

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen  
**Band:** 30 (1978)  
**Artikel:** Strengthening of masonry buildings  
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**DOI:** <https://doi.org/10.5169/seals-24174>

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## STRENGTHENING OF MASONRY BUILDINGS (+)

by

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## SUMMARY

The problem of strengthening buildings made by bricks and stone-work is considered at two principal levels: (a) invention, description and evaluation of various techniques for strengthening; (b) cost-benefit analysis and design methods. Level (a) is introduced in the paper as a survey presentation. Level (b) consists in the statement of the problem of design as an optimization with logical (yes-no) variables. The problem is translated into a graph and solved by a method of critical path.

## RESUME

Le renforcement des bâtiments en pierre et/ou briques est considéré à deux niveaux: (a) invention, description et évaluation des techniques. (b) analyse des coût-profit et décisions du projet.

Le niveau (a) est donné par une revue. Le niveau (b) est ici présenté par la formulation du projet dans la forme d'une minimisation à variables logiques. La solution est indiquée par la méthode du parcours critique d'un graph orientée.

## ZUSAMMENFASSUNG

Das Problem bezüglich der Verstärkung von Stein- und Ziegel-Bauten ist hier aus zwei Standpunkten betrachtet: (a) Erfindung, Beschreibung und Wertung von verschiedenen Versärkungstechniken; (b) Analyse von den Kosten-Ersparungen und Zeichnungsmethoden. Niveau (a) wird als Quellenverzeichnis dargelegt. Niveau (b) das Problem der Zeichnungsmethoden ist erklärt als Optimierung von logischen Variablen (ja-nein). Das Problem ist einem "graph" gegeben und durch die Methode "critical path" aufgelöst.

(+) Research carried out in the frame of C.N.R.'s Italian Geodynamics Project.

## 1. INTRODUCTION

In many countries of the world situated in seismic areas various urban settlements include old buildings made up by various techniques and materials among which bricks and stones are more commonly used. Quite often these buildings have a poor resistance against horizontal forces generated by earthquakes. This is due to many factors such as (a) the poor quality of mortars, (b) the inadequate bonds between orthogonal walls, (c) the high in-plane deformability of horizontal diaphragms, what prevents horizontal forces to be transferred to vertical resisting elements, (d) the poor bonds between slabs and walls. An important role is played by functional changes and manipulations which frequently old buildings experienced during their life: this causes either the weakening of bearing walls due to openings not accounted for in the original design or the addition of "new" parts to the buildings which give rise to planar dissymetries which in turn originate torsional effects during the seismic shock.

These elements point out the importance of the problem connected with the definition of strengthening methods for masonry buildings. In the Author's opinion, the problem can be splitted in different stages:

a) Invention, testing, and practical implementation of techniques to add resistance to buildings of the above mentioned type. The following chapter 2 is devoted to a survey of the literature and of the current practice in this field. Attention is paid to the evaluation of the additional resistance that can be obtained by different techniques, although quantitative results are **scarse**.

b) Statement of design methods for strengthening. A decision method in earthquake engineering rests on cost-benefit analysis; refs. [1] , [2] , [3] are examples of this approaches. In Chapter 3 of the present paper, cost-benefit analysis is implemented to deal with practical design. In the case of strengthening old buildings structural decisions to be taken are often quantified by logical rather than by scalar design variables. For example: the design for strengthening a masonry building may deal with the decisions of re-building or not slabs, or/and prestressing or not the wall... while it is not very important to define "to what extent" the new slab must bear or "to what extent" the prestressing should be. The decisions regarding "to what extent" often are not structural variables since they depend on technological and practical constraints. While decisions regarding "the what extent" are expressed clearly by scalar design variables, "to build or not to build" is expressed by a logical (yes-no) variable.

In Chapter 3 the traditional statements of the design seen as minimization problem are adopted. The functions to be minimized contain the cost of strengthening, the non-structural benefits due to the works of strengthening, the expected future monetary damages and the expected number of victims. In traditional cost-benefit analysis the minimization is carried out with respect to continuous design variables. Constrained minimization give rise to the concept of marginal cost, useful to incorporate non monetary aspects of the problem, such as the loss of lives. As it was stated earlier, in this case we have to deal with discrete (yes-no) design variables. As a consequence new minimization techniques have to be implemented. This will be done by representing the design space as a graph and adopting the critical path technique to minimize the object function. In addition the nature of implied variables makes the concept of marginal cost to be no more pregant. Two different uses of cost-benefit analysis may take shape. The first one (sec. 3.2) consists in the determination of the minimum cost of strengthening, considering also expected future damages but ignoring losses in human lifes. The second (sec.3.3) consists in

including the risk to human life as a penalty term. The two corresponding optimal design will bracket the range of reasonable solutions for practical design. Ref. [2] shows that in some instances this range is very narrow. Thus the use of the two above procedures allows to identify a sort of "feasible" region for strengthening design.

## 2. MAIN STRENGTHENING TECHNIQUES

Basically strengthening operations carried out on old buildings aim to give rise to a box-type structural behaviour. Continuous vertical elements need thus to be properly connected each other and to horizontal diaphragms which in turn have to transmit horizontal forces to resisting vertical walls proportionally to their stiffnesses. Moreover an appropriate distribution of shear walls has to be obtained in such a way that torsional effects are avoided.

These targets may be pursued in various manners: the essential features of the main techniques which are usually adopted will be shortly described in what follows.

### 2.1 Vertical plates

The basic idea of this procedure is to overlap to original walls new continuous resisting vertical structures. This can be made in several ways, i.e.:

(1) With reinforced concrete plates laid on the two sides of the wall and sewed together by transversal steel passing through the wall. These plates are usually more than 5 cm. thick, it turns out that the original walls become considerably bigger and heavier.



FIG. 1 A

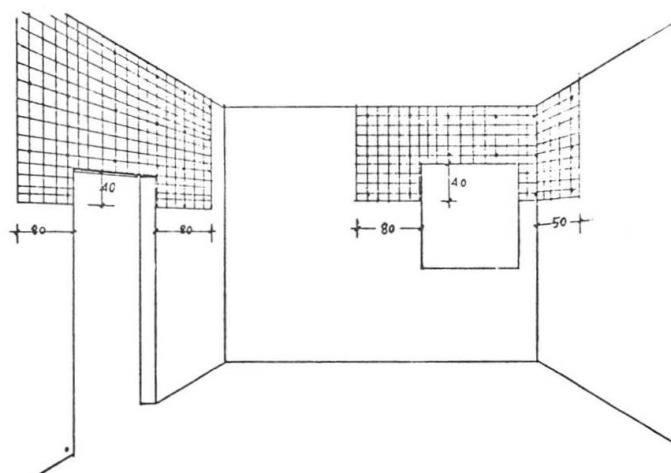


FIG. 1 B

A development of this technique [7] consists in the use of steel nets with modular shape of 15 X 15 cm. placed on both sides of the wall and mutually connected through the wall (fig.1). Concrete is spread over the net thus obtaining vertical plates about 3 cm. thick. A difficulty connected with the use of vertical plates lies in the poor continuity between the old and the "new" wall due to incomplete adhesion between the pre-existing and strengthening structure and due to shrinkage.

(2) These difficulties may be somehow overtaken by the use of gunite (or shotcrete). This is a method of applying a cement sand mix with an impact which assures a good bond. It is a mix with a rather good water-cement ratio for good strength and minimum shrinkage. Moreover this method of application provides

excellent freedom of shape. A 1.5 - 2 cm. thickness may be obtained. The use of gunite requires high-pressure equipments, what results into practical limitations of the method especially when one has to deal with walls of a very poor quality.

## 2.2 Horizontal runners

A traditional strengthening method consist in the use of horizontal r.c. bonds at roof, lintel and plinth level. Under a structural point of view such runners improve bending characteristics of walls transverse to the direction of the horizontal force by supporting them at fixed points and reducing bending tensions in the horizontal plane which occur when transverse walls behave as slabs due to seismic action. Good results may be obtained by coupling horizontal and vertical r.c. runners with appropriate connections among them. In this way a sort of framed system is achieved which exhibits a good resistance to seismic forces. This technique however produces strong changes in the original look of the building and this fact may constitute a restriction to its use.

## 2.3 Steel reinforcement

Reinforcing bars may be inserted in drilled cores which are then sealed by cement grouting.

This technique may achieve excellent results and shows the merit of not producing changes in the original look of the building. However drilling may take place successfully only in systems which already have enough strength: when the quality of the building is very poor it is advisable to proceed to an improvement of quality of the walls (e.g. by grouting) before inserting bars.

Steel reinforcement may be used both to achieve a bond between orthogonal walls and to increase the lateral of single walls (figs. 2-3-4). In the first case diagonal drills are performed on either vertical edges of the two walls. As far as the second problem is concerned different possibilities of placing reinforcement exist. In ref. [4] three different solutions were examined with reference to simple models (see fig.5) i.e. steel at vertical corners (fig. 5a), steel at jambs (fig. 5b) and steel both at vertical corners and jambs (fig. 5c). Experimental ultimate loads (defined as the load causing the first crack in each pier) show the following ratios in the three cases stated above:

$$(UL)_a : (UL)_b : (UL)_c = 1:0.89:1.56$$

As far as ductility is concerned tests show that when reinforcement exists anywhere in a pier this can take additional shear force after cracking. This does not happen with unreinforced wall, where failure is sudden.

In refs. [5] and [6] tests carried out at Roorkee school on models of brick buildings strengthened in various ways are reported. The following table shows the strengthening methods which have been investigated and the improvement of lateral resistance. Reference is made to the lateral resistance of the unreinforced house.

TABLE 1

Type	Ultimate Load
1) Unreinforced house	1
2) With lintel band	1
3) Lintel and plinth bands	1.25
4) Vertical steel at corners	2.95
5) " " at jambs	1.4
6) " " and corners	4.1
7) " " at corners + lintel band	3.2
8) " " at jambs + lintel band	1.6
9) " " at jambs + corners + lintel band	4.4

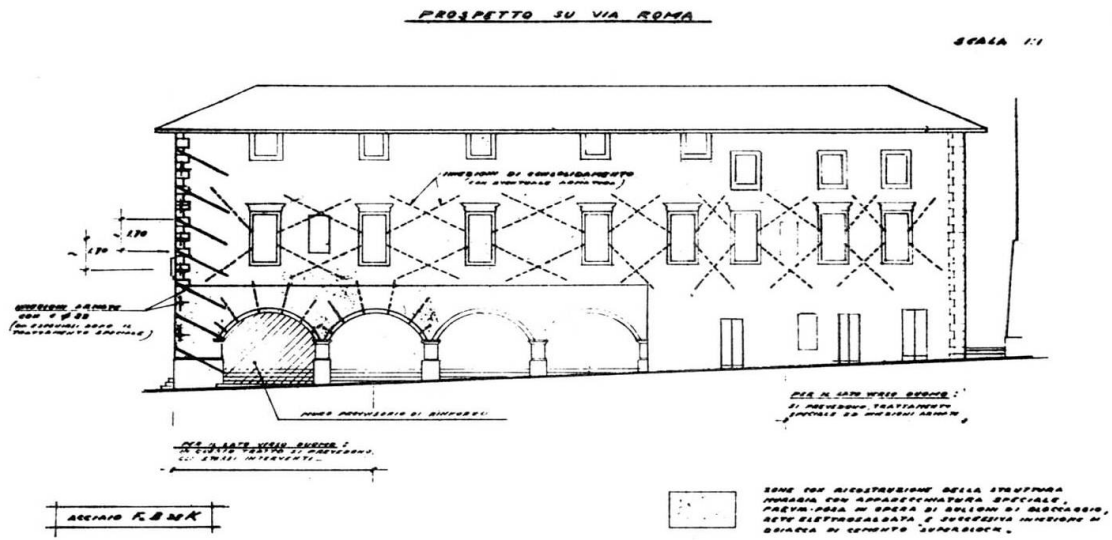


FIG. 3

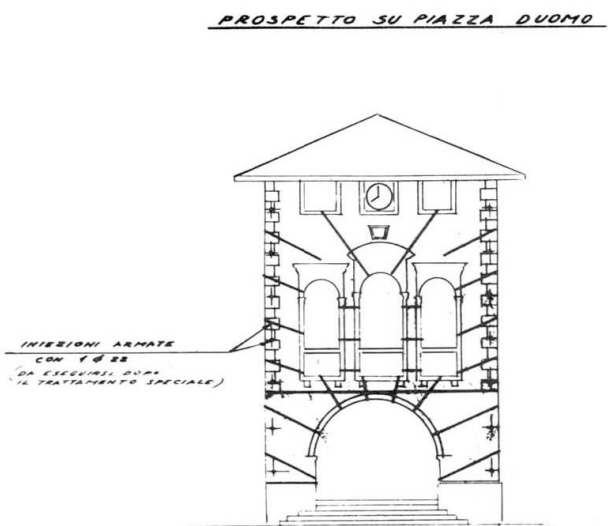


FIG. 2

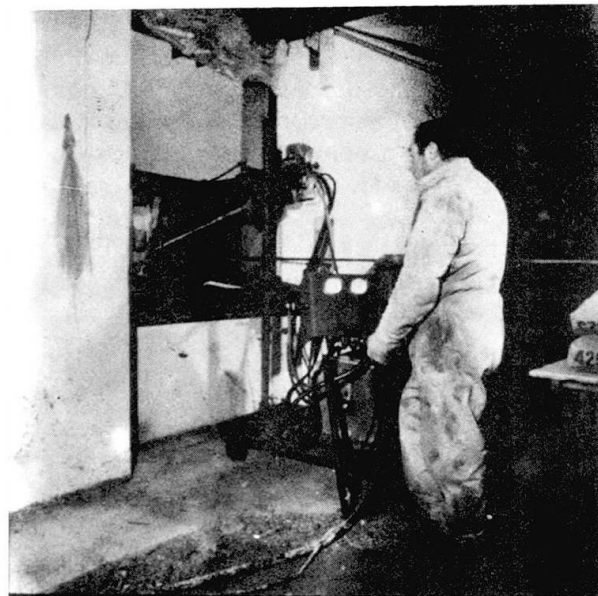


FIG. 4

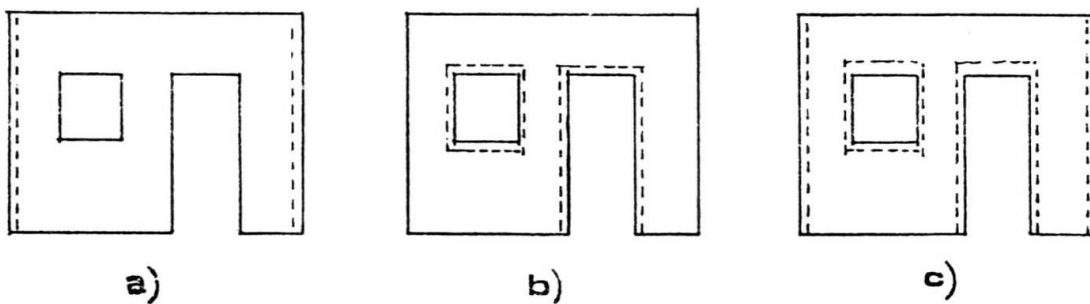


FIG. 5

Comments to above table reported in [5] point out that horizontal steel at lintel level does not contribute to lateral resistance, since failure occurs at the plinth level. This feature is confirmed by the comparison of the cases (4) and (5) with the cases (7) and (8) respectively which differ from the previous ones due to the lintel band whose effect towards lateral strengthening is seen to be of the order of the 12% as a minimum.

This respect it is worth noting however that the insertion of lintel bands improves the connection between orthogonal walls. In the case of originally poor tie between such walls horizontal steel may result in an increasing of lateral resistance. Moreover above results make clear the considerable importance of vertical steel. Steel at jambs is relatively less important with respect to the ultimate load: the overall resistance of the structure is however increased due to the better defense of corners resulting from the reinforcement.

#### 2.4 Prestressing

Prestressing of walls may be obtained by the use of vertical (fig.6) and horizontal (fig. 7,8) tendons which can be either inserted in drilled cores or placed on both faces of the wall. In the case of vertical rods they are threaded into foundations anchorages. Horizontal tendons are connected to vertical edges of walls by means of steel plates which distribute pressures over a portion of wall. Usually bars of 14-18 cm. of diameter are employed to this aim. It should be noted that the use of prestressed tendons may produce changes in the original statics of the building which might not be suffered by poor quality structures; it is thus advisable to previously undertaken strengthening operations which enable structure to withstand tendons. In some instances horizontal tendons are lied down on slabs connecting opposite walls or corners (fig.9). The basic aim of prestressing is to induce into the wall a biaxial state of compression in order to reduce tensions due to lateral load. Note that brickwork is especially suited for prestressing due to its limited creep and shrinkage characteristics [9]. In ref. [8] the following expression is given to represent the increase  $h$  of lateral resistance of a prestressed wall by means of horizontal rods:

$$\frac{\left( \frac{\sigma_o - \sigma}{3 \tau_K} + 1 \right)^2 - \left( \frac{\sigma_o - \sigma}{3 \tau_K} \right)^2}{1 + \frac{3}{2} \frac{\sigma_o}{\tau_K}} = h^2$$

being:

$\sigma_o$  = average vertical compression stress

$\sigma$  = horizontal compression stress

$\tau_K$  = ultimate shear stress with no vertical overloaded on the wall (self weight only).

Tests reported in [8] shown an increase of lateral resistance which is bigger for lower values of  $\tau_K$  (poor quality wall).

#### 2.5 Grouting

Intrusions of cement grout into wall interstices is frequently used. This technique shows the advantage of producing no change in the original look and in the original statics of buildings. For this last reason it is frequently employed before the use of other strengthening techniques (such as drilling or prestressing) in order to assure enough strength.



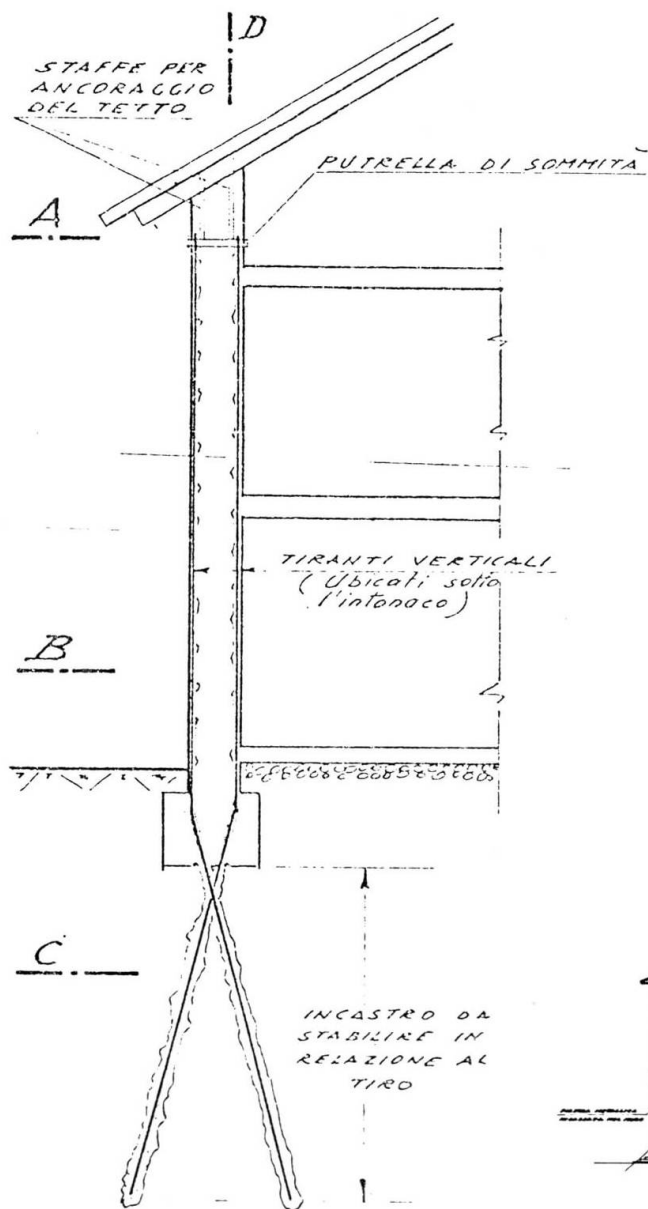


FIG. 6

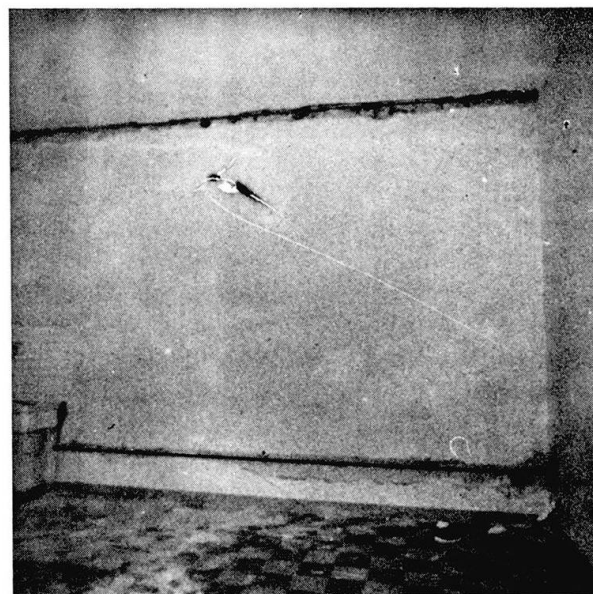


FIG. 7

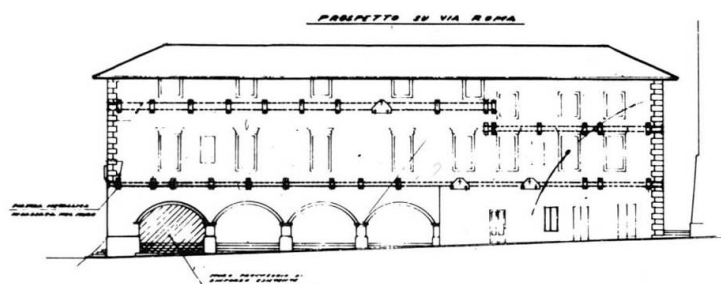


FIG. 8

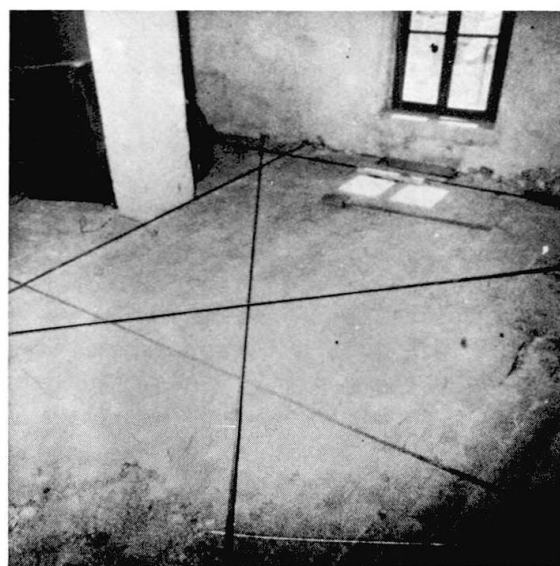


FIG. 9



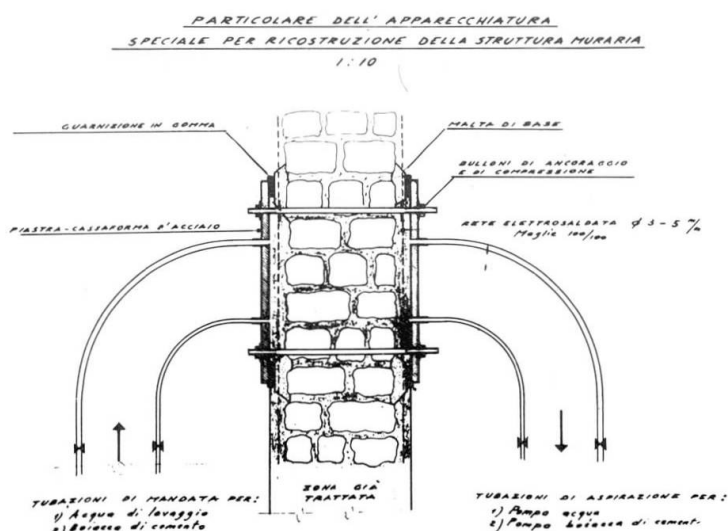


FIG. 10



FIG. 11

The efficiency of grouting is conditioned by the initial quality of wall, the type of cement mix used and by level to diffusion within the wall of the mixture. The last two factors play an important role in the determination of the overall cost of the operations.

Moreover grouting is not very effective with respect to the improvement of connections between orthogonal walls. Intrusions are often performed by drilling 4 cm. diameter cores with a spacing of 40-100 cm. into the wall (see fig. 10-11). Intrusions are made at a low pressure (3 - 4 Kg/cm<sup>2</sup>)

level so that exceeding water may be properly drained. Under a broad point of view it may be stated that by grouting the wall, which quite often is of a poor quality, may achieve a lateral strength of the same order of a well-made unreinforced wall. If an increase of lateral resistance is desired, as it may happen if a seismic provision is enforced to old buildings, grouting has to be coupled to other strengthening methods. Tests reported in ref. [8] show that the poorer in the wall the greater increase in lateral strength may be obtained.

### 3. DESIGN DECISIONS FOR STRENGTHENING

A cost-benefit statement of the problem of design deals with the following four items:

i) the cost of the strengthening:

$$C = \sum_{i=1}^n C_i \quad (1)$$

where  $C_i$  is the cost for the  $i$ -th type of strengthening work. For instance:  $C_1$  is the cost of re-building the slabs,  $C_2$  the cost of grouting, etc... In the following  $C$  and  $C_i$  are costs per year, and can be related to an unique investment via the concept of constant investment rate (or amortization)

ii) The benefit derived from each strengthening work, with the exception of benefit for structural (seismic-proof) consequences:

$$B = \sum_{i=1}^n B_i \quad (2)$$

For instance:  $B_1$  is the benefit for the new slabs and floor,  $B_2$  is the benefit for a water-proof external wall, etc... A criterion can be to evaluate  $B_1$  in terms of variation in rentability. Again  $B_1$  is referred to one year.

iii) The future monetary damages due to earthquakes. Assuming the intensity of the earthquake (such as peak ground acceleration) as an independent variable, a damage function of the type of fig. 12 for a single building is often [3] assumed. The value  $\bar{a}$  is the ground acc. corresponding to collapse of the building,  $\bar{a}$  marks the initial cracks,  $\bar{C}$  the monetary value of the building (in yearly units) and  $\phi$  is a factor of amplification due to the event "collapse" with reference to "total unserviceability" of the building.

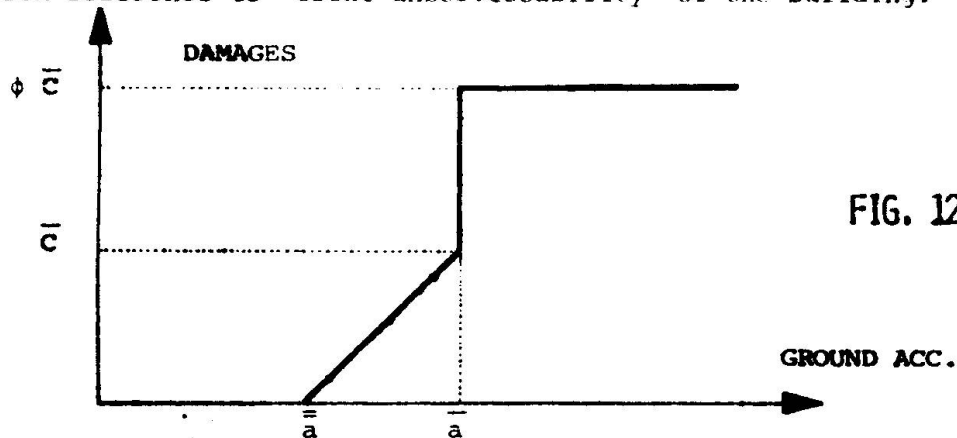


FIG. 12

In the present case  $\bar{C} = C_p + C$ , where  $C_p$  is the yearly cost of the building before strengthening. Ref. [3] shows that the future monetary damages can be expressed by

$$D = (1 + f) \cdot \phi \cdot \bar{C} \cdot N(\bar{a}) \quad (3)$$

where  $N(\bar{a})$  is the expected number of earthquakes per year having a peak ground acceleration greater than  $\bar{a}$ ;  $f$  is a parameter depending basically on the ratio  $\bar{a}/a$ . See ref. [3] for analytical expressions.

iv) The expected number of victims per year can be expressed by

$$V = \epsilon \cdot \bar{n} \cdot N(\bar{a}) \quad (4)$$

where  $\bar{n}$  is the number of people living in the building and  $\epsilon < 1$  is a factor taking into account absence of inhabitants, warning....

### 3.1 Statement of the optimization problem

In the literature the design problem derived from cost benefit-analysis is stated in different ways:

$$\min (C - B + D + \mu V) \quad (5)$$

$$\text{or} \quad \min (C - B + D); \quad V \leq K_1 \quad (6)$$

$$\text{or} \quad \min (V); \quad (C - B + D) \leq K_2 \quad (7)$$

The relationships among (5), (6), (7) are discussed in ref. [2], [3], together with the meaning of  $\mu$ . Minimization is carried out with respect to the design variables.

In the present case, as pointed out in the introduction, the design variables are expressed more properly by "yes-no" type decision. Therefore any particular point of the design space corresponds to a particular combination of

"presence" or "absence" of indices  $i$  in the terms  $C$  and  $B$ . A particular design corresponds to a certain value of the collapse acceleration of the building which is expressed in terms of the same indices  $i$  occurring in  $C$  and  $B$ :

$$\bar{a} = a_p \cdot \prod_{i=1}^n r_i \quad (\Pi = \text{serial product}) \quad (8)$$

where  $a_p$  is the collapse acceleration of the building before strengthening and

$$r_i = 1 + p_i \quad (9)$$

where  $p_i = 0$  if the  $i$ -th reinforcing has not be included in the design.  
 $p_i$  is the percentage of additional resistance due to the  $i$ -th work of strengthening included in the design.

It must be noted that the format of eq.(8) is suggested by table 1, sec.2.3, however other formats are compatible with the sequel. The evaluation of  $a_p$  can be a very serious problem. Since it is outside the scope of this paper, reference is made to the survey of refs. [10][11], and to the methodology discussed in ref. [12].

In conclusion: knowing the seismicity  $N(a)$ , once  $\bar{a}$  is evaluated through (8), a value of  $D$  and  $V$  can be also associated to it, through (3) and (4).

The expression appearing in (5), (6), (7) can now computed in principle. Therefore the constrained (6), (7) or unconstrained (5) minimizations can be carried out, provided  $\mu$ ,  $K_1$ ,  $K_2$  are given.

In what follows a technique to solve above optimizations is shown.

### 3.2 The minimum-cost strengthening

As stated in the introduction, the minimum cost design is such that

$$\min W = \min (C - B + D) \quad (10)$$

This corresponds to the problem (5) with  $\mu = 0$  and to problem (6) with  $K_1 = \infty$ .

In order to solve problem (10) let us draw a graph as in fig.13. The points of the graph are: a) a zero design corresponding to the not strengthened existing building, b) a row of points each corresponding to one strengthening work, (three in the example of fig. 13), c) other rows of points corresponding to works to be done in alternative: in the example of fig. 13 horizontal tendons into existing slabs is alternative to the complete re-building of the slab.

The arcs between the points are such that any point is connected to zero and to all the following points, with the exception of column-arcs (arc 2-3 in the example). Any design can be represented by a path starting from zero and ending to any point. For example, the path 0-2-4 means a strengthening with horizontal tendons and vertical prestressing.

The minimization problem is a problem of critical path: find the shortest way "d" from zero to any point. The length  $d_{i,j}$  is defined as

$$d_{i,j} = W_j - d_i$$

where  $d_i$  is the minimum value of  $(C - B + D)$  when only the works from zero to  $i$  are considered as design variables and  $W_j$  is the value of  $(C - B + D)$  when the work  $j$  is added to such optimum design.

The above statement and eqs.(3)-(8) point out that the lengths of all the arcs

cannot be calculated before the minimization (like in the classical critical path problem [13]). This is due to the nonadditive nature of the term  $D$  in eq.(10), see eqs.(3) and (8).

On the other hand, since the classical algorithm for critical path proceeds backwards, the classical minimization procedure can be used and the length  $d_{i,j}$ , calculated at any step. In fig.13 the steps for the sequential optimization are written for the example given. In general:

$$d_{o,i} = C_i - B_i + D_{o,i}$$

where  $D_{o,i}$  is the function  $D$  (eq.(3)), with

$$\begin{aligned}\bar{a} &= a_p \cdot r_i \\ \bar{C} &= C_p + C_i\end{aligned}$$

also

$$\begin{aligned}d_{i,j} &= C_j - B_j + D_{i,j} \\ D_{i,j} &= D_j - D_i\end{aligned}$$

$D_i$  is the function  $D$ , eq.(3), with  $\bar{a} = \bar{a}_i$  and  $\bar{C} = \bar{C}_i$  corresponding to the critical path design from zero to  $i$ ;  $D_j$  the same with

$$\begin{aligned}\bar{a} &= \bar{a}_i \cdot r_j \\ \bar{C} &= \bar{C}_i + C_j\end{aligned}$$

It is useful to take record of the values  $\bar{a}_i$ ,  $\bar{C}_i$  at each step, as fig.13 shows.

The recursive relation is:

$$d_i = \min \{ d_{o,i} ; \min_j (d_j + d_{j,i}) \} \quad (11)$$

where  $j$  ranges over all the arcs incident in point  $i$ .

The critical path is obtained as the design for which

$$d = \min_i \{ d_i \} \quad (12)$$

where  $i$  ranges over all the points.

### 3.3 Design including non monetary damages

We refer now to the general cases of eqs.(5), (6), (7). Ref. [2] defines  $\mu$  as the "maximum price the community is willing to pay in order save one life", and it is suggested to evaluate it by considering the other (rather than earthquake-induced) risks that the community has to face.

In the case of strengthening,  $\mu$  can be assumed to be equal to the same value associated to the definition of the seismic coefficient for new buildings. When  $\mu$  is given as a number, the solution of the problem (5) can be obtained by the same technique of problem (10). The only change consists in the addition of one term  $\mu V$  (via eq.4) in the computation of  $d_i$ ,  $d_{i,j}$ ... Indeed we have still a problem of unconstrained minimization.

Ref. [2] points out that in some instances the design is rather insensible to

changes in  $\mu$  : therefore it can be advisable first to solve the problem of sec. 3.2 ( $\mu = 0$ ) and then the problem (5) with a very large  $\mu$  (some million dollars). The two optimal designs will bracket the reasonable design solution.

In ref. [14] the solution of the problem for strengthening and replacement of building in urban areas produces an optimal value for the design collapse acceleration  $\bar{a}^*$  of the buildings to be strengthened. If this result is available, it may be taken into account in the problem of sec. 3.2. The only difference lies in the minimization of eq.(12) where only the  $d_i$  for which  $\bar{a}_i \geq \bar{a}^*$  must be considered.

It is also clear that problem (6) can be solved by dropping the terms  $d_i$  (in eq.(12)) for which  $\varepsilon \bar{n} N(\bar{a}) > K_1$ .

Problem (7) too can be solved by a similar technique.

#### 4. CONCLUSIONS

The nature of the problem of strengthening requires, as it has been shown, that structural choices pass through a minimization with discrete design variables. The techniques of solution are available and are presented and worked out for this particular problem herein.

Technical inputs to this problem, which have been outlined in chapter 2, are however rather scarce and need further research. The present paper points out the kind of experimental and theoretical information which need to be assessed for a rational choice of a strengthening design.

#### 5. ACKNOWLEDGEMENTS

The research has been sponsored by C.N.R. Thanks are due to ICOS, Milan, for supplying the picture of chapter 2.

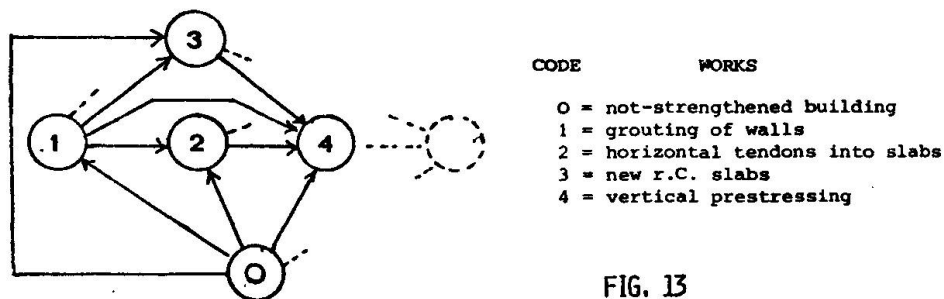


FIG. 13

$d_i$	path	$\bar{c}_i$	$\bar{a}_i$
$d_1 = d_{0,1}$	0 - 1	$c_p + c_1$	$a_p \cdot r_1$
$d_2 = \min \{d_{0,2} ; (d_1 + d_{1,2})\} = d_{0,2}$	0 - 2	$c_p + c_2$	$a_p \cdot r_2$
$d_3 = \min \{d_{0,3} ; (d_1 + d_{1,3})\} = d_1 + d_{1,3}$	0 - 1 - 3	$c_p + c_1 + c_3$	$a_p \cdot r_1 \cdot r_3$
$d_4 = \min \{d_{0,4} ; (d_3 + d_{3,4}) ; (d_2 + d_{2,4}) ; (d_1 + d_{1,4})\} = d_2 + d_{2,4}$	0 - 2 - 4	$c_p + c_2 + c_4$	$a_p \cdot r_2 \cdot r_4$
$d = \min \{d_1, d_2, d_3, d_4\} = d_4$	0 - 2 - 4		

REMARK: in the above table possible solutions (and related paths,  $\bar{c}_i$ ,  $\bar{a}_i$ ) are given as a matter of example.

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