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# THE MODELLING OF SPECIAL WATER FILLED STRUCTURES UNDER SEISMIC LOADS

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**ABSTRACT**

Special water filled structures (water-towers) need special rules for the design against earthquakes. The effects of water, the plastification of the structure and the soil-structure-interaction have to be taken into account. The corresponding design rules are discussed considering a watertower, destroyed during the Friuli earthquake of May 6, 1976.

**ABSTRAIT**

Les réservoirs d'eau sur hautes piles exigent des méthodes de calculation spéciaux pour le design seismic. Les effects de l'eau, la plastification de la structure et l'interaction du sol avec la structure doivent être considérés. Les règles correspondentes pour le design seismic sont traitées à l'exemple d'un réservoir d'eau qui a été détruit pendant le tremblement de terre en Friuli du 6 mai 1976.

**AUSZUG**

Wasserbehälter auf hohen Sockeln (Wassertürme) verlangen nach besonderen Berechnungsmethoden für die seismische Auslegung. Insbesondere sind die Wassereffekte, das Plastifizieren der Struktur sowie die Bodenflexibilität in Erwägung zu ziehen. Am Beispiel eines während des Friauler Erdbebens vom 6. Mai 1976 zerstörten Wasserturmes werden die wichtigsten Bemessungsregeln beleuchtet.

## 1. INTRODUCTION

Among the buildings destroyed during the Friuli Earthquake of May 6, 1976 was a watertower at the Gemona railway station, which apparently had not been sufficently designed against earthquake forces (Figure 1).

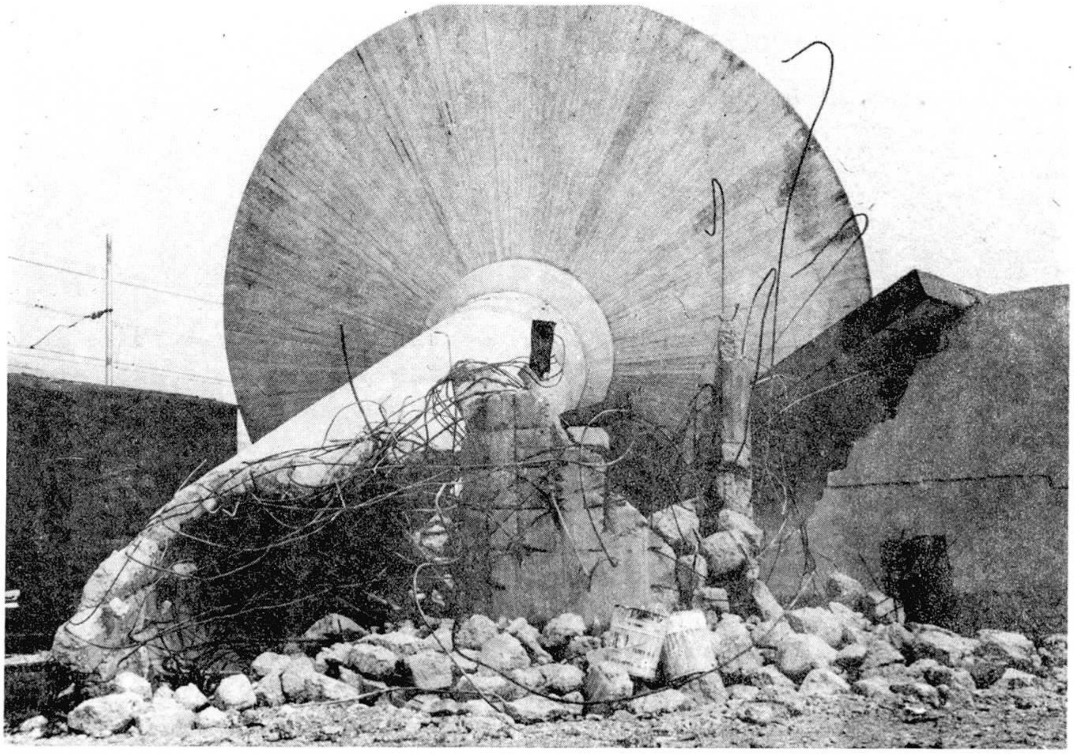


Figure 1: Destroyed watertower at Gemona railway station

Buildings of this type are of special interest because they cannot be designed by normal rules of a building code. Moreover, they belong to lifelines which demand special design considerations.

The purpose of this paper is to discuss very simple rules for modelling and designing such watertowers from a practical point of view, and besides give an answer to why the watertower at Gemona railway station failed.

Taking the hydrodynamic effects into account, Housners method [1] is expanded to conical and composite containers. Structural nonlinearities as plastic flow and P-$\Delta$-effect are considered too.

## 2. DESIGN PROBLEMS

Some of the most important questions a designer of a watertower has to answer are the following:

a) How is the water pressure on the container wall to be calculated? Has the water-structure-interaction to be taken into consideration?
b) Which is the effect of the water on the frequencies of the whole structure and on the bending moment at the base of the tower?
c) How has plasticity to be taken into account; which model would be appropriate?

d) How would the vertical force (weight of water and container) influence the response and the failure?
e) Does the nonsymmetric opening (door) at the base of the tower produce torsion, resulting in stiffness-degrading or the column?
f) Which is the influence of the foundation flexibility on the response?
g) Which form of seismic input would be appropriate to this type of problem?

## 3. BASIC CONCEPT FOR DESIGN

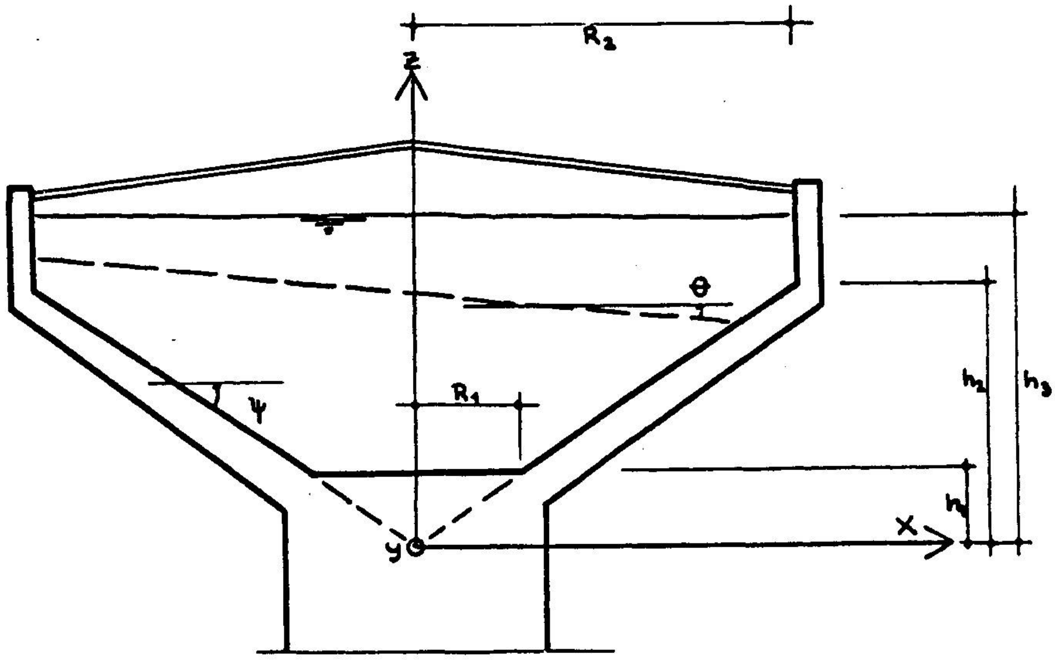
The watertower has to be designed in such a manner that it will withstand the specified earthquake without being fully destroyed but it may be severely damaged. In other words the behaviour of the structure during the specified earthquake may go into the plastic range, however the structure's ultimate resistance should not be exceeded.

For ease of handling the design by the Structural Engineer it should be tried to decouple the water-structure-interaction problem and to solve the hydrodynamic problem by a simple method.

## 4. HYDRODYNAMIC FORCE

As mentioned before it will be tried to decouple the hydrodynamic problem from the determination of the overall structural response. This can be accomplished if

- the container walls are assumed to be rigid,




$h_1$ = 0.74 m $\quad$ $R_1$ = 1.30 m
$h_2$ = 2.54 m $\quad$ $R_2$ = 4.55 m
$h_3$ = 2.89 m $\quad$ $tg\Psi$ = 0.558

Figure 2: Geometry of the container of the considered watertower (Gemona railway station)

- the water is considered to be incompressible,
- Housner's concept [1] of added masses and springs for modelling the water is employed

It may be shown that in the present case the first two assumptions lead to reasonable results. It is recommended to use the added masses concept since until now there exists no exact direct solution of the hydrodynamic problem in containers of the given shape (Figure 2).

In the following, the methods to determine these added masses and springs are discussed. Housner [1] destinguished the pressures on the container wall in respect to impulsive and convective pressures. In the present case, where part of the container is of conical shape (Figure 2), the convective pressures are of special importance.

## 4.1. Convective pressures

When the walls of the fluid container are subjected to horizontal accelerations, the fluid itself is excited into oscillations and this motion excerts pressures on the walls of the container. To examine the first mode of vibration of the fluid, constraints to be provided by horizontal, rigid membranes, free to rotate are considered as in [1] and as shown in Figure 2 (the rigid membranes are inclined by the angle ). Following the same procedure as in [1], a velocity field which satisfies the boundary conditions at the rigid wall is assumed which means that the water particles move parallel to the wall. Such a velocity field is found in the conical part of the cylinder to be of the form

$$\begin{aligned} u &= \frac{1}{3} b^2 \dot{\theta}_{,z} + \frac{1}{a} r \dot{\theta} \\ v &= -\frac{1}{3} xy \dot{\theta}_{,z} \\ w &= x \dot{\theta} \end{aligned} \tag{1}$$

and in the cylindrical part (from [1])

$$\begin{aligned} u &= \frac{1}{3} b^2 \dot{\theta}_{,z} \\ v &= -\frac{1}{3} xy \dot{\theta}_{,z} \\ w &= x \dot{\theta} , \end{aligned} \tag{2}$$

where r, b, x, y, and $\theta$ are defined in figures 2 and 3, $a = tg\Psi$, a dot means differentiation with respect to time and $(\ )_{,z} = \partial(\ )/\partial z$. u, v, w are the components of the velocity $\underline{v}$. The equations (1) differ from the equations (2) only in the component u, where in (1) the second summand is due to the conical shape of the container.

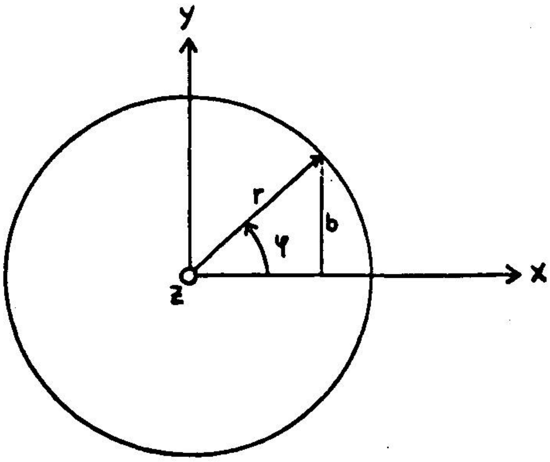



**Figure 3:** Plan view of the container (z = const.)

Assuming the distinct solutions $\theta_1$ and $\theta_2$ in the conical part and the cylindrical part of the container respectively, and by applying Hamilton's Principle

$$\delta \int_{t_1}^{t_2} (T-V)\, dt = 0,$$

T : kinetic energy of the fluid,
V : potential energy of the fluid,

the following set of differential equations and boundary conditions for the unknown $\theta_1$ and $\theta_2$ are derived:

$$(2\alpha - \beta_{,z})\,\dot{\theta}_1 - 2(\gamma\,\dot{\theta}_{1,z})_{,z} = 0, \qquad h_1 \leq z \leq h_2$$

$$\varepsilon\,\dot{\theta}_2 - \gamma\,\dot{\theta}_{2,zz} = 0, \qquad h_2 \leq z \leq h_3$$

$$\dot{\theta}_1 \quad \text{finite or} \qquad z = h_1 = 0$$

$$\dot{\theta}_1 = 0, \qquad z = h_1 > 0 \tag{3}$$

$$\dot{\theta}_1 = \dot{\theta}_2, \qquad z = h_2$$

$$\beta\,\dot{\theta}_1 + 2\gamma\,(\dot{\theta}_{1,z} - \dot{\theta}_{2,z}) = 0, \qquad z = h_2$$

$$\gamma\,\ddot{\theta}_2 + g\,\varepsilon\,\theta_2 = 0, \qquad z = h_3$$

where

g : gravity acceleration

$$\alpha = \left(\frac{1}{4} + \frac{1}{a^2}\right)\pi r^4$$

$$\beta = \frac{\pi}{2a} r^5$$

$$\gamma = \frac{2\pi}{27} r^6$$

$$\varepsilon = \frac{\pi}{4} r^4 .$$

Solving for $\dot{\theta}_1$ and $\dot{\theta}_2$ :

$$\dot{\theta}_1 = (a_1 z^{\lambda_1} + a_2 z^{\lambda_2}) C \sin \omega t \tag{4}$$

$$\dot{\theta}_2 = (\coth \mu z + a_3 \sinh \mu z) C \sin \omega t .$$

These equations are quite general and may be taken to solve similar problems for containers with different shape.

With the dimensions of Figure 2,

$$\omega = 3.22 \ \sec^{-1}$$

$$a_1 = 1.293 \qquad a_3 = -0.6438$$

$$a_2 = -0.3920 \qquad \mu = 0.4038 .$$

The pressure p in the fluid is given by

$$\text{grad } p = -\rho \dot{\underline{v}} ,$$

which leads to the horizontal force $P_x$ on the wall. $P_x$ is calculated in the considered example as

$$P_x = -89.66 \rho \pi \omega C \cos \omega t .$$

The moment M exerted on the wall turns out to be

$$M = -733.2 \rho \pi \omega C \cos \omega t ,$$

so that the elevation of the applied force $P_x$ is given by

$$z = 8.18 \text{ m} .$$

This is an important result because it shows that containers of conical shape lead to much higher moments than comparable cylindrically shaped containers. Consequently, in the case of conical and combined containers as in the considered example, application of Housner's added masses for cylinders would lead to unsatisfactory results. For this example, the added mass and spring may easily be calculated. By comparing the exerted force on the wall and the kinetic energy of the added mass with the respective amounts of the fluid, the added mass $m_1$ and the spring k are given by

$$m_1 = 0.602\ m_o \qquad m_o\text{: mass of water}$$
$$k = 4.78 \cdot 10^5\ \text{kg/sec}^2.$$

The respective figures for a cylindrical container of equal volume and radius as for the considered combined container are

$$m_1 = 0.545\ m_o$$
$$k = 7.33 \cdot 10^4\ \text{kg/sec}^2$$
$$w = 1.33\ \text{sec}^{-1}.$$

As mentioned, the combined container leads to a significantly higher frequency for the first mode of the fluid than for a comparable cylindrical container.

Since an exact solution of the hydrodynamic problem in the case of the considered combined container is not available as of today, a direct check of accuracy is not possible. However, a comparison of the frequency of a rectangular conical container with the exact solution given by Troesch [2] shows, that the velocity field (1) leads in this case to the exact frequency.

## 4.2. Impulsive Pressures

No model exists in the sense of Housner's concept [1] , which allows to calculate the impulsive pressures for combined containers. Therefore it is suggested to choose an added mass, rigidly fixed to the container, calculated for a comparable cylindrical container following Housner's formulas. Since the added mass will be slightly overestimated by this procedure, the design proves to be conservative.

# 5. STRUCTURAL RESPONSE

## 5.1. Force-Deflection-Relationship

Considering now as an example the watertower at Gemona station (Figure 4), the structural response taking into account nonlinear effects and soil flexibility is discussed.

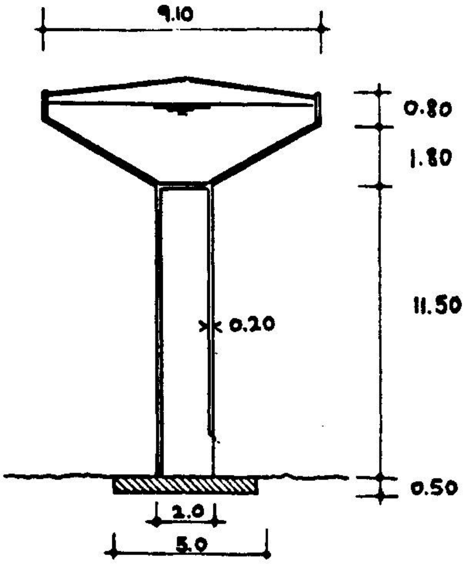



Figure 4: Watertower at Gemona railway station

First, the momentum-curvature-relationship of the reinforced concrete column is evaluated. As may be seen (Figure 5), the existing normal force increases the ultimate moment at ductile failure considerably.

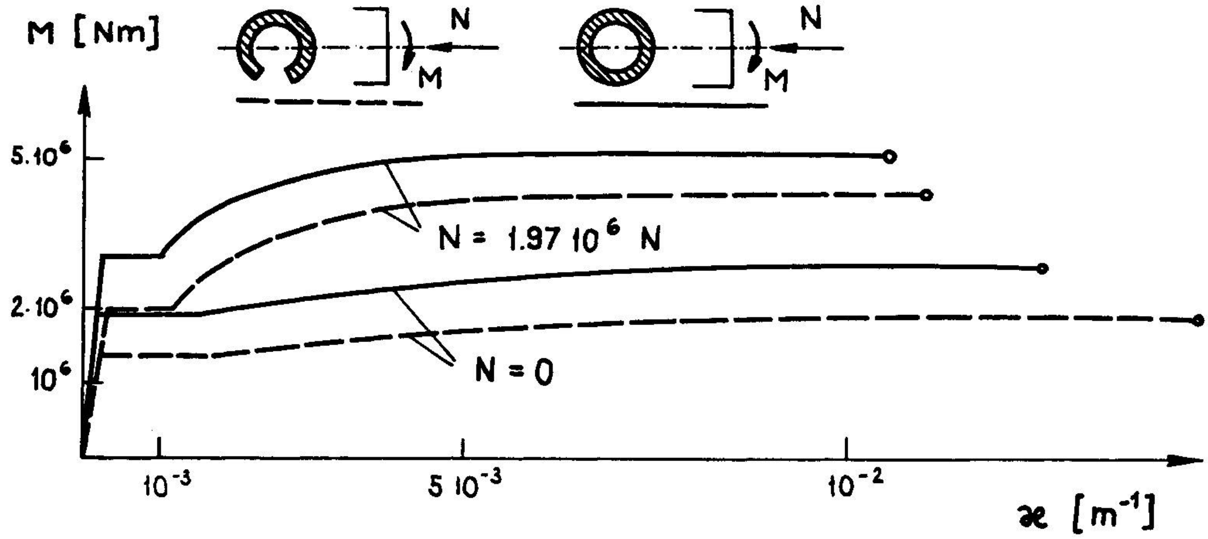



Figure 5: Momentum-curvature-relationship for the reinforced concrete column. The normal force $N = 1.97 \cdot 10^6$ N is the result of water and structure weight.

To get the deflection $\delta$ of the column due to the horizontal force F (Figure 6), the curves of Figure 5

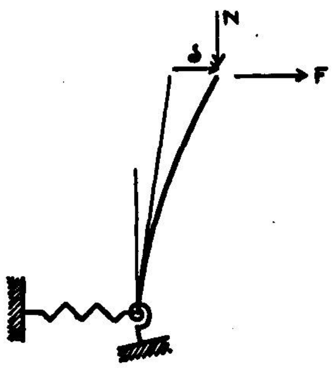



Figure 6: Structural model to calculate the force-deflection-relationship

have to be integrated over the length of the column for different forces F. This leads to the nonlinear force-deflection-relationship indicated in Figure 7.

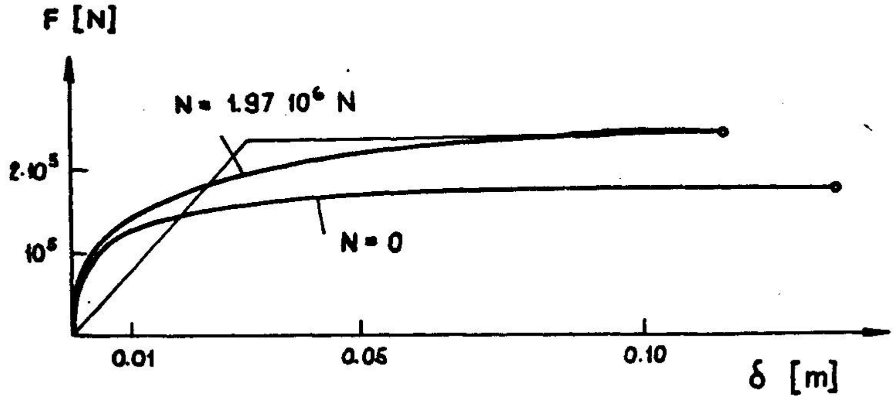



Figure 7: Force-deflection-relationship for the structural system of Gemona watertower

While the P-Δ-effect, which reduces the ultimate force F by approximately 10%, cannot be neglected, the decrease of stiffness due to the shear force may be disregarded.

Consequently, the vertical force influences the ultimate bearing capacity of the structure considerably. On the other hand, stiffness degrading which appears under high cyclic loading above failure deformation, has not to be taken into account here because the response above this limit is not considered.

For a simplified elastic analysis, an appropriate linear stiffness of the column may be chosen as indicated in Figure 7.

## 5.2. Elastic Analysis

An elastic analysis of the water-structure-system can be performed to study the coupling mechanisms of water and structure. For the purpose of determining and discussing the response of Gemona watertower a two-degree-of-freedom-system (Figure 8) is chosen.

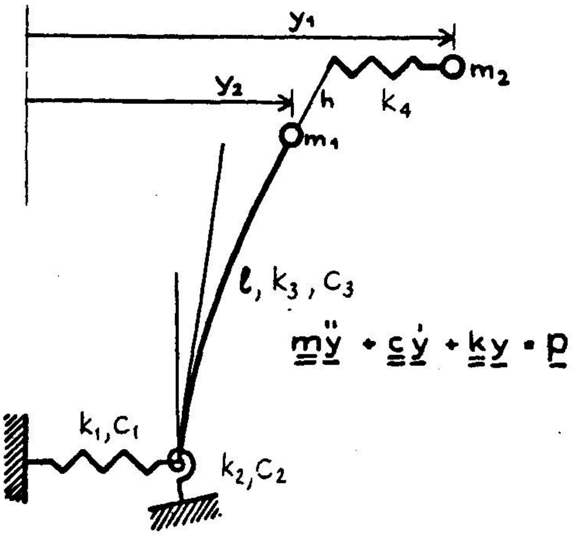



Figure 8: Structural system of Gemona watertower for elastic response analysis; $m_1$ is the sum of the weight of the structure and Housner's added mass for impulsive pressure; $m_2$ and $k_2$ account for the added mass-spring-system resulting from the convective pressure in the composite combiner.

For a cylindrical container with flat bottom, h would be near zero, while in the present case h is not negligible and influences the response considerably. The input parameters for an elastic analysis are (see Figure 8):

| | |
|---|---|
| $k_1 = 1.56\ 10^9$ kg/sec$^2$ | $C_1 = 26$ % |
| $k_2 = 9.52\ 10^9$ kg/m$^2$/sec$^2$ | $C_2 = 7$ % |
| $k_3 = 4.10\ 10^6$ kg/sec$^2$ | $C_3 = 7$ % |
| $k_4 = 4.78\ 10^5$ kg/sec$^2$ | |
| $l = 13.9$ m | $m_1 = 1.32\ 10^5$ kg |
| $h = 5.84$ m | $m_2 = 4.61\ 10^4$ kg. |

The soil stiffness and damping parameters were derived from usual elastic half-spare theory, and the column stiffness $k_3$ was reduced by considering the fact, that the centre of gravity of mass $m_1$ lies above the upper end of the column.

A response spectrum analysis was performed, using a provisional response spectrum of ENEL, recorded at Tolmezzo (Italy) during the Friuli earthquake of May 6, 1976. Local soil conditions at Gemona were considered by introducing an increasing factor of 1.4, which leads to a maximum free field acceleration $a_o = 0.43$ g.

The eigenfrequencies of the system were found to be

$$f_1 = 0.425 \text{ Hz}$$
$$f_2 = 1.02 \text{ Hz}$$

while

$$P_{x\,max} = 7.8\ 10^4 \text{ N}$$
$$M_{max} = 4.1\ 10^6 \text{ Nm}$$
$$\delta_{max} = 0.072 \text{ m.}$$

Two important conclusions may be drawn from these results and the respective mode shapes: A strong coupling between the water and the structure mode is noticed. Thus, the water influences the response of the whole system considerably. This fact is in direct connection with the parameter h. If h vanishes as e.g. for cylindrically shaped containers, this coupling effect becomes insignificant.

Secondly it is shown that the Gemona watertower had to fail because the calculated maximum moment at the base exceeds the ultimate moment by about 20 %.

In this example the rocking spring contributes to the response of the whole system by the amount of 10 % and therefore may not be neglected, while the horizontal spring turns out to be of little influence.

## 5.3. Torsion

A nonsymmetrical opening (door, 2.0 x 0.6 m) at the base of the watertower gives rise to the question if torsion may impair the bearing capacity of the structure. Considering a twisting moment which results from the maximum horizontal force exerted on the structure, it can be shown, that the ultimate bending moment (Figure 5) is reduced by about 3 % and the overall stiffness of the column (Figure 7) by 6%. Furthermore, a comparative elastic response spectrum analysis shows that the deflections due to twisting of the lowest part of the column is of no significance. Therefore it is concluded that torsional effects usually can be ignored.

## 5.4. Nonlinear response

In paragraph 5.2. a strong mode coupling was found. This mode coupling leads to both moment and horizontal force loading at the top of the column. If the moment was absent as e.g. for cylindrical containers, one might use the simplified bilinear force-deflection-relationship of Figure 7 directly and integrate step by step over time by applying e.g. the linear acceleration method. This method is easy to apply and gives reasonable results. However, to get the nonlinear response of watertowers with conical containers, this method is not acceptable. In this case, where the moment at the top of the column is not absent, at every time step the moment-curvature-relationship of Figure 5 has to be integrated over the length of the column to give its deflection for a given pair of M and F at every time step. Alternatively a nonlinear finite element procedure may be chosen.

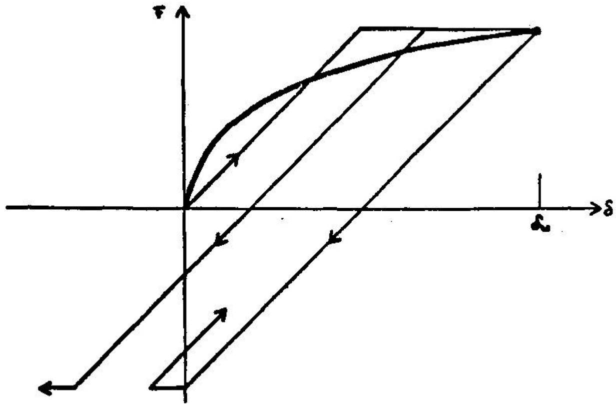



Figure 9: Simplified bilinear hysteretic behaviour of the Gemona watertower column.

## 6. CONCLUSION

In evaluating the response of watertowers it is possible to decouple the hydrodynamic problem from the determination of structural response by using the added mass concept. Fluid oscillations produce much higher moments in conical containers than in cylindrically shaped conbiners with flat bottom. Therefore it is recommended to construct only watertowers with vertical walls and flat bottom in seismic active zones.

In evaluating the seismic structural response, the weight of the water-structure-system and the rocking mode are to be considered while the twisting moment due to small nonsymmetries, and the shear force may usually be neglected. An elastic response spectrum analysis may be performed to evaluated the coupling behaviour of the modes. However, this method is not recommended for design purposes. Since the ultimate strength of the column is equal to the bearing capacity of the whole system and the design shall be such that the ultimate resistance is reached, a nonlinear analysis is required. This nonlinear analysis can be performed easily by small computer routines in case of a cylindrical containment with flat bottom; in case of a conically shaped containment a nonlinear finite element analysis would be appropriate.

Up to date, usual building code recommendations cannot be taken as design basis. Therefore, it is recommended to incorporate in the codes rules for the design of watertowers as developped in this paper.

## ACKNOWLEDGEMENT

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## REFERENCES

[1] Housner, G.W.: Dynamic Pressure on Accelerated Fluid Containers: Bulletin of the Seismological Society of America, Vol. 47, No.1, Jan. 1957

[2] Troesch, B.A.: Boundary Problems in Differential Equations; Proceedings of a Symposium Conducted by the Mathematics Research Center at the University of Wisconsin, Madison, April 20-22, 1959, ed. by R.W. Langer, Madison, The University of Wisconsin Press., 1960