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**Structural Design Implications of Analytical Techniques in Computing**

**Influence des techniques analytiques dans la programmation sur le projet des structures**

**Auswirkungen von analytischen Techniken in der EDV für die Berechnung von Tragwerken**

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**Summary**

This paper reviews the advantages and limitations of current analytical approaches used in the computer analysis of structures. Some specific sources of error are indicated. The techniques covered include harmonic methods, the grillage method and the finite element method. The division of responsibility between: 1) computer program writer, 2) user manual writer, 3) computer bureau, 4) designer (and program user) is discussed in the light of the characteristics of the techniques covered in the paper.

**Résumé**

L'article passe en revue les avantages et les limites des approches analytiques conventionnelles utilisées dans le projet des structures à l'aide de l'ordinateur. Quelques sources d'erreur typiques sont mentionnées. Les techniques considérées comprennent les méthodes harmoniques, la méthode du grillage et la méthode des éléments finis. La répartition de la responsabilité entre: 1) l'auteur du programme d'ordinateur, 2) l'auteur du manuel d'utilisation du programme, 3) le centre de calcul, 4) le projeteur (et l'utilisateur du programme) est envisagée en fonction des caractéristiques et des techniques présentées dans l'article.

**Zusammenfassung**

Der Artikel beschreibt die Vorteile und Grenzen von gewöhnlichen analytischen Annäherungen, welche im komputergestützten Entwurf von Tragwerken benutzt werden. Einige typischen Fehlerquellen werden aufgezeigt. Die betrachteten Techniken sind die harmonische Methode, die Gittermethode und die Finiteelementen-Methode. Die Teilung der Verantwortung zwischen: 1) Autor des Computerprogramms, 2) Autor des Programmhandbuchs, 3) Rechnungszentrum, 4) Entwerfen (und Programmbenützer) wird anhand der im Artikel dargestellten Eigenschaften und Techniken besprochen.



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## INTRODUCTION

This paper reviews the advantages and limitations of current analytical approaches used in the computer analysis of structures. Some specific sources of error are indicated.

The techniques covered include harmonic methods, the grillage method and the finite element method. The division of responsibility between

- 1) computer program writer
- 2) user manual writer
- 3) computer bureau
- 4) designer (and program user)

is discussed in the light of the characteristics of the techniques covered in the paper.

## HARMONIC METHODS

The use of methods based on Fourier series solutions have been quite widespread particularly in the analysis of bridge decks. The main techniques have been

- a. Orthotropic plate theory. This usually involves the solution of the fourth order differential equation for elastic orthotropic plates by the Levy-Nadai method<sup>(1)</sup>. Guyon<sup>(2)</sup> and Massonnet<sup>(3)</sup> set up a simple tabular technique in which the first term only of the sine series representing the elastic deflection curve was used to characterise the distribution of bending moments due to wheel loads on bridge decks. The advent of the computer has permitted more accurate solutions using a relatively simple program. The Highway Engineering Computer Branch of the British Department of Transport has issued a program ORTHOP<sup>(4)</sup>, based on work by the author, for application to right slab and pseudo-slab decks with edge-stiffening beams. This enables the calculation of bending and twisting moments and shear and reactive forces.
- b. Finite strip method. Cheung<sup>(5)</sup> originated this hybrid method which is useful for bridge and roof structures of uniform cross-section. The structure is divided into longitudinal strips running the full length of the structure. The longitudinal variation of displacement is characterised by trigonometric series functions and the transverse variation by polynomial

functions (as conventionally used for the finite element method). The method has considerable advantages in terms of simplification of input data and economy of computer storage and run time as compared with the finite element method.

- c. Folded plate method. The matrix formulation by Scordelis<sup>(6)</sup> of the Goldberg and Leve analysis for folded plates has been programmed and applied to bridge and roof structures. It is a series solution but is less versatile than the finite strip method which has largely replaced it.
- d. Limitations and practical difficulties. The harmonic methods are all based on functions of the form

$$p(x) = \sum_{n=1}^r (H_n \sin \frac{n\pi x}{L}) A_n$$

where  $p(x)$  represents the load distribution

$H_n$  is a load function (in trigonometric form)

$A_n$  is a function embodying geometric and stiffness parameters and hyperbolic and trigonometric functions

Hence the displacement

$$w = \sum_{n=1}^r \frac{L^4}{n^4 \pi^4} (H_n \sin \frac{n\pi x}{L}) B_n$$

where  $\frac{\partial^4 B_n}{\partial x^4} = A_n$

Because the series for  $w$  converges with  $\frac{1}{n^4}$  it converges very rapidly. The series for bending moments depend upon terms in  $\frac{\partial^2 w}{\partial x^2}$  and thus converge less rapidly with  $\frac{1}{n^2}$ . The series of shear force converges very slowly with  $\frac{1}{n}$ . These essential differences are not always understood by users of programs based on this method.

#### e. Examples

1. Fig. 1 illustrates the relationship between the shearing force diagram and the number of harmonic terms used for the finite strip solution of one span of a continuous box beam under uniform loading. As the number of terms increases, the harmonic solution approaches the correct linear distribution. However values of reaction at the support are very significantly lower than true reaction values.

It is possible to obtain reasonable assessments of shear force at the supports by considering a position near rather than at the support point but users who fail to understand this characteristic of the method can make errors.

2. In a development of orthotropic plate theory the ORTHOP2 program is currently being developed to analyse right slab and pseudo-slab bridge decks with unequal edge beams. The edge beam stress parameters are developed in terms of the deflection profile at the slab-beam boundary. The expression for bending moment in the edge beam develops a tendency to oscillate about the true solution if more than a small number of harmonic terms are considered. This is clear from Fig. 2, which shows comparisons with finite element and finite strip results for the same problem. Fig. 3 illustrates the divergence which occurs above 6 harmonic terms. Here then is a circumstance which is the opposite of Example 1, i.e. now, a large number of terms does not improve accuracy.

These two examples provide an object lesson to users on the necessity to understand the characteristics of analytical techniques before applying them to design situations.

#### GRILLAGE ANALYSIS

The representation of a slab or pseudo-slab structure by a grid or grillage of beams interconnected at rigid joints has always been a popular technique with bridge engineers and it has been given new impetus by the availability of the computer.

The method is relatively simple in terms of data preparation and economical in computer storage and run time.

##### a. Choice of beam spacing

Each beam of the grillage replaces a finite width of the deck. Thus the choice of the relative positions of the beams is important as also is the allocation of bending and torsional rigidity to each beam. West<sup>(7)</sup> recommends that there should be odd numbers of longitudinal and transverse beams and that as far as possible they should be at equal spacing and of equal stiffness. The gross torsional rigidity of the deck should be assigned in equal parts to the longitudinal and transverse beams. An orthogonal pattern of beams is desirable even for a skew planform - even though this conflicts with the recommendation for equal beam spacing.

##### b. Limitations and practical difficulties

1. The grillage method is relatively insensitive to concentration of stress and, for example, will underestimate the peak stress below a small patch of load.
2. The grillage method often underestimates torsional moments and overestimates values of bending moment at positions remote from a load concentration.

These two points are illustrated by Figs. 4 and 5, which show values for four alternative skew grillage arrangements for the analysis of a model skew deck tested by Rusch and Hergenroder.

3. Results are difficult to interpret for decks which have non-parallel edges because the beams now represent variable widths.
4. Results are unreliable for curved decks where the angle is greater than about 20 degrees between supports.

#### FINITE ELEMENT METHOD

The finite element method is now well-known to most structural engineers through the work of practitioners such as Zienkiewicz<sup>(8)</sup> and others. It is the most general of the methods available for structural analysis and large packages (e.g. PAFEC and NASTRAN) are now available with a variety of alternative elements for two- and three-dimensional stress analysis.

#### Limitations and practical difficulties

1. The finite element method is both costly and complex to use. Its cost makes it particularly desirable to avoid errors and abortive computer runs, but its very complexity makes it prone to misunderstandings between design engineer and analyst. The method should only be employed where simpler techniques are inappropriate, e.g. for non-uniform members, non-standard geometry, or inelastic materials.
2. The volume of input data for a finite element analysis is usually large and errors are often difficult to spot.
3. Hinton<sup>(9)</sup> points out the difficulties of interpreting stress distribution from a finite element program output because of discontinuities between elements. Interpretation is largely subjective and can therefore be inconsistent and irrational.
4. Simple equilibrium checks should always be made to ensure that gross errors or misunderstandings are not present. A recent case is known to the author, of a skew bridge deck analysed using a large finite element package, where elements were chosen which for skew axes imposed a degree of restraint at the nominally simple line supports. This effectively reduced the mid-span bending moment and design of the bridge was well-advanced before a perceptive engineer made a simple check of equilibrium and discovered the mistake.

#### RESPONSIBILITY

There are four groups of personnel who carry responsibility for the computer program and its use in the analysis and design of a structure. These are:

- 1) Computer program writer
- 2) User manual and program manual
- 3) Computer bureau staff
- 4) Engineer designer (program user)

Frequently one individual will have written both the program and the manuals and this is a desirable state of affairs. At least there must be very close collaboration between the program writer and the author of the manuals.

Computer bureaux usually take over proven standard programs and their staff may have a very limited knowledge of the structural principles underlying the program. However they should gain a thorough knowledge of the manuals and the program input and output. They should be ready to seek advice from the originator of the program in cases of difficulty.

The structural designer chooses (or sanctions the choice) of the program to be used for a particular problem. He must therefore be aware of the general characteristics, limitations and costs of each of the alternatives. He must make extensive checks to ensure that the results are structurally valid.

If shearing stresses are likely to be critical in a particular structure then the finite strip method is a poor choice. If local moment peaks are important (for example in fatigue situations), the grillage method is not the best choice; the finite element method will only give reasonable results with a very fine mesh arrangement; on the other hand a harmonic method, if applicable, would be accurate and inexpensive.

Most major computer programs used in structural analysis have been written by engineers whose understanding of the implications of the technique used is beyond question. However errors of logic do occur in programs and can often lie undetected until a particular problem solution or a change of computer system brings them to the surface. These must be guarded against and although they might be said to be primarily the responsibility of the original program writer, the bureau staff and program user must also be on their guard against such an occurrence.

In general the designer inevitably bears the main responsibility over the use of computer programs. He chooses the program (which implies knowledge of the underlying method of analysis); he must be able to know if results are substantially in error. Computer bureaux staff have responsibility as sub-contractors to ensure that the program is working as intended and that input data are checked. If they recommend a particular program they must be familiar with the limitations of the program. Manual writers have the responsibility of ensuring that both the capabilities and the limitations of a program are clearly stated. They must provide guidance to aid avoidance of



errors and misunderstandings but cannot be held responsible for users who ignore manual instructions. Program writers are responsible for translating a method of analysis or a design procedure into a logical and unambiguous computer program. They cannot be held responsible for subsequent misuse of programs which they have written. However if their program is written under contract for a particular purpose subsequently not fulfilled, or if they are extracting a royalty for use of a program, there is an obligation to provide help and advice to users.

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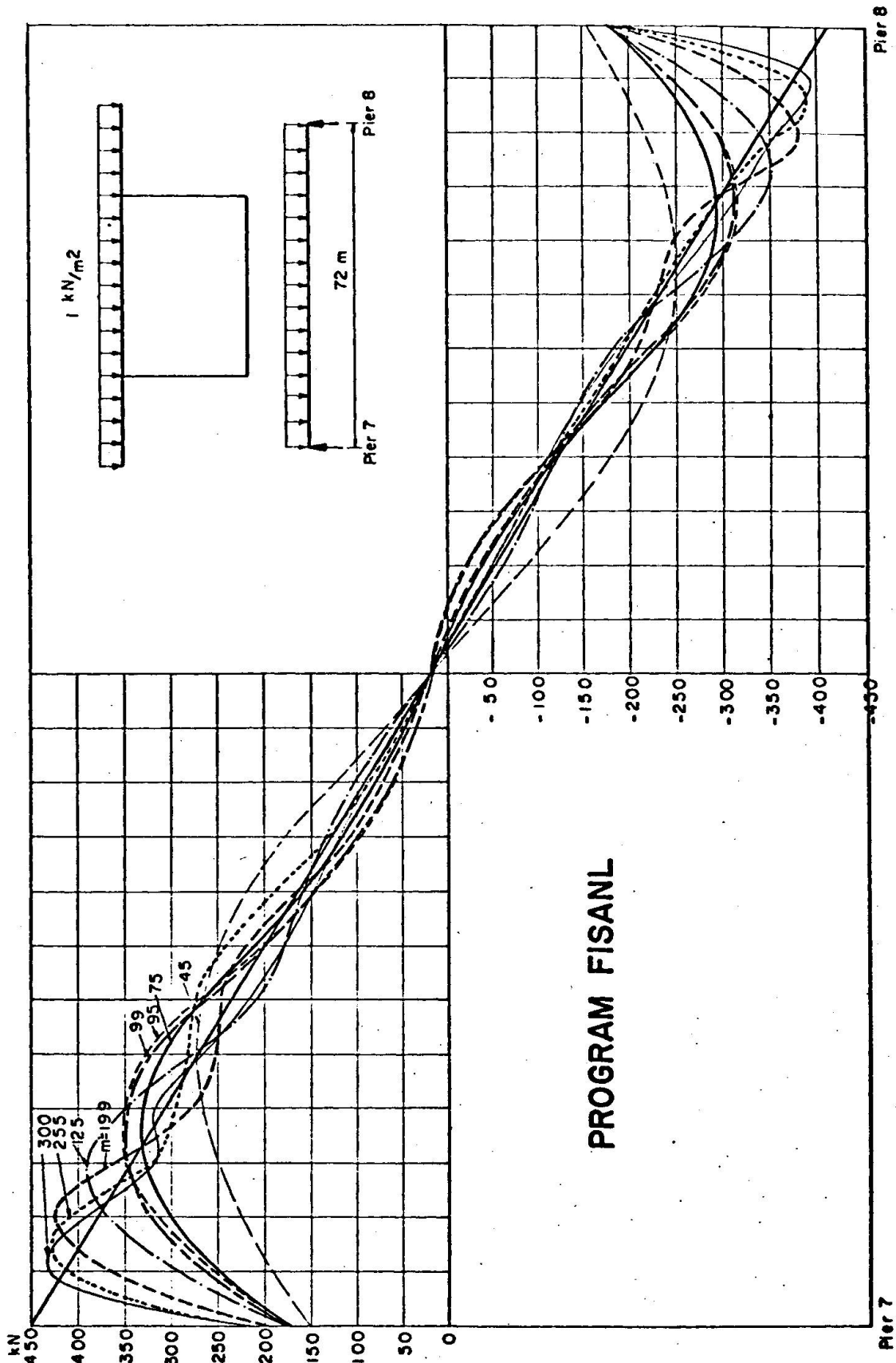


FIG. 1 SHEARING FORCE DISTRIBUTION ALONG ONE SPAN OF CONTINUOUS BEAM

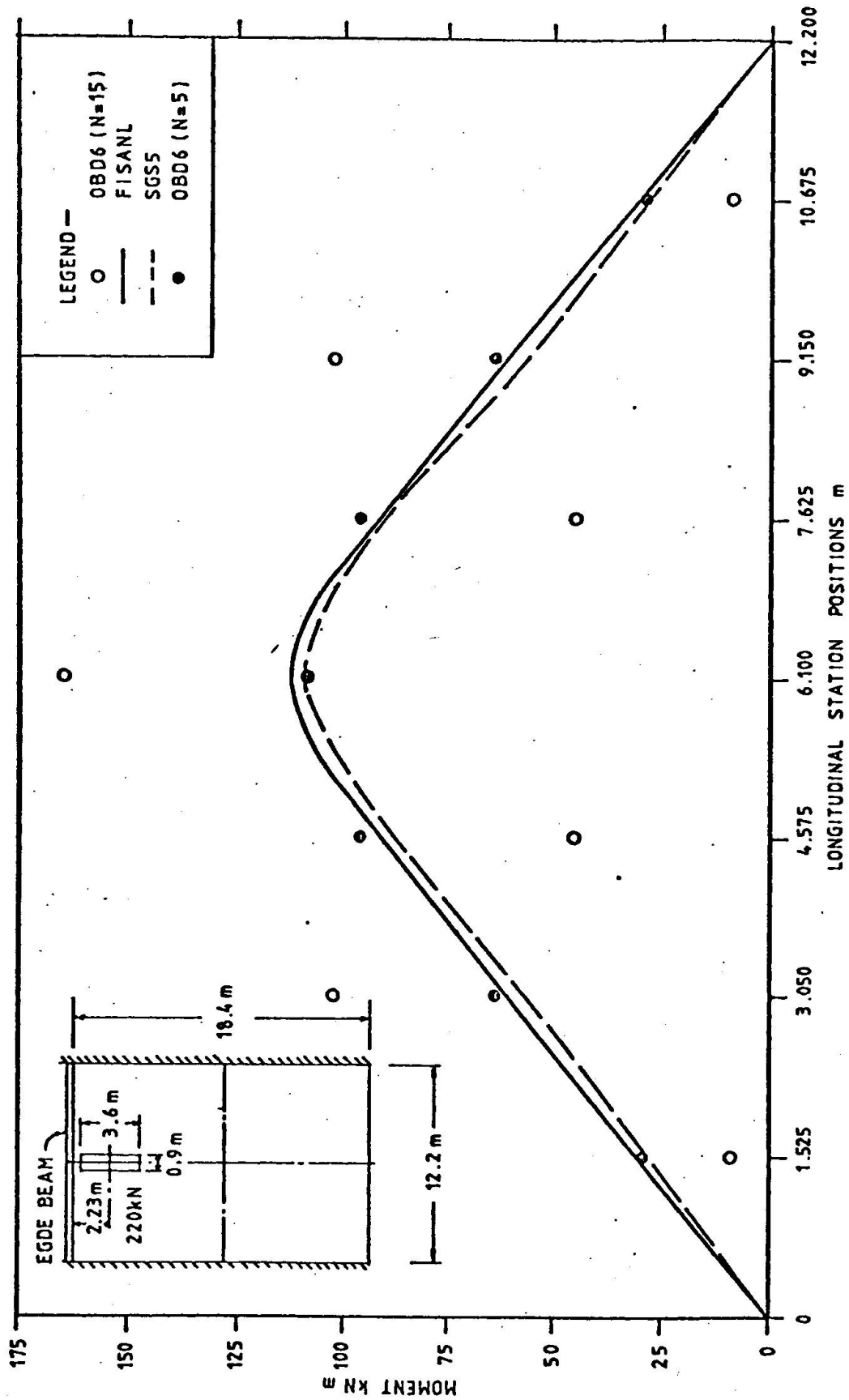


FIGURE 2 EDGE BEAM BENDING MOMENTS 220kN LOAD AT 2.23 m FROM THE EDGE

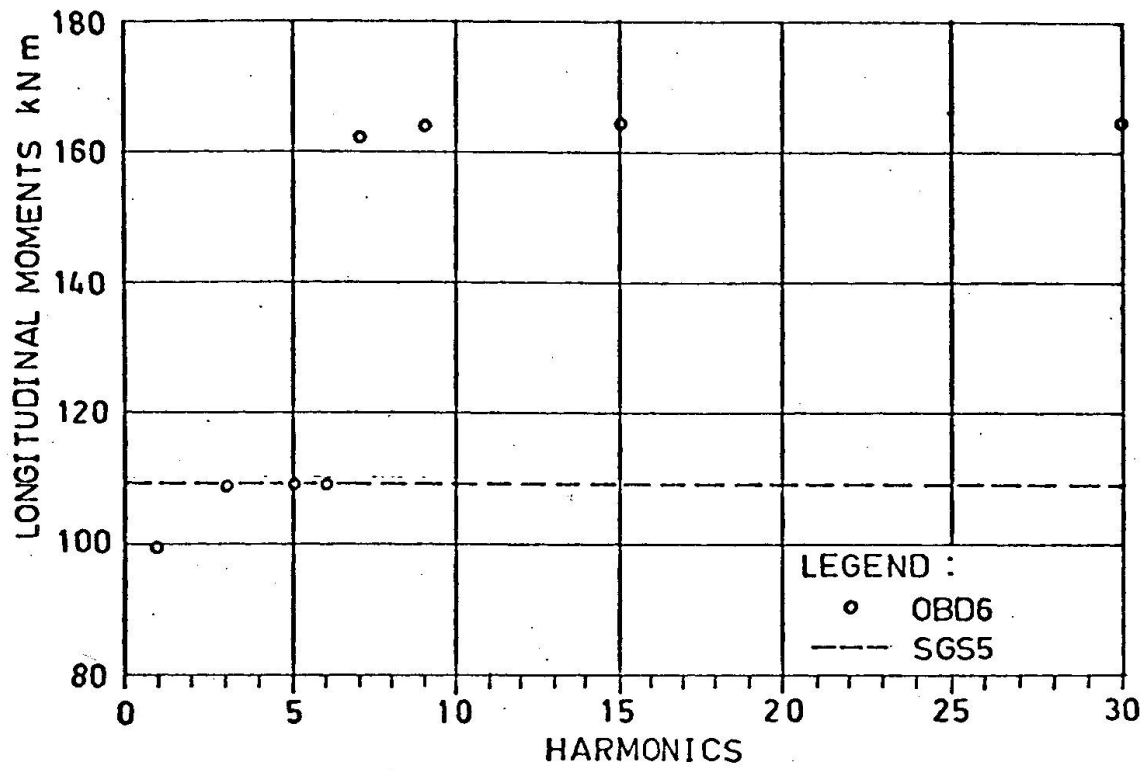


FIGURE 3 VARIATION OF MID-SPAN EDGE BEAM BENDING MOMENT VALUES WITH HARMONIC SUMS



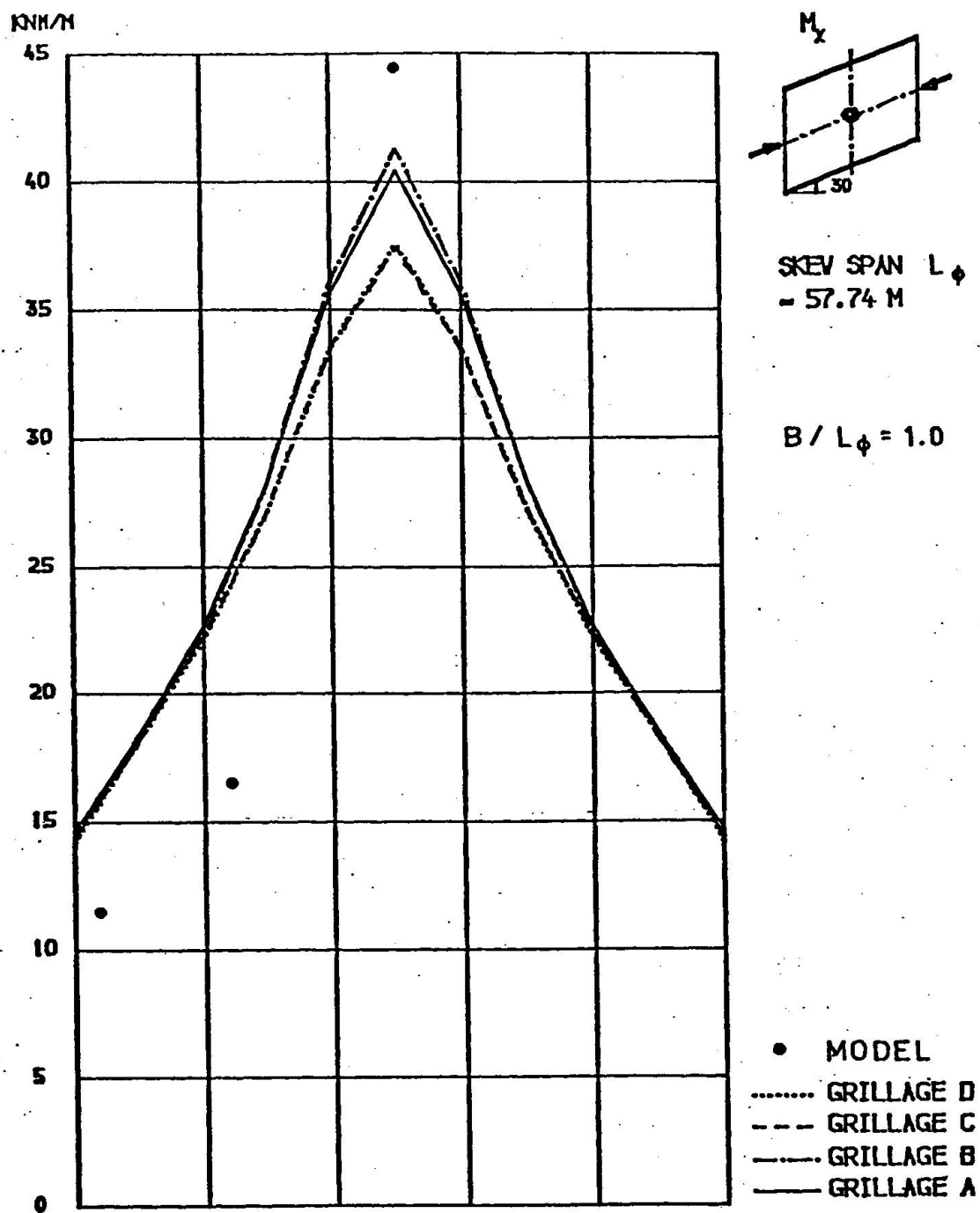


FIG. 4 LONGITUDINAL MOMENT PROFILE AT MID-SPAN

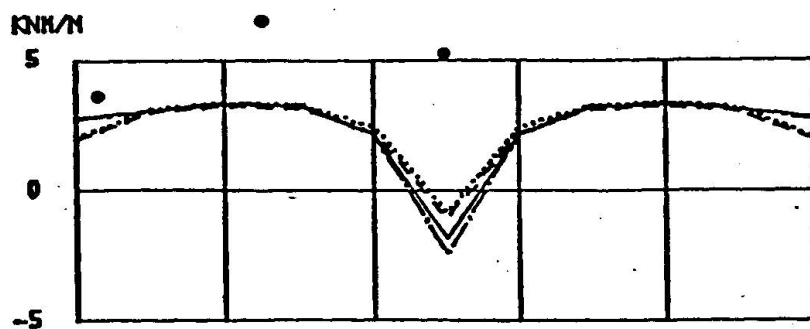


FIG. 5 TWISTING MOMENT PROFILE AT MID-SPAN