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**On Dynamic Analysis of Thermally Cracked Concrete Structures**

Analyse dynamique de structures en béton présentant des fissures d'origine thermique

Dynamische Berechnung von thermisch belasteten Betonbauwerken

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**Summary**

The safety analysis of a concrete structure requires taking into account the crack distribution, in particular when thermal and dynamic loads are present. A considerable spreading of the design parameters may occur and consequently an increasing of the building cost. Two broad categories of reinforced concrete structures have been investigated in this respect: shear walls of buildings and cylindrical containment structures. A mathematical model for computing their lateral rigidities is commented.

**Résumé**

Pour une analyse de sécurité d'une structure en béton il faut tenir compte de répartition des fissures surtout lorsqu'il y a des charges thermiques et dynamiques. Une extension considérable des paramètres du projet peut avoir lieu et en conséquence une augmentation du coût de la construction. On a examiné en ce sens deux grandes catégories de structures en béton armé: des bâtiments à parois verticales et des structures à réservoirs cylindriques. Un modèle mathématique permettant de calculer les rigidités latérales est illustré.

**Zusammenfassung**

Die Sicherheitsberechnung eines Betonbauwerkes muss die Rissverteilung berücksichtigen, besonders wenn thermische und dynamische Belastungen dabei sind. Es kann daraus eine entsprechend erhöhte Anzahl von Entwurfsparametern resultieren, was die Kosten des Bauwerks beeinflusst. Zwei Haupttypen von Bauwerken aus Stahlbeton werden untersucht: Schubwände in Gebäuden und zylindrische Tanks. Ein mathematisches Modell, welches die laterale Steifigkeit prüft, wird kommentiert.

1. Premises

As cracks in reinforced concrete are more the rule than the exception, one has to think in terms of an anisotropic material when defining the stiffness matrix.

As to the flexural behaviour of beams and columns, such model can be supported by a large amount of both experimental and theoretical knowledge but this is not the case for a two-dimension structure, a part from the complexity of the involved calculus.

The need of reliable mathematical models for cracked concrete is typical for building shear walls, for containment structures and for chimneys. Their vertical sections carry only a limited amount of compression due to dead loads and so they are likely to be cracked by thermal or shrinkage loads. A limit case, finally is the one of precast large panel buildings, where in practice all the vertical members are shear walls and the construction joints, at the boundary of each panel, act as natural cracks.

Cracking could be avoided, but only through the adoption of a very dense mesh of reinforcing bars: for a 20 cm thick shear wall, a 12 mm bar each 10 cm both vertically and horizontally, for instances. This means an impressive consumption of steel (say about 200 kg/m<sup>3</sup>) for structures that, until now, have been built practically with no steel inside.

The fact is that shrinkage or thermal cracks do not mean neither the collapse of a structure nor the loss of serviceability of it. What is requested to the designer is simply to limit the width of the cracks with reference to the corrosion dangers or to the leakage specifications.

Thus when entering in the evaluation of the dynamic response of reinforced concrete structures, in most of the cases, one should think in terms of a cracked structure and should properly evaluate the loss of stiffness due to the cracks which obviously means an increased local deformability of the member under consideration.

To face the above mentioned situation one normally assumes suitable upper and lower bounds for the concrete rigidities and envelopes internal stresses related to either bound. The upper bound for rigidities is obtained by assuming uncracked concrete. The lower bound by minimizing the concrete stress transfer across the cracks. This procedure gives up to a spreading of the stress analysis results, and consequently to a possible increased building cost.

This procedure does involve not only sophisticated structures, like the containment of a nuclear reactor, but, implicitly, conventional structures too. Their design in fact is often made depending directly from the more severe situation relying on either the lower or the upper bound of concrete rigidities, which in general are far from the real rigidities. Examples of this spreading will be shown for the containment of a nuclear power plant, and for a building shear wall.

Both the ACI and CEB Codes [1,6] give suggestions on the Shear Strength of Reinforced Concrete beams taking care of the basic mechanisms of shear transfer that is:

- a) Shear transfer by concrete shear stress (on the uncracked parts of the member).
- b) Interface shear transfer (aggregate interlock through a crack).
- c) Dowel shear (dowelling forces in the bars crossing a crack).
- d) Arch action (as significantly possible in deep beams).
- e) Shear reinforcement (stirrups and bent bars).

Special rules are not given instead for two dimensional members. In any case reference is always made to the strength of the structural element but not to the deformability of it. That is if one has to build the stiffness matrix of a cracked two-dimensional structure is really in trouble.

On the subject an interesting document is nevertheless the ACI-ASCE Committee 426 Report n° 70-46 [16] as here many papers dealing with various aspects of shear strength and behavior are collected.

Most of the experimental work has been done on relatively small specimens and so the scale effect, going to full scale structure, must be evaluated. But, as we know, this is the only source of information on the stress-strain relation for cracked structures that we have at present.

## 2. Reinforced Concrete Containment Structures

Some of the leading engineering firms at the IV SMIRT [5,9,11,19] declared that to cope with the combination of thermal and earthquake loads, three separate dynamic assumptions are currently enveloped in the design of a containment structure:

- 1) undamaged wall;
- 2) vertically cracked wall;
- 3) both vertically and horizontally cracked wall.

Moreover, to cope with the combination of thermal load and internal pressure - generally included in the loading of a containment structure - the cracks are considered as involving the entire thickness of the wall.

The most rough way cracks can be included is by neglecting any stress transfer across the two surfaces of the crack but for the reinforcing crossing it. This means disregarding the mechanisms of "aggregate shear transfer" and that of "concrete tension stiffening". An overevaluation of the crack effects may occur, depending on the deepness of the crack within the wall. The kind of load also deserves its importance.

As an example, fig. 1 and 2 show some aspects of the dynamic analysis of the reactor building, at Caorso Power Plant. The primary containment wall has been modelled under two assumptions: undamaged wall and cracked wall. The cracks involve one half of the thickness, both vertically and horizontally. The comparison between normal modes shapes under these two assumptions is shown.

From these figures no meaningful difference is apparent between the two approaches, especially for the first mode of vibration, consisting of a nearly rigid rotation of the structure on the soil. Significant differences on the dynamic behaviour of the structure would be apparent only if higher modes of vibration could be meaningful, but this is not the case of seismic excitation.

On the other hand, when seismic internal actions are superimposed to pressure and thermal loads, the combined effects result fairly apart each from the other within the two approaches. Let us in fact regard the M-N interaction domain of the horizontal section at the foundation level - fig. 3 - where the representative points are shown, as related to the two opposite approaches. Not only the reinforcing steel percentage but also its distribution is largely affected by the crack hypothesis and, therefore, by the way it has been dealt with.

About the distribution of the reinforcing, note that the Caorso containment used both an orthogonal (hoop, vertical) and inclined (diagonal) reinforcing system. The inclined system was designed to resist the tangential shear. Only one horizontal earthquake component, 0.24 g peak ground acceleration, was combined with a vertical 0.16 g peak acceleration. The absolute summation of internal stresses due to the two components was assumed.

This kind of reinforcing was common to all U.S. containment designed approximately 10 years ago. In recent times among the major goals of improving structural design for containment structures there is either to verify the need for inclined reinforcing or to provide information to substantiate their elimination

### 3. Shear walls of a Building

#### 3.1. On the need of thinking in terms of cracked shear walls

Suppose to look at a shear wall 20 cm thick reinforced with Ø 10 mm each 20 cm vertical and horizontal corrugated bars near each face.

Let the characteristic strength  $f_{ck}$  of the concrete be 30 MPa and the one  $f_{yk}$  of the steel 440 MPa.

The steel to concrete ratio, longitudinally and transversally is:

$$\rho_l = \rho_t = \frac{A_s}{b \cdot d} = \frac{0,78 \times 5 \times 2}{20 \times 100} = 0,39\%$$

and the steel consumption (overlapping included) is about 70 kg/m<sup>3</sup>.

If cracks must not appear the design shear stress allowed by the CEB Code is:

$$\tau_{Rd} = 0,25 f_{ctd} = 0,25 \cdot 1,36 = 0,34 \text{ MPa}$$

for plane concrete and

$$\tau_{Rd}^* = (1 + 50 \rho_l) \tau_{Rd} = 1.195 \tau_{Rd} = 0,406 \text{ MPa}$$

when taking advantage of the dowelling effect of the existing longitudinal reinforcement. The improvement therefore is less than 20%.

The above evaluated design shear stress is rather low as the value 0,406 MPa can very easily be overcome through shrinkage and thermal effects acting together with modest transversal loads

Infact if the floors can in some way avoid the design thermal elongations, the decrease of temperature needed to crack the wall is only:

$$\Delta t = \frac{\sigma_{ctd}}{\alpha \cdot E} = \frac{1,36}{10^{-5} \cdot 32.000} = 4,25 \text{ } ^\circ\text{C}$$

and the corresponding crack width is

$$w = \alpha \Delta t l = 4,25 \cdot 10^{-5} l$$

that is, for a 6 m long wall

$$w = 0,255 \text{ mm}$$

Therefore both the shear weakness of the uncracked structure and the sensitivity to thermal and shrinkage effects ask the designer to think in terms of cracked shear walls when designing them.

If one thus thinks in terms of a cracked wall, the allowable design stress goes from 0,406 MPa to

$$\tau_{Rd3} = \beta_1 \cdot \rho_t \frac{f_{yk}}{s} + \beta_2 \tau_{Rd}$$

The coefficients  $\beta_1$  and  $\beta_2$  are a given function of  $\rho_t = 0.39\%$ :

$$\beta_1 = 0,57 \text{ and } \beta_2 = 2,04$$

Therefore in our case we get:

$$\tau_{Rd3} = 0,57 \frac{0,39}{100} \frac{440}{100} + 2,04 \cdot 0,34 = 0,85 + 0,69 = 1,54 \text{ MPa}$$

and the total design shear strength is now four times greater than the one of the uncracked wall.

The contribution of the vertical reinforcement appears to be of the same order of the one of concrete. It must be noticed, moreover, that, according to the CEB Code, the longitudinal reinforcement does not affect the result.

The crushing of the wall due to diagonal compression is surely avoided as the corresponding design shear stress  $\tau_{Rd2}$  is very high:

$$\tau_{Rd2} = 0,30 \frac{f_{ck}}{\gamma_c} = 0,30 \frac{30}{1,5} = 6 \text{ MPa}$$

### 3.2. Influence of cracks on the lateral stiffness

Not only lateral resistance, but also adequate lateral rigidity has to be proven for shear walls. The hypothesis of cracks weakening the wall's lateral rigidity has been not normally taken into consideration. If it were, it would significantly alter the structural organization of the building and would give rise to an overriding increase in the design stresses in the columns. In fact, when a given shear wall distribution against lateral forces is assumed in the design of the building, the dimensions of adjacent beams and columns are largely based on the presence of these shear walls. Usually these shear walls provide almost all the resistance to lateral shear; in this case for thermal load, current design practice is based on the assumption that, if present, it does not weaken the lateral rigidity, or, better, it must not damage the shear walls.

In this case thus the upper bound rigidity is assumed for the wall, leading to a conservative reinforcing steel distribution.

An opposite approach has been pursued for precast panel buildings in presence of vertical joints. Notice that when vertically cracked, a shear wall is likely to act as a monolithic cantilever if, along the vertical section, an adequate shear transfer can be accommodated, in spite of the presence of cracks. Such transfer can be contributed, in

the case of a vertical joint, by shear friction, and by the dowel action provided by horizontal reinforcement, if present.

Nevertheless, in the practice, this shear transfer is neglected unless a special joint design is performed. So that the two adjacent panels are modeled as independent cantilevers and the stiffness matrix is obtained through a lower bound approach. Fig. 4,5,6 and 7, on the other hand show that although if the shear transfer is associated with a finite slip between the two adjacent panels, nevertheless the structure behaves as a monolithic cantilever.

A shear stiffness reduction of the joint by a factor 0,1 is allowed without altering the lateral rigidity  $K$  - fig. 5 - by more than 10%.

Some experimental tests on this subject have been reported in [2].

#### 4. Incorporation of the Available Experiences in Mathematical Models

##### 4.1. Shear Transfer

Several researches have documented experimental studies on shear transfer across open cracks in precracked concrete [8,11,14,15]. These experiments confirm that combined dowel action and interface shear transfer is an efficient mechanism for shear transfer at a slightly open crack. Mean while, under cyclic loading, bond deterioration promotes relative displacement at the crack faces, causing aggregate interlock cracking and dowel cracking parallel to the longitudinal reinforcement: a loss of stiffness thus occur [15].

According to [19], the load - slip characteristic at the crack, including degradation during cyclic loading, are sufficiently known so that they can be embedded in a nonlinear analysis to predict the effect of deformations at the crack on the dynamic response.

As to authors' knowledge, at least one computer code, commercially available and of a general purpose, includes a "concrete model": Adina Code [3]. This is able to switch from the isotropic behaviour of uncracked concrete to the orthotropic behaviour of cracked element, and can take into account any kind of shear transfer, but its degradation during cyclic loading. Only a few special-purpose computer codes do this: for instances fig. 8 represents the shear stress vs. shear slip at crack which has been incorporated into the dynamic response analysis program [19].

Let now remind that the main parameter of the dynamic response obviously is not the maximum shear stress locally transmitted. Also the shear strain accumulated during half a cycle is no more of interest, because, in general, the motion is reversed during the next half cycle and so part of the strain may be recovered. The total shear strain cumulated across the crack at the end of the dynamic excitation is among the important parameters.

To this purpose in the authors' opinion at least two comments may be suggested, about the shear stress - shear slip curve.

- 1) Any idealized curve as that of fig. 8, is necessarily symmetric around both axes, while symmetry is more or less lacking in practice. Moreover, idealized dynamic loads are often symmetrized too, see for instances artificial time histories representing seismic excitations. Under these premises the net strain cumulated during a dynamic excitation may underestimate substantially the real strain, and may be even zero.
- 2) As to vertical cracking due to pressure or thermal loads in a cylindrical containment, the numerical approach shows cracks as straight lines, but in practice they are fairly irregularly shaped,

so that a slip surface does not appear. Any irregularity acts as the aggregate interlock, providing a further contribution to shear transfer.

In the first case, therefore, the random nature of the crack leads to a more severe strain than expected by analysis; in the second case it leads to a less severe stiffness loss than expected. In both cases, in conclusion, however simple be the structures to which such models apply, nevertheless the sign of the involved errors is quite unpredictable.

#### 4.2. Concrete Tension Stiffening

Consider fig. 9 where a concrete specimen is shown of length  $l$ . The stiffness  $F/\Delta l$  is not only contributed by the reinforcing bar, but also by the concrete itself, however cracked, provided that a suitable bond allows stress transfer from the steel to the surrounding concrete. Let call this contribution "concrete tension stiffening".

The existing practical proposals for dealing with this phenomenon are founded on tensile tests on bars surrounded by concrete, and checked by bending tests.

In particular, according to Beeby's tests on bars surrounded by concrete [4], the tension stiffening can be represented by an average stress distribution in the concrete, linearly distributed from a value of zero at the neutral axis and a value of  $10 \text{ kg/cm}^2$  at the centroid of tension steel. This distribution does not change sensibly by increasing the applied stresses on the cross section. Such a distribution is noticeably non linear versus the applied forces, and the average strain for steel surrounded by concrete can be reproduced to a given stress level by assuming a suitably reduced modulus of elasticity for concrete in tension, apart from the presence of reinforcing steel.

The tension stiffening is in general of a limited effect in the moment-rotation diagram for a beam. It is of noticeable importance in the problem here considered only when axial stiffness of a beam or a wall is concerned, mainly for underreinforced concrete.

When the crack width has to be evaluated sophisticated friction elements are required to represent the steel-concrete slip: see for instance [7]. This element connects two separate nodes occupying the same physical position: see fig. 10.

This second approach may be applied to analyse local situations of small extent: in fact the mesh size for truss elements representing steel and for plane elements representing concrete needs to be very refined otherwise the constant stress truss element cannot reproduce suitably the mechanism of stress transfer from steel to concrete, and so cannot reproduce adequately the structure stiffness. Typically the mesh dimension needs to be  $1/10 \div 1/5$  of the crack separation, i.e., of the order of 10 cm in a large number of cases.

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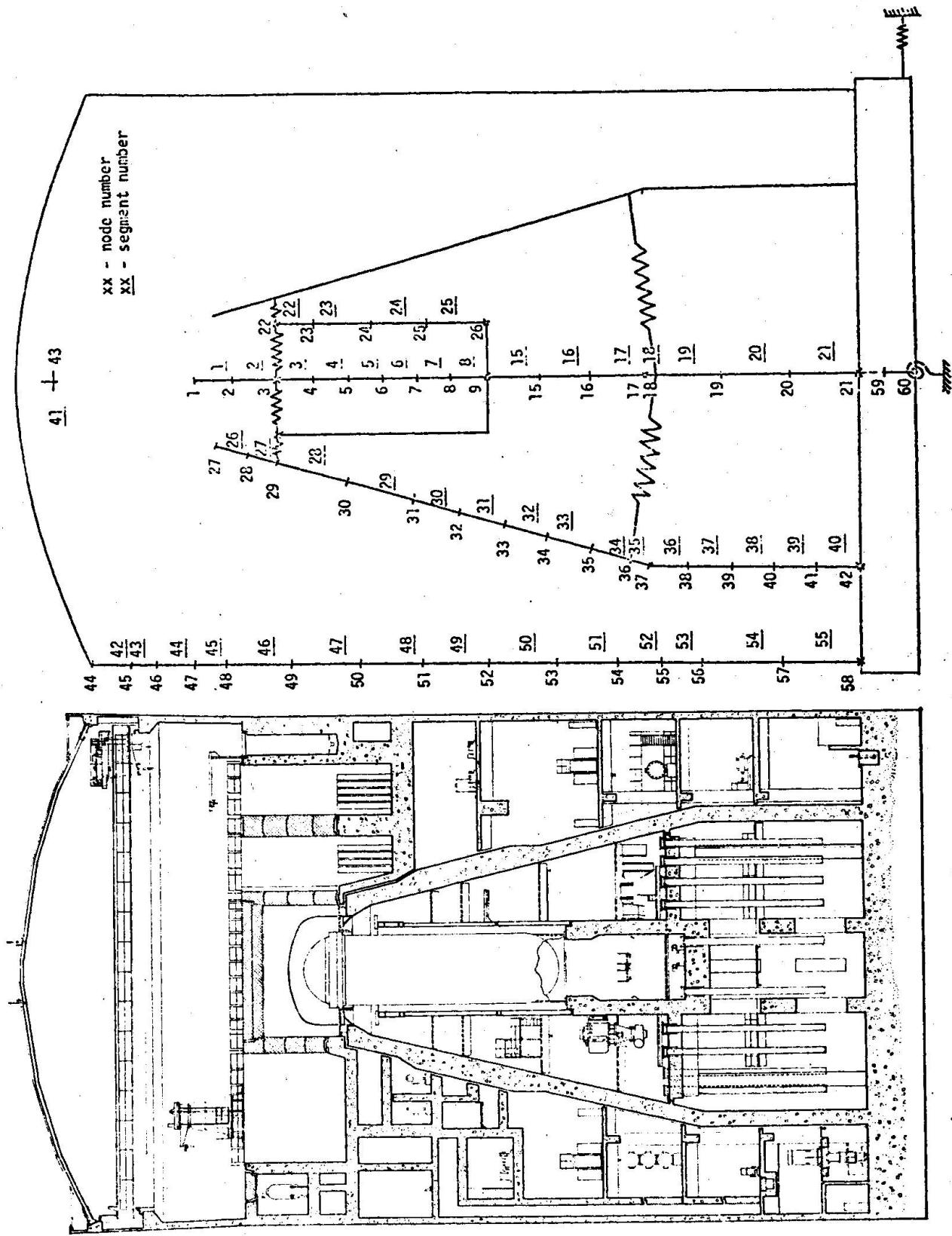
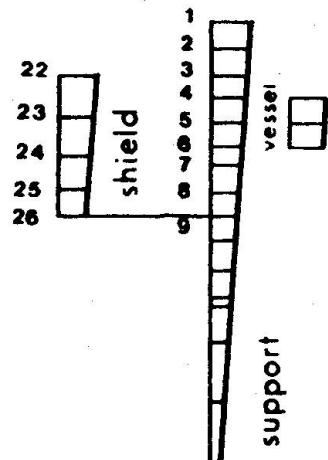
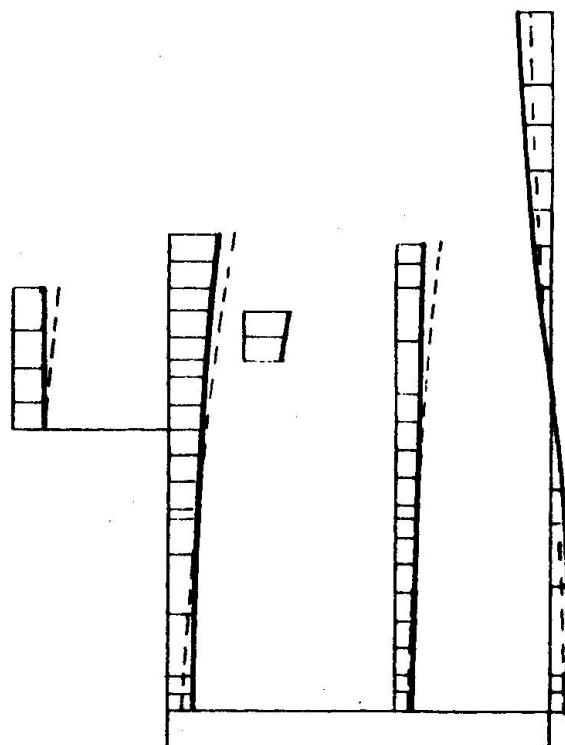


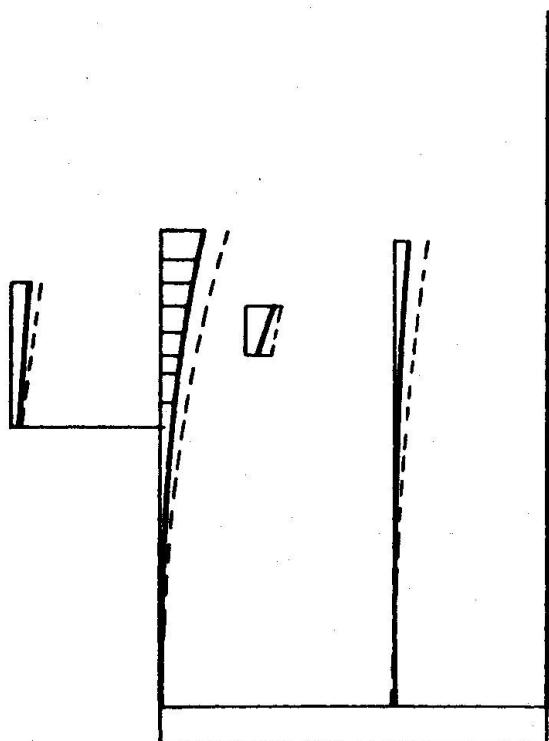
Fig. 1 Caorso reactor building: model for dynamic behaviour analysis.



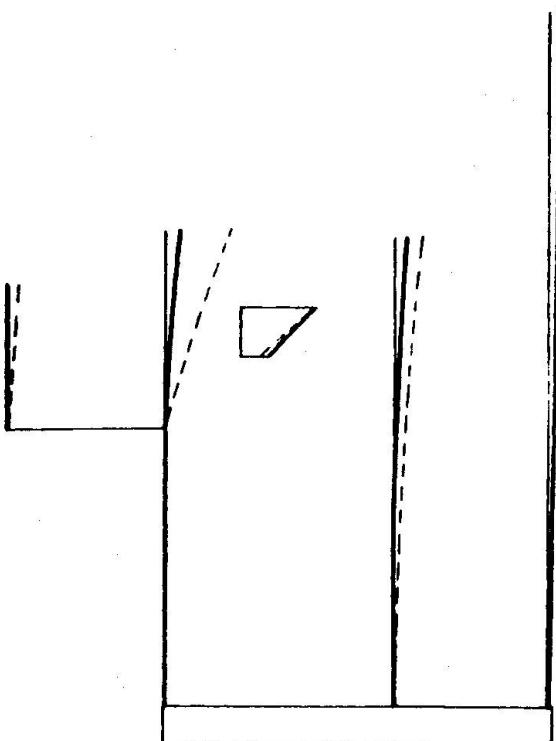
MODE 1



MODE 2

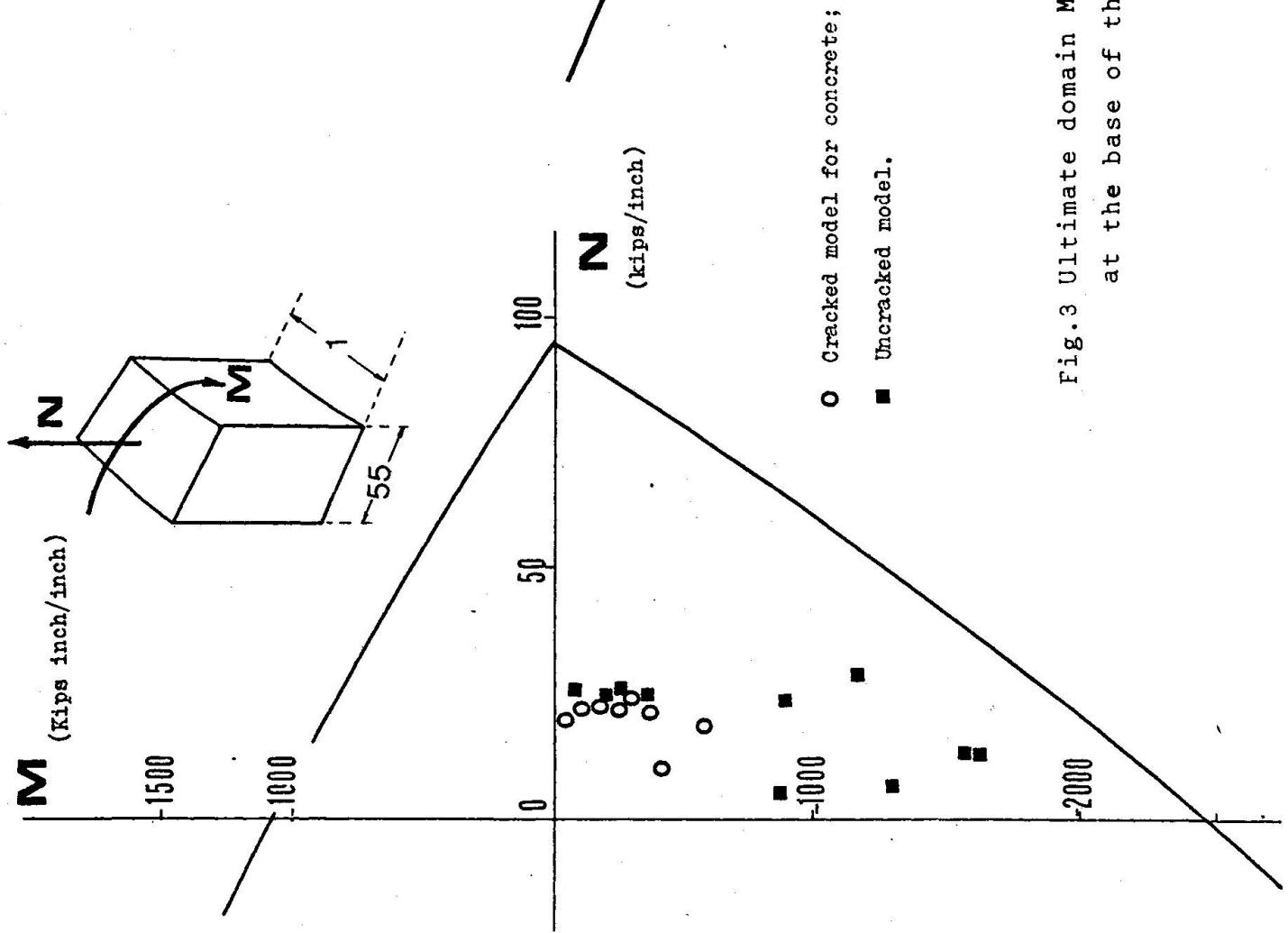
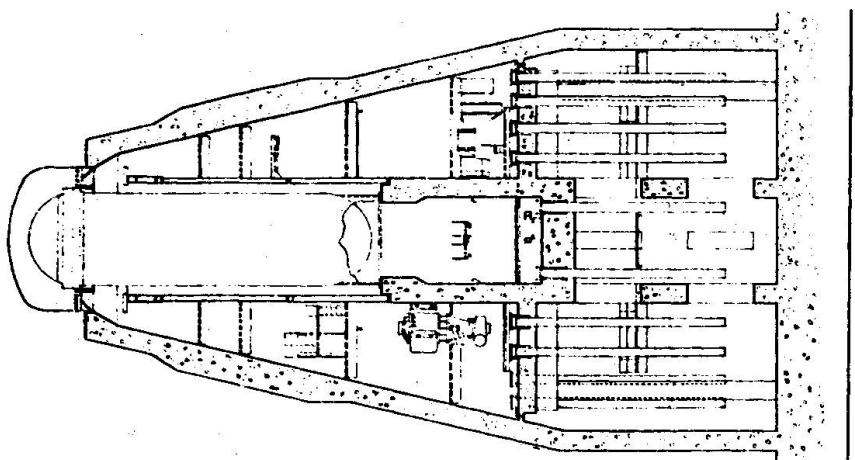


MODE 3



MODE 6

Fig. 2 Normal modes shapes of Caorso reactor building. Continuous lines refer to uncracked primary containment. Dashed lines to cracked primary containment.



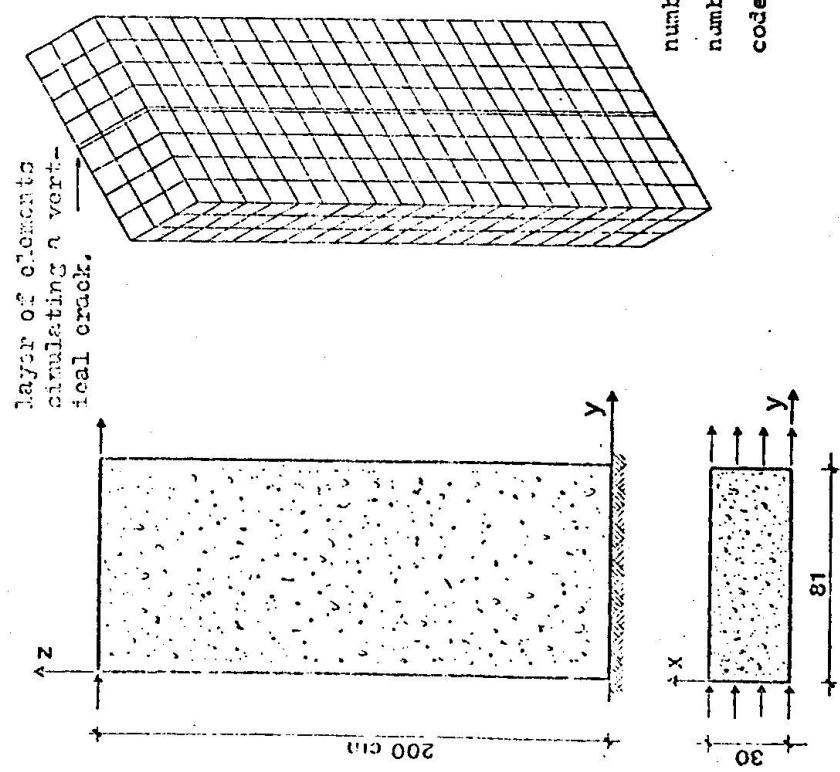


Fig. 4 Numerical model for the shear wall

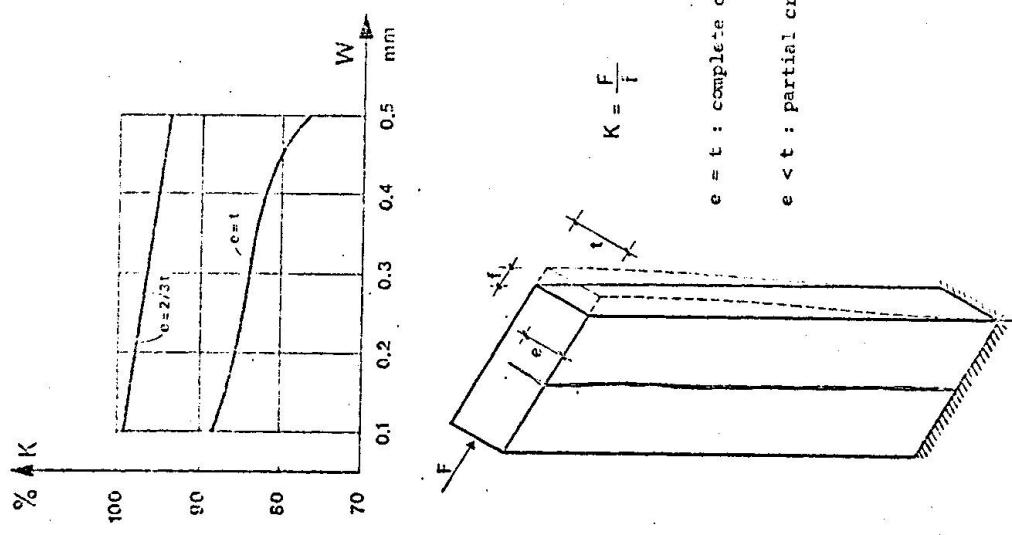


Fig. 5 Shear wall rigidity as a function of the crack width (aggregate interlock is considered).

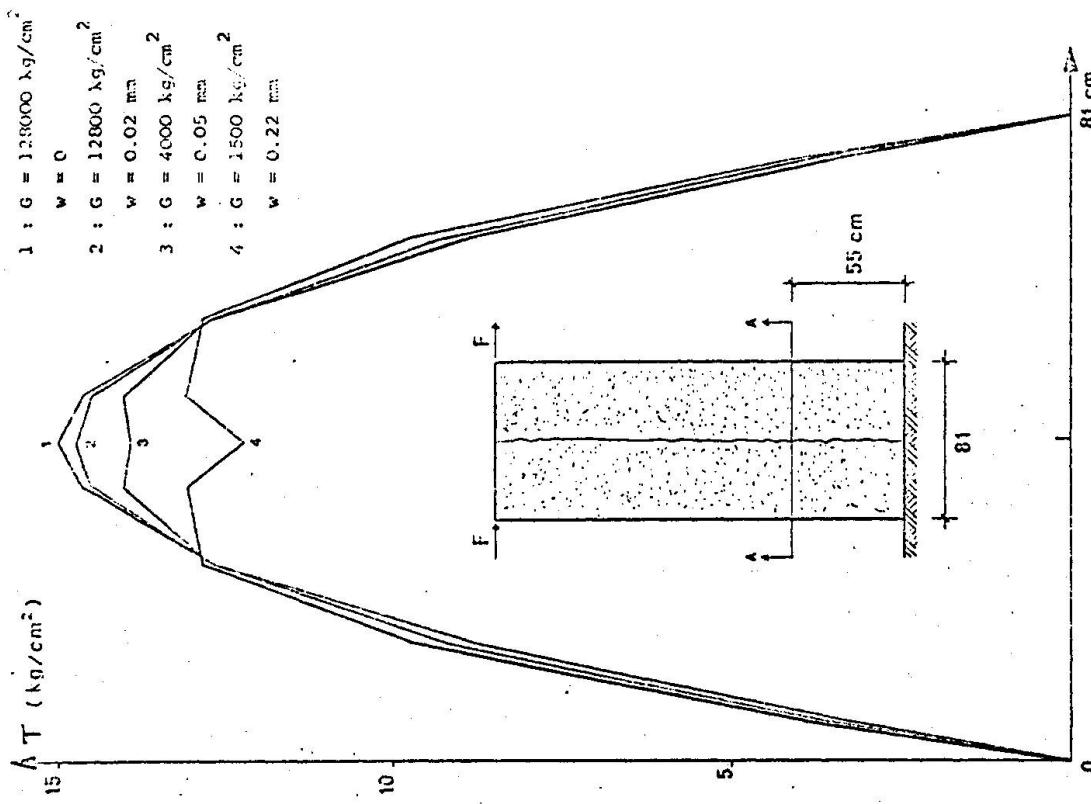
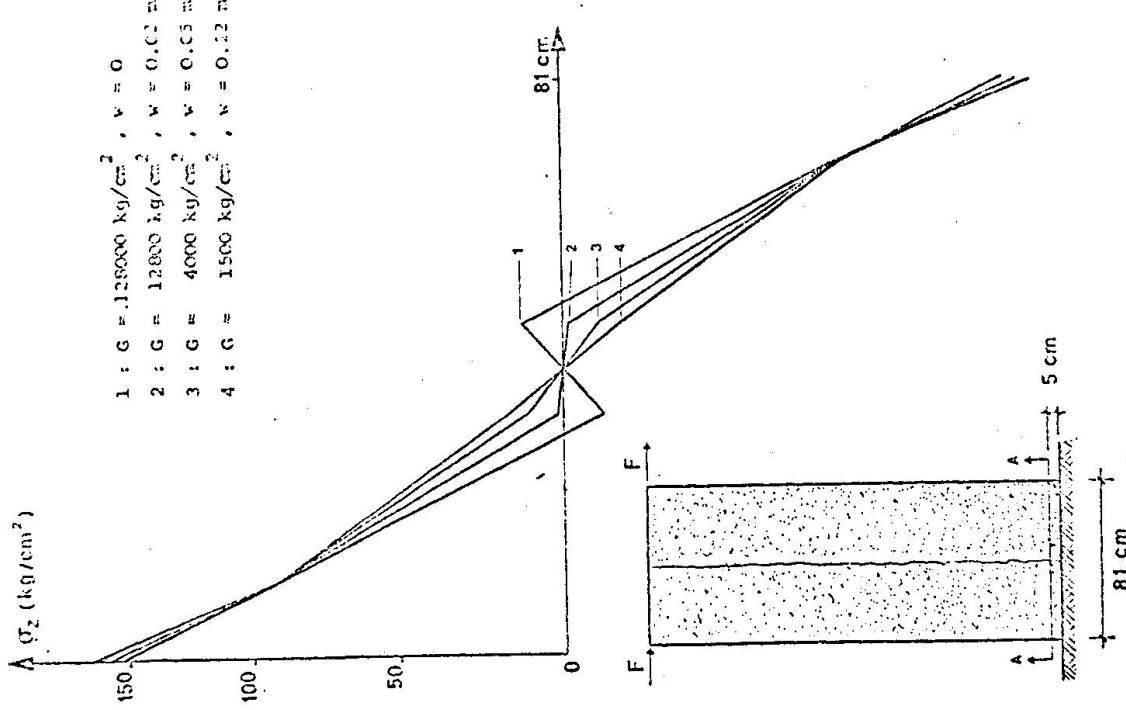


Fig. 6 Vertical normal stresses on section AA: for comparison, if no shear transfer is permitted through the crack the maximum  $\sigma_z$  is twice the peak stress of the uncracked wall.

Fig. 7 Tangential stresses on section AA as function of the tangential modulus of elasticity  $G$  assigned to the layer of crack elements.

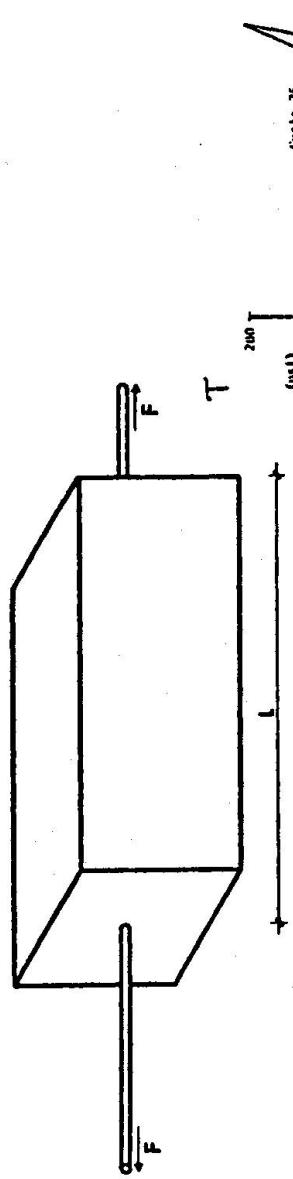


Fig. 9  
Fig. 8 Idealized shear stress-shear strain relationship.

Fig. 10 Simulation of a crack and of the bond slip between steel and concrete.

