Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 31 (1978)

Artikel: Optimization of structures using decomposition

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DOI: https://doi.org/10.5169/seals-24898

IABSE COLLOQUIUM on:
AIPC "INTERFACE BETWEEN COMPUTING AND DESIGN IN STRUCTURAL ENGINEERING"
IVBH August 30, 31 - September 1, 1978 - ISMES - BERGAMO (ITALY)

Optimization of Structures Using Decomposition

Optimisation des structures au moyen de la méthode de décomposition

Optimierung von Tragwerken mittels Dekompositionsmethode

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Summary

The application of the decomposition method to the optimization of structural systems consists in dividing the system into parts and coordination of them to assure adequacy to the reality. After general formulation of optimization the conditions for applying decomposition have been discussed. A case study - the optimization of the steel structure of industrial sheds - illustrates the successful application of the decomposition method. The computer system OSY which performs this optimization, based on the decomposition princimple, has been mentioned.

Résumé

L'application de la méthode de décomposition pour l'optimisation des systèmes de construction consiste en une division du système en parties, en leur coordination pour assurer leur concordance avec la réalité. Après une formulation générale de l'optimisation, on a discuté les conditions de l'application de la décomposition. L'optimisation de la structure métallique de toitures en shed est presentée comme un exemple de l'application de la méthode de décomposition. On mentionne aussi le système informatique OSY avec lequel cette optimisation, basée en principe de la décomposition, est faite.

Zusammenfassung

Die Anwendung der Dekomposition zu der Optimierung von Tragwerksystemen besteht in der Teilung des Systems in Subsystem und ihre Koordination zwecks der Beibehaltung der Verträglichkeit des Systems mit der Realität. Nach allgemeiner Formulierung der Optimierung werden Voraussetzungen für die Anwedung der Dekomposition erörtert. Eine Fallstudie - Optimierung der Stahlkonstruktion von Industriehallen - zeigt eine erfolgreiche Anwedung der Dekomposition. Das Computer-System OSY, das diese Optimierung auf der Grundlage des Dekompositionsprinzips vornimmit, wird erwähnt.

1. INTRODUCTION

Let us start by general formulation of the optimization problem. Given:

1/ a decision space X' , consisting of the elements (decisions) $x: x \in X'$;

2/ a set of permissible decisions X, defined as

$$X' \supset X = \{x : g(x) \leq 0, \ h(x) = 0\}$$

3/ a functional f optimum criterion , which maps the decision space X' into the space of real numbers R, i.e.

$$f : X' \rightarrow R$$

The task of the optimization is to find such an element

$\hat{x} \in X$ that $(\forall x \in X) \ f(\hat{x}) \leq f(x)$

In this way we can formulate many optimization problems, in particular the problem of optimization of systems. It does not mean, however, that we are able to solve every mathematically formulated optimization problem.

The question arises, how to solve large problems occuring in practice? The concept of solving such problems is to apply decomposition, which consists in dividing the problem into parts and solving them parallelly and independently by assuring the existing connections between these parts by so called coordination.

2. DECOMPOSITION

Large optimization problems occuring in practice are, as a rule, the problems of the optimization of systems. The main feature of systems is that they consist of distinct components, called subsystems, and - because of the conflict of interests arising between the subsystems and the systems as a whole - the optimum of the system is not the sum of the optima of subsystems. The partitioning of the large optimization problems, which is necessary for applying decomposition, goes on in natural way in the case of the optimization of systems, because the system is composed of distinct parts, i.e. subsystems.

Let us now formulate the decomposition mathematically. For simplification let us consider the optimization problem in the Euclidian space. The primary problem is:

$$\inf_{x \in X} f(x) , \qquad X \subset E \qquad (1)$$

To solve the problem (1) by decomposition we divide the vector x into 2 parts (y,z), where y are coordination variables and z are the decision variables of the decomposed parts of the whole problem. The decomposed problem can be then formulated as follows [1] :

$$(2)\quad \inf_{x=(y,z)\in X} f(x) = \inf_{\substack{y\in Y \\ z\in Z \\ (Y,Z\subset X)}} f(y,z) = \inf_{y\in Y} \varphi\left[\, p_1(y,z^1)_{z^1\in Z^1(y)},\ldots,p_N(y,z^N)_{z^N\in Z^N(y)}\right]$$

where: $z^1, z^2, \ldots, z^N$ are separate parts of the vector z,
$Z^i(y)$ for $i = 1,2,\ldots,N$, is the set of permissible vectors z^i depending on y,
f is the objective function of the not decomposed problem,
φ is the global objective function of the decomposed problem,
p_i, for $i = 1,2,\ldots, N$, are the subsystem-objective functions.

If certain conditions are fullfilled, the solution of the decomposed problem (2) is at the same time solution of the primary problem (1).

Let us now discuss the conditions which are to be fullfilled in order to apply the decomposition principle. Let us assume that there is a set X' (decision space) of the discrete decision variables which is decomposed into 2 parts. In order to apply the decomposition, the objective function must be separable with respect to decomposed parts so that elements of one part of the set must not be included into the objective function of the second part. Some of the restrictions which are imposed upon the decision variables of the first decomposed part concern also certain variables of the second part. These decision variables which have common restrictions will be treated as the coordination variables. Let us present it in the matrix form (fig. 1). Let us assume that objective function is additive, which is the premise of the decomposition of the problem. Let us consider the restriction matrix with 1 lines, each of them representing an restriction R_i $(i = 1,2,\ldots,1)$ imposed on decision variables. When the restriction matrix is completely filled up (fig. 1a), any decomposition is impossible. The problem must be then treated as one inseparable whole. The opposite extreme case arises, when the restriction matrix has the block-diagonal structure (fig. 1b). In that case one has not to do with one general problem but with separate problems which can be solved independently.

In the case of systems there always exists partly overlapping of the restrictions relating several subsystems (fig. 1c). These decision variables which occur in more than one restriction groups, let us call them y, do the coordinating and controlling with regard to the decomposed parts of the problem. The remaining decision variables of the decomposed parts will be called z.

In order to solve the decomposed problem the method of the two--stages-optimization(parametric optimization)can be sucesfully applied [1]. It consists in optimizing in two levels. In the lower level a multiple optimization of the decomposed parts is done with regard to their decision variables for consecutive values of the coordination variables which are assigned in the upper level and - in the lower level - treated as paramaters. The results of the lower level optimization $(\hat{z}(y))$ are availed in the upper (coordination) level for searching optimum values of the coordination variables y. As optimization procedures the searching methods are used in both levels.

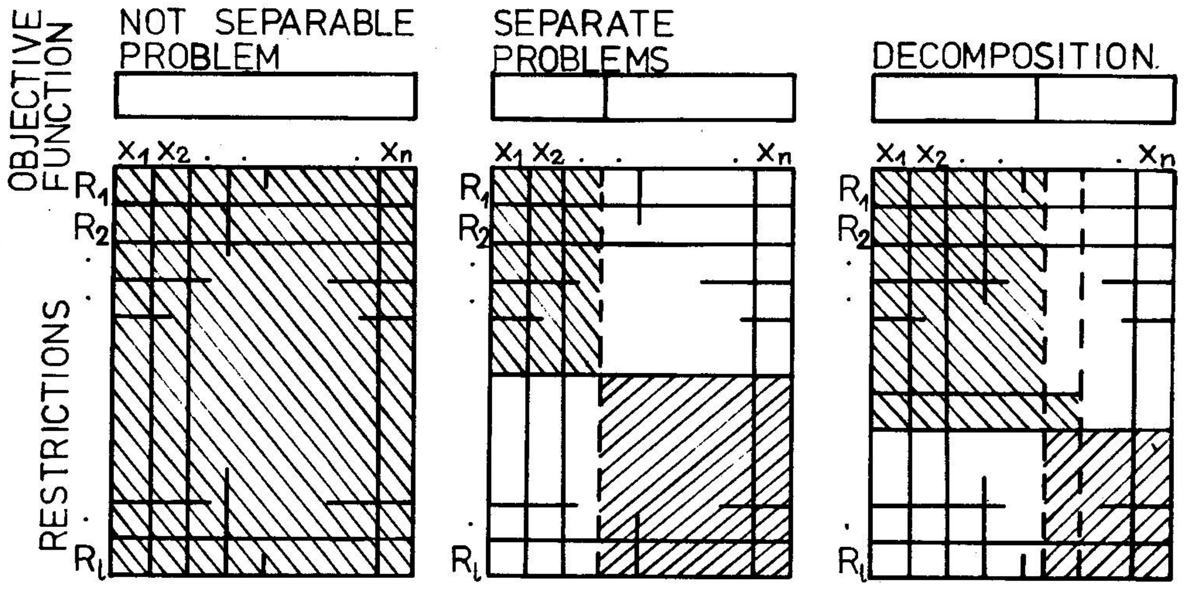

Fig. 1 Matrix representation of optimization problems

The condition for applying the two-level-optimization is that in the upper (coordination) level the restrictions imposed upon the coordination variables y must not include the variables z. In the lower level, i.e. in the decomposed parts, the restrictions can concern as well the variables z and y.

3. CASE STUDY

An example of the application of the decomposition to the optimization of a system is the optimization of the steel structure of industrial sheds (fig. 2).

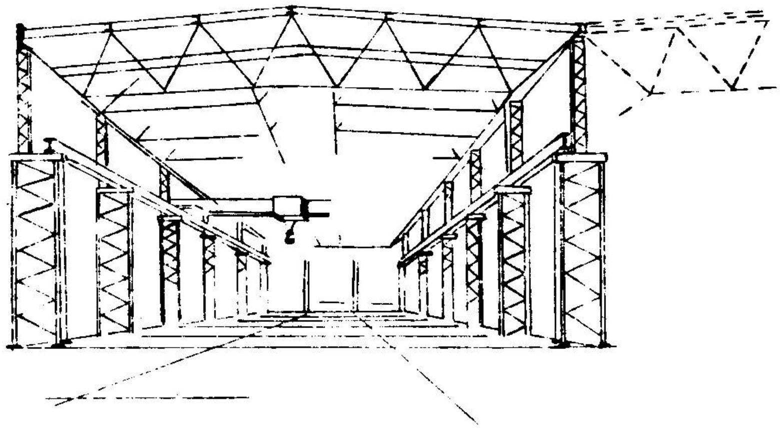

Fig. 2 Steel structure of an industrial shed

The structure of an industrial shed is a system which consists of a number of structural elements (subsystems) , see fig. 3. If each element is optimized separately this can lead to contradiction from the view point of optimum of the system as a whole. E.g. the optimum height of the cross-section of the gantry beam, found for the beam as a separate structure, can lead to the enlarging of the cubage of the shed and to the greater use of steel in higher colums. The optimization of a system cannot be therefore done on just assembling optimum solutions of the subsystems.

For solving the problem of the optimization of the structure of industrial sheds the author applied the decomposition method [2 - 6] .

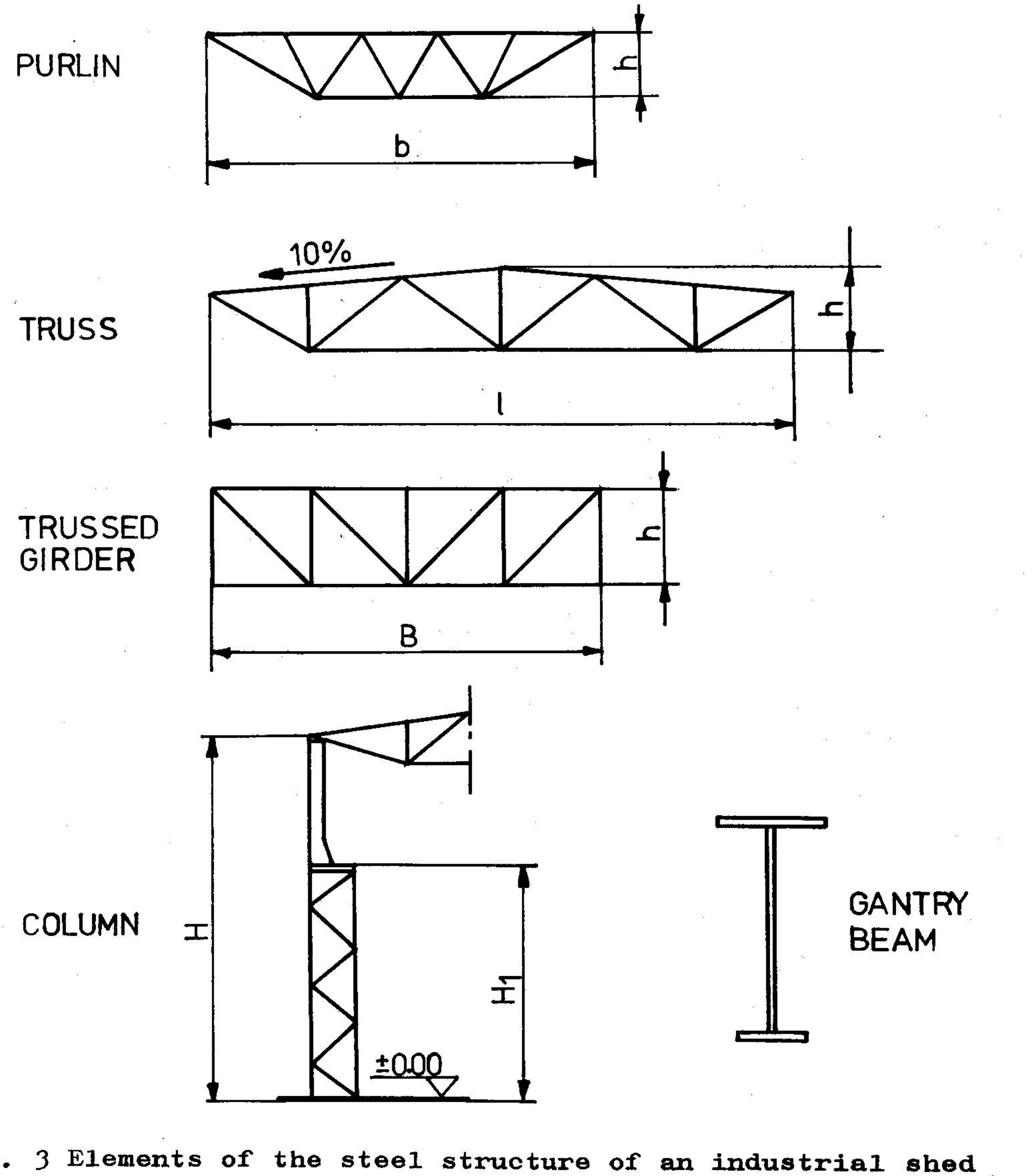

Fig. 3 Elements of the steel structure of an industrial shed

As the decomposed parts following subsystems were separated:

1. purlin,
2. truss,
3. trussed girder,
4. gantry beam,
5. column.

The optimum criterion was the cost of the carrying structure of the shed, objective function was therefore the sum of the execution cost of the subsystems. This function was separable with respect to the subsystems. The decomposition was therefore possible.

In the restrictions three decision variables refer to more than one subsystem. These variables stand for coordination variables. The mathematical model of the optimization problem of the shed structure - after decomposition - is:

$$\inf_{\substack{x \in X \\ y \in Y}} Q\,(x,y) = \inf_{y \in Y} \sum_{i=1}^{n} \inf_{x^i \in X^i(y)} Q_i\,(x^i, y) \qquad (3)$$

where: Q - the objective function of the whole system,
Q_i - the objective functions of i-subsystem,
x - vector of all subsystem-decision variables,
x^i - vector of the decision variables in the i-subsystem,
y - vector of the coordination decision variables,
X - permissible set of x-vectors,
Y - permissible set of y-vectors,
$X^i\,(y)$ - permissible set of x^i-vectors. This set is a function a function of y-vector, $\left(\bigcup_{i=1}^{n} X^i = X \right)$.

The substance of the formula (3) can be expressed in words as follows: The constrained infimum of the vectors x and y, is equal - after decomposition - to the constrained infimum (with regard to the coordination variables y) of the sum of the constraines infima of the subsystem objective functions Q_i, defined respectively for their decision variables x^i and suitable coordination variables y.

In the decomposed problems of finding infima of the subsystem--objective functions Q_i the decision variables are parameters.

The decision space of the optimization structure of a shed is rather a general kind. It was not possible to reduce it to the Euclidian space, because several decision variables were of topological kind (e.g. truss type, profile type) and they could not be represented by numbers. This fact limited substantially the amount of suitable optimization procedures. To solve the problem of the optimization of the shed structure the exhaustive enumeration was mainly applied.

The problem of the optimization of the steel structure of one aisle industrial shed is the 33 - dimensional problem. Using the decomposition the dimension of the problem was reduced to the threedimensional coordination problem and 5 parallelly solved decomposed problems from 6 to 7 dimensions.

A computer system called OSY, developed by the author and his team, was based on the above presented decomposition principle. This system performs the optimization of the steel structure of industrial sheds. The system OSY was implemented on Polish ODRA and British ICL computers.

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