for the
plate element. Although there are now three variable parameters (the angle ϕ of
the yield line plus the previous two), these points correspond to the restricted
type of plastic flow which can occur on a yield-line, and therefore only give a
limited part of the true yield surface in six dimensions.

It would appear that explicit expressions for the true general yield surface can
only be derived if further restrictions are introduced. Thus Morley [18] obt-
ained an upper bound by considering a generalised yield-line but giving the con-
crete infinite compressive strength: the resulting approximate expression can
only be useful for tensile or small compressive membrane forces, and the limits
of applicability are not known.

Special cases of the generalised yield line have been considered. Symmetry of
a slab can often make $\dot{\delta}_{nt} = 0$, simplifying the resulting expressions for inter-
action between N_n and M_n. In slabs with no lateral restraint at the boundaries,
$\dot{\delta}_n$ and $\dot{\delta}_{nt}$ are often assumed to take such values that the membrane forces N_n and
N_{nt} vanish: in this case the moment M_n developed on a yield line closely approx-
imates the value (dependent on $\cos^2\phi$) given by Johansen [2].

It may be objected that at a concentrated yield line some of the basic assumpt-
ions listed in section 2 are unlikely to be fulfilled. The bond between steel
and concrete is very likely to break down at a single wide crack. Also, the
large theoretical strains are certain to exceed the strain-capacity of concrete
in compression. We simply regard the theoretical concentrated yield line as an
idealisation of what happens in practice, and trust that experiments will show
that the idealisation is useful for many practical purposes, and reveal any limits
on its use.

Another objection which has been much raised is that, due to dowel action and
distortion of the bars, the reinforcement at a yield line may well not just carry
stress in its original direction, but may contribute some extra strength. Some
experimenters [19] have found substantial extra strength on yield lines at 45°
to orthotropic reinforcement, others [20] have found negligible extra strength,
and some [21] consider the strength-increase due to 'kinking' to be related to
bar diameter and crack width. In view of these uncertainties, and of the dif-
ficulty of incorporating it into a plastic theory, 'kinking' of reinforcing
bars across cracks is conservatively ignored here.

5.   EXPLICIT YIELD CRITERIA

In contrast to kinematic analysis, where a yield criterion in terms of stress-
resultants is not strictly required, such a criterion is essential if a lower
bound on the collapse load of a plated structure is to be obtained by consid-
ering statically-admissible fields of stress-resultants. This yield criterion
can itself be generated by considering statically-admissible distributions of
stress within the slab element - an approach which leads to useful explicit
formulae for the yield surface in some important special cases.

Thus Nielsen [22, 8] considered orthotropic reinforcement symmetrically dis-
posed about the median plane, and treated the case of membrane forces only,
using statically-admissible stress distributions to derive equations for the

various parts of the yield surface in N_x, N_y, N_{xy} space. For example, consider both sets of reinforcing bars at yield in tension, and the concrete stress point on the side AB of the yield criterion of Fig. 4, with the principal direction at some angle α to the x-axis. N_x, N_y and N_{xy} can be expressed in terms of α and the principal stress in the concrete, and if these are eliminated we obtain the expression

$$(f_y A_x - N_x)(f_y A_y - N_y) = (N_{xy})^2 \qquad (2)$$

which is the equation of a cone in N_x, N_y, N_{xy} space. Here A_x, A_y are the areas of steel in the x and y directions per unit slab width. The complete yield surface, Fig. 6, for membrane forces only consists for $N_{xy} \geq 0$ of two conical sur-
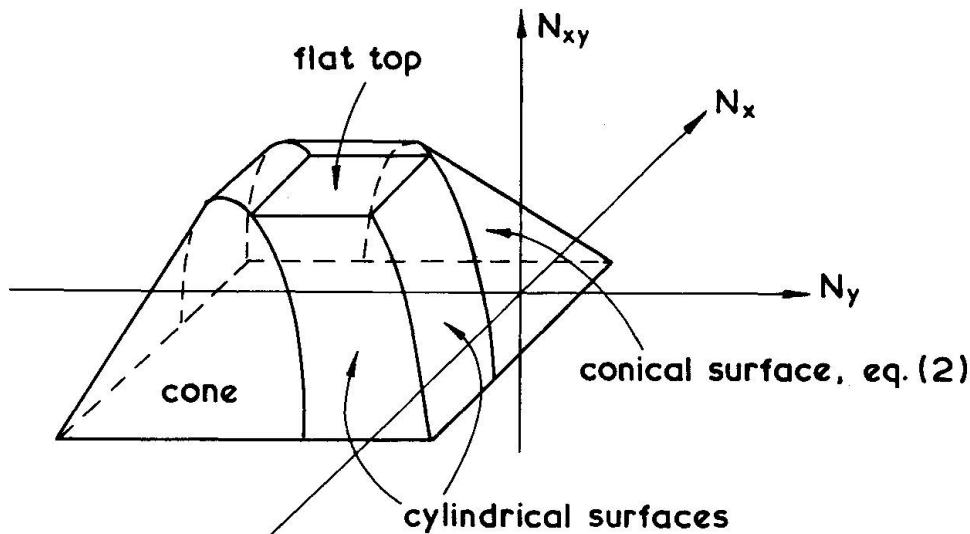

Fig. 6   Yield surface for orthogonal reinforcement, membrane forces only

faces, four cylindrical surfaces and a flat summit plateau. Sims   has recently found a mechanism of plastic flow for each region, to show that the surface found by considering statically-admissible stress distributions is the true yield surface in this case.

Statically-admissible stress distributions have also been used by Nielsen [3] to explore the yield criterion for orthotropic slabs under moments only. Again explicit equations for the yield surface are obtained and Nielsen shows that Johansen's original criterion is a close approximation for practical reinforcement proportions. There does not yet seem to have been much work done on statically admissible stress distributions which could lead to a yield criterion for the more general case of combined moments and membrane forces.

An alternative method has been used by Rajendran and Morley [23] to generate the point on the yield surface corresponding to any given combination of stress-resultants N_x ..... M_{xy} . In essence, a vector $\dot{\varepsilon} = \dot{\varepsilon}_x$ ..... $\dot{k}_{xy}$ of strain-increments is assumed and used to obtain an upper bound λ on the factor Y by which the given stress-resultant vector must be multiplied so as to end on the yield surface: the kinematic 'upper-bound' theorem is being applied to the slab element. By varying $\dot{\varepsilon}$ so as to minimise λ, a close approximation to Y is found. The method requires a computer to do the minimisation, and results in any number of points on, rather than an explicit equation for, the yield surface. Thus the method is of limited use in lower-bound calculations for complete plate structures: indeed, few such calculations have been made except in cases of

very limited interaction between the stress-resultants.

6. OPTIMAL REINFORCEMENT

A question which often arises in design is how best to reinforce a slab element against a given combination of stress-resultants at ultimate load. The overall concrete dimensions are assumed to be given, and the object is to minimise the total volume of reinforcement required. In practice, the reinforcement layers will often be restricted to lie at certain levels only, with bars in given perhaps skew directions. The optimal problem for the slab element is a vital part of the wider problem of how best to reinforce a complete concrete structure to collapse under some specified load.

It is easy [24] to adapt the approach of Drucker and Shield [13], with a few additional assumptions,to show that an arrangement of reinforcement which allows collapse with constant magnitude of strain-increment along all the bars will be optimal. (Strain-increments where steel could have been provided must be smaller.)

With membrane forces only and orthogonal steel disposed symmetrically about the median plane, equal tensile strain-increments in the bar directions x, y (Fig. 2a) can be achieved by having the principal directions of concrete stress at 45° to the bars. Taking the concrete stress-point on the side AB of the yield criterion (Fig. 4), it is then straightforward to derive the formulae

$$f_y \cdot A_x = N_x + |N_{xy}|$$

$$f_y \cdot A_y = N_y + |N_{xy}| \tag{3}$$

for the optimum reinforcement. These equations were derived by Nielsen in 1963 [22], starting from his yield criterion, equation (2). For skew steel, Fig. 2b, the principal stress directions in the concrete in the optimal situation are along the bisector axes r, s. Notice that equations (3) can be derived, without any appeal to plastic theory, simply by assuming that concrete can carry no tension.

Unfortunately, equations (3) for the optimal reinforcement only apply for a certain range of combinations of applied stress-resultants. In other cases the optimum arrangement may have compression steel, or no steel at all, in one or both of the prescribed directions. Some combinations of forces cannot be taken, however much reinforcement is provided: thus for orthogonal steel $|N_{xy}| \leqslant f_c \, D/2$ where D is the slab thickness. A full presentation of the expressions for optimum reinforcement for all the various possible force combinations is given by Clark [25], who treats both orthogonal and skew reinforcement.

Expressions analogous to (3) for slab elements carrying bending moments only have been derived by Nielsen [3], Wood [26] and Armer [27]. Again, there is an upper limit on the twisting moment which an orthogonally-reinforced element can carry, however much reinforcement is provided. Also, there are again a number of expressions for the optimum reinforcement, each valid for a certain range of combinations of bending-moment. Attempts have been made [28] to display the regions of validity of the various formulae in charts such as Fig. 7 - but these are perhaps rather too complicated for routine use in design. More recently, computer programmes embodying the expressionshave been written, to process the output from finite element analyses of plated structures and calculate the amount of reinforcement required.

$$A : f_y h A_{1x} = M_x + |M_{xy}|$$
$$f_y h A_{1y} = M_y + |M_{xy}|$$
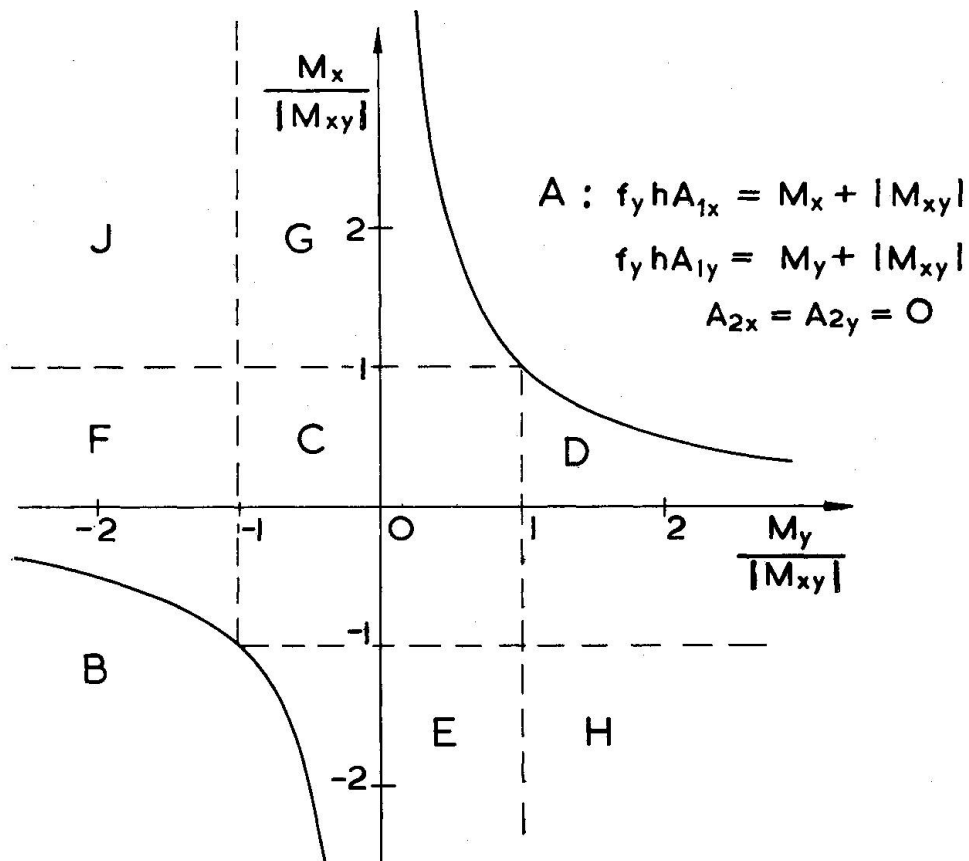$$A_{2x} = A_{2y} = O$$

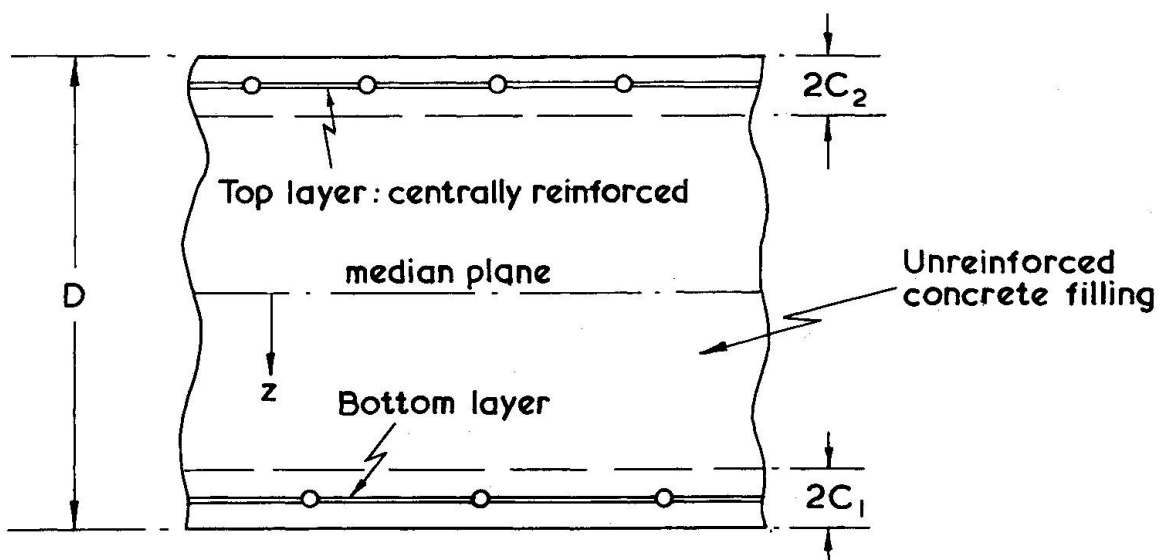Fig. 7   Regions of validity of expressions for optimal orthogonal steel

Fig. 8   Filled-sandwich model of a slab element

Some effort has recently been devoted to optimising the reinforcement in slab elements subjected to combinations of moments and membrane forces. A direct application [29] of the Drucker and Shield theorem proved useful for those force combinations which require tension steel in all four possible positions, but difficult to program for other cases. A more promising approach, pioneered by Brøndum-Nielsen [30],is to regard the slab element as a sandwich, whose outer layers take only membrane forces and so can be designed using the equations derived by Nielsen [22] or Clark [25]. A recent computer program [31] is based on a filled sandwich, Fig. 8. Nielsen's or Clark's equations are applied to the assumed outer layers, whose thickness is related to the cover to the steel. The computer is programmed to make use of the compressive strength of the unreinforced concrete filling, if it is advantageous to do so. Various refinements can be included, such as a minimum 'nominal' steel percentage,design for one loading case taking account of steel already provided to cope with some other loading, and devaluation of the strength of the filling, perhaps to allow for transverse shear forces.

All these methods of reinforcement design are based on pure plasticity theory. There has been relatively little work done on the behaviour of a plastically-designed element at working load, to check crack widths etc. Some attention has beendevoted [25] to what limitations must be imposed on the plastic design to ensure that problems due to limited strain-capacity are avoided, but this matter has not yet been dealt with convincingly.

7.   EXPERIMENTAL EVIDENCE

The best experimental evidence in support of the plate yield criteria outlined above is the success of plastic methods of analysis based on them, e.g. yield line theory for slabs, in predicting the collapse loads of complete structures. One must of course beware that defects in the yield criteria, due perhaps to strain-capacity problems, may be masked by beneficial large-deflection or boundary-restraint effects which simple plastic theory takes no account of.

In addition to work on kinking of steel [19,20,21], a number of experiments have been carried out on slabs in bending, to explore the yield criterion [5,32,33]. For problems where the plate properties are dominated by the steel reinforcement, as in pure bending, the yield criteria based on plastic theory seem to be adequate. In these cases, the 'effectiveness factor' ν for the concrete is of little significance, and may be taken as 0.6 on the cube strength, as is usual in beams.

Strain-capacity problems may mean that the plastic theory is suspect when membrane forces are large and compressive. Considerable attention has recently been given to push-off specimens (Fig. 9) of the type first tested by Hofbeck et al [34]. The specimens are designed to study transfer of shear across the central plane EF, but modifications can be introduced to vary the stress conditions on this plane [34,23,36]. For initially uncracked specimens, the plastic yield criteria outlined above seem to agree quite well with the experimental results, but Mattock and his co-workers find that specimens which are initially cracked on EF have appreciably less strength. Plastic theory in its simple form cannot explain this reduction in strength.

The problem seems to be that plastic theory assumes that the concrete can take a shear stress on EF which depends only on the compressive strength and the principal stress direction. This shear stress can only be carried across a crack by aggregate interlock, a phenomenon whose strength and ductility depend on crack width and aggregate size [37,38]. In unfavourable circumstances an

Fig. 9   Assumed stress conditions in 'push-off' test specimens

effectiveness factor of appreciably less than 0.6 may then be required to
fit the experimental results. A similar situation was encountered by Cookson
[17] in work on microconcrete models with central reinforcement, where crack
widths at ultimate load were comparable to the aggregate size. More work on this
matter is needed.

8.   CONCLUSIONS

The paper has surveyed the various methods available for deriving yield criteria
for reinforced slab elements, and considered the optimal design of reinforcement.
Plastic theory for such elements seems to be reasonably well developed, except
perhaps for complex combinations of moment and membrane force. More work is
needed on strain-capacity problems and effectiveness factors for large membrane
forces in compression or shear, especially for concrete which is already cracked.
The behaviour at working load of slab elements designed plastically for specified
ultimate strength also needs attention.

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