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**Autor:** Uherkovich, I.

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### Elasto-Plastic Stage by Stage Analysis

Analyse elasto-plastique des stades de construction

Elastoplastische Bauvorgangsberechnung

I. UHERKOVICH

Head of the Design Office

Losinger Ltd - VSL International

Berne, Switzerland

#### Summary

The latest knowledge of the physical properties of materials and of increasingly complicated construction methods is fairly soon mentioned in design codes. The paper analyses the possible ways of solution. The suitability of an extensive simulation of the physical processes in an engineering calculation is discussed. The requirements with regard to the capacity of the computer and therefore the cost of the calculations may become very high, whereas the possibilities of checking the results remain small.

#### Résumé

Les connaissances nouvellement acquises sur les propriétés physiques des matériaux ainsi que les méthodes de construction de plus en plus complexes sont mentionnées assez rapidement dans les normes de calcul. L'exposé analyse l'opportunité de disposer d'un modèle de calcul reproduisant avec fidélité les processus physiques. Ces calculs exigent des ordinateurs de grande capacité et impliquent par conséquent des coûts relativement élevés, alors que les possibilités de contrôle des résultats restent faibles.

#### Zusammenfassung

Die Kenntnisse über die physikalischen Eigenschaften der Baustoffe wie auch die Anwendung von ständig komplizierteren Bauvorgängen finden heute einen sehr schnellen Niederschlag in den Entwurfsvorschriften. Im Beitrag wird anhand eines Beispiels über die Zweckmässigkeit der treuen Nachbildung der physikalischen Vorgänge bei Ingenieurberechnungen diskutiert. Die Anforderungen an die Kapazität der Rechanlage und damit auch die Kosten der Berechnung können aber sehr gross sein, die Kontrollierbarkeit der Resultate klein.

## 1. INTRODUCTION

There is a very close correlation between the possibilities of calculation and the designed structures.

In the past, when we had recognized the influence of certain executional solutions or material properties, we chose such a constructive detailing in the design so as to limit this influence to a negligible value in order to avoid calculations which, at that time, were practically not executable. Today we have computers which, according to the opinion of many people, seem to be means of unlimited power.

This leads to the design of structures in which the influences we neglected in the past have to be considered as accurately as possible. This, on the other hand, leads to the development of computer programmes which not only represents a quantitative increase and acceleration of the existing calculation algorithms developed for hand calculating, but it also forms an absolutely new and different working level.

In this contribution I would like to try to clarify the development having occurred by means of an example, as I think that the findings from the analysis of a particular problem will lead to general conclusions.

## 2. SOME WORDS ON CONCRETE CREEP AND ITS EFFECTS

As already mentioned in the introduction, certain material properties can highly influence the volume of a calculation. This, for instance, is true for the stress-strain behaviour of concrete which does not have a linear, time-independent character, as it does for steel, but which is very much dependent on the environment and time. In the last two decades the research of this behaviour, generally known as shrinkage and creep of concrete, has become a favorite field of activity for numerous scientists to whom we also owe the quick introduction of new findings into the standards.

In the new "CEB/FIP Recommendations for the design and construction of concrete structures" for example, the concrete deformation from a stress increase  $\sigma_{(t_0)}$  in the period  $t_0$  to  $t$  is given as:

$$\epsilon_{(t,t_0)} = \sigma_{(t_0)} \left\{ \frac{1}{E_{c(t_0)}} + \frac{1}{E_{c(28)}} \left[ B_{o(t_0)} + \varphi_d B_{d(t-t_0)} + \varphi_f (B_{f(t)} - B_{f(t_0)}) \right] \right\}$$

where:

$$B_{o(t_0)} = 0,8 \left( 1 - \frac{f_{c(t_0)}}{f_{c\infty}} \right)$$

$$\varphi_f = \varphi_{f_1} \cdot \varphi_{f_2}$$

$$\varphi_d = 0,4$$

The individual coefficients are found from the following figures (diagrammes) and tables:

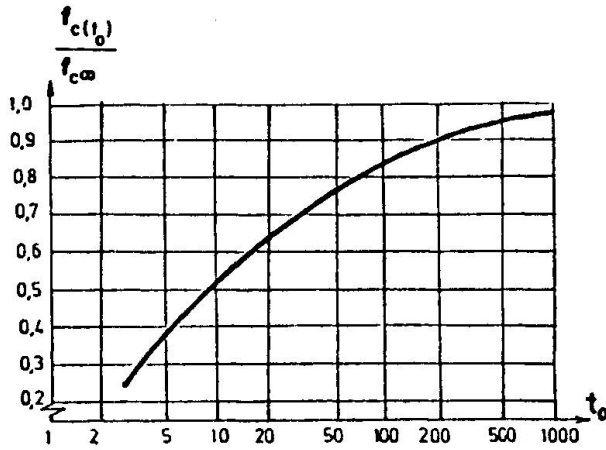


Fig. 1: Coefficient  $\frac{f_c(t_0)}{f_{c\infty}}$

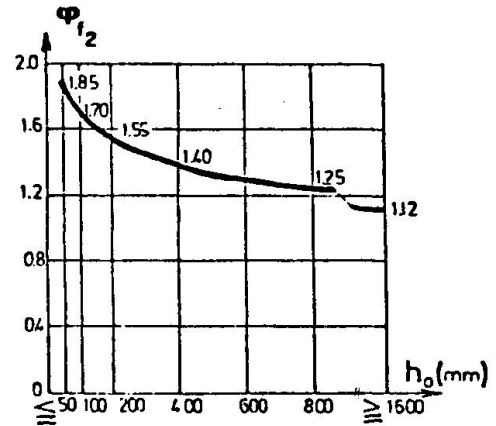


Fig. 2: Coefficient  $\varphi_{t_2}$

Environment	Relative humidity	$\varphi_{t_1}$
Water	100 %	0,8
Very humide atmosphere	90 %	1,0
Exterior in general	70 %	2,0
Very dry atmosphere	40 %	3,0

Fig. 3: Coefficient  $\varphi_{t_1}$

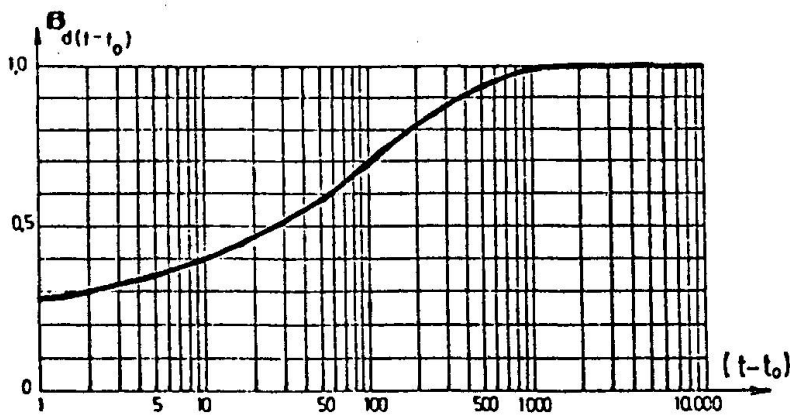


Fig. 4: Coefficient  $B_{d(t-t_0)}$

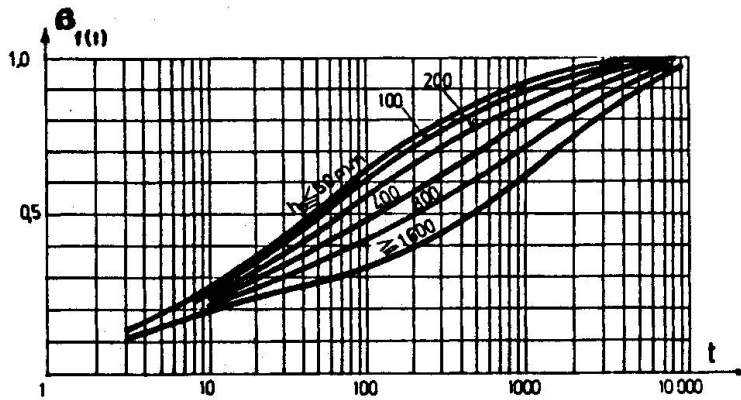


Fig. 5: Coefficient  $B_{(t)}$

I do not intend to elaborate on concrete creep but I would like to emphasize a fact which is of definite importance for my further considerations.

The deformation of an element in a time period is not only given by the stress present in the element in this period, the time origin of the stress also plays a role. This means that in the superposition of several stress stages of different time origins in one element, (i.e. a structural part of homogeneous deformation behaviour), a different creep coefficient has to be introduced for every one of these stress stages. If we pursue the deformation of this element for a series of several time periods we have to elaborate a matrix of coefficients in which the number of elements is equal to the product of the "(number of stress stages) and (the number of time periods considered)".

If a structure consists of parts with different properties (which in general applies) - different properties here mean for example different age, dimensions, reinforcement content a.s.o. - then for each of these parts ("elements") a separate matrix of creep coefficients is to be established.

Without going into details, two of the numerous existing rheological models will be shown for completeness and for a better understanding of what a creep function is:

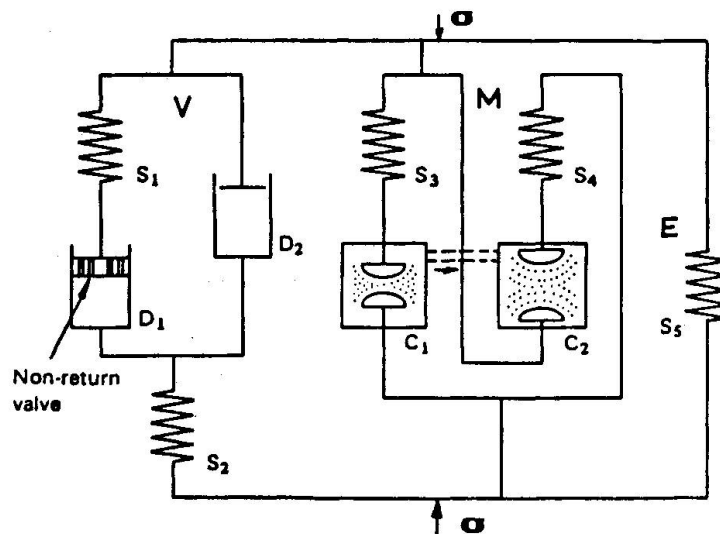


Fig. 6: The Gopalakrishnan-Neville-Ghali model

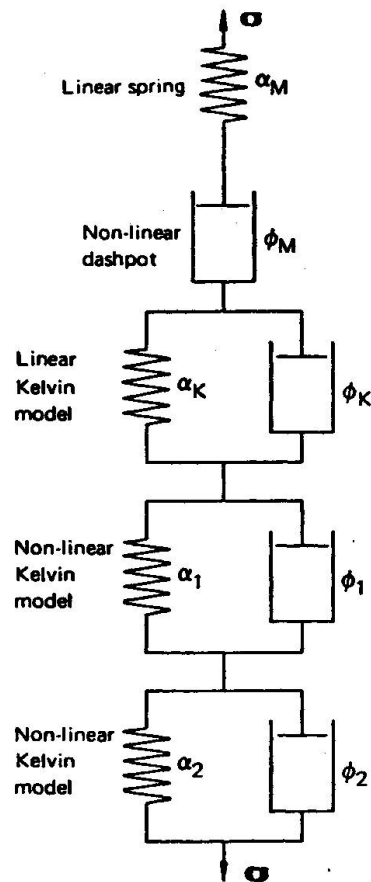


Fig. 6: Freudenthal Roll's model

A digital simulation of such processes is fairly difficult. To complicate the matter, it must be mentioned that independent of the stress-dependent deformation (creep) there is also a purely time-dependent deformation of the concrete (shrinkage) which superposes with the creep deformation. The total time-dependent deformation of an element in the actual structure (I am mainly thinking of post-tensioned concrete) leads to different but coupled effects:

- 2.1. By axially distorting the structural parts, the elongation-dependent post-tensioning force changes ("loss due to creep"); this change of the post-tensioning force and the resulting change of the stress distribution then influence the distortion itself.
- 2.2. With elements composed of parts with different deformation properties (so-called "composite sections" = i.e. concrete of different age plus steel) the stress shifts from elements of high deformability to elements of lower deformability (for example from young concrete to older concrete or from concrete to steel). This, however, influences the deformation properties of the whole element and thereby also the magnitude of the deformation. We therefore talk of a so-called "inner shifting of the stresses due to creep".

2.3. With structures where the boundary conditions vary with time (e.g. due to changing bearing conditions) secondary forces due to time-dependent deformation properties of the elements develop without a change of the active load condition. They in turn influence the deformations through changes of the stress distribution.

Here we are dealing with the so-called "outer shiftings of the bearing forces due to creep".

### 3. STRUCTURAL DESIGN PROGRAMMES TAKING INTO ACCOUNT THE CREEP OF CONCRETE

From the above presentation of the problem which probably was not short but the shortest possible, we come back to our theme which concerns the possibilities of calculating such processes.

If we intend to use a fully automatic processing, the programme must consist of the following main parts:

- structural design (e.g. by finite elements)
- calculation of stresses in the cross section including computation of post-tensioning
- calculation of creep and shrinkage

These parts must together form one system and run under a superimposed programme control. The Fig. 7 schematically represents such a programme system.

All data concerning the time of construction of the individual structure parts, the time of loading or unloading a.s.o. are stored in the part called "programme control". This part then regulates the sequence of calculation processes. After each time period a stress calculation has to be done for each element. This is used as a starting point for the creep calculation which must also be done for each element in each time period. Together with shrinkage the element deformations are computed which then serve as a starting point for calculating the changes of the prestressing force and the redistribution of the stresses in the section. The calculation of the change of secondary forces again has to be done by a new structural calculation (the calculation of compatibility). When the mathematical bases of this programme are elaborated, attention must be paid to the fact that the inner and outer force equilibrium conditions are strictly observed.

This brings considerable difficulties as there is no longer a direct relationship between deformations and forces.

As already mentioned, every stress stage in every element has to be stored as an independent cause of creep during the whole calculation in order to allow a creep computation to be made.

From the processing point of view some problems arise in the run, especially the long computation time and the need for large storage. It may be mentioned here that, in addition, for a highest possible accuracy in calculations, the (elastic) E-Modulus should be introduced as a function of concrete age and

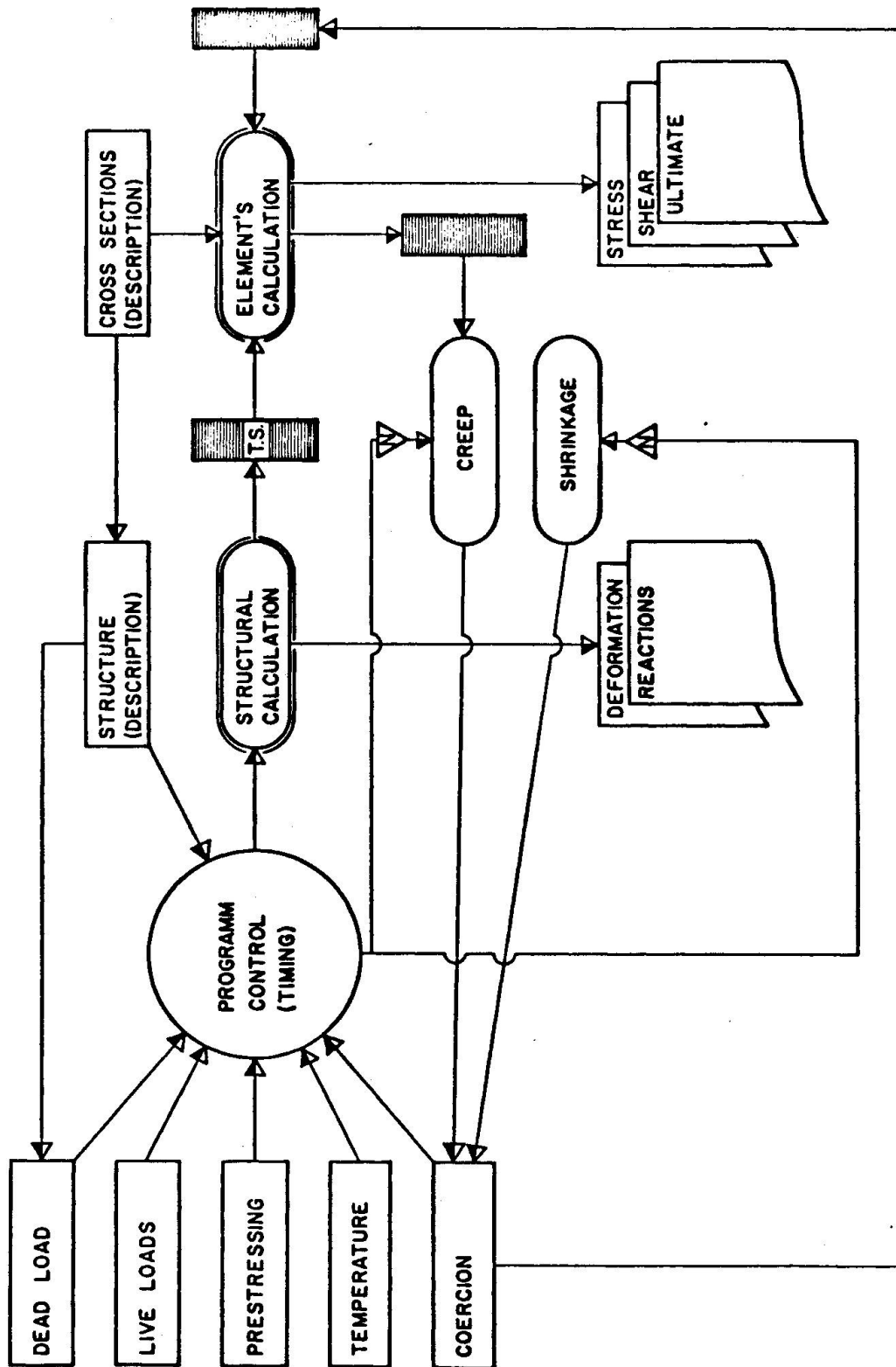


Fig. 7: Programme system for stage by stage calculation

that the real construction times have to be adjusted in the calculation according to the seasonal climatic conditions.

We tried to establish such a programme by strictly applying the creep formula mentioned in the introduction, with some simplifications in the process given in Fig. 7. We did not consider the "inner shifting of the stresses due to creep" for example, and the prestressing losses were calculated from the same initial data, and not from the deformations of the elements. The processing also was not fully automatic.

We neither considered a time-variable E-Modulus nor the adjustment of the time data from temperature influences. In the programme we made use of the fact that the elastic structural calculation forwards inherent element deformations; it therefore was sufficient to multiply these by the corresponding creep coefficients and to reintroduce the element deformations so obtained into the structural calculation as a loading case again in order to produce the compatibility of the deformations.

Hence the constraints ("outer shifting of the bearing forces") could be calculated which appear as a new loading case as they are also subjected to "decreasing" (relaxation).

The programme block scheme is given in Fig. 8.

The calculation of a fairly complicated case with more than 40 time periods could thus be done. Each time period practically led to a change of the structure. As "heart piece" the member programme "STATIK" (developed at the Swiss Federal Institute of Technology in Zurich under the direction of Prof. Anderheggen) was used. The proper elaboration of the full creep programme was done by Thomas Friedrich, Eng., in the surprisingly short period of 3 months.

#### 4. DESIRABLE AND OBTAINABLE ACCURACY

The question whether such a high precision in calculations is reasonable, is very interesting. I would like to point out first that a highest possible accuracy, e.g. in prestressed concrete bridge construction, is not so much of interest for the stability design than for the deformation considerations of a structure. A calculation of the forces in the structure to an accuracy of  $\pm 5\%$  or  $\pm 10\%$  allows for a justified comparison with the allowable values. This is not the case for the form of a structure where not limiting values but real and most accurate possible values have to be considered. With a free cantilever bridge, for example, an up to 100 m long cantilever steadily moves up and down during construction. A calculation of accuracy usually required for statical calculations would lead to deformations that cannot be undone later and thus the bridge surface would become uneven.

For illustration I would like to present the following values: Thanks to the accurate manner of calculations in the previously mentioned example we were able to obtain all movements of a bridge cantilever with an accuracy of 20 mm compared to the effective values obtained on site (with respect to the centre of the 108 m long span). A hand calculation of reasonable volume (based on

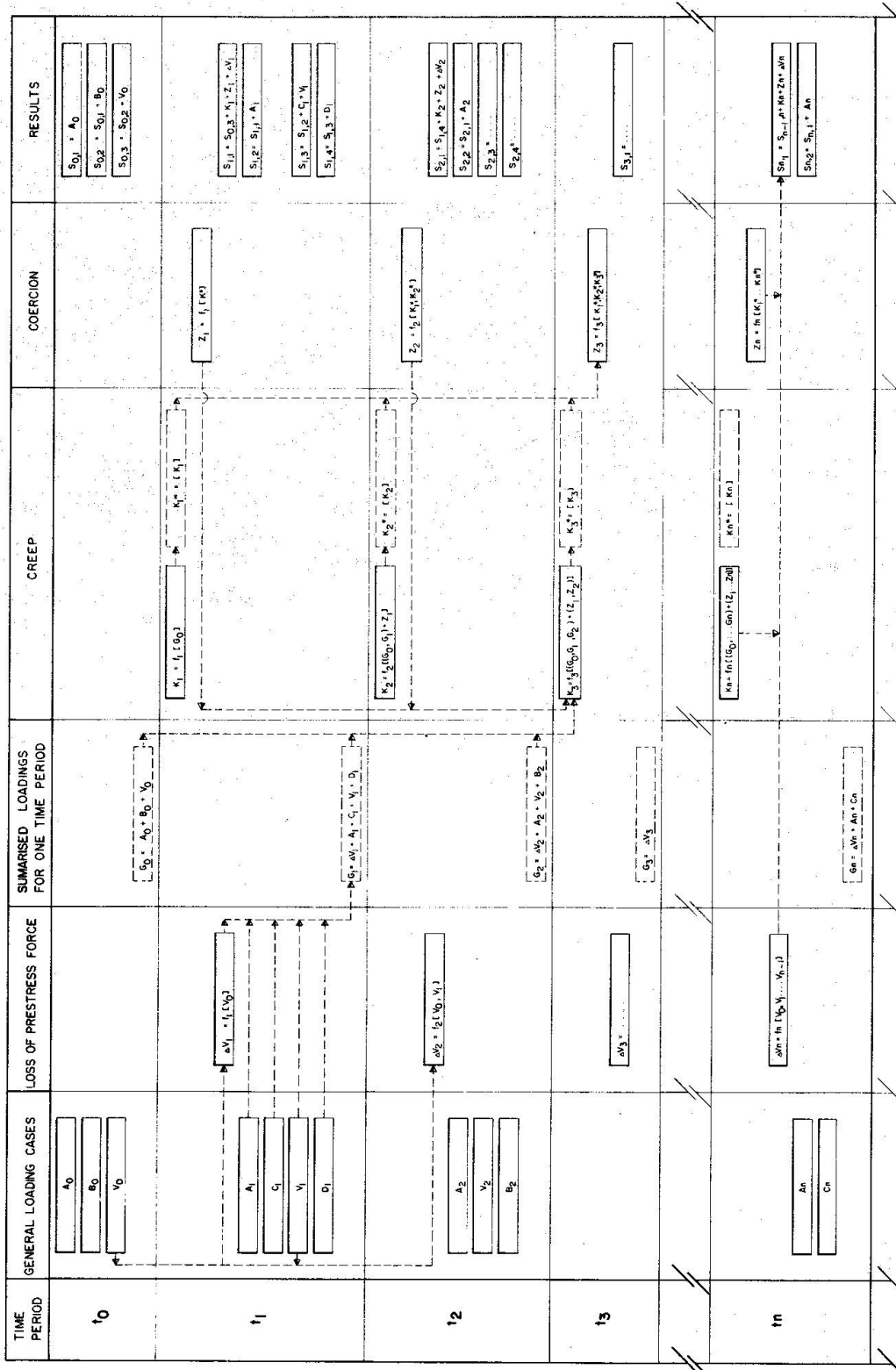


Fig. 8: Block scheme of realized programme

elastic deformations calculated with a computer) gave a deviation of 60 mm. With an increasing span length this value would rise with the power product of the span ratios.

One can imagine that a calculation without simplifications would bring the accuracy to below 10 mm in this case.

Considering increasing span lengths, more slender sections (and thus bigger deformations) and increasingly complicated construction procedures which take only little of the calculations into account, we can answer the question about the right of a highest possible accuracy in the affirmative. Obviously a calculated accuracy only makes sense if it is not beyond the possibilities of the actual conditions. In the mentioned case of the prestressed concrete bridge not only the effective material properties but also the influence of the atmospheric conditions had to be captured. This today is not only possible but generally done on the sites.

The obtainable accuracy, last but not least, is of economic and not of technical interest. The first prediction of the material properties and the construction times is always a first approximation only. A higher degree of accuracy in the calculation than that in the first adoption is only justified when a new run can be made with known new correction values. If such a new run is not economically acceptable, the accuracy required in the first run was of small utility. In other words the limits of obtainable accuracy are determined more by the calculation costs than by outside circumstances. This, however, means that as the hardware is becoming cheaper and cheaper, a temptation is created for a higher accuracy in the results by repeating runs, especially when no or only small programme costs are involved.

## 5. CONCLUSIONS

I have tried to demonstrate the problems of a coupled calculation of complicated, technical and physical processes by means of an actual example of the calculation of the "Computer Stage Analysis" used in bridge design, taking into account the elastoplastic properties of materials.

Of greatest importance, it seems to me, are the following facts:

- 5.1. The progress in practical construction, as well as applied research, forces the engineer to make more and more complicated calculations.
- 5.2. These calculations are made practically possible only by using computer programmes.
- 5.3. Due to the enormous progress in computer manufacturing and the consequent important price reductions of the calculators, such calculations today are possible at low costs and therefore the demand for them will certainly increase.
- 5.4. The possibilities of controlling the results, however, are very limited.

The real problems of such a development therefore are not in the technical mastering but in the general consequences, especially with regard to the responsibility for the product and the allowance for an action without sufficient possibilities of control.

This, however, is the theme of another session.

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