

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

**Band:** 30 (1978)

**Artikel:** Aseismic design of multi-story space frames

**Autor:** Raghavan, T.M.S. / Nigam, N.C.

**DOI:** <https://doi.org/10.5169/seals-24166>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 13.03.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## ASEISMIC DESIGN OF MULTI-STORY SPACE FRAMES

T. M. S. Raghavan

Graduate Student, Department of Civil Engineering  
Indian Institute of Technology, Kanpur, India

and

N. C. Nigam

Professor in Aeronautical Engineering  
Indian Institute of Technology, Kanpur, India

## SUMMARY

The inelastic behavior of a six-story space frame is investigated under the simultaneous action of the two horizontal components of El-Centro-1940 earthquake. The response is compared with elastic and inelastic behavior of constituent plane frames subjected to individual components. The results indicate that inelastic interaction has a significant effect on the response.

## 1. INTRODUCTION

The inelastic behavior of structures under dynamic loads has received considerable attention during the last two decades. In the design of structures subjected to occasional loads, such as the dynamic loads due to strong motion earthquakes and blasts, it is now generally accepted that the excursion of structural material into inelastic range must be permitted to achieve economical and safe design. This realization has led to extensive research in the inelastic behavior of structures for such loads. Early investigations in this area were based on the considerations of energy input, the elastic energy capacity and the energy dissipated through inelastic deformations, without obtaining a detailed response of the structure [1,2,3]. The availability of high speed digital computers, permitting step-by-step integration of large systems, stimulated detailed studies by several investigators [4,5,6,7,8,9,10]. In these and other similar investigations, the inelastic response of framed structures is obtained under the following simplifying assumptions:

- (i) A three-dimensional framed structure is treated as an assemblage of plane frames along each of its principal directions.
- (ii) The yield behavior at a section is assumed to depend only on flexure, neglecting the effect of axial and shear forces.
- (iii) The response is obtained on the basis of a preassumed moment-rotation (or curvature) relationship of either general yielding [9,10], bilinear [7,8] or elastic-perfectly-plastic type [4].

Since the behavior of a structure during inelastic excursions is nonlinear, the principle of superposition is not applicable, and a response analysis treating a space frame as an assemblage of plane frames subjected to in-plane ground motion is not valid. It is necessary to model a framed structure as a space frame subjected to simultaneous action of ground motion components. A general theory, incorporating the effects of inelastic interactions on the dynamic response of space frames, was developed and applied to a simple space frame in 1967 [11,12]. Since then, several investigators have studied inelastic response of space frames, including the effects of work-hardening [13,14,15,16,17]. The work has also been extended to R/C frames [18].

In this paper, the behavior of a six-story building frame is investigated under the simultaneous action of two horizontal components of El. Centro, 1940 earthquake. The response is compared with the elastic and inelastic behavior of constituent plane frames subjected to individual components investigated by PENZIEN [4]. The results indicate that inelastic interaction has a significant effect on the response.

## 2. THEORY OF INELASTIC INTERACTION

A framed structure consists of an assemblage of discrete one-dimensional elements interconnected at their ends. Consider a

section of such an element and assume that:

- (i) Stress-strain relation is elastic-perfectly-plastic.
- (ii) The yield behavior at the section is described by an yield surface [11]:

$$\phi(\bar{Q}) = 1$$

such that

the section is elastic if  $\phi(\bar{Q}) < 1$  (1)

$$\text{or if } \phi(\bar{Q}) = 1 \text{ and } \dot{W}^P < 0 \quad (2)$$

the section is yielding if  $\phi(\bar{Q}) = 1$  and  $\dot{W}^P \geq 0$  (3)

where  $\bar{Q}$  is the generalised force vector at the section,  $\bar{q}$  is the generalised displacement vector at the section, and

$$\dot{W}^P = \langle \bar{Q}, \dot{\bar{q}}^P \rangle \quad (4)$$

representing the rate of plastic work.

Under above assumptions it can be shown that at a regular point on the yield surface [11]

$$\bar{Q} = [K] (\bar{q} - \bar{q}_0) \quad (5)$$

if the section is elastic, and

$$\bar{Q} = [K] \left( \dot{\bar{q}} - \frac{\langle K \dot{\bar{q}}, \frac{\partial \phi}{\partial \bar{Q}} \rangle}{\langle K \frac{\partial \phi}{\partial \bar{Q}}, \frac{\partial \phi}{\partial \bar{Q}} \rangle} \frac{\partial \phi}{\partial \bar{Q}} \right) \quad (6)$$

if the section is yielding.

$\bar{q}_0$  denotes the current position of equilibrium and  $K$  is the stiffness matrix.

The force-displacement relations for elastic and yielding behaviors of a one dimensional element can be derived on the basis of equations (5) and (6) [12]. If the effects of inelastic interaction are neglected, Equation (6) reduces to

$$\dot{Q}_i = 0 \quad \text{if } |Q_i| = Q_{yi} \text{ and } \dot{W}_i^P \geq 0 \quad (7)$$

where  $Q_{yi}$  is the yield level of  $Q_i$ .

### 3. SIX-STORY SPACE FRAME

Consider a six-story space frame shown in Figure 1. The frame is identical in directions 1-1 and 2-2. The floors are assumed to rigid and remain parallel during lateral deformation. The elastic stiffness is such that the fundamental mode of vibration of the plane frames is triangular in shape. The damping is assumed to be viscous and proportional to story stiffness. The entire mass of the structure is concentrated equally at each floor level and the story heights are equal. The yield strength of each plane-frame is specified by a parameter,  $\theta$ , defined as the ratio of the

yield value of the base shear to the total weight of the structure based on idealised elastoplastic behavior shown in Figure 2(b). The yield value of the story shear,  $(Q_i)_y$ , is assumed to be proportional to story stiffness  $k_i$ . Table 1 gives the stiffness, damping and yield strength characteristics of the frame.

Under the simplifying assumptions stated above, yielding in a story occurs at the top and bottom sections of the columns simultaneously. The forces acting at these sections are the axial force and bending moments and shear forces in directions 1-1 and 2-2. The yield behavior is governed by the interaction between these forces. If the effect of the axial force and shear forces is neglected, the yield behavior at a section is governed by the interaction between  $M_{i1}$  and  $M_{i2}$ , the bending moments in the directions 1-1 and 2-2. Since the story shear

$$Q_{ij} = \frac{8M_{ij}}{h_i} \quad \begin{matrix} i = 1, 2, \dots, 6 \\ j = 1, 2 \end{matrix} \quad (8)$$

the yield behavior of a story is identical to the yield behavior at the end sections and may be expressed in terms of the story shears  $Q_{i1}$  and  $Q_{i2}$ . Figure 2(a) shows the yield surface in the two dimensional force space. If the effects of interaction are neglected, the yield behavior reduces to idealised elasto-plastic behavior as shown in Figure 2(b).

### 3.1 Equations of Motion

The equations of motion of the space frame due to base excitation during an earthquake can be written in terms of story shear,  $Q_{ij}$ , and lateral displacement,  $u_{ij}$ , using Equations (5), (6) and (7). For the  $i$ th mass the equations of motion are:

$$\begin{aligned} m_i \ddot{u}_{1i} + c_i (\dot{u}_{1i} - \dot{u}_{1i-1}) - c_{i+1} (\dot{u}_{1i+1} - \dot{u}_{1i}) + (Q_{1i} - Q_{1i+1}) \\ = m_i \ddot{z}_1(t) \\ m_i \ddot{u}_{2i} + c_i (\dot{u}_{2i} - \dot{u}_{2i-1}) - c_{i+1} (\dot{u}_{2i+1} - \dot{u}_{2i}) + (Q_{2i} - Q_{2i+1}) \\ = m_i \ddot{z}_2(t) \end{aligned} \quad (9)$$

where subscripts 1 and 2 denote directions 1-1 and 2-2 and  $i$  denotes the story as shown in Figure 1. Further

- $m_i$  -  $i$ th mass;
- $c_i$  - inter floor viscous damping coefficient in the  $i$ th story;
- $u_{1i}, u_{2i}$  - the lateral displacements of mass  $m_i$  relative to the base;
- $Q_{1i}, Q_{2i}$  - story shears in the  $i$ th story;
- $\ddot{z}_1, \ddot{z}_2$  - horizontal components of the ground acceleration during an earthquake.

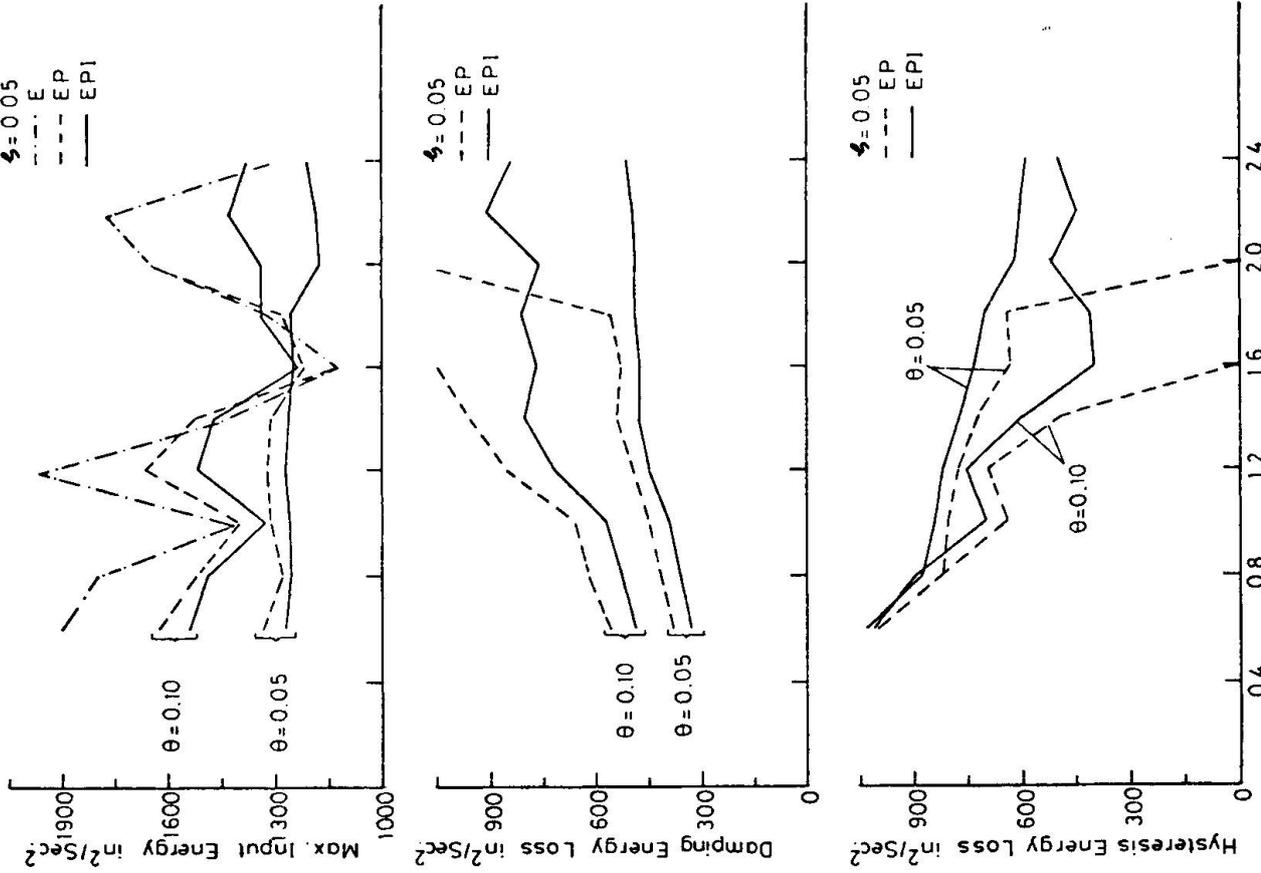


Fig.3 Energy Input and Dissipation.

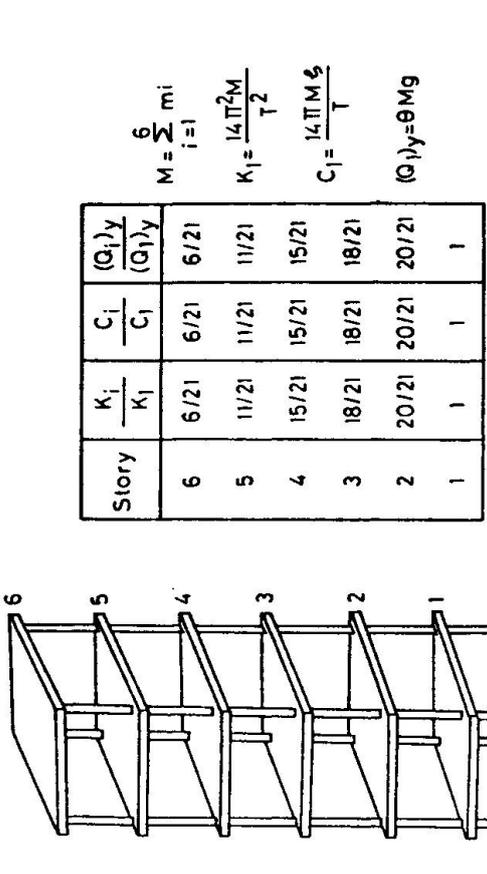


Table.1 Structural Properties of Space Frame

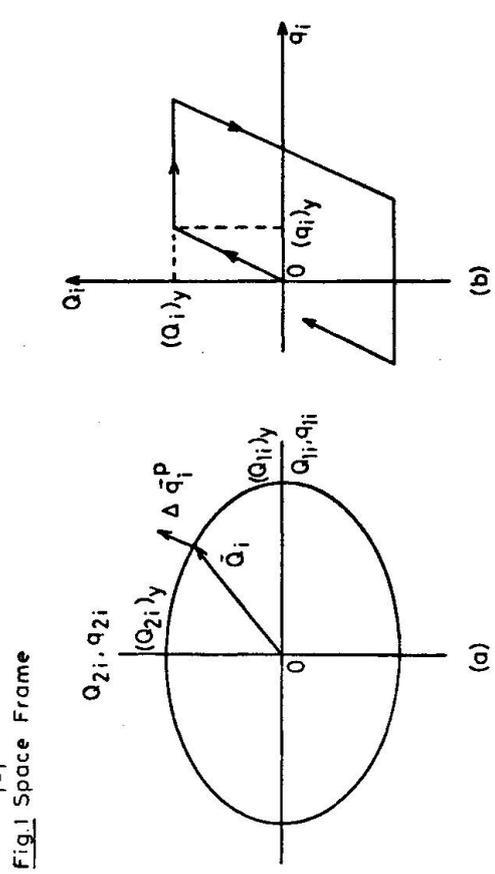


Fig.2 Yield Surface for i-th Story (a) With Interaction (b) Without Interaction

The relationship between story shears and story displacements depends upon whether the story is elastic or yielding. Further during yielding it depends upon whether the effects of inelastic interaction are included or ignored. For each of these three cases, the story shear-displacement relations can be derived using Equations (5), (6) and (7) as under:

#### Elastic Behavior (E)

$$\begin{aligned} Q_{1i} &= k_i q_{1i} \\ Q_{2i} &= k_i q_{2i} \end{aligned} \quad (10)$$

where

$$\begin{aligned} q_{1i} &= (u_{1i} - u_{1i-1}) \\ q_{2i} &= (u_{2i} - u_{2i-1}) \end{aligned}$$

#### Elasto-Plastic Behaviour Without Interaction (EP)

$$\begin{aligned} Q_{1i} &= k_i [q_{1i} - (q_{1i})_o] \quad \text{if } |Q_{1i}| < (Q_{1i})_y \\ &\quad \text{or if } |Q_{1i}| = (Q_{1i})_y \text{ and } \dot{w}_{1i}^p < 0 \\ |Q_{1i}| &= (Q_{1i})_y \quad \text{if } \dot{w}_{1i}^p \geq 0 \\ Q_{2i} &= k_i [q_{2i} - (q_{2i})_o] \quad \text{if } |Q_{2i}| < (Q_{2i})_y \\ &\quad \text{or if } |Q_{2i}| = (Q_{2i})_y \text{ and } \dot{w}_{2i}^p < 0 \\ |Q_{2i}| &= (Q_{2i})_y \quad \text{if } \dot{w}_{2i}^p \geq 0 \end{aligned} \quad (11)$$

#### Elasto-Plastic Behavior with Interaction (EPI)

$$\begin{aligned} Q_{1i} &= k_i [q_{1i} - (q_{1i})_o] \\ Q_{2i} &= k_i [q_{2i} - (q_{2i})_o] \\ \text{if } \phi(Q_{1i}, Q_{2i}) &< 1, \text{ or if } \phi(Q_{1i}, Q_{2i}) = 1 \\ &\quad \text{and } \dot{w}_i^p < 0 \end{aligned} \quad (12)$$

and

$$\begin{aligned} \dot{q}_{1i} &= \frac{k_i \left[ \left( \frac{\partial \phi}{\partial Q_{2i}} \right)^2 \dot{q}_{1i} - \frac{\partial \phi}{\partial Q_{1i}} \frac{\partial \phi}{\partial Q_{2i}} \dot{q}_{2i} \right]}{\left[ \left( \frac{\partial \phi}{\partial Q_{1i}} \right)^2 + \left( \frac{\partial \phi}{\partial Q_{2i}} \right)^2 \right]} \\ \dot{q}_{2i} &= \frac{k_i \left[ - \frac{\partial \phi}{\partial Q_{1i}} \frac{\partial \phi}{\partial Q_{2i}} \dot{q}_{1i} + \left( \frac{\partial \phi}{\partial Q_{1i}} \right)^2 \dot{q}_{2i} \right]}{\left[ \left( \frac{\partial \phi}{\partial Q_{1i}} \right)^2 + \left( \frac{\partial \phi}{\partial Q_{2i}} \right)^2 \right]} \end{aligned}$$

$$\text{if } \phi(Q_{1i}, Q_{2i}) = 1 \quad \text{and} \quad \dot{W}_i^p \geq 0 \quad (13)$$

It may be noted that for elastic (E) and elasto-plastic behavior (EP), the equations of motion are uncoupled in the directions 1-1 and 2-2 and can be integrated independently. For elasto-plastic behavior with interaction (EPI), the equations of motion are coupled.

### 3.2 Response Computation

The equations of motion of the space frame for elastic, elasto-plastic and elasto-plastic behavior with interaction are integrated using the fourth order Runge-Kutta method of step-by-step integration. First 30 seconds of the E-W and N-S components of El-Centro-1940 earthquake record are used as ground acceleration  $Z_1$  and  $Z_2$  respectively. The interval of integration  $\Delta t = T/50$ , where  $T$  is the fundamental natural period of the frame. The response is computed for yield strength parameter  $\theta = 0.05, 0.1, 0.2$ ; damping ratio  $\zeta = 0.05$  and fundamental period  $T$  varying from 0.6 to 2.4 secs. For elasto-plastic behavior with interaction, the yield surface is expressed by

$$\left[ \frac{Q_{1i}}{(Q_i)_y} \right]^2 + \left[ \frac{Q_{2i}}{(Q_i)_y} \right]^2 = 1 \quad (14)$$

The response parameters computed are the energy input and energy dissipated due to damping and hysteresis during yielding; absolute displacements and ductility ratios; permanent set and number of inelastic excursions and plastic energy ratio for each story.

## 4. RESULTS AND DISCUSSION

Figure 3 shows the maximum energy input per unit mass to the space frame during the earthquake and the energy loss due to damping and hysteresis during yielding for elastic, and elasto-plastic behavior with and without interaction. The curves show that, in general, inelastic deformations reduce the energy input and interaction reduces it further, the reduction depending significantly on the yield strength parameter  $\theta$ . The inelastic deformations reduce the energy dissipation due to damping and interaction reduces it further. This fact is significant in modern high-rise buildings with low damping as it implies less dependence on damping to limit the response. It is further seen that if the interaction is ignored, elasto-plastic analysis would indicate that the space frame remains elastic for ( $T > 1.6, \theta = 0.1$ ) and ( $T > 2.0; \theta = 0.05$ ), whereas, the frame will actually yield due to interaction.

Figure 4 shows the absolute displacements of the space frame in the two directions for the fundamental periods  $T = 0.6$  and 1.2. It is seen from the curves that for  $T = 0.6$ , interaction causes a large increase in displacement over elastic and elasto-plastic behavior in both directions. For  $T = 1.2$ , there is a large increase in the direction 1-1 due to interaction, but the dis-

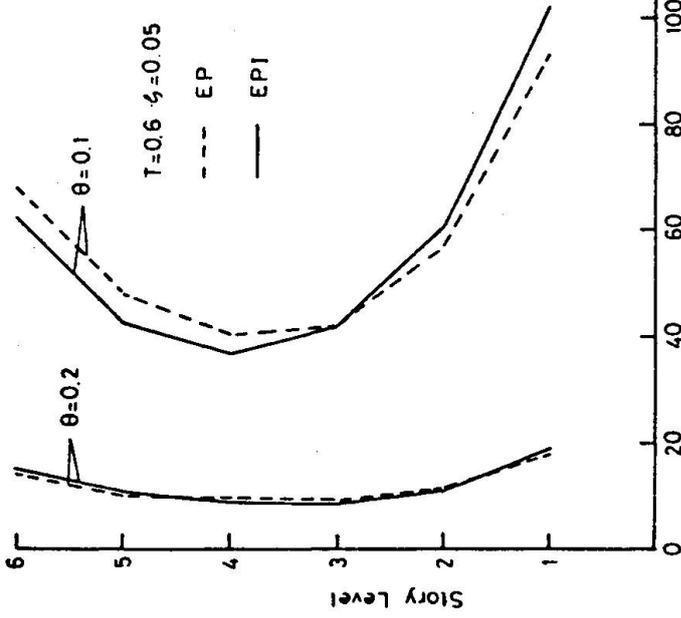


Fig. 5 Story Plastic Energy Ratio

Story	Permanent Set (ins)			No. of inelastic excursion		
	B - P 1 - 1	E P I	E P I	B P 1 - 1	E P I	E P I
1	0.036	0.656	0.280	30	55	50
2	0.03	0.280	0.0403	29	48	47
3	0.0033	0.0403	-0.0198	29	50	50
4	-0.0426	-0.0198	-0.112	34	59	59
5	0.00248	-0.112	-0.329	39		
6	0.099	-0.329				

Table 2: Permanent Set and Number of Inelastic Excursions.  $\theta = 0.1, T = 0.6$ .

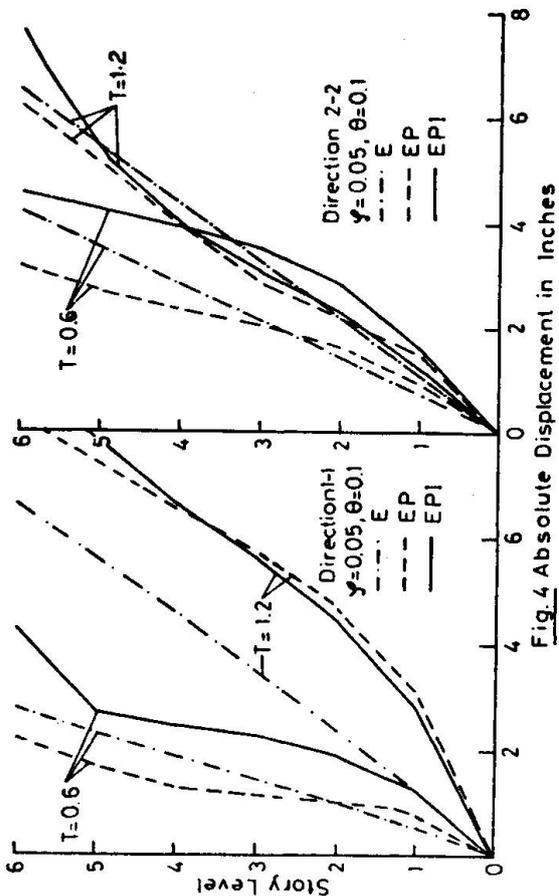


Fig. 4 Absolute Displacement in Inches

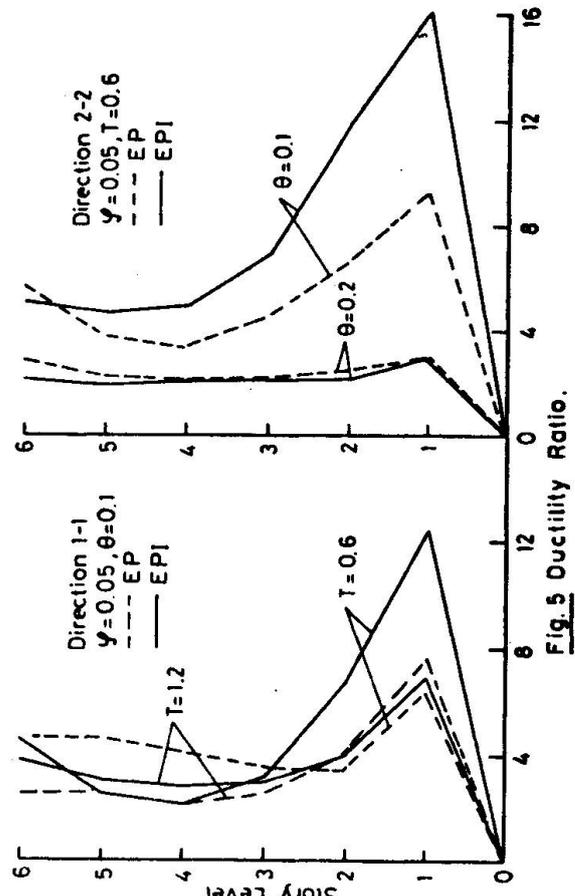


Fig. 5 Ductility Ratio.

placements are nearly same in the direction 2-2. Large interfloor displacements occur in the top and bottom storeys due to inelastic behavior and interaction causes further increase suggesting the need to strengthen these storeys. If the interaction effects are ignored, as in conventional plane frame elasto-plastic analysis, displacements may be grossly under estimated.

Figure 5 shows the ductility ratio in each story for  $T = 0.6, 1.2$  and  $\theta = 0.1$  in direction 1-1 and  $T = 0.6$  and  $\theta = 0.1$  and  $0.2$  in direction 2-2. It is seen that for  $T = 0.6$  and  $\theta = 0.1$ , the ductility requirement is large in the top and bottom storeys. For  $T = 0.6$  and  $\theta = 0.2$ , the ductility requirement is significantly reduced and interaction effects are insignificant. Comparison of curves for  $\theta = 0.1$  and  $0.2$  also indicates, what may happen to a structure designed for  $\theta = 0.1$ , if it is subjected to an earthquake of twice the intensity of the design earthquake. For  $T=1.2$  sec and  $\theta = 0.1$ , the ductility requirement is low and interaction effects are significant only in the top story. On the basis of idealised elasto-plastic analysis of plane frames, PENZIEN [4] recommended optimum values of  $\theta = 0.3, 0.2$  and  $0.1$  for  $T = 0.3, 0.6, 0.9$  and more, respectively. Due to large computer time required, it has not been possible to study the response for several values of  $\theta$ . However, on the basis of limited results and the fact that the general nature of response for elasto-plastic behavior, with and without interaction, is same, it appears that the above recommendations shall also apply to space frames. The large ductility requirements in top and bottom storeys suggest strengthening of these storeys to remain within acceptable limits for safe aseismic design.

Plastic energy ratio has been suggested as a criterion for inelastic design [11,12]. Figure 5 shows the plastic energy ratio in each story for  $T = 0.6$  and  $\theta = 0.1$  and  $0.2$ . It is seen that unlike ductility ratio the effect of interaction on plastic energy ratio is small. Increase in  $\theta$  from  $0.1$  to  $0.2$  decreases the ductility requirement significantly and makes it nearly uniform over the entire frame.

Table 2 gives the number of excursions in the inelastic range and the permanent set at the end of the earthquake in the direction 1-1. The behavior in direction 2-2 is similar. It is seen that elasto-plastic analysis without interaction grossly under estimates the number of excursions. This result is important because strength degradation during repeated yielding increases with the number of excursions in the inelastic range. The interaction significantly increases the permanent set in the top and bottom storeys, which may determine the serviceability of the structure after an earthquake.

The inelastic behavior of the space frame has been analysed in terms of parameters governing inelastic design, such as, energy input and energy dissipated, lateral displacement, ductility ratio, plastic energy ratio, number of excursions into the inelastic range and permanent set. The overall behavior is consistent with the general findings of the earlier investigations based on simple space frames [11,13]. The results clearly show that the conventional elasto-plastic analysis, treating a space frame as an assemblage of plane frames, may significantly underestimate the ductility requirements in the low period range for

small values of yield strength. The inelastic behavior is sensitive to the yield strength parameter,  $\theta$ , which must be chosen carefully. Several codes specify stiffness distribution to give a triangular first mode. The results indicate the need to strengthen the top and bottom stories of such frames. The investigation did not take into account the P-A effect and the effect of work-hardening which may alter some of the conclusions.

#### REFERENCES

1. HOUSNER, G.W.: Limit Design of Structures. Proceedings of the First World Conference on Earthquake Engineering, San Francisco, June 1956, pp.5:1-5:11.
2. HOUSNER, G.W.: Behavior of Structures During Earthquakes. Proceedings ASCE, Vol. 85, Oct. 1959, pp. 109-129.
3. BLUME, J.A.: A Reserve Energy Technique for Earthquake Design and Rating of Structures in the Inelastic Range. Proceedings of the Second World Conference on Earthquake Engineering, Vol. II, Tokyo and Kyoto, Japan, 1960, pp.1061-1084.
4. PENZIEN, J.: Elasto-Plastic Response of Idealised Multi-Story Structures Subjected to a Strong Motion Earthquake. *ibid*, pp.739-760.
5. HOUSNER, G.W.: The Plastic Failure of Frames During Earthquakes. *ibid*, pp. 997-1012.
6. CLOUGH, R.W., *et al*: Inelastic Earthquake Response of Tall Buildings. Proceedings of the Third World Conference on Earthquake Engineering, Vol. II, Auckland and Wellington, New Zealand, Jan. 1965, pp.68-89.
7. SAUL, W.E. *et al*: Dynamic Analysis of Bilinear Inelastic Multiple Story Shear Buildings. *ibid*, pp.533-551.
8. GIBERSON, M.F.: The Response of Nonlinear Multi-Story Structures Subjected to Earthquake Excitation. Ph.D. Thesis, California Institute of Technology, 1967.
9. JENNINGS, D.C.: Response of Simple Yielding Structures to Earthquake Excitation. Ph.D. Thesis, California Institute of Technology, 1963.
10. GOEL, S.C.: Inelastic Behavior of Multi-Story Building Frames Subjected to Earthquake Motion. Ph.D. Thesis, The University of Michigan, 1967.
11. NIGAM, N.C.: Inelastic Interactions in the Dynamic Response of Structures. Ph.D. Thesis, California Institute of Technology, 1967.
12. NIGAM, N.C.: Yielding in Framed Structures Under Dynamic Loads. J. Engg. Mech. Divn., ASCE, EM5, Oct. 1970, pp.687-709.

13. WEN, R.K. and FARHOOMAND, F.: Dynamic Analysis of Inelastic Space Frames. J. of Engg. Mech. Divn., ASCE, EM5, Oct. 1970, pp. 667-686.
14. NIGAM, N.C.: Aseismic Design of Structures in the Inelastic Range. Proceedings of the Fourth Symposium on Earthquake Engineering, University of Roorkee, India, Nov. 1970, pp. 262-265.
15. PORTER, F.L. and POWELL, G.H.: Static and Dynamic Analysis of Inelastic Frame Structures. Earthquake Engineering Research Centre, University of California, Berkeley, Report No. EERC 71-3, June, 1971.
16. KOBORI, T. et al: Earthquake Response of Frame Structures Composed of Inelastic Members. Proceedings of the Fifth World Conference on Earthquake Engineering, Vol. II, Rome, 1974, pp.1772-1781.
17. PADILLA-MORA, R. and SCHNOBRICH, W.C.: Non-linear Response of Framed Structures to Two-dimensional Earthquake Motion. Department of Civil Engineering, University of Illinois. Report No. YILU-ENG-74-2015, July 1974.
18. TAKIZAWA, H. and AOYAMA, H.: Biaxial Effects in Modelling Earthquake Response of R/C Structures. Earthquake Engineering and Structural Dynamics, Vol.4, No.6, Oct.-Dec. 1976, pp. 523-552.

Leere Seite  
Blank page  
Page vide