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Punching Shear in Concrete Slabs

Poinçonnement des dalles en béton

Durchstanzen von Betonplatten

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SUMMARY
The failure mechanism is examined, and various theories and design rules for centrical punching shear are reviewed. Based upon the classical theory of plasticity, an analytical solution is presented, describing the punching phenomenon in agreement with experimental evidence. Test results are compared with strength predictions of building codes and of plastic analysis. It is concluded that, in spite of completely different basic concepts, the two methods are not incompatible. Excentrical punching is briefly treated.

RESUME
Le mécanisme de rupture est étudié et diverses théories et règles de calcul pour le poinçonnement centrique sont évaluées. Fondée sur la théorie classique de plasticité, une solution analytique est présentée, décrivant avec fidélité le phénomène de poinçonnement. Des résultats d'essais sont comparés avec les prévisions de normes et de l'analyse plastique. Il y a lieu de constater que les deux méthodes ne sont pas incompatibles, bien que basées sur des notions complètement différentes. Le poinçonnement excentrique est enfin traité brièvement.

ZUSAMMENFASSUNG
Der Bruchmechanismus für zentrisches Durchstanzen wird untersucht, und verschiedene dafür entwickelte Theorien und Berechnungsverfahren werden besprochen. Eine auf die klassische Plastizitätstheorie sich stützende analytische Lösung wird angegeben, die das Durchstanzphänomen der Versuchserfahrung entsprechend beschreibt. Versuchsergebnisse werden mit rechnerischen Voraussagen von Bemessungsvorschriften und von plastischer Berechnung verglichen. Es wird gefolgert, dass die zwei Methoden nicht unvereinbar sind, obwohl die Grundlagen sehr verschieden sind. Exzentrisches Durchstanzen wird kurz behandelt.

1. INTRODUCTION

Punching shear failure may occur in concrete slabs - prestressed or conventionally reinforced - subjected to highly concentrated loads, e.g. impact loads or wheel loads on bridges, or at slender columns supporting flat slabs. The failure is located in a surface running through the slab from the loaded area to the opposite face (cf. Figure 1). The concrete body limited by the failure surface is simply punched out. This type of failure is not much impeded by the main reinforcement, and will therefore tend to reduce the ultimate load to a value below the flexural capacity of the slab.

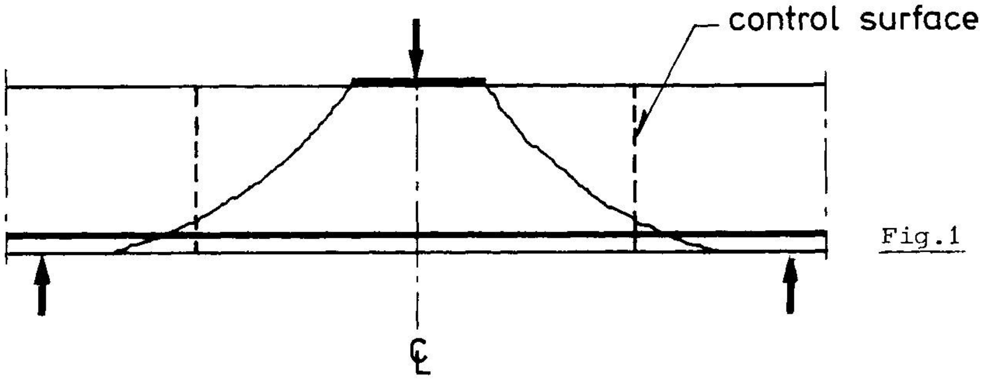


Fig.1 Punching shear failure

A decade of research has shown that the classical theory of plasticity may be used as an efficient tool in the analysis of shear problems in concrete structures, cf. NIELSEN & al. [78.3] and BRAESTRUP & al. [78.1].

In the present paper, we shall briefly review some design rules and theories for punching shear, and we shall present a theoretical approach based upon the theory of plasticity. The design rules and the predictions of plastic analysis are compared with some test results reported in the literature.

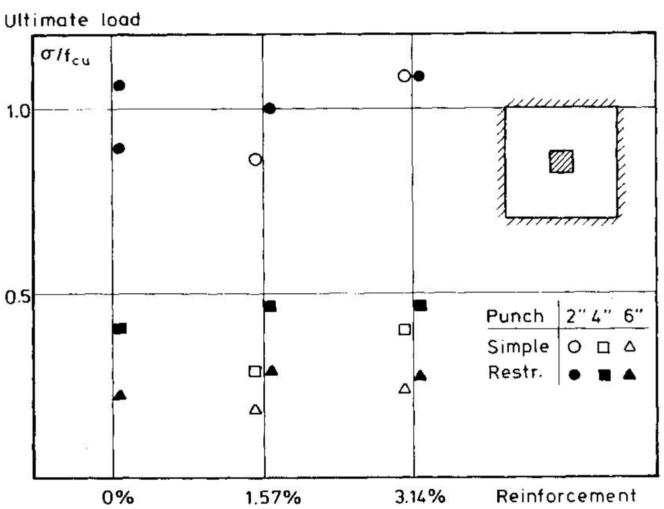


Fig.2 Punch load vs. main reinforcement. Simple and restrained slabs tested by TAYLOR & HAYES [65.2]

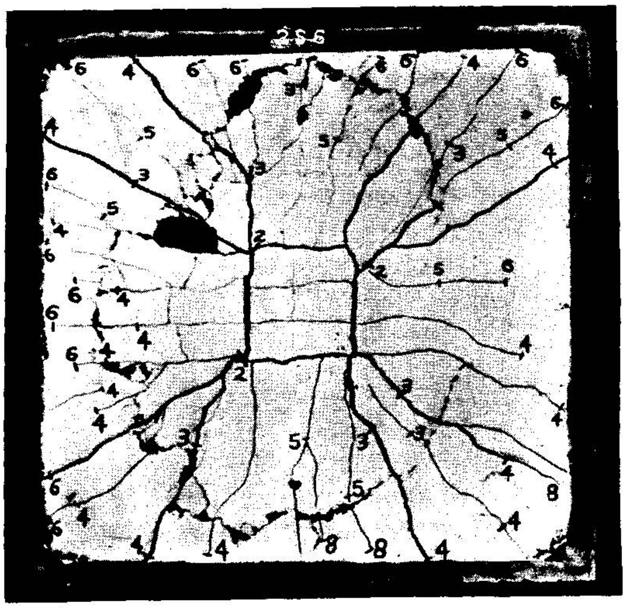


Fig.3 Failure of lightly reinforced simple slab with 6" punch. (From TAYLOR & HAYES [65.2])

2. THE MECHANISM OF FAILURE

The failure of slabs subjected to concentrated loading is very dependent upon the support conditions, especially the degree of restraint against in-plane edge movements. Thus the question of punching shear can hardly be separated from that of compressive membrane action (dome effect). This point is borne out nicely by a test series carried out by TAYLOR & HAYES [65.2].

They tested three series of square slabs, centrally loaded by square punches of varying size. The flexural reinforcement of the series corresponded to 0%, 1.57%, and 3.14%, respectively. The slabs were either simply supported or laterally restrained by a heavy welded steel frame. The unreinforced specimens, however, were tested in the restrained condition only.

The results corresponding to punches of sizes 2", 4", and 6" are shown on Figure 2. We have plotted the applied ultimate pressure σ, rendered nondimensional through division by the cube strength f_{cu}, against the percentage of reinforcement. It appears from the figure that not only are the strengths of the restrained specimens generally higher, they are also virtually independent of the amount of flexural reinforcement, which is not the case for the simply supported slabs.

In a real slab subjected to punching at an interior point, lateral movements will be restrained by the surrounding structure. Unfortunately, at most punching tests, care has not been taken to ensure similar conditions. Consequently, the ultimate load P_{test} may be expected to be approximately equal to the flexural capacity P_{flex}. Indeed, TAYLOR & HAYES [65.2] report that their unrestrained slabs with weak reinforcement were close to flexural failure.

The strength in flexure may be estimated by yield line theory, as done by GESUND & KAUSHIK [70.1]. They calculated the ratio P_{flex}/P_{test} for 106 alleged punching tests and found an average of 1.015 with a standard deviation of 0.248. Later, GESUND & DIKSHIT [71.3] developed punching shear formulas based upon yield line theory. The applicability of yield line theory was questioned by CLYDE & CARMICHAEL [74.5], who introduced considerations of the moment field (lower bound approach). The latter authors criticized the conventional test procedures, as did also CHRISWELL & HAWKINS [74.4].

The fact that so many flexural failures have been regarded as punching shear is probably due to the deceiving aspect of the collapse mode. However, one lesson to be learned from plastic analysis is that the actual appearance of the failure is not important for the strength. In fact, the ultimate load can often be predicted quite as well or even better by a completely different failure mechanism.

For weakly reinforced unrestrained slabs, the failure is accompanied by radial cracks and yielding of the main reinforcement, cf. Figure 3. The crushing of the concrete around the load and the spalling at the opposite face may be explained as secondary phenomena related to the rotational capacity in connection with the flexural failure mechanism. Consequently, failures involving yielding of the main reinforcement shall not be regarded as punching.

Heavily reinforced thin slabs may collapse before yielding of the reinforcement by a mode involving crushing of the concrete without the formation of a punching cone. Such slabs might be treated as overreinforced in flexure, and again the failure is related to lack of rotational capacity.

As a consequence, we shall consider as punching shear failures, only collapse modes characterized by the punching out of a concrete body in the direction of the loading, the remainder of the slab remaining comparatively rigid.

3. DESIGN RULES AND THEORIES

Most attempts to analyse the punching shear resistance of slabs are based upon the procedure as follows:
First a nominal shear stress is calculated, dividing the ultimate load P by the area of a socalled control surface around the loaded area, cf. Figure 1. The control surface is a cylinder, the section being a certain critical perimeter of length p, and the height being defined as either the total slab depth h, the effective depth d, or the internal moment lever arm z. The shear stress τ is then compared with a strength parameter for the concrete, usually some tensile strength measure.

The method sketched above was introduced by TALBOT [13.1] early in the century. TALBOT tested square footings, centrally loaded by square colomns of size a. As critical perimeter he used a square with the side length $a+2d$, and the depth of the control surface was taken as z. TALBOT concluded from the tests that the ultimate shear stress $\tau = P/4(a+2d)z$ was of the same order of magnitude as that of simple beams without shear reinforcement.

TALBOT's method forms the basis of most building code regulations concerning punching shear. In Section 7, we shall briefly review some examples, and compare the predictions with test results.

The code rules differ considerably in the definition of the control surface and in the choice of concrete strength measure. Some of the codes have modified the formula by introducing empirical factors depending upon the slab depth and the amount of flexural reinforcement. Many such empirical design rules have been formulated, and skall not be discussed here. References, reviews, and comparisons may be found in the reports from the ACI-ASCE Committee 326 [62.1], the Comité Européen du Béton [66.2], and the ASCE-ACI Committee 426 [74.2], as well as in the report by MOE [61.1], the paper by BERNAERT [65.1], and the thesis by ZAGHLOOL [71.5]. Further contributions are contained in the ACI Shear Symposium Volume [74.1]. Some authors, e.g. AOKI & SEKI [71.2], take account of compressive membrane effects, cf. also [74.2]. Excentrical punching is treated in Section 9.

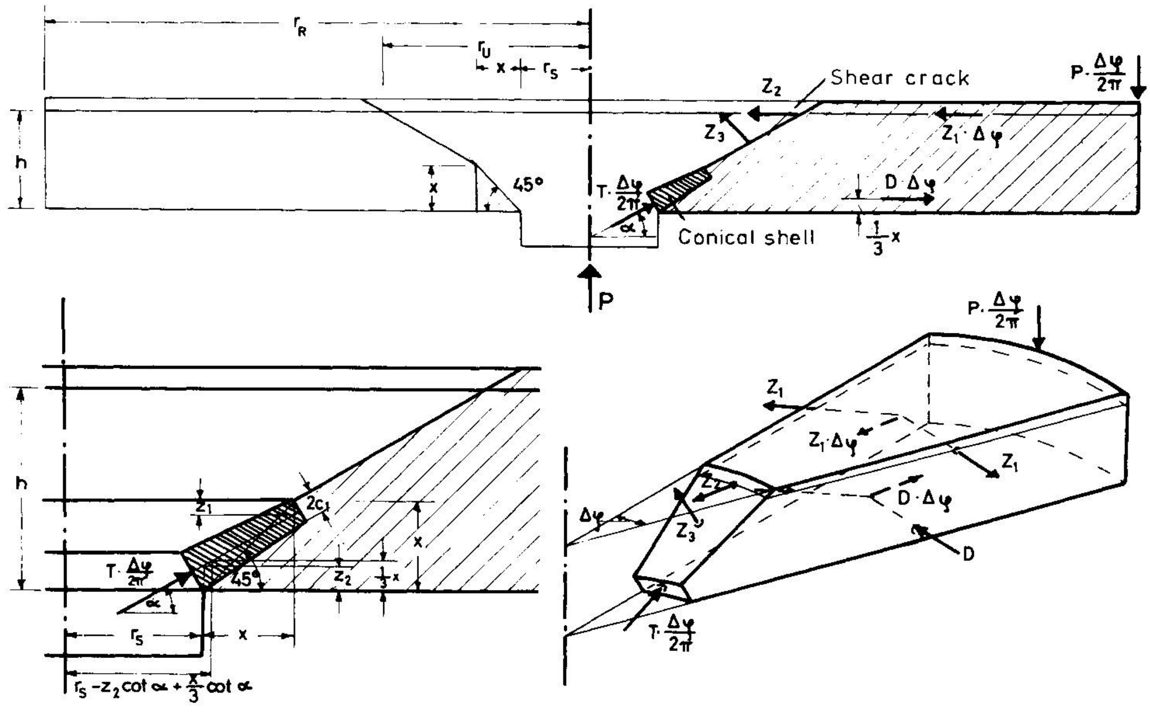


Fig.4 Mechanical model of KINNUNEN & NYLANDER. (From REIMANN [63.3])

A derivation of a punching strength formula based upon a rational mechanical model was attempted by KINNUNEN & NYLANDER [60.1]. They tested circular slabs, loaded at the free edge, and centrally supported on a circular column. The reinforcement was axisymmetrically disposed. Based upon test observations, an idealized model of the cracked state was proposed, cf. Figure 4. It consists of a rigid central truncated cone, confined by a shear crack, and segmental slab portions separated by yield lines. The segments are supposed to be carried by a compressed conical shell between the column and the root of the shear crack. Punching failure is assumed to occur when the tangential compressive concrete strain at the column face reaches a certain critical value.

The model was modified for orthogonal, two-way reinforcement by KINNUNEN [63.2], and extended to slabs with shear reinforcement by ANDERSSON [63.1]. The theory forms the basis for the Swedish code regulations [64.1], and has had a profound influence upon European code considerations as well. Thorough reviews with design examples are found in a CEB bulletin [66.2] and in the report by SCHAIDT & al. [70.3]. The analysis leads to an iterative procedure which tends to be quite complicated, and a simplified version has recently been given by NYLANDER & KINNUNEN [76.7].

The collapse mode considered by KINNUNEN & NYLANDER is basically that of flexural failure. Indeed, for the tests of [60.1], GESUND & KAUSHIK [70.1] found $P_{flex}/P_{test} = 1.075$ with a standard deviation of 0.159 . This is bound up with the fact that the test slabs were provided with little or no lateral restraint. The theory has been modified by HEWITT & BATCHELOR [75.2], who introduce a "restraint factor", depending upon the boundary conditions, to take account of dome effects. The authors report excellent agreement with test results.

The model of KINNUNEN & NYLANDER was critically reviewed by REIMANN [63.3], who proposed a theory based upon a similar failure mechanism, replacing the compressed conical shell by a yield hinge around the column. As critical parameter, REIMANN considered the concrete stress (rather than the strain) in the circumferential direction at the column face.

Another theoretical approach to the punching problem is due to LONG & BOND [67.1]. The bending moments in the vicinity of the column are found by elastic analysis. Punching is considered to take place when the concrete stress reaches a critical value, corresponding to a failure envelope for concrete under biaxial compression. When various "correction factors" are introduced, the predicted punching loads agree well with test results. MASTERSON & LONG [74.9] extended the theory to cover the effect of compressive membrane action. A simplified version of the method has been presented by LONG [75.4].

A common feature of the mechanical models reviewed above, is that they involve yielding of the reinforcement, combined with crushing of the concrete around the punch (column) Thus they are related to flexural collapse.

The fact that no attempt was made to analyse a proper shear failure mechanism is probably due to the lack of a simple constitutive description of the resistance of concrete to shearing deformation. The problem was attacked by BRAESTRUP & al. [76.2], using the modified Coulomb failure criterion, introduced by CHEN & DRUCKER [69.1] and described in the section below. The analysis is reviewed in Section 5, cf. also NIELSEN & al. [78.3].

MARTI & THUERLIMANN [77.4] (see also MARTI & al. [77.5]) also apply the Coulomb failure criterion, but without a tension cut-off, cf. Sections 4 and 5.

4. CONSTITUTIVE MODEL FOR CONCRETE

A plastic analysis of the punching problem may be based upon the failure mechanism sketched on Figure 1 and discussed in Section 2: A concrete plug is punched out in the direction of the load, the rest of the slab remaining rigid. The deformations are located in a rotationally symmetrical failure surface, in which the concrete is in a state of plane strain.

To calculate the ultimate punching load, we apply the work equation, i.e. equate the external work done by the punching force with the internal work dissipated in the failure surface. To determine the internal work, we introduce a constitutive model for the concrete as follows:

> The concrete is assumed to be a rigid, perfectly plastic material with the modified Coulomb failure criterion as yield condition and the deformations governed by the associated flow rule (normality condition).

This assumption complies with the requirements of limit analysis. This means that a load found by equating the rates of external and internal work is an upper bound solution for the ultimate punching force.

The modified Coulomb failure criterion consists of two conditions:

$$\tau = c - \sigma\tan\varphi \qquad (1a),$$

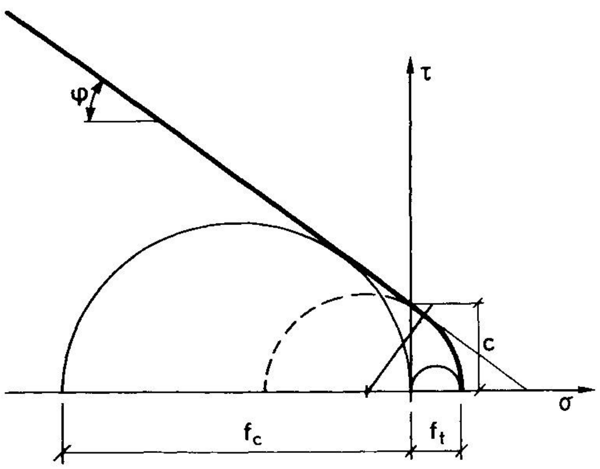
and $\sigma = f_t$ (1b)

Here τ and σ are shear and normal stress, respectively, on an arbitrary section in the material. c is the cohesion, φ is the angle of internal friction, and f_t is the uniaxial tensile strength. The uniaxial compressive strength f_c is related to the cohesion through the formula

$$f_c = 2c\sqrt{k} \qquad (2),$$

where the parameter k is determined by the angle of friction:

$$k = \frac{1+\sin\varphi}{1-\sin\varphi} \qquad (3)$$



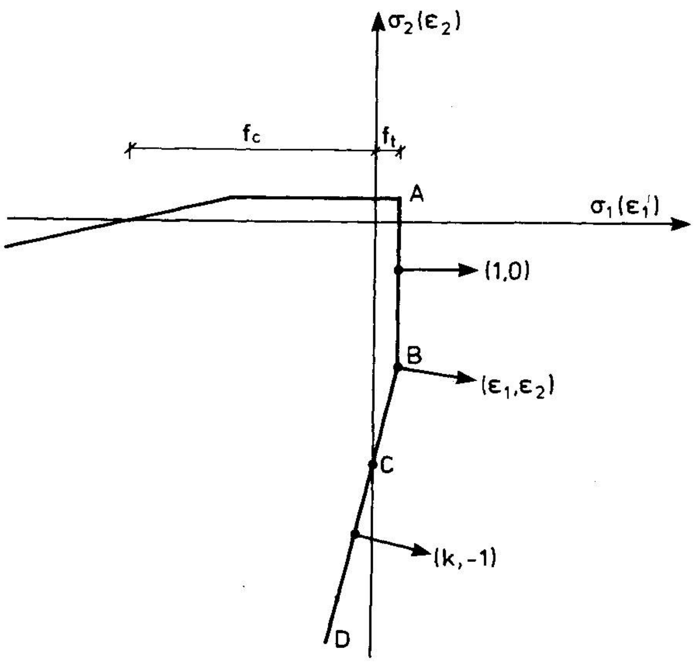


Fig.5 Modified Coulomb criterion
a) Failure envelope
b) Yield locus in plane strain

The failure criterion is sketched in a σ,τ-diagram on Figure 5a. The straight line, equation (1a), corresponds to Coulomb sliding failure. The circular cut-off, representing equation (1b), is a modification introduced to take account of the limited tensile strength of concrete.

The yield locus in the case of plane strain is shown on Figure 5b. The principal stresses are σ_1 and σ_2, and the principal strain rates are termed ε_1 and ε_2. The associated flow rule requires the vector $(\varepsilon_1,\varepsilon_2)$ to be an outwards directed normal to the yield locus at the corresponding stress point (σ_1,σ_2). At a corner, the vector $(\varepsilon_1,\varepsilon_2)$ is confined by the normals to the adjacent parts of the locus.

This simple constitutive model describes the strength properties of concrete, and the deformations at failure, by means of only three material parameters: The tensile strength f_t, the compressive strength f_c, and the angle of friction φ. The elastic deformations are neglected, and unlimited ductility at failure is assumed. The latter assumption is rather unrealistic, and necessary modifications are discussed in Section 8.

The unmodified Coulomb criterion, equation (1a), used by MARTI & THUERLIMANN [77.4], contains only two parameters, e.g. the compressive and the tensile strengths. Then the angle of friction is given by equation (3) with $k = f_c/f_t$. Thus the criterion leaves no possibility of varying f_t independently of f_c without affecting the value of φ. Further, with a reasonably small tensile strength, the angle of friction must be fairly great (cf. Figure 5a), which is not realistic in the presence of hydrostatic compression. For instance, a tensile strength equal to 10% of the compressive strength corresponds to an angle of friction $\varphi \approx 55^{\circ}$.

Consider a kinematical discontinuity (failure surface) in the concrete. Figure 6b shows the intersection of the failure surface with the plane determined by the surface normal n and the relative velocity vector v, inclined at the angle α to the surface. The discontinuity is an idealization of a narrow region of depth Δ with a high, homogeneous strain rate v/Δ (Figure 6a). The rate of internal work W_ℓ dissipated per unit area of the failure surface is:

$$W_\ell = \Delta(\varepsilon_1\sigma_1+\varepsilon_2\sigma_2)$$

The principal strain rates are found to be:

$$\varepsilon_1 = \frac{v}{2\Delta}(1+\sin\alpha) \quad \text{and} \quad \varepsilon_2 = -\frac{v}{2\Delta}(1-\sin\alpha)$$

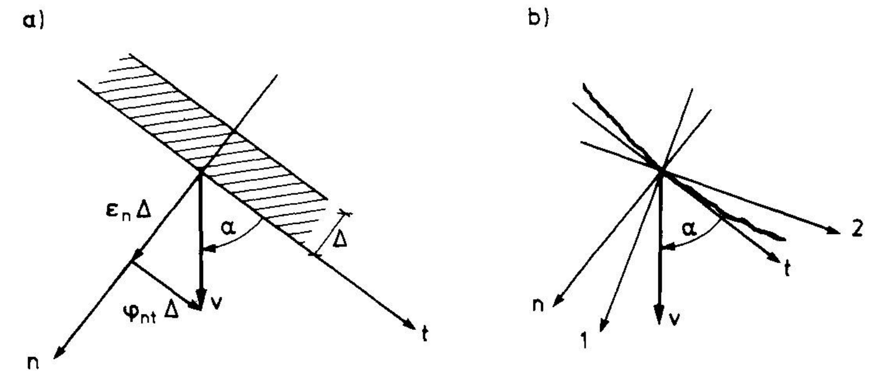


Fig.6 Failure surface in plain concrete
a) Narrow zone with high straining b) Kinematical discontinuity

According to the flow rule, the corresponding stresses are determined by point B on the yield locus, Figure 5b. Inserting, we find W_ℓ as a function of v and α:

$$W_\ell = \frac{1}{2} v f_c(\ell - m\sin\alpha) \quad \text{for} \quad \varphi \leq \alpha \leq \pi/2 \tag{4}$$

The parameters ℓ and m are defined as:

$$\ell = 1 - (k-1)f_t/f_c \quad \text{and} \quad m = 1 - (k+1)f_t/f_c \tag{5}$$,

where k is given by equation (3).

Equation (4) reduces to

$$W_\ell = v f_t \quad \text{for} \quad \alpha = \pi/2 \tag{6}$$

and $$W_\ell = \frac{1}{2} v f_c(1-\sin\varphi) \quad \text{for} \quad \alpha = \varphi \tag{7}$$

The assumed yield criterion and the associated flow rule does not allow the situation $\alpha < \varphi$. To describe such failure mechanisms,it will be necessary to introduce a more sophisticated constitutive model for concrete.

A more detailed derivation of equation (4) is given in reference [76.2] or [78.3].

5. PLASTIC ANALYSIS

Consider a concrete slab of depth h , supported along a circular perimeter with diameter D , and centrally loaded by a circular punch (or column) of diameter d_0 . The applied load is P , and the slab is supposed to be reinforced in such a manner that flexural failure is prevented. We assume that the reinforcement is able to resist axial forces only, i.e. dowel action is neglected. Then the reinforcement does not contribute to the internal work, and the work equation yields:

$$P v = \int W_\ell \, dA \tag{8}$$

Here W_ℓ is given by equation (4), and the integral is taken over the entire failure surface, sketched on Figure 7a. The generatrix is described by the function $r = r(x)$, and we have:

$$dA = 2\pi r \frac{dx}{\cos\alpha} \quad \text{and} \quad \tan\alpha = \frac{dr}{dx} = r'$$

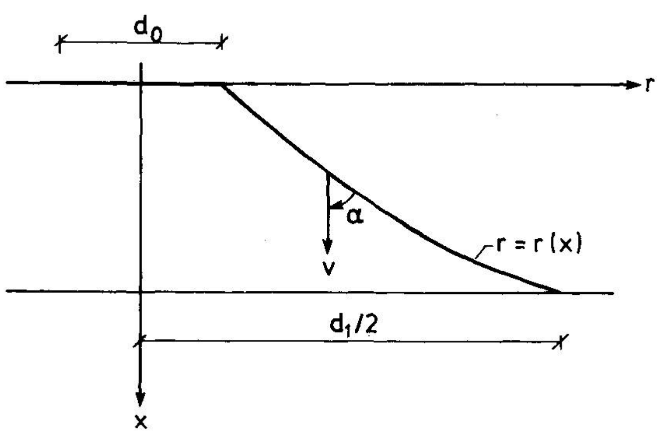


Fig.7a Failure surface

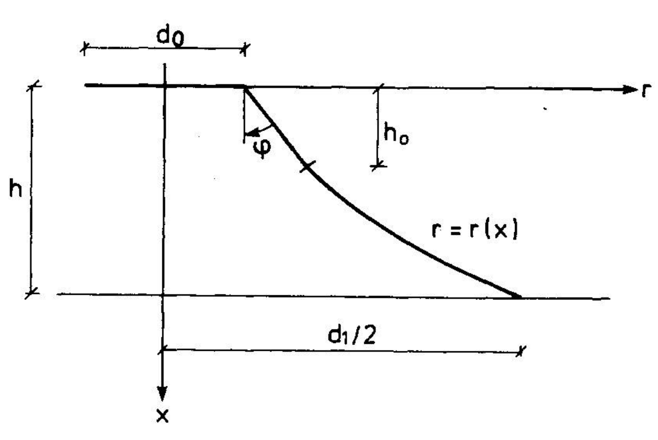


Fig.7b Optimal generatrix

Inserting into equation (8), we find the upper bound:

$$P = \pi f_c \int_0^h (\ell\sqrt{1+(r')^2} - mr')\, r\, dx \tag{9}$$

The lowest upper bound solution is found by variational calculus. The problem is to determine the function $r = r(x)$ which minimizes the functional (9), subject to the condition:

$$\alpha \geq \varphi \quad , \quad \text{i.e.} \quad r' \geq \tan\varphi \tag{10}$$

As shown by BRAESTRUP & al. [76.2], the solution is:

$$r = \frac{d_0}{2} + \tan\varphi \qquad \text{for } 0 \leq x \leq h_0 \tag{11a}$$

$$r = a\cosh\frac{x-h_0}{c} + b\sinh\frac{x-h_0}{c} \qquad \text{for } h_0 \leq x \leq h \tag{11b}$$

Here $c = \sqrt{a^2-b^2}$. The optimal generatrix is sketched on Figure 7b. It consists of a catenary curve (11b), joined by a straight line (11a) at the level $x = h_0$. The three constants $a, b,$ and h_0 are determined by the boundary conditions:

$$a = \frac{d_0}{2} + h_0\tan\varphi \tag{12}$$

$$\frac{b}{c} = \tan\varphi \tag{13}$$

$$\frac{d_1}{2} = a\cosh\frac{h-h_0}{c} + b\sinh\frac{h-h_0}{c} \tag{14}$$

Here $d_1 \leq D$ is the diameter of the intersection of the failure surface with the bottom face of the slab (Figures 7). This diameter is determined so as to give the lowest upper bound. For certain values of d_0, d_1, and h , the optimal solution has $h_0 = 0$, i.e. the failure surface contains no conical part. In this case, equation (13) reduces to the inequality:

$$\frac{b}{c} \geq \tan\varphi \tag{15}$$

and equations (12) and (14) determine the constants a and b .

The corresponding lowest upper bound for the ultimate load is (cf.[76.2]):

$$P = \frac{1}{2}\pi f_c[h_0(d_0+h_0\tan\varphi)\frac{1-\sin\varphi}{\cos\varphi} + \ell c(h-h_0) + \ell\left(\frac{d_1}{2}\sqrt{\left(\frac{d_1}{2}\right)^2-c^2}-ab\right) - m\left(\left(\frac{d_1}{2}\right)^2 - a^2\right)] \tag{16}$$

where d_1 is given by equation (14).

In order to satisfy the condition (10), we must require that $D \geq D_0$, where

$$D_0 = d_0 + 2h\tan\varphi \tag{17}$$

If we choose $D = D_0$, the failure surface degenerates to a truncated cone, equation (11a). Then we have $\alpha = \varphi$ at all points, which means that the failure takes place as pure sliding (stress regime BD on Figure 5b). In this case, the failure load is independent of the tensile strength. Indeed, with $h_0 = h$, equation (16) yields:

$$P = \frac{1}{2}\pi f_c\, h(d_0+h\tan\varphi)\frac{1-\sin\varphi}{\cos\varphi} \tag{18}$$

Reference [76.2] describes an iterative procedure which determines the optimal upper bound solution for given geometrical quantities h, d_0, and D, and for given material properties f_t, f_c, and φ. The result is not very dependent upon the angle of friction, and the conventional value $\varphi = 37^\circ$ (corresponding to $\tan\varphi = 0.75$ and $k = 4$) is used throughout. The solution, however, is very sensitive to the ratio f_t/f_c. For $f_t = 0$, the lowest upper bound decreases with increasing d_1, which means that the optimal failure surface will extend all the way to the support. If we introduce a non-zero tensile strength, then the upper bound will be a minimum for a finite value of d_1. Figure 8 shows examples of optimal generatrices corresponding to various relative punch diameters d_0/h, plotted for different values of f_t/f_c. The support diameter D is chosen sufficiently great so as to not affect the solution. It appears that even a very small tensile strength results in a considerable contraction of the failure surface around the punch.

As a non-dimensional load parameter, we may take the quantity τ/f_c, where

$$\tau = \frac{P}{\pi(d_0+2h)h} \tag{19}$$

is the average shear stress on a control cylinder of depth h, circumscribing the loaded area in the distance h (cf. Section 3).

For $f_t = 0$, the theoretical load parameter is a function of the support diameter D and the punch diameter d_0. When the support diameter increases towards infinity, the ultimate load approaches zero asymptotically.

For a finite tensile strength and a sufficiently great support diameter, the failure takes place within the support ($d_1 < D$). Then the load parameter is a function of the punch diameter only. In reference [78.3], the load parameter is plotted as a function of d_0 and D for zero and non-zero tensile strength (cf. also Figure 10 of the section below).

a) $f_t = f_c/400$

b) $f_t = f_c/250$

c) $f_t = f_c/100$

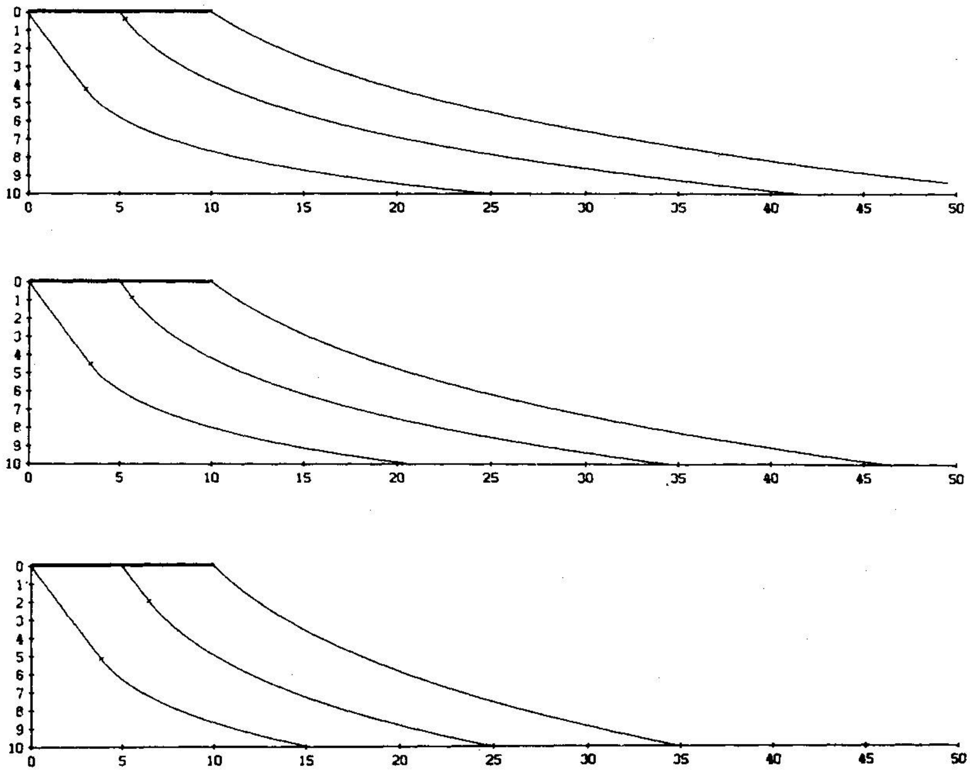


Fig.8 Optimal failure surface generatrices for $d_0/h = 0, 1$, and 2

The analysis shows that if we have some tensile strength and a support diameter which is not too close to D_0, given by equation (17), then the load parameter is almost constant. Thus a shear stress τ, defined by an expression similar to equation (19), seems to be an appropriate choice as a design variable.

MARTI & THUERLIMANN [77.4] consider a conical failure surface with the half angle φ. They find the upper bound solution:

$$P = \frac{1}{2}\pi f_c h\left(\frac{d_0}{\sqrt{k}} + \frac{1}{2}h\left(1 - \frac{1}{k}\right)\right)$$

Inserting equation (3), this is seen to be equivalent with equation (18).

In reference [78.2], conical failure surfaces with half angles $\alpha > \varphi$ were analysed. The solution was improved by adding a truncated cone with half angle φ, in such a way that the generatrix becomes a broken line. Still, a substantial improvement is obtained with the optimal failure surface derived above (cf.[78.2]).

In the plastic analysis described in this section, it has been assumed that the punching load was balanced by an annular reaction only. At a column supporting a flat slab, the punching force is due to loads on the slab. A distributed counterpressure can easily be taken into account in the analysis, see [76.2]. The effect is very similar to that of a tensile strength of the same magnitude, cf. Figure 8.

The presence of a uniformly distributed shear reinforcement would have the same effect as a counterpressure. However, generally any shear reinforcement will be concentrated in one or several rings around the punch. Then it will have no influence upon the shape of the generatrix, unless a more dangerous failure surface can be found which does not activate all or part of the reinforcement. The yield force of the active shear reinforcement will simply have to be added to the punching load, equation (16).

6. EXPERIMENTAL VERIFICATION

The plastic analysis developed in the preceding section seems to offer a satisfactory description of punching failure, whether this is achieved by actual punching of slabs, or by pulling out of a disc imbedded in a concrete block. Similar phenomena are also observed at the popouts produced by internal pressure near a concrete surface, e.g. due to alkali-aggregate reactions, cf. BACHE & ISEN [68.1].

A striking feature of such failures is the extension of the failure surface, and the thin, even razor-sharp edge of the punched-out body. This is most easily appreciated at pull-out tests, where the failure surface is not disturbed by the presence of reinforcement. Figure 9a shows a failure piece produced by HESS [75.1]. The failure was obtained without any annular counterpressure, tension being applied simultaneously to two bolts imbedded in opposite faces of the specimen.

A test of this kind highlights the influence of the tensile strength. For $f_t = 0$, the theory predicts a splitting failure at the level of the imbedded disc, at an applied force equal to zero. This is obviously at variance with experience. However, as shown in reference [76.2], a tensile strength equal to only 0.25% of the compressive strength is sufficient to ensure realistic failure surfaces. This extremely low value indicates that the effectiveness of the tensile strength is very small (cf. Section 8).

Figure 9b shows the generatrix corresponding to $f_t = f_c/400$ and the same relative punch diameter ($d_0/h = 0.72$) as for the specimen of Figure 9a. The agreement between predicted and observed shape is excellent.

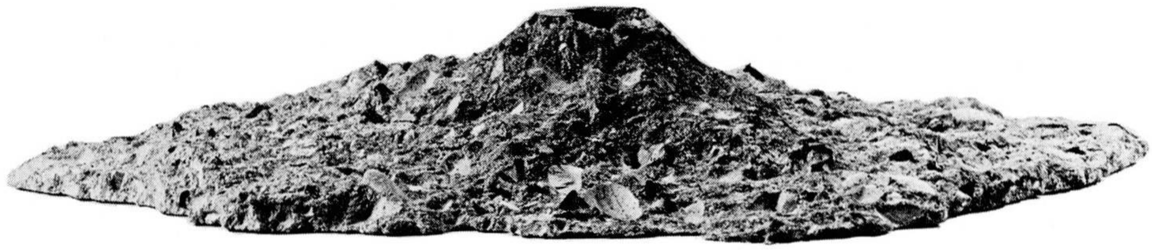
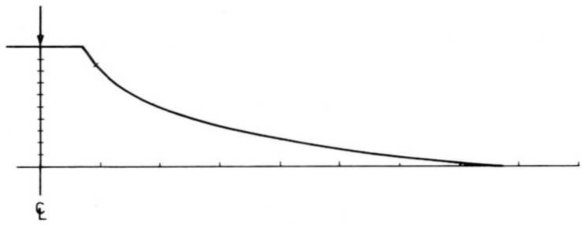


Fig.9 Pull-out test, $d_0/h = 0.72$
a) Failure piece (HESS [75.1])
b) Predicted shape, $f_t = f_c/400$

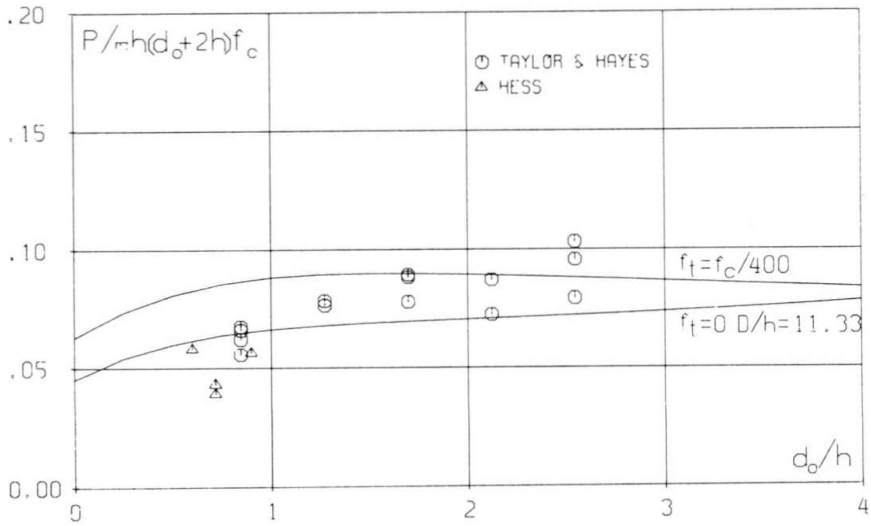


The fact that we have to introduce a diminutive tensile strength is of course not satisfactory. For the punching of slabs, where there is a well-defined maximum extension of the failure surface, we may (conservatively) neglect the tensile strength altogether. Provided the support diameter is not very great, this does not have any great influence upon the predicted ultimate load.

On Figure 10, we have calculated the load parameter for the restrained slabs tested by TAYLOR & HAYES [65.2], and plotted it against the relative punch diameter d_0/h (d_0 is taken as the diameter of the circle with the same perimeter as the square punch). For comparison are shown the theoretical curves corresponding to $f_t = f_c/400$ and to $f_t = 0$. In the former case, the tensile strength is sufficient to ensure that the failure takes place within the support. In the latter case, the support diameter D is put equal to the span of the square slabs. Figure 10 also shows the results of the pull-out tests of HESS [75.1]. For these points, the curve corresponding to $f_t = 0$ is without meaning, as the predicted load would be zero in that case.

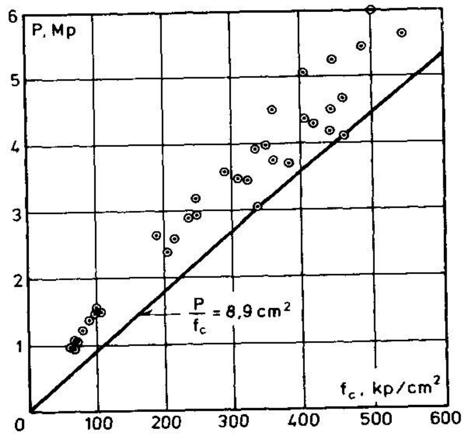
The plot shows that the load parameter does not vary much with the punch diameter, a fact which is reflected by the common design rules (cf. Section 3). What little variation there seems to be, is to some extent described by the plastic analysis.

Fig.10
Load parameter as function of punch diameter. Tests compared with theory.



If we choose the support diameter $D = D_0$, as given by equation (17), then the load is determined by equation (18), i.e. independent of the tensile strength. Consequently, it is possible to measure the compressive concrete strength by means of pull-out tests. This is the idea behind the Lok-test, developed by KIERKEGAARD-HANSEN [75.3]. The geometry of the test rig is very close to satisfying equation (17), but was designed empirically to give a good correlation between pull-out force and compressive strength. The success is demonstrated on Figure 11, showing the results of some tests carried out at the Structural Research Laboratory [74.3]. The solid line represents the relationship predicted by equation (18), if we take $\tan\varphi = 0.60$ to satisfy equation (17). As the angle of friction for concrete is slightly higher, the formula underestimates the pull-out strength somewhat (cf. JENSEN & BRAESTRUP [76.5].

Fig.11
Results of pull-out tests (Lok-strengths) compared with concrete cylinder strengths (References [74.3] and [76.5])



The applicability of the Lok-test to concrete quality control has been confirmed by field investigations, see LEKSOE & JENSEN [77.3].

The plastic analysis of the preceding section can also be applied to the radial punching of circular cylinders. A preliminary investigation of this problem indicates excellent agreement between the predicted and experimental loads, cf. HESS & al. [78.2].

7. PREDICTIONS BASED ON BUILDING CODES

Below we shall briefly review four typical codes of practice for punching design. They are all based upon the notion of a control surface (cf. Section 3).

CEB-FIP Model Code

The Comité Euro-International du Béton and the Féderation Internationale de la Précontrainte recently completed a Model Code [78.4] for reinforced and prestressed concrete structures. The design shear stress is calculated as:

$$\tau = \frac{P}{\kappa(1+50\rho)pd}$$

The critical perimeter p is defined as the length of the shortest, convex curve which nowhere is closer than 0.5 d to the loaded area. The depth factor $\kappa \geq 1$ is calculated as $\kappa = 1.5 - d$, d being inserted in meters. The reinforcement factor $1+50\rho$ is determined by inserting $\rho \leq 0.008$ as the mean proportional of the reinforcement ratios in the two orthogonal reinforcement directions. The 1976 draft [76.3] for the Model Code contained the same formula, but the upper limit for the benificial influence of the reinforcement was considerably higher.

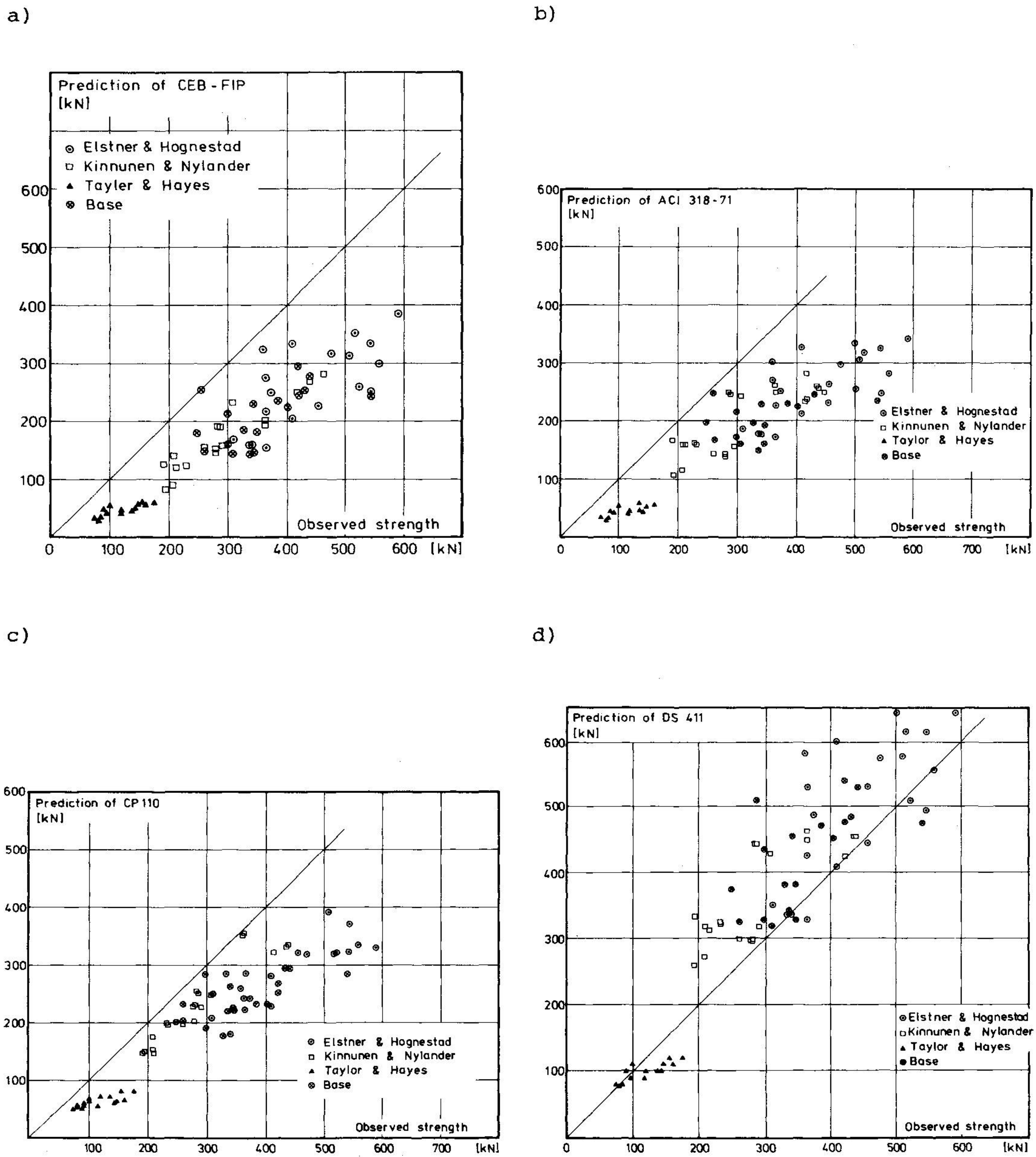


Fig.12
a) European Model Code
b) American building code
c) British code of practice
d) Danish code of practice

Punching test results compared with building code predictions

The shear stress τ is required to be inferior to $1.6\ \tau_{Rd}$, τ_{Rd} being the design concrete shear strength, tabulated as a function of the characteristic compressive strength f_{ck} (proportional to $f_{ck}^{2/3}$).

ACI 318-71

The American building code ACI 318-71 [71.1] puts:

$$\tau = \frac{P}{0.85\ pd}$$

where p is the minimum perimeter which approaches no closer than $0.5\ d$ to the loaded area. Thus obviously the critical perimeter must have rounded corners, like in the case of CP 110 (below) and the CEB-FIP Model Code (above). Nevertheless, the Commentary on the building code shows critical perimeters with sharp corners, in the manner of DS 411 (below). The comparison calculations (cf. below) are carried out using the minimum perimeter with rounded corners.

The shear stress τ must be inferior to the shear strength of the concrete, which is calculated as a function of the compressive strength f_c (proportional to $\sqrt{f_c}$).

CP 110

The British code of practice CP 110 [72.1] has

$$\tau = \frac{P}{\xi_s pd}$$

Here p is the smallest perimeter which nowhere is closer than $1.5\ h$ to the loaded area. The factor $\xi_s \geq 1$ depends upon the slab depth, according to a table in the code. The shear stress τ is required to be inferior to the concrete shear strength, which is tabulated in the code as a function of the compressive strength and of the ratio of reinforcement.

DS 411

The Danish building code DS 411 [76.4] puts

$$\tau = \frac{P}{ph} ,$$

where p is the perimeter of a figure similar to the loaded area in the distance d . The shear stress τ must be inferior to the tensile concrete strength which is tabulated as a function of the compressive strength f_c (proportional to $\sqrt{f_c}$).

As mentioned in Section 2, many tests reported in the literature as punching may just as well be described as due to flexure. HESS [77.1] has made a critical assessment of a great number of tests in order to exclude all the flexural failures. On Figures 12, some of the remaining results are compared with the strength given by the building codes described above.

The plots comprise some typical test series, viz.: ELSTNER & HOGNESTAD [56.1], KINNUNEN & NYLANDER [60.1], TAYLOR & HAYES [65.2], and BASE [66.2]. The latter series is reported on page 83 of the CEB Bulletin. In the analysis, average strengths are used, rather than characteristic values. Cube strengths f_{cu} are converted to cylinder strengths f_c by the formula $f_c = 0.8\ f_{cu}$.

The figures show that DS 411 slightly overestimates the load-carrying capacity, whereas the other codes are rather conservative, especially ACI 318-71 and the CEB-FIP Model Code. The latter represents a change from the draft [76.3], which was more liberal, cf. HESS & al. [78.2].

8. PREDICTIONS BASED ON PLASTIC ANALYSIS

The upper bound solution derived in Section 5 is based upon the simplified constitutive model introduced in Section 4, which assumes unlimited ductility of the concrete. In reality, however, concrete is not a perfectly plastic material. Particularly in tension, the behaviour is quite brittle. When applying plasticity to concrete, it is therefore prudent to neglect the tensile strength. As explained in Section 6, this leads to unrealistic results for punching (pull-out) without a support. The tensile strength which is necessary to explain the observed phenoma is very small indeed, of the order of $f_t = f_c/400$. This value is by no means indicative of the true tensile concrete strength, which is approximately 10% of the compressive strength. This shows that what tensile strength the concrete may possess, it is very little effective, due to the brittleness and possibly a "zipper" effect at failure. Consequently, we shall as a rule take the tensile concrete strength to be zero.

Also in compression, the ductility of concrete is quite limited, and we even have a falling branch on the stress-strain curve. Hence the redistribution of stresses which may be necessary to obtain the ultimate load predicted by plastic analysis can only take place at the expense of losing strength. This observation suggests that we might take account of the lack of ductility simply by reducing the concrete strength measure. Consequently, we replace the uniaxial compressive concrete strength by the effective strength f_c^*, where:

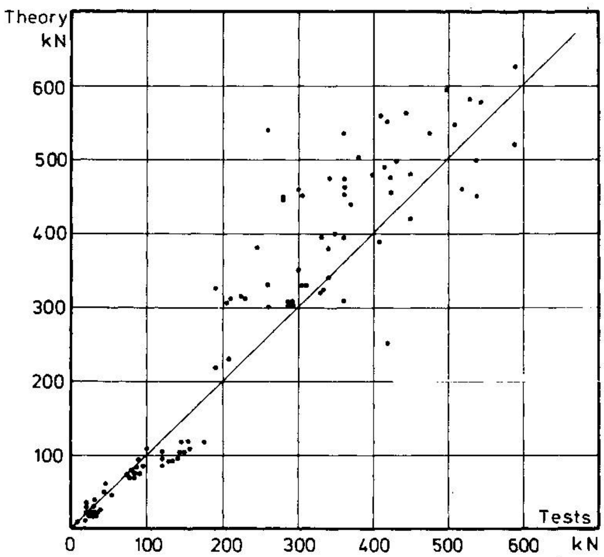
$$f_c^* = \nu\, f_c \tag{20}$$

Here f_c is the conventional strength, measured e.g. by the standard cylinder test, and ν is an empirical effectiveness factor describing the ductility of the concrete. The effectiveness factor is evaluated by comparison with experimental evidence.

In addition to expressing the concrete ductility, the effectiveness factor will have to describe all effects not explicitly accounted for in the theory, e.g. the influence of the neglected elastic deformations.

As seen from equation (16), the theoretical ultimate load is proportional to the concrete strength f_c. The value of ν for a given test can therefore be calculated as the ratio between the observed and the predicted strengths. The result will of course depend upon the amount of tensile strength assumed in the analysis.

Fig.13
Predictions based on plastic analysis compared with punching test results of
ELSTNER & HOGNESTAD [56.1]
BASE [66.2]
KINNUNEN & NYLANDER [60.1]
TAYLOR & HAYES [65.2]
DRAGOSAVIC & van den BEUKEL [74.6]
KAERN & JENSEN [76.6]
(References [77.1] and [78.2])



HESS [77.1] (cf. also HESS & al. [78.2]) analysed 101 punching tests carried out by ELSTNER & HOGNESTAD [56.1], BASE [66.2], KINNUNEN & NYLANDER [60.1], TAYLOR & HAYES [65.2], DRAGOSAVIC & van den BEUKEL [74.6], and KAERN & JENSEN [76.6]. Assuming $f_t = 0$ he found an average value of $\nu = 0.86$ with a coefficient of variation of 28%. For $f_t = f_c/400$, the average is $\nu = 0.69$, the coefficient of variation again being 28%.

BRAESTRUP & al. [76.2] analysed 54 tests, mainly pull-out tests reported by KIERKEGAARD-HANSEN [75.3]. Assuming $f_t = f_c/400$, the average was found to be $\nu = 0.83$ with a coefficient of variation of 16%. Thus it seems that in pull-out tests, the concrete is more effective, probably due to the greater stiffness of the specimen.

There appears to be a significant variation of ν with the concrete strength level: the stronger the concrete, the smaller the effectiveness factor. This trend is to be expected, since ν is principally a measure of ductility. The variation may be described empirically by assuming f_c^* to be proportional to $\sqrt{f_c}$. Interestingly, the same empirical relationship is often used between the tensile and compressive concrete strengths (cf. Section 7).

For all the 101 test results taken together, the best agreement is obtained with the formula $\nu = 4.22/\sqrt{f_c}$, where f_c is measured in MPa. Introducing this expression, the variance for the effectiveness factor is reduced by 12%, cf. HESS & al. [78.2]. For many of the individual test series, the variance is only half as great, indicating that much of the remaining scatter is due to difficulties in comparing different concrete strength measures.

For some of the experimental investigations, the concrete quality is given by the cube strength f_{cu}, and the results are analysed putting $f_c = 0.8\ f_{cu}$. Still, the values of f_c cover substantial variations in test procedures, regarding size of specimen, conditions of curing, and rate of loading.

On Figure 13, the predicted strengths of all the 101 test slabs are plotted against the values actually obtained. The analysis is carried out assuming $f_t = 0$ and $f_c^* = 4.22\sqrt{f_c}$. For tests with square punches, the punch diameter d_0 is taken as the diameter of the circle with the same perimeter. The support diameter for square slabs is put equal to the span length. The slab depth is inserted as the total depth h. In [78.2], similar plots are shown for the individual test series. Comparing Figure 13 with Figures 12, we note that the scatter is of the same order of magnitude as for the predictions based upon building code rules.

9. EXCENTRICAL PUNCHING

The heading actually covers two different problems: that of punching accompanied by moment transfer, and that of punching near an edge or corner of the slab.

Many attempts have been made to modify the empirical formulas based upon a control surface to take account of load excentricities. In addition to the reports cited in Section 3, reference is made to the papers by HERZOG [74.8], DRAGOSAVIC & van den BEUKEL [74.6], and van den BEUKEL [76.1]. The idea is to introduce additional shear stresses on the control surface, calculated assuming a linear variation in analogy with the normal stresses in a beam due to the bending moment. The validity of this approach is to some extent supported by analysis based upon elastic thin plate theory, cf. MAST [70.2]. Still, the method is purely formal, and any elaborate calculation of additional stresses is hardly justified, considering the rather arbitrary choice of the control surface.

Another semi-empirical method is the beam type analogy, cf. HAWKINS [74.7]. The slab sections framing into the column are idealized as beam sections, presumed capable of delivering bending moment, torque, and shear force at the control surface. A more rational approach is that of LONG [73.1], which is based upon elastic-plastic thin plate theory (cf. LONG & BOND [67.1]).

The plastic solution of Section 5 remains a valid upper bound also if the punching force at an interior slab point is accompanied by a bending moment. The question is whether or not the presence of the moment will significantly reduce the ultimate punching load. The moment is most effectively resisted by the main reinforcement, rather than by the concrete stresses in the failure surface. Therefore it would seem most reasonable to design the flexural reinforcement accordingly, and leave the punching design unaffected.

In contrast, at edges and corners the mechanism of failure is completely different. Due to the lack of symmetry, the deformation is no longer constrained to be perpendicular to the slab. Thus the main reinforcement will generally contribute to the internal work. Consequently, we would expect the resistance at edge and particularly corner columns to be governed by the flexural capacity. Indeed, ZAGHLOOL [71.5] found that punching at edge columns is a secondary phenomenon, developing after yielding of the reinforcement at the slab-column interface. Strength expressions depending primarily upon the amount of flexural steel were derived by beam type analogy. ANDERSSON [66.1] also found the shear stresses by beam type analogy and the theory of elasticity. The failure criterion was related to the theory of KINNUNEN & NYLANDER for interior columns (see Section 3), cf. also KINNUNEN [71.4].

Corner columns were investigated by INGVARSSON [77.2], who found that the shear failure was analogous to diagonal tension failure in beams. A similar failure mechanism was studied by ZAGHLOOL & de PAIVA [73.2].

Punching at edge and corner columns is the subject of a research project just started at the Structural Research Laboratory. The theoretical approach is based upon the constitutive model outlined in Section 4, and due account is taken of the flexural reinforcement, cf. above. Some tests will be carried out to complement the existing experimental evidence, which is rather meagre.

10. SUMMARY AND CONCLUSIONS

In Section 2, we define punching as a proper shear failure mechanism, distinguished from flexural collapse modes. The importance of lateral restraints is emphasized.

The commonly applied analyses of punching shear are reviewed in Section 3. Broadly speaking, two separate lines are followed. One considers the shear stress on a nominal control surface around the loaded area. This is a purely empirical method which has little relation to the actual punching phenomenon. The other approach is more rational, in the sense that it starts out from the collapse mode observed during tests. However, the considered failure mechanism is basically flexural.

Section 4 describes a constitutive model which may be used in the plastic analysis of a proper shear failure. The concrete is assumed to be a rigid, perfectly plastic material with the modified Coulomb failure criterion as yield condition and the associated flow rule.

An upper bound solution is derived in Section 5. The optimal shape of the failure surface is determined by variational calculus. The solution agrees well with experimental evidence, as shown in Section 6. The theory also explains the fact that with a suitable design of the test rig, it is possible to measure the compressive concrete strength by means of pull-out tests.

In Section 7 and 8, the results of punching tests are compared with the strength predictions of building codes and of plastic analysis. Although the latter is based upon concepts entirely different from those of the former, it confirms the applicability of the nominal shear stress on a control surface as a design variable. The main difference is that the compressive and not the tensile concrete strength is the governing material parameter. However, the effective concrete strength depends upon the cylinder strength in much the same way as does the tensile strength. In both cases, it is the ductility of the concrete which is the decisive factor. Whether this is expressed through an effective strength or through a tensile strength is to some extent a matter of taste.

Finally, Section 9 treats excentrical punching. It is suggested that moment transfer at internal columns be considered separately from the punching. On the other hand, edge and corner columns require a different approach, mainly because of the lack of lateral restraint. Indications are that plastic analysis may provide solutions to these problems as well.

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