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ENGINEERING DECISIONS AND SEISMIC RISK PREVENTION[1]

by

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Abstract - A mathematical model leading to the marginal cost of a saved life at a site exposed to seismic risk is presented. The site is assumed included in an ideal seismic zone, with uniform distribution of epicenters and with constant depth of focuses. The anlysis is carried out on the basis of four different magnitude-frequency laws: "linear", "truncated-linear", "quadratic" and "truncated-quadratic".

Resume' - On présent un modèle mathématique pour le calcul du cout marginal d'une vie sauvée dans un lieu expose' au risque sismique. On suppose que le site soit compris dans une zone idéalisée dans la quelle la distribution des épicentres est uniforme et la profondeur des hypocentres est constante. Le calcul est conduit sur la base de quatre different hypothèses à propos de la corrélation entre la magnitude et la fréquence: "lineaire", "lineaire-tronquée", "parabolique", "parabolique-tronquée".

Zusammenfassung - Ein mathematisches Modell für die Berechnung des Lebensbewahrungzusatzkostenpreis in einem erdbebengefährdeten Ort vorgeschlagen wird. Mit der Annahme dass der Ort in einer Region liegt wo die Epizentrenverteilung gleichmässig ist und die Epizentrentiefe konstant ist, die Rechnung entwickelt wird mit vier verschiedenen Magnitude-Frequenz Zusammenhängen (linear, linear mit Beschränkung, parabolisch und parabolisch mit Beschränkung).

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1. INTRODUCTION

The occurrence of earthquakes in a given zone is generally represented by means of the average number N(m) of earthquakes with magnitude greater than m in a year. A classic assumption about the function N(m) is:

$$\log_{10} N(m) = a - bm, \tag{1}$$

with a, b constant coefficients depending on the zone.

It has been observed by many Authors that the use of eq. (1) in elaborating statistical data normally overestimates the occurrence of large events. An improvement of the "linear" magnitude-frequency law (1) can be obtained either assuming a "truncated-linear" law (i.e. imposing an upper bound m_1 on m) or assuming a "non-linear" law leading to lower values of N(m) for large events in respect of eq. (1). In this second case the "non-linear" law can also be truncated. Shlien and Toksöz [1] used a "quadratic" form for the magnitude frequency law:

$$\log_{10} N(m) = a + b'm + b''m^2. \tag{2}$$

Merz and Cornell [2] carried out an analysis with both a linear and a quadratic magnitude-frequency law for a fault-site configuration, concluding that the difference in the prediction of local seismicity is significant in the high-ground-acceleration region.

The aim of the present paper is to develop the model contained in [2] in order to include in it the effects of the earthquakes on the buildings, taking into account both the economical aspects of the problem and the expected number of victims. The model thus obtained can be useful when the differences in local seismicity, depending on the alternative assumptions about the magnitude-frequency law, must be discussed from the point of view of engineering decisions in the field of seismic risk prevention.

The first step is the calculation of the local seismicity at a site contained in an ideal seismic zone, with uniform distribution of epicenters and with constant depth of focuses, starting from four different mangitude-frequency laws: linear, truncated linear, quadratic, truncated-quadratic. Thus four different expressions giving the local seismicity at the considered site are obtained. For this first step the mathematical treatment is essentially the same as in [2], adapted to the particular hypothesis about the distribution of potential earthquake sources.

The second step is based on the following assumptions: 1) the decisions regarding seismic risk prevention are mostly condensed in the design value of the lateral force coefficient C; 2) the consequences of alternative designs are well represented by the marginal cost of a saved life, $\Delta D/\Delta L$, which is a function of C [3]. Therefore the function $\Delta D/\Delta L(C)$ is calculated for the considered site starting from the different expressions of the local seismicity previously defined.

2. LOCAL SEISMICITY

2.1 First Case

Consider an ideal seismic zone with uniform distribution of epicenters, with constant depth of focuses, and where the peak ground acceleration y at the distance r from the epicenter of an earthquake of magnitude m depends only on r and m. Consider the earthquakes with $m>m_o$ (m_o being the value below which earthquakes are not of engineering importance and/or the statistical data are not reliable). Then the linear magnitude-frequency law (1) can be written:

$$N^{(1)}(m) = \begin{cases} \lambda & ; \quad m \leq m_o \\ \lambda e^{-\beta(m-m_o)} & ; \quad m > m_o \end{cases} \tag{3}$$

Let $F_M(m)$ be the distribution function of the random variable M. Then the assumption (2) implies for a single earthquake:

$$P[M>m] = 1 - F_M^{(1)}(m) = \begin{cases} 1 & ; \quad m \leq m_o \\ e^{-\beta(m-m_o)} & ; \quad m > m_o \end{cases} \tag{4}$$

The corresponding probability density is:

$$f^{(1)}(m) = \begin{cases} 0 & ; \quad m \leq m_o \\ \beta e^{-\beta(m-m_o)} & ; \quad m > m_o \end{cases} \tag{5}$$

Observe that the coefficient β defines the distribution function $F_M(m)$: it does not depend on the total average number of earthquakes λ.

Consider one site sufficiently far from the border of the zone and assume that for an earthquake with M = m at distance R = r from the site, the peak ground acceleration at the site is:

$$y = \begin{cases} b_1 e^{b_2 m} r_o^{-2} & r \leq r_o \\ b_1 e^{b_2 m} r^{-2} & r > r_o \end{cases} \tag{6}$$

Then for an earthquake at distance R = r, from eq. (4) and (6) we obtain the probability of exceeding the acceleration y at the site:

$$P[Y>y/R=r]=p_{y/r} = 1-F_M^{(1)}[m(y,r)]=\begin{cases}\left(\frac{y}{b_1}r_0^2\right)^{-\beta/b_2} e^{\beta m_0} & ; r\le r_0 \\ \left(\frac{y}{b_1}r^2\right)^{-\beta/b_2} e^{\beta m_0} & ; r>r_0\end{cases} \tag{7}$$

and for an earthquake at random distance $R \leq r_1$ (r_1 being the distance at which an earthquake is not of engineering importance):

$$p_y = \int_0^{r_1} p_{y/r} \cdot f(r)\,dr \tag{8}$$

where f(r) is the probability density of the random variable R; i. e., due to the uniform distribution of the epicenters:

$$f(r)dr = \frac{2\pi r\,dr}{\pi r_1^2} \tag{9}$$

Assume now that the number of earthquakes in a year is a Poisson distributed random variable with mean λ. The seismicity at the site is then represented by:

$$P[Y>y]_{one\ year} = 1-F_Y(y) = 1-e^{-\lambda p_y} \simeq \lambda p_y = N(y) \tag{10}$$

where the approximation is valid for small probabilities and N(y) is the annual average number of earthquakes with Y>y. For the present first fase, from eq. (7), (8), (9), (10) we obtain:

$$N^{(1)}(y) = 1-F_Y^{(1)}(y) = A\,y^{-\beta/b_2} \tag{11}$$

where

$$A = \frac{\lambda \cdot e^{\beta m_0}\, b_1^{\beta/b_2}}{(1-\beta/b_2)\, r_1^2}\left(r_{max}^{\,2-2\beta/b_2} - \frac{\beta}{b_2}\, r_0^{\,2-2\beta/b_2}\right) \tag{12}$$

In eq. (12) the distance r_{max} coincides with r_1. The different symbol has been choosen because in the analogous expressions for the second and the fourth cases, in which m is truncated, r_{max} will be a function of y, and hence different from r_1.

The probability density $f^{(1)}(y)$ is given by:

$$f^{(1)}(y) = A\,\frac{\beta}{b_2}\,y^{-\frac{\beta}{b_2}-1} \tag{13}$$

2.2 Second Case

If an upper bound m_1 is imposed on m the probability density f(m) must be changed. Assuming that the new probability density $f^{(2)}(m)$ is proportional to $f^{(1)}(m)$ in the range $m_o<m\leq m_1$, the normalization of $f^{(2)}(m)$ leads to:

$$\int_{m_o}^{m_1} K f^{(1)}(m)\,dm = 1,$$

and hence:

$$K = \frac{1}{1-e^{-\beta(m_1-m_o)}}$$

This implies

$$1-F_M^{(2)}(m) = \begin{cases} 1 & ;\ m\leq m_o \\ K\left[1-F_M^{(1)}(m)\right]+1-K & ;\ m_o<m\leq m_1 \\ 0 & ;\ m>m_1 \end{cases} \tag{14}$$

Hence:

$$N^{(2)}(m) = K N^{(1)}(m) + \lambda(1-K) \tag{15}$$

As regards the calculation of $N^{(2)}(y)$ we can start from the equation:

$$1-F_Y^{(2)}(y) = \lambda\int_0^{r(y)}\left\{1-F_M^{(2)}[m(y,r)]\right\}f(r)\,dr \tag{16}$$

where the limit r(y) of the integral is given by:

$$r(y) = b_1^{1/2}\,e^{b_2 m_1/2}\,y^{-1/2} \tag{17}$$

In fact, as observed by Cornell [4], earthquakes beyond a distance defined by solving

$$y = b_1 e^{b_2 m_1} r^{-2}$$

cannot possibly cause a peak acceleration at the site greater than y since their magnitude cannot exceed m_1.

From eq. (11), (12), (16), (17) we obtain:

$$1-F_Y^{(2)}(y)=N^{(2)}(y)=K\left[N^{(1)}(y)\right]_{r_{max}=r(y)}+\lambda(1-K)\frac{r^2(y)}{r_1^2} \qquad (18)$$

The probability density $f^{(2)}(y)$ is given by:

$$f^{(2)}(y)=-\lambda\frac{d}{dy}\int_0^{r(y)}\{1-F_H^{(2)}[m(y,r)]\}f(r)dr=$$
$$=-\frac{\lambda}{r_1^2}\int_0^{r_0}\frac{\partial}{\partial y}K\{1-F_M^{(1)}[m(y,r_0)]\}2rdr-\frac{\lambda}{r_1^2}\int_{r_0}^{r(y)}\frac{\partial}{\partial y}K\{1-F_H^{(1)}[m(y,r)]2rdr+ \qquad (19)$$
$$-\frac{\lambda}{r_1^2}\{1-F_H^{(2)}[m(y,r(y)]]\}2r(y)\frac{dr(y)}{dy}-\frac{\lambda}{r_1^2}(1-K)\frac{dr^2(y)}{dy}.$$

Taking into account that

$$m[y,r(y)]=m_1$$

and $K[1-F_M^{(1)}(m_1)]+1-K=P[M>m_1]=0,$
the eq. (19) becomes:

$$f^{(2)}(y)=K[f^{(1)}(y)]_{r_{max}=r(y)} \qquad (20)$$

2.3 Third Case

Assume:

$$N^{(3)}(m)=\lambda\, e^{\beta_1(m-m_0)+\beta_2(m^2-m_0^2)} \qquad (21)$$

This implies that, for a single earthquake:

$$P[M>m]=1-F_M^{(3)}(m)=\begin{cases}1 & ;\quad m\le m_0\\ e^{\beta_1(m-m_0)+\beta_2(m^2-m_0^2)} & ;\quad m>m_0\end{cases} \qquad (22)$$

Maintaining the remaining hypotheses as in the first case, the seismicity at the site for the present third case is:

$$N^{(3)}(y) = 1 - F_Y^{(3)}(y) = B\, y^{\beta_1/b_2}\left[r_0^{2+2\beta_1/b_2}\, e^{\varphi(y,r_0)} + 2\int_{r_0}^{r_{max}} r^{2\frac{\beta_1}{b_1}+1}\, e^{\varphi(y,r)}\, dr\right] \tag{23}$$

where

$$B = \frac{\lambda}{r_1^2}\, b_1^{-\beta_1/b_2}\, e^{-\beta_1 m_0 - \beta_2 m_0^2} \tag{24}$$

$$\varphi(y,r) = \frac{\beta_2}{b_2^2}\, \ln^2\left(\frac{y}{b_1} r^2\right) \tag{25}$$

The probability density $f^{(3)}(y)$ is:

$$\begin{aligned} f^{(3)}(y) = -B\, y^{\frac{\beta_1}{b_2}-1}\Big\{ & r_0^{2+2\frac{\beta_1}{b_2}}\, e^{\varphi(y,r_0)}\left[\frac{\beta_1}{b_2} + \frac{\beta_2}{b_2^2}\, 2\ln\left(\frac{y}{b_1} r_0^2\right) - 1\right] + \\ & + r_{max}^{2\frac{\beta_1}{b_2}+2}\, e^{\varphi(y,r_{max})} - 2\int_{r_0}^{r_{max}} r^{2\frac{\beta_1}{b_2}+1}\, e^{\varphi(y,r)}\, dr \Big\} . \end{aligned} \tag{26}$$

In eq. (23), (26), as for the first case, $r_{max} = r_1$.

2.4 Fourth Case

If the magnitude-frequency law (21) is truncated at m_1 we get:

$$1 - F_M^{(4)}(m) = \begin{cases} 1 & m \leq m_0 \\ K_1\left[1 - F_M^{(3)}(m)\right] + 1 - K_1 ; & m_0 < m \leq m_1 \\ 0 & m > m_1 \end{cases} \tag{27}$$

where

$$K_1 = \frac{1}{1 - e^{\beta_1(m_1 - m_0) + \beta_2(m_1^2 - m_0^2)}} . \tag{28}$$

Moreover, the equations (18), (20) become:

$$N^{(4)}(y) = K_1 \left[N^{(3)}(y) \right]_{r_{max}=r(y)} + \lambda(1-K_1)\frac{r^2(y)}{r_1^2}, \tag{29}$$

$$f^{(4)}(y) = K_1 \left[f^{(3)}(y) \right]_{r_{max}=r(y)} \tag{30}$$

3. MARGINAL COST OF A SAVED LIFE

The mathematical model leading to the calculation of the marginal cost of saved life $\Delta D/\Delta L$ is based on the following assumptions:

- The amount of damage due to an earthquake with peak ground acceleration y is obtained from the total cost of the building multiplied by a factor d(y, C) which depends on y and on the design lateral force coefficient C:

$$d(y,C) = \begin{cases} 0 & ; \quad y \le y_i \\ \dfrac{y-y_i}{y_c-y_i} & ; \quad y_i < y \le y_c \\ 1 & ; \quad y_c < y \end{cases} \tag{31}$$

where y_i is the value of y for which damage caused by an earthquake will begin to be appreciable and is given by:

$$y_i = \gamma + C \tag{32}$$

y_c is the value of y corresponding to the collapse of the structure and is given by:

$$y_c = \xi C + \zeta . \tag{33}$$

- The total monetary damage is supposed to be the one just described multiplied by 1.5 in order to take into account the indirect damage. Thus the cost of damage in liras/year person is given by:

$$D_1 = 1.5\,\varrho \int_0^{y_{max}} d(y,C) f(y)\, dy , \tag{34}$$

where ρ is the initial cost of the building per person, and y_{max} is the maximum possible peak ground acceleration at the site:

$$y_{max} = b_1 e^{b_2 m_1} r_0^{-2} \tag{35}$$

which becomes ∞ in the first and in the third case.

- The additional construction cost due to seismic design (expressed in liras/year/person taking into account the interest on the inves̲ted capital) is given by:

$$D_2 = \rho(hC+\theta), \qquad C \geq 0.01 \tag{36}$$

- The expected number V of victims/year/person is proportional to the number of failures:

$$V = \mu N(y_c) \tag{37}$$

Under the foregoing assumptions, the marginal cost $\Delta D/\Delta L$ can be expressed in terms of the derivatives of the total cost $D = D_1+D_2$ and of the expected number of victims V in respect of C. Assuming D_m as new symbol for the marginal cost of a saved life, we get:

$$D_m = \frac{\dfrac{d(D_1+D_2)}{dC}}{-\dfrac{dV}{dC}} ;$$

or also:

$$D_m = D_{m,1} + D_{m,2} ,$$

where $D_{m,1}$ and $D_{m,2}$ are the marginal costs of a saved life calcu̲lated, respectively, only with the cost of damage D_1 and only with the additional cost for seismic design D_2.

Taking into account that:

$$\frac{d}{dC}\int_{y_i}^{y_{max}} d(y,C)f(y)dy = \int_{y_i}^{y_c} \frac{\partial}{\partial C} d(y,C)f(y)dy + \int_{y_c}^{y_{max}} \frac{\partial}{\partial C} d(y,C)f(y)dy +$$

$$+ d(y_c,C)f(y_c)\xi - d(y_i,C)f(y_i) - d(y_c,C)f(y_c)\xi ,$$

and that:

$$\frac{\partial}{\partial c} d(y,c) = 0 \qquad \text{when} \quad y > y_c$$

$$d(y_i, c) = 0,$$

we get:

$$\frac{d}{dc}\int_{y_i}^{y_{max}} d(y,c) f(y)\, dy = \int_{y_i}^{y_c} \frac{\partial}{\partial c} d(y,c) f(y)\, dy.$$

Hence the marginal cost $D_{m,1}$ is given by:

$$D_{m,1} = \frac{1.5\rho \int_{y_i}^{y_c} \frac{\partial}{\partial c} d(y,c) f(y)\, dy}{p\, \xi\, f(y_c)}, \tag{38}$$

while the marginal cost $D_{m,2}$ is obviously:

$$D_{m,2} = \frac{\rho\, \vartheta}{p\, \xi\, f(y_c)} \tag{39}$$

It is interesting to observe:

- that in force of eq. (10) the marginal cost $D_{m,1}$ (due to damage) depends on the earthquakes with y contained in the interval $[y_i, y_c]$, but is does not depend on the earthquakes with $y > y_c$;
- that the marginal cost $D_{m,1}$ does not depend on the total number of earthquakes λ ; as both D_1 and V are proportional to λ;
- that when $D_m^{(1)}$ and $D_m^{(3)}$ are calculated for the first and the third case, the marginal costs $D_m^{(2)}$ and $D_m^{(4)}$ for the corresponding truncated cases are simply given by:

$$D_m^{(2)} = \left[D_{m,1}^{(1)}\right]_{r_{max}=r(y)} + \frac{1}{K}\left[D_{m,2}^{(1)}\right]_{r_{max}=r(y)},$$

$$D_m^{(4)} = \left[D_{m,1}^{(3)}\right]_{r_{max}=r(y)} + \frac{1}{K_1}\left[D_{m,2}^{(3)}\right]_{r_{max}=r(y)}$$

Just to give an example of a complete numerical application of the mathematical model, the linear and the quadratic magnitude-frequency laws contained in [2] have been considered (without truncation in both cases):

$$\log_{10} N(m) = a_o - 0.98\,(m - m_o)$$

$$\log_{10} N(m) = a_o + 1.076\,(m - m_o) - 0{,}218\,(m^2 - m_o^2)$$

The coefficient a_o referred to the area πr_1^2 has been derived from an assumed value a=6 referred to 10^6 km^2.

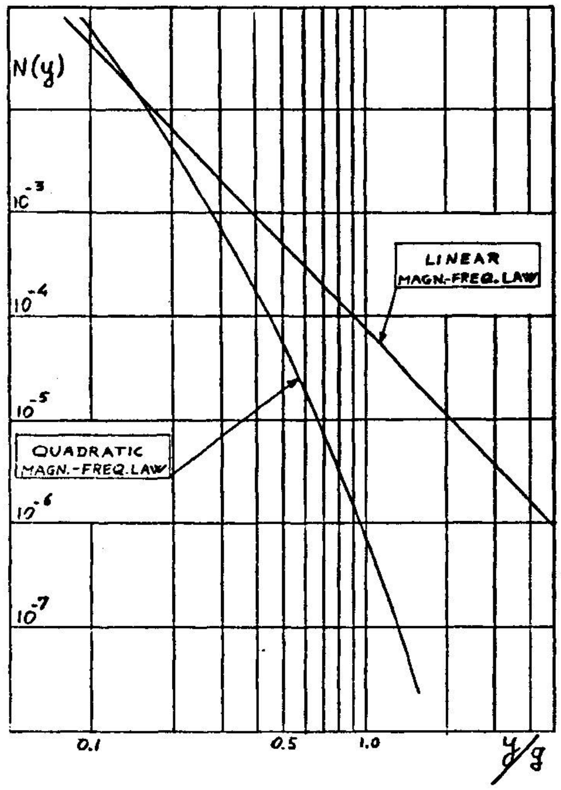

Fig. 1 - Seismicity at a site included in an ideal homogeneous seismic zone with two different magnitude-frequency laws taken from Merz and Cornell [2].

The remaining coefficients of the model have been assumed as follows:

eq. 32	$\gamma = 0{,}05$
eq. 33	$\xi = 6 \quad , \quad \zeta = 0.375$
eq. 34	$\rho = 7.5 \cdot 10^6$ liras/person
eq. 35	$b_1 = 1200/981$ km^2 $\quad b_2 = 0.8$, $r_o = 25$ km
eq. 36	$h = 0.0778$ year^{-1}, $\vartheta = -0{,}000778$ year^{-1}
eq. 37	$\mu = 0.3$

The results of the calculation are shown in the figures 1 and 2. Fig. 1 shows the seismicity at the site. Fig. 2 shows the influence of the alternative seismicities on the marginal cost of a saved life. The influence, in this case, is very large. However it must be observed that a quantitative discussion will be possible only on the basis of linear and quadratic magnitude-frequency laws, derived from the same set of statistical data possibly referred to an approximately homogeneous distribution of potential earthquake sources.

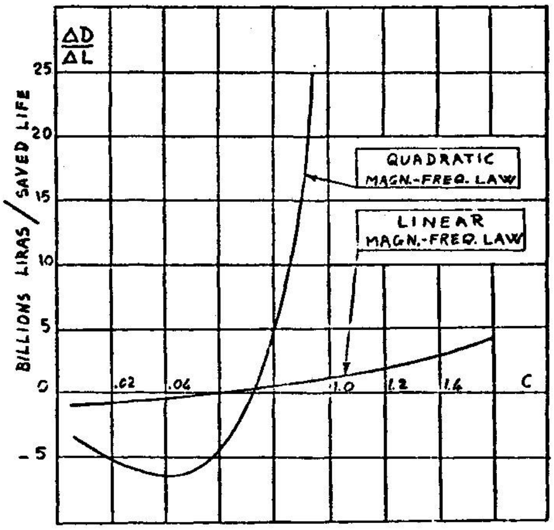

Fig. 2 - Marginal cost of a saved life versus design coefficient C for the site of fig. 1.

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