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EFFECT OF INITIAL STRESSES ON PLATE BUCKLING AND BUCKLING OF BOX COLUMNS

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ABSTRACT

The plate buckling is studied in Appendix 1, presented as a contribution at the IABSE-Congress, Amsterdam 1972.

When dealing with the column buckling of the welded box columns with quadratic cross sections it is assumed that the effective cross section consists of four angles with the flange width b_m (= b_e in Figure 3 in Appendix 1). b_m is the effective width of the compressed plate at the failure load in plate buckling.

The failure criterion at the column buckling is assumed to be that failure occurs when the compressive stress in the two most loaded angle flanges exceeds the value $\sigma_{kb} \cdot b/b_m$, where σ_{kb} is the value of $\sigma_0 = N/2bd$ at plate buckling failure.

Results of the calculations are given in Appendix 2. The diagrams show that the influence of the initial stresses is of great importance.

FAILURE LOAD AND EFFECTIVE WIDTH OF COMPRESSED STEEL PLATES WITH INITIAL STRESSES AND INITIAL DEFLECTIONS

Column buckling is influenced by the local plate buckling. The local plate buckling is dependent on initial stresses due to welding and initial deflections of the plates.

The author has studied the plate buckling in the overcritical range using a model of calculation, which enables to consider the initial stresses and the initial deflection in a relatively simple manner.

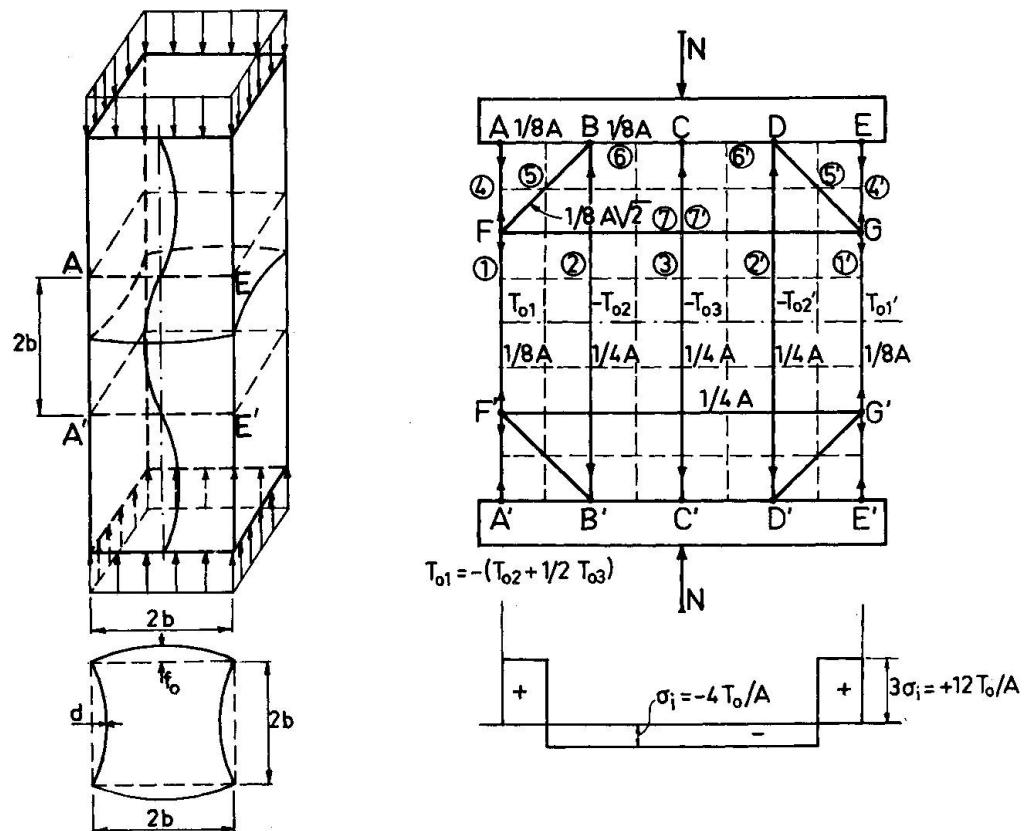


Fig. 1a and 1b Model of Calculation

The investigation is part of a research project regarding the carrying capacity of welded hollow columns, built up by thin plates. The project is carried out at the Department of Building Statics and Structural Engineering at the Royal Institute of Technology, Stockholm and at the Swedish Institute of Steel Construction, Stockholm.

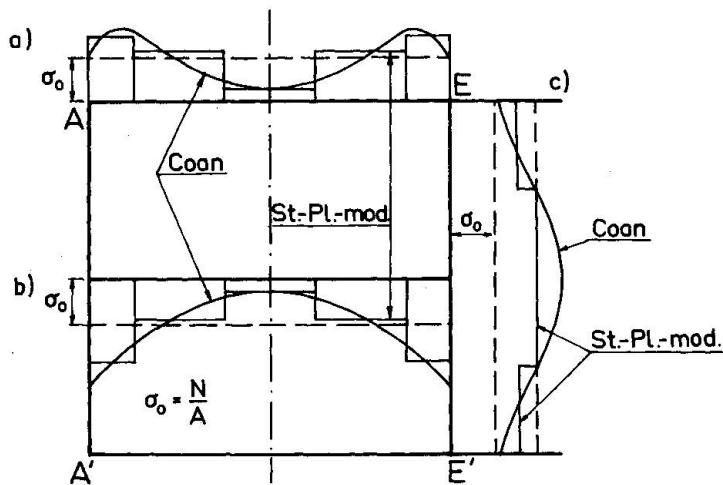


Fig. 1c
Distribution of compressive stresses in the direction of the load N at the edge A-E (a)) at the line of symmetry (b)) and the edge E-E' (c)). Comparison with solution by Coan $\sigma_0/\sigma_{el} = 1,74$.

The model of calculation consists of a plate acting only in plate bending and of the strips 1-7 and 1'-7', taking the membrane stresses only. (Fig. 1) The strips are connected to the plate at the points A-G and A'-G'. The areas of the strips are shown in Fig. 1, where $A = 2b \cdot d$ is equal to the area of the cross section of the plate. The normal forces in the strips are caused by 1) the initial stresses, 2) the normal force N in the plane of the plate, which gives forces in the different strips in proportion to their areas and 3) of forces which are caused by the changes of length of the strips as the bending deformation of the strips follows the bending deformation of the plate.

The mathematical treatment is omitted in this connection. It is the author's intention to publish the theory and the rather comprehensive results in a near future.

A treatment of the problem starting from the fundamental Eq. by von Kármán and Marguerre adjusted to take into account the influence of the initial stresses is in the author's opinion very difficult. In Fig. 1c a comparison is made with a solution by Coan [1] for a case where $\sigma_i=0$. The membrane stresses in the direction of the compressive load N at the supports at the middle of the plate and along a free edge are considered. It is seen from the Figure that it is a good agreement between Coan's results and the results from the calculations for the model in Fig. 1b both regarding the maximum values and the distributions of stresses.

It is hardly possible to precise adequate criteria of failure for the highly statically indeterminate system in question where the elasto-plastic state of stresses must be considered. The author has instead of trying to give a complex theory started from a relatively detailed study of the stresses in different parts of the elastic plate caused by bending and torsional moments and the normal forces. Then that load has been determined at which total yielding (yield stress over the whole cross section) will occur at the point considered, if the bending and torsional moments as well as the normal forces have the values calculated

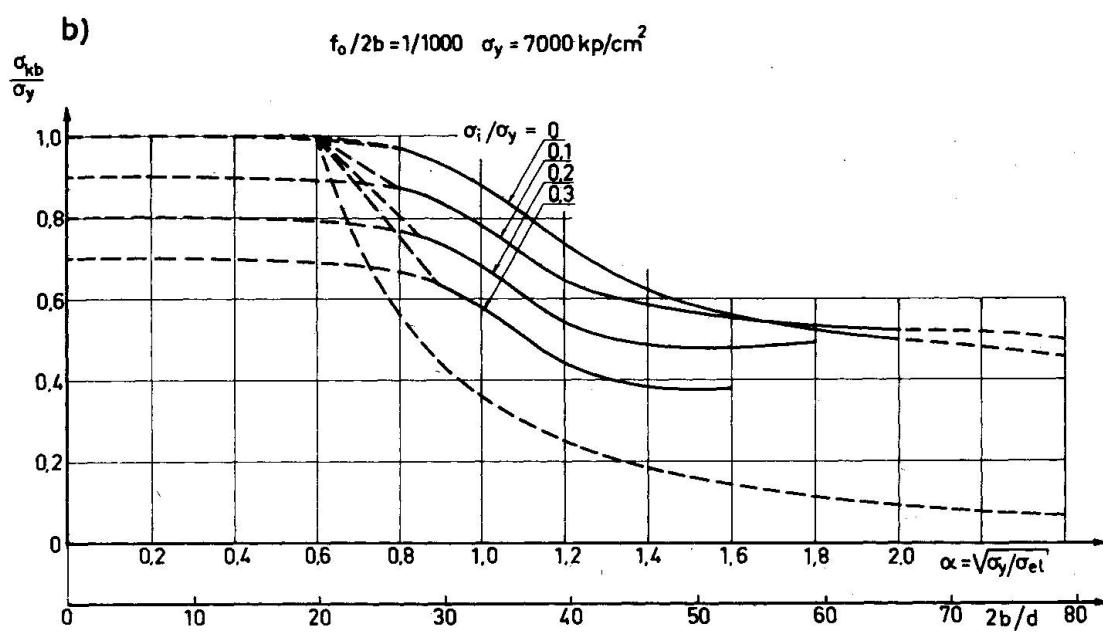
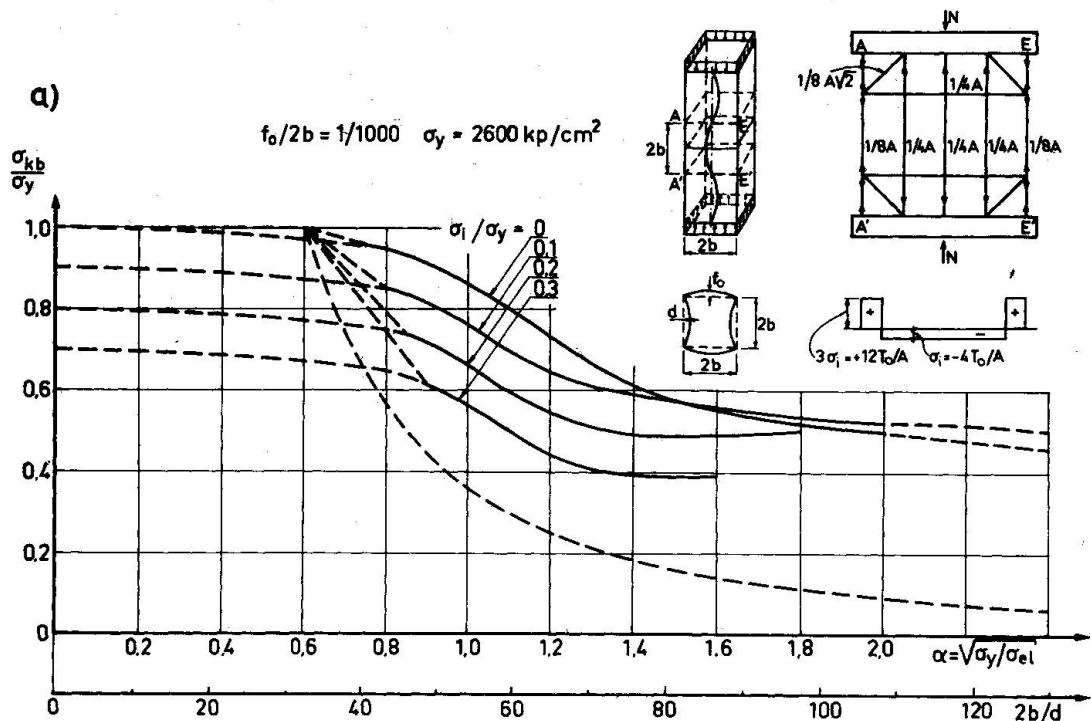


Fig. 2a and 2b Values of $\sigma_{kb} = N/2bd$ at buckling failure divided by σ_y at different σ_i/σ_y as function of $\alpha = \sqrt{\sigma_y/\sigma_{el}}$

a) $\sigma_y = 2600 \text{ kp/cm}^2$ b) $\sigma_y = 7000 \text{ kp/cm}^2$

from the theory of elasticity. At the judgement of the failure load the following points have been considered:

1. The midpoint of strip (A-A', Fig. 1a). Yielding due to normal force (compression) in the direction of the load N .
2. The midpoint of the strip 2 (B-B', Fig. 1a). Yielding due to bending moment and normal force in the direction of the compressive load N .
3. The centre of the plate (midpoint of strip 3). Yielding due to bending moment and normal force in the direction of the compressive load N .
4. The corner points. Yielding due to torsional moment and normal force in the direction of the load N .

The results are given in Fig. 2 for two values of the yield stress: 2 600 kp/cm² and 7 000 kp/cm² and for the ratio initial deflection over plate width $f_0/2b = 1/1 000$. For most of the calculated points of the diagrams the alternatives 2) and 3) above were most dangerous and the failure loads were for these points calculated as the average values of the failure loads for the alternatives 2) and 3).

For $\sigma_i/\sigma_y = 0$ and $1,2 < \alpha < 2,0$ and for $\sigma_i/\sigma_y = 0,1$ and $\alpha > 1,8$ alternative 4) was most dangerous. For $\sigma_i/\sigma_y = 0$ and $\alpha > 2,0$ alternative 1) was most dangerous.

The effective width b_e (see Fig. 3) is of importance for the column buckling. Calculated values at failure load are given in Fig. 3 for different σ_i/σ_y ($f_0/2b = 1/1 000$; $\sigma_y = 2 600$ kp/cm²). It is seen from the Figure that the initial stresses highly affect the values of b_e/b .

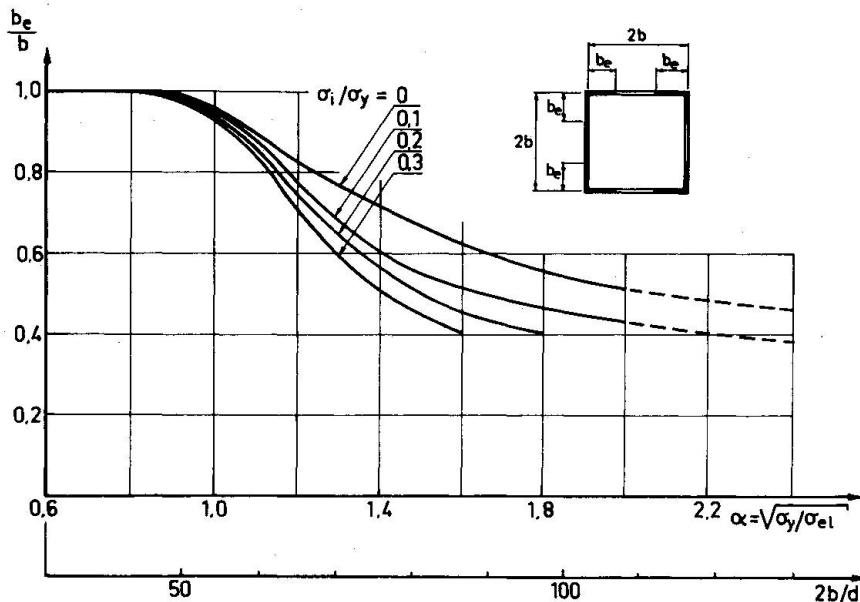


Fig. 3 Ratio b_e/b at the failure load as a function of α .
($\sigma_y = 2 600$ kp/cm²)

It is seen from Fig. 2 that the initial stresses have a very important negative influence on the critical buckling stresses especially for $0,8 < \alpha < 1,6$. The initial stresses have a negative effect on the effective width b_e (see Fig. 3). Both these effects reduce the column buckling load. The applied distribution of the initial stresses is unfavourable. Calculations of a case where $\sigma_i = 0$ in the strip 3 have given higher failure loads. It is therefore a need of studying the influence of the fabrication methods on the distribution of initial stresses. Finally the author among investigations will remind of those by Nishino, Ueda, Tall [2]; Dwight, Moxham [3] and Dwight, Ractcliffe [4] of buckling of welded columns of hollow sections, where it was pointed out that the initial stresses have a large unfavourable effect on the failure load.

SUMMARY

The behaviour of compressed steel plates in the overcritical range is studied. A simplified model of calculation, Fig. 1, which enables to consider initial stresses and deflections is used. The results are intended to serve as a basis for design rules. It is shown that the initial stresses reduce the failure load especially for the dimensions corresponding to $0,8 < \alpha < 1,6$, Fig. 2. Furthermore the effective width is reduced by the initial stresses, Fig. 3. Here omitted results for other distributions of the initial stresses are more favourable.

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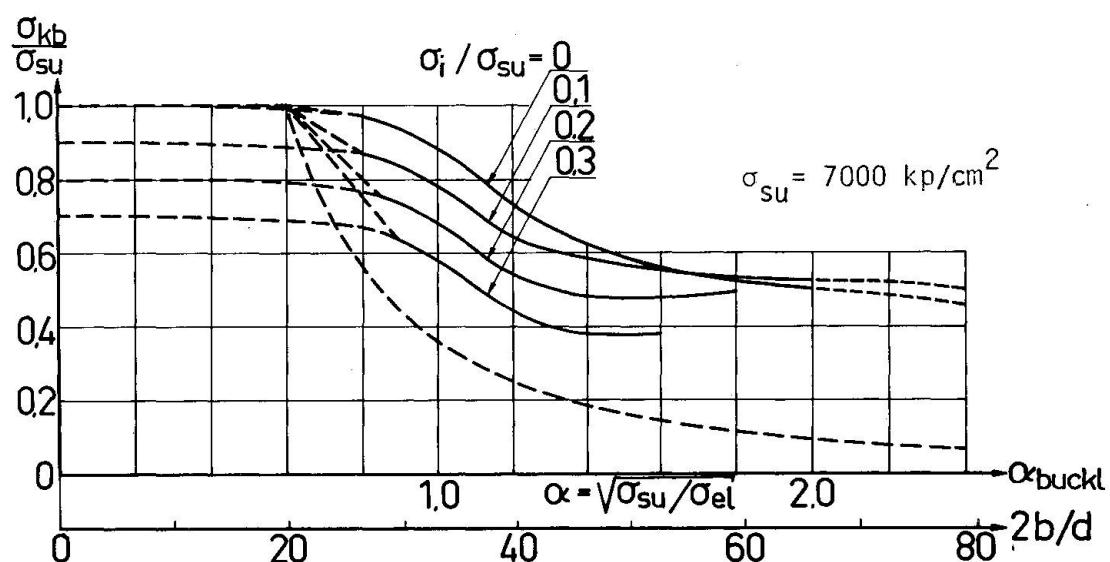
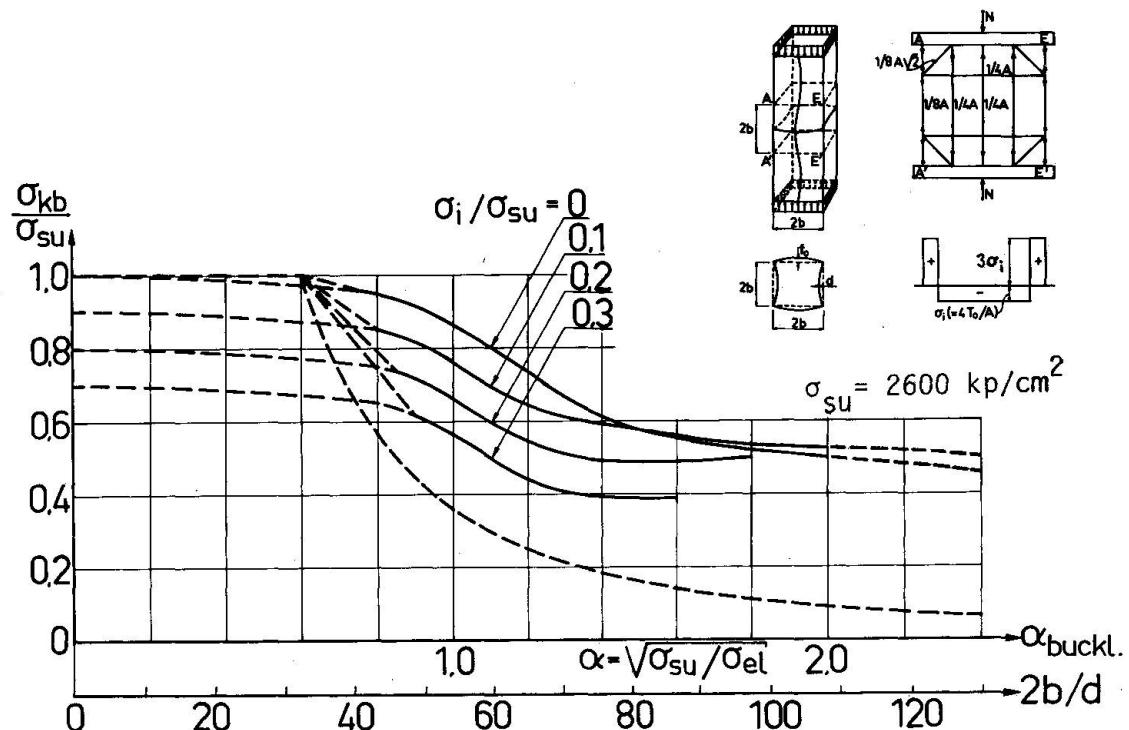


Fig. 1 Critical stress $\sigma_{kb} = N_{kb}/2bd$ at plate buckling, divided by σ_{su} at different σ_i/σ_{su} as a function of $\alpha_{buckl.} = \sqrt{\sigma_{su}/\sigma_{el}}$. Initial stress distribution 1. ($\sigma_{su} = \sigma_y$) $f_o/2b = 1/1000$;

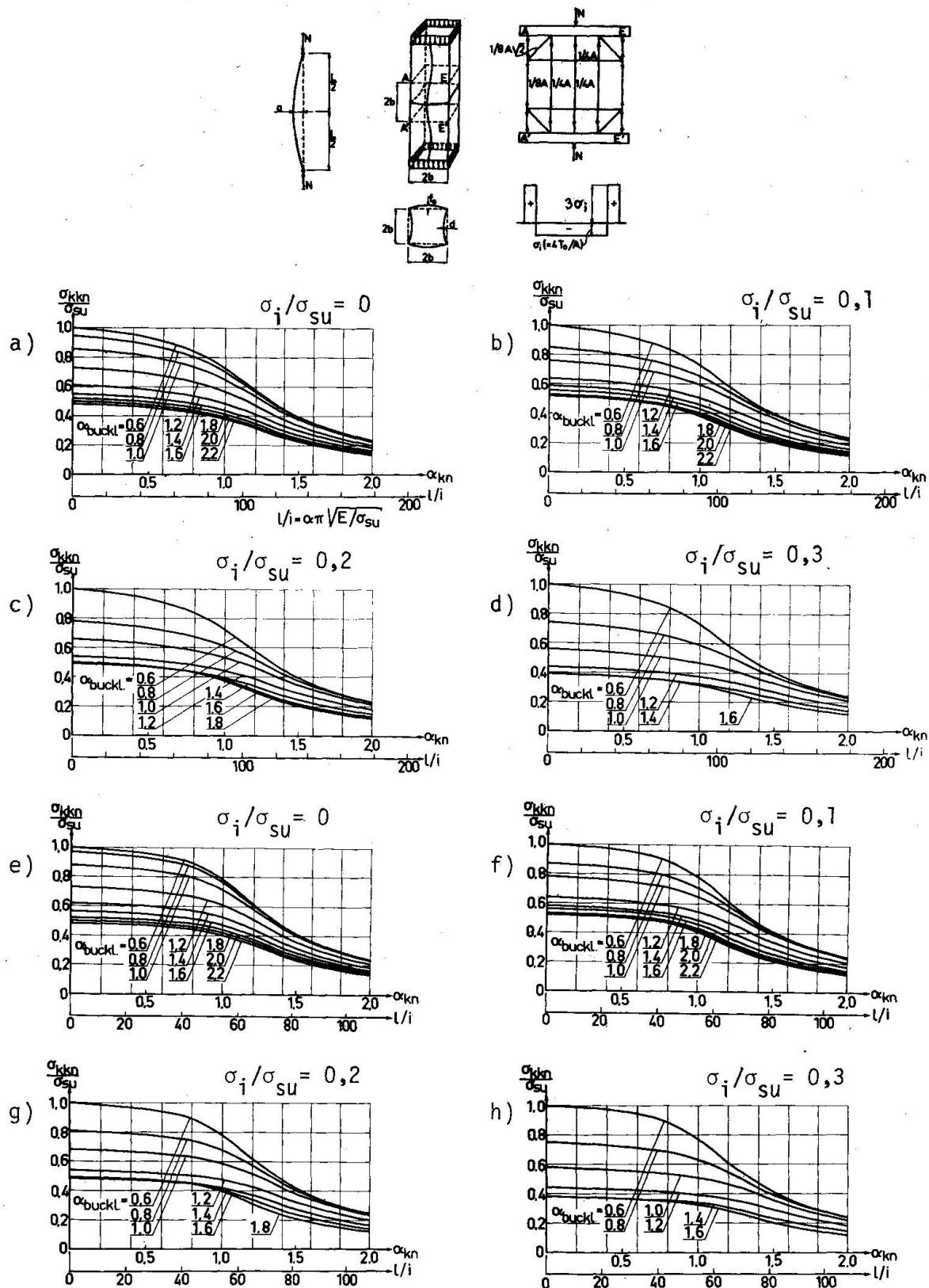


Fig. 2 Critical stress $\sigma_{kkn} = N_{kkn}/2bd$ at column buckling, divided by σ_{su} at different α_{buckl} as a function of $\alpha_{kn} = \sqrt{\sigma_{su}/\sigma_{el}}$. Initial stress distribution 1. ($\sigma_{su} = \sigma_y$) $a/l = 1/1000$; $f_0/2b = 1/1000$;
 a)-d): $\sigma_{su} = 2600 \text{ kp/cm}^2$; e)-h): $\sigma_{su} = 7000 \text{ kp/cm}^2$.

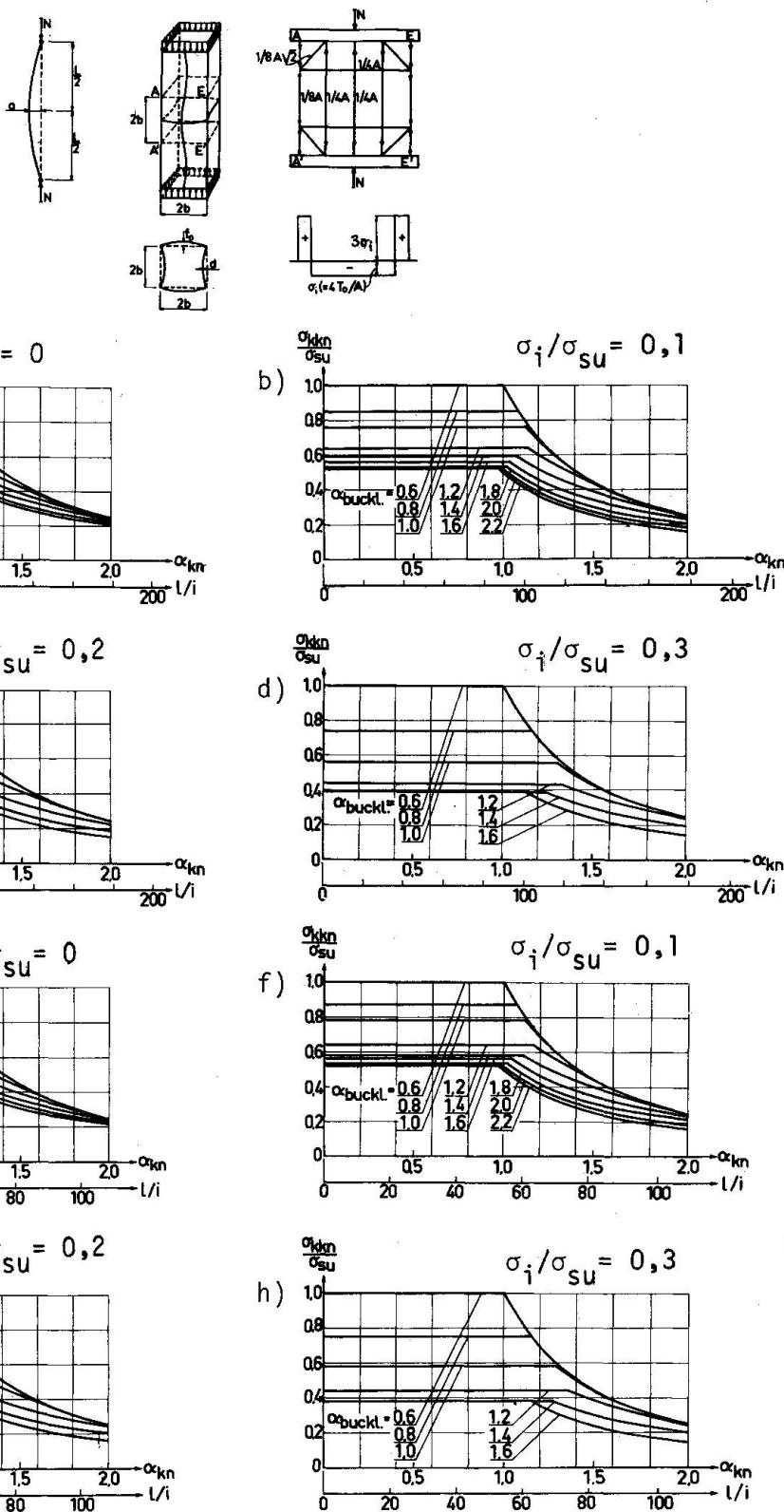


Fig. 3 Critical stress $\sigma_{kkn} = N_{kkn}/2bd$ at column buckling, divided by σ_{su} at different α_{buckl} as a function of $\alpha_{kn} = \sqrt{\sigma_{su}/\sigma_{el}}$. Initial stress distribution l . ($\sigma_{su} = \sigma_y$); $a/l = 0$; $f_0/2b = 1/1000$;
 a)-d): $\sigma_{su} = 2600 \text{ kp/cm}^2$; e)-h): $\sigma_{su} = 7000 \text{ kp/cm}^2$.

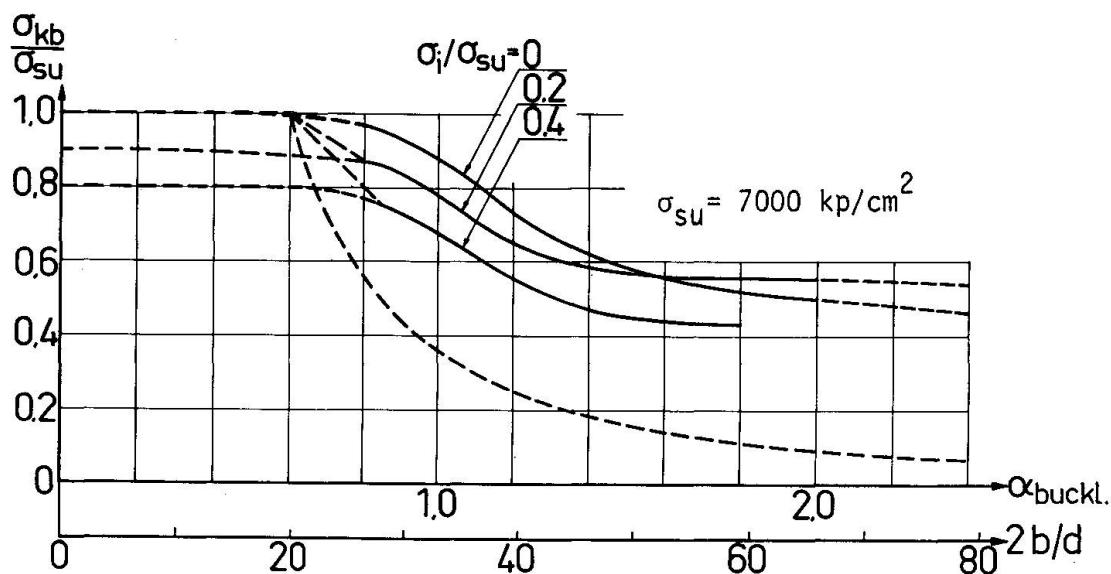
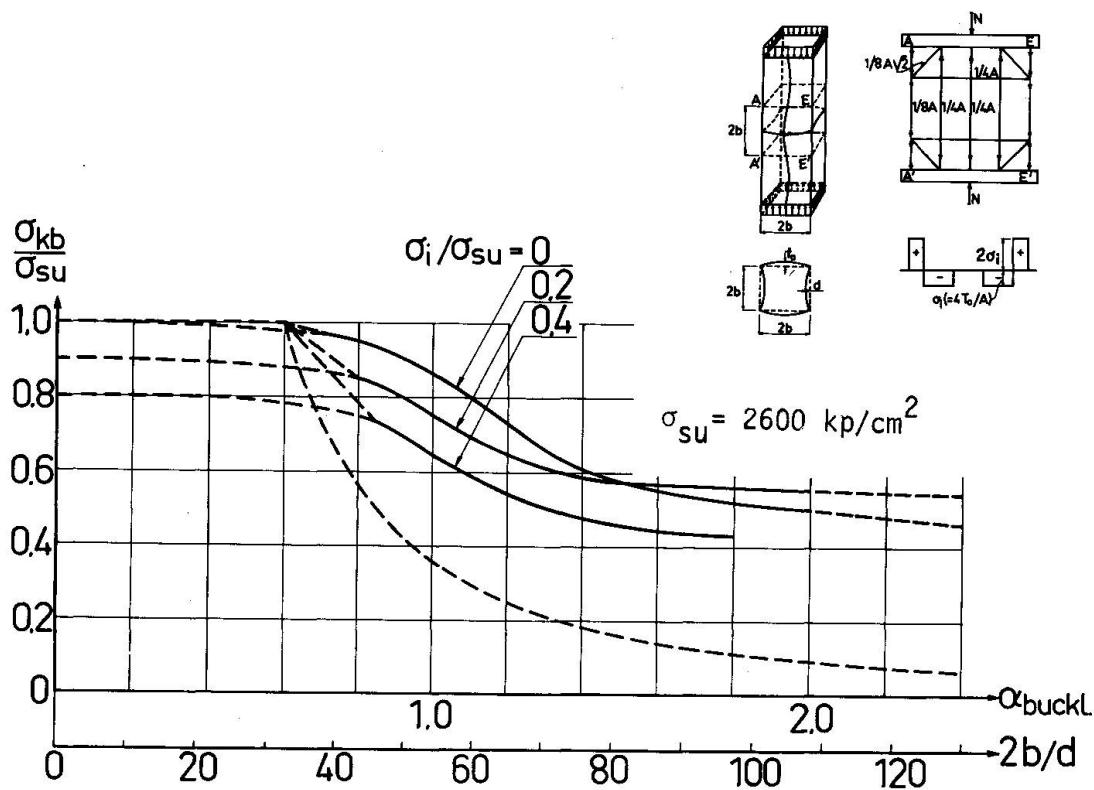


Fig. 4 Critical stress $\sigma_{kb} = N_{kb}/2bd$ at plate buckling failure, divided by σ_{su} at different σ_i/σ_{su} as a function of $\alpha_{buckl} = \sqrt{\sigma_{su}/\sigma_{el}}$. Initial stress distribution 2. ($\sigma_{su} = \sigma_y$). $f_0/2b = 1/1000$.

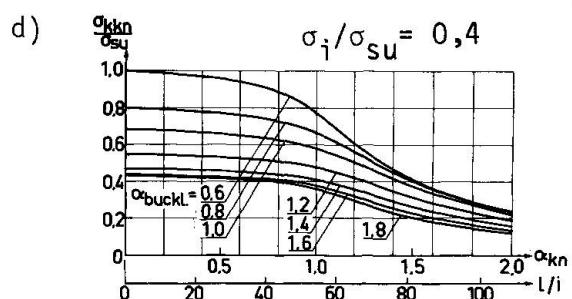
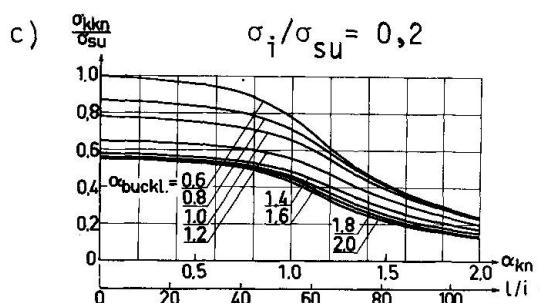
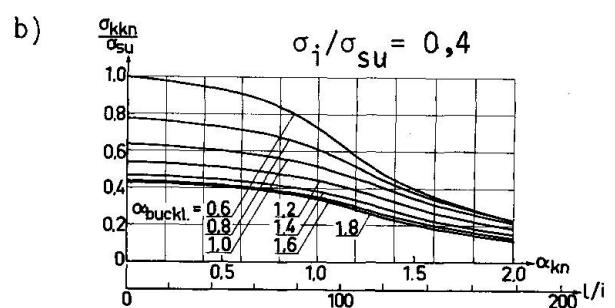
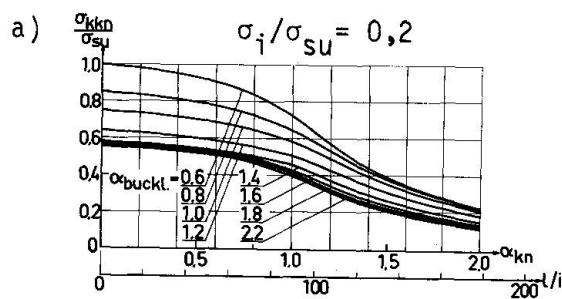
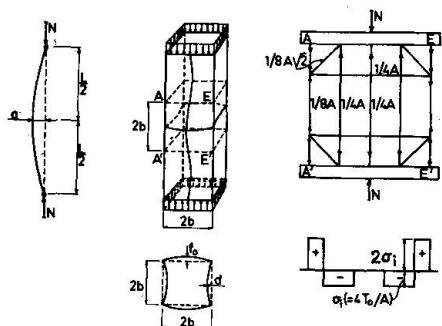


Fig. 5 Critical stress $\sigma_{kkn} = N_{kkn}/2bd$ at column buckling divided by σ_{su} at different α_{buckl} as a function of $\alpha_{kn} = \sqrt{\sigma_{su}/\sigma_{el}}$. ($\sigma_{su} = \sigma_y$).
 Initial stress distribution 2. $a/l = 1/1000$; $f_0/2b = 1/1000$;
 a) - b): $\sigma_{su} = 2600 \text{ kp/cm}^2$; c) - d): $\sigma_{su} = 7000 \text{ kp/cm}^2$.

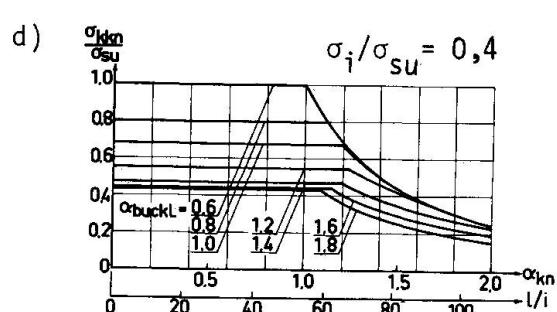
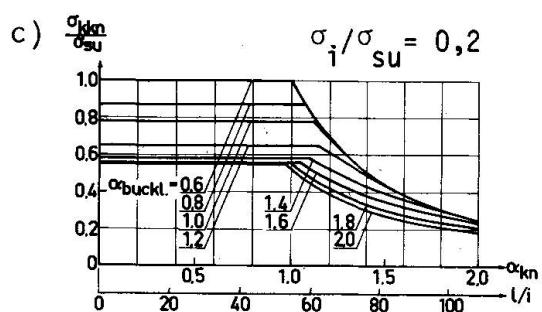
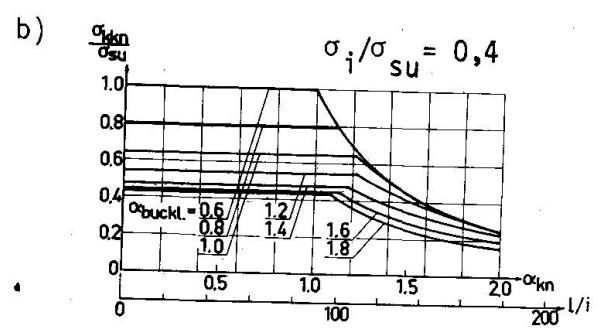
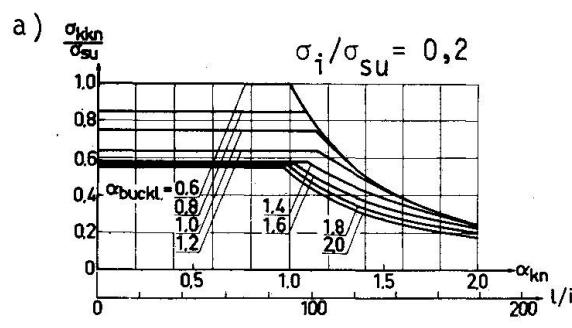
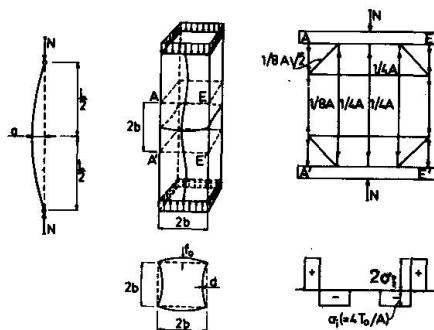


Fig. 6 Critical stress $\sigma_{kkn} = N_{kkn}/2bd$ at column buckling divided by σ_{su} at different α_{buckl} as a function of $\alpha_{kn} = \sqrt{\sigma_{su}/\sigma_{el}}$. ($\sigma_{su} = \sigma_y$). Initial stress distribution 2. $a/l = 0$; $f_0/2b = 1/1000$;
 a) - b): $\sigma_{su} = 2600 \text{ kp/cm}^2$; c) - d): $\sigma_{su} = 7000 \text{ kp/cm}^2$.

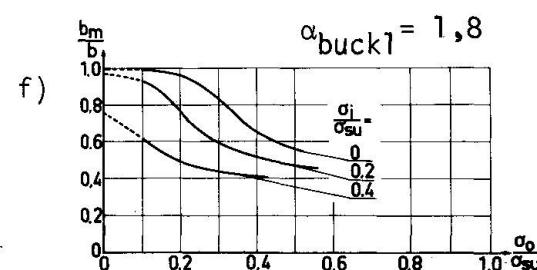
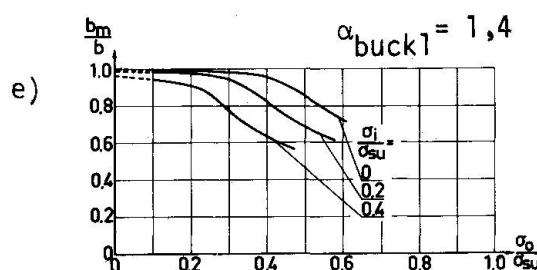
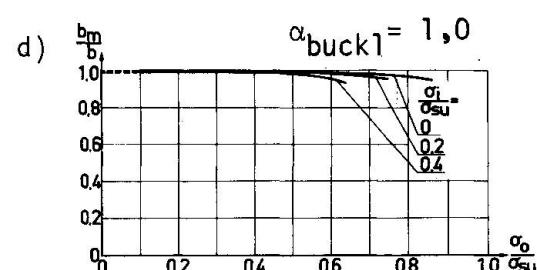
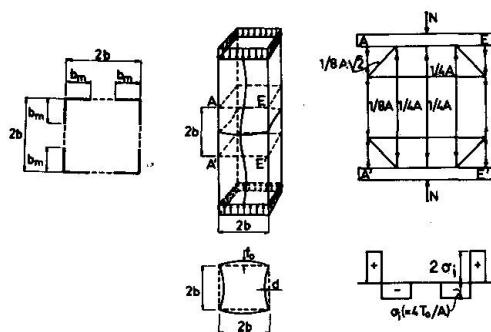
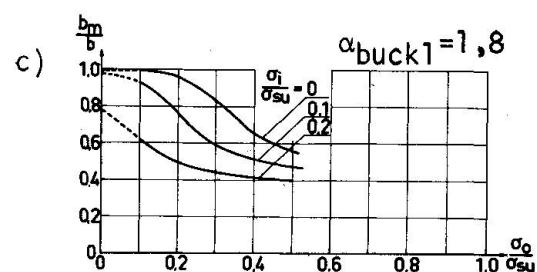
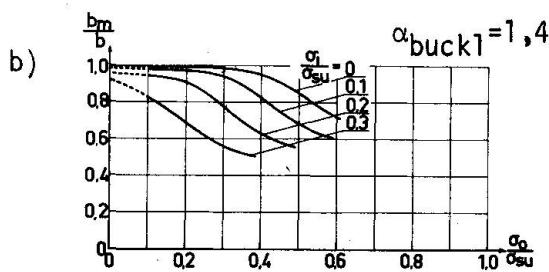
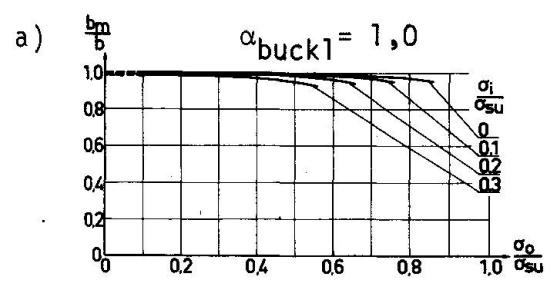
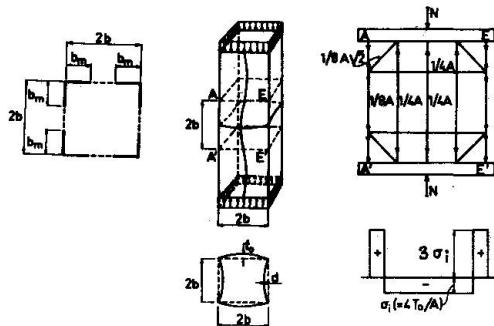


Fig. 7 Effective width b_m divided by b at different σ_i/σ_{su} as a function of $\sigma_0 = N/2bd$, divided by σ_{su} . ($\sigma_{su} = \sigma_y$). $\sigma_{su} = 2600 \text{ kp/cm}^2$; $f_0/2b = 1/1000$;

a) - c): Initial stress distribution 1

d) - f): Initial stress distribution 2.

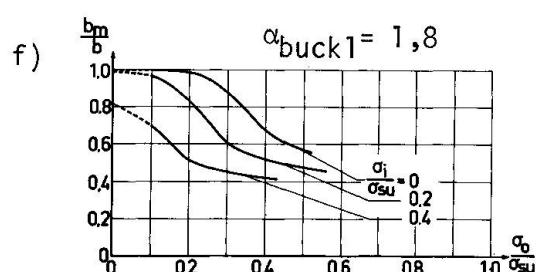
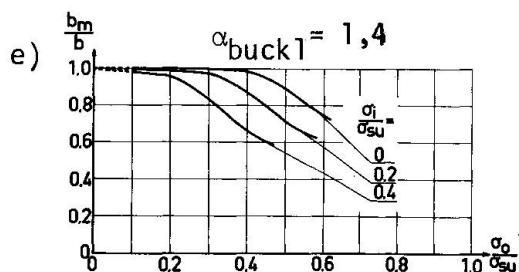
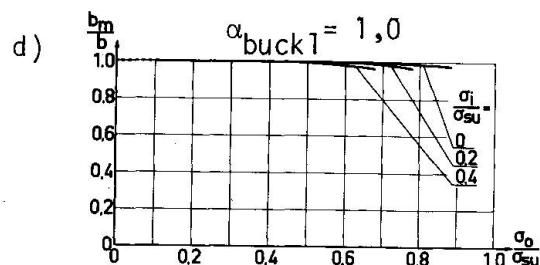
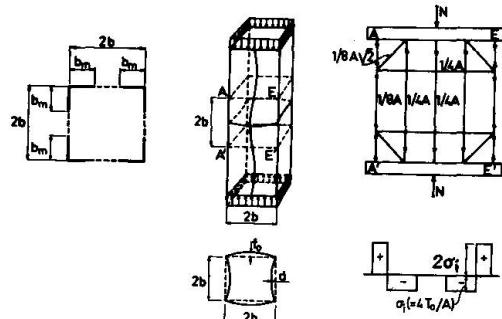
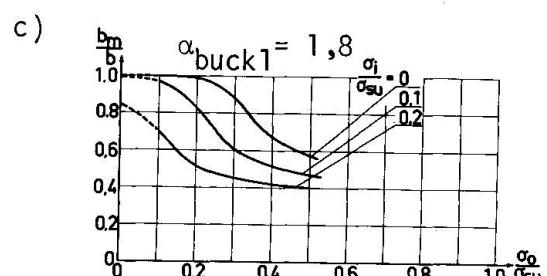
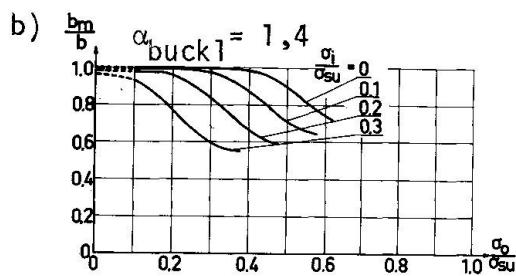
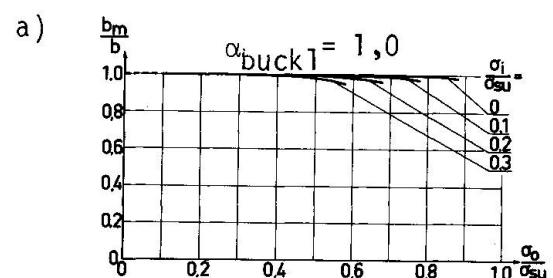
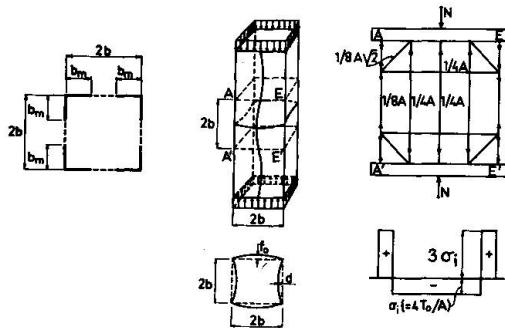


Fig. 8 Effective width b_m divided by b at different σ_i/σ_{su} as a function of $\sigma_0 = N/2bd$, divided by σ_{su} . ($\sigma_{su} = \sigma_y$). $\sigma_{su} = 7000 \text{ kp/cm}^2$; $f_0/2b = 1/1000$;

a) - c): Initial stress distribution 1

d) - f): Initial stress distribution 2.

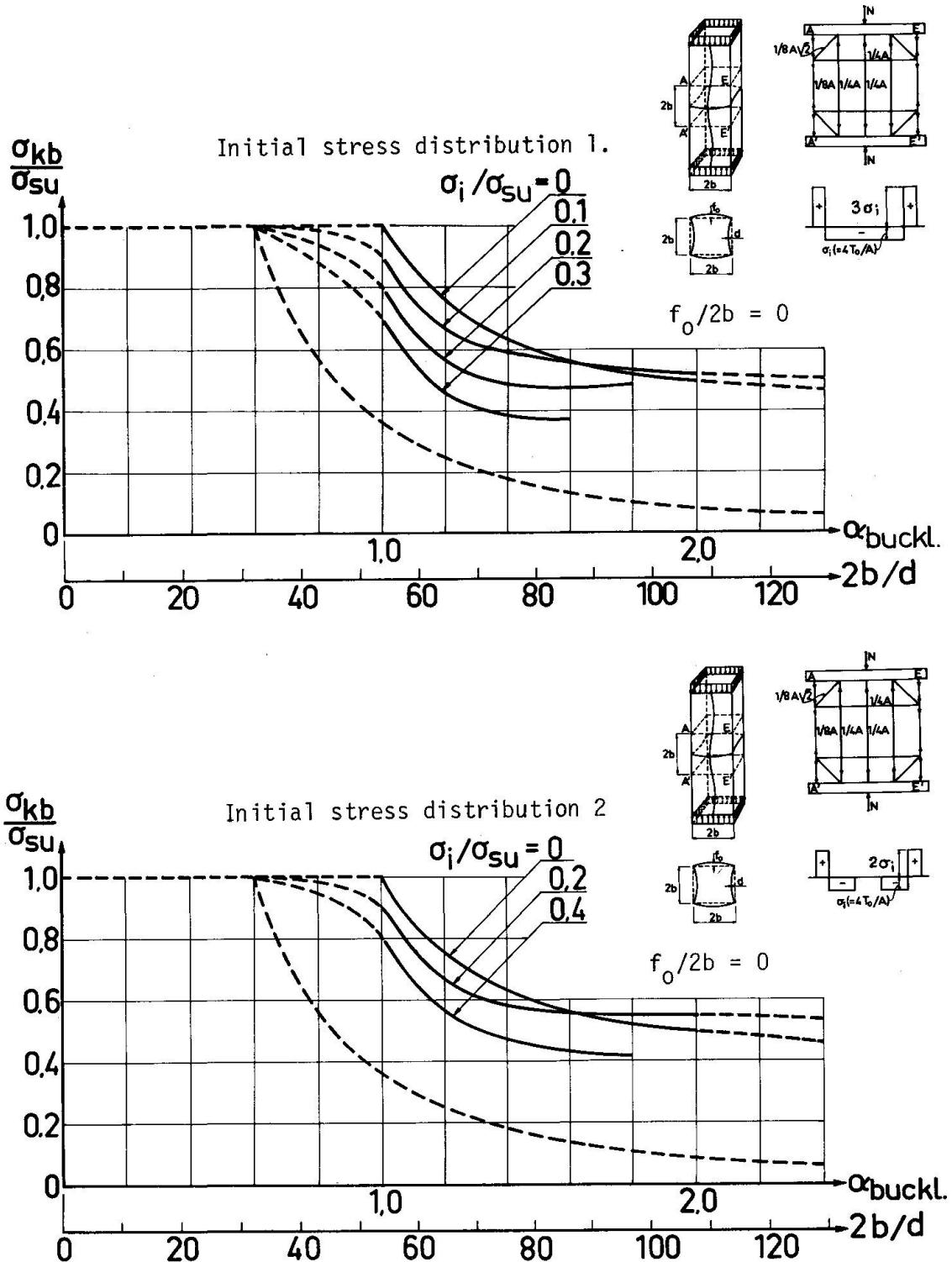


Fig. 9 Critical stress $\sigma_{kb} = N_{kb}/2bd$ at plate buckling divided by σ_{su} at different σ_i/σ_{sy} as a function of $\alpha_{buckl} = \sqrt{\sigma_{su}/\sigma_{el}}$. ($\sigma_{su} = \sigma_y$) $f_0/2b = 0$; $\sigma_{su} = 2600 \text{ kp/cm}^2$ and $\sigma_{sy} = 7000 \text{ kp/cm}^2$.

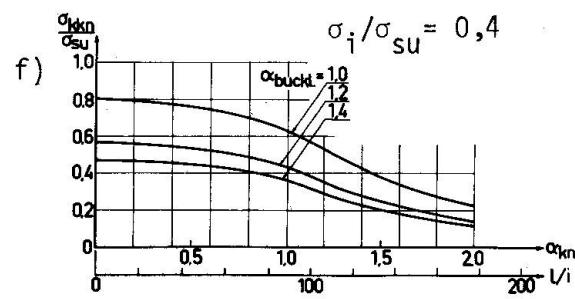
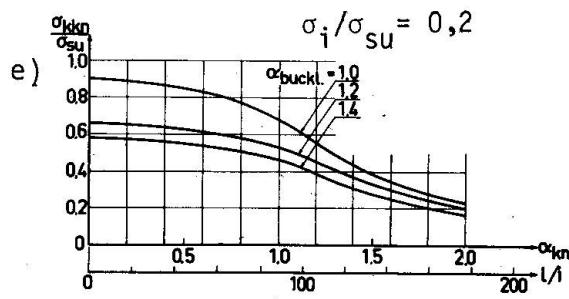
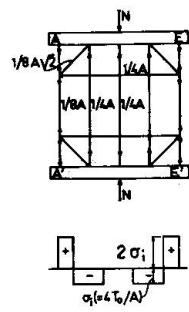
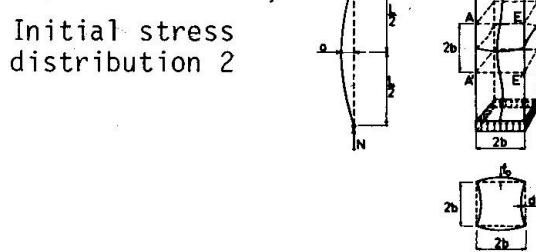
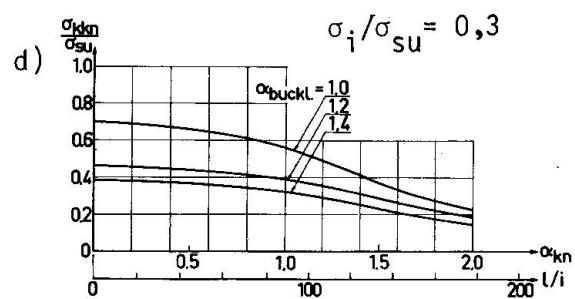
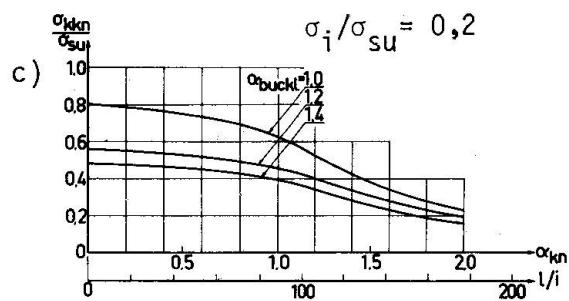
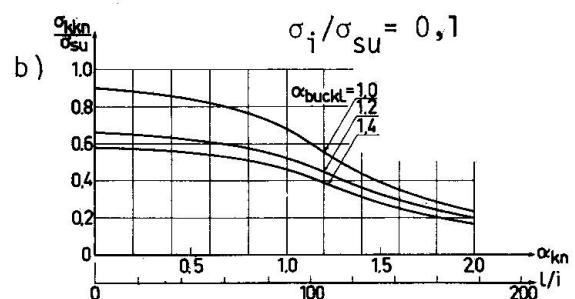
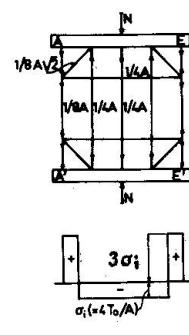
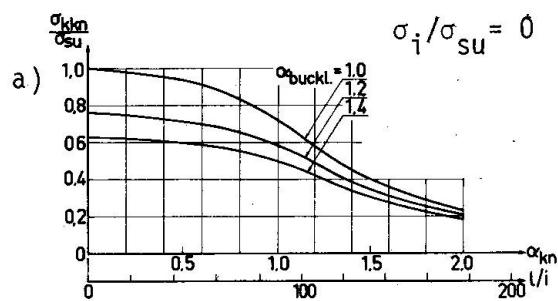
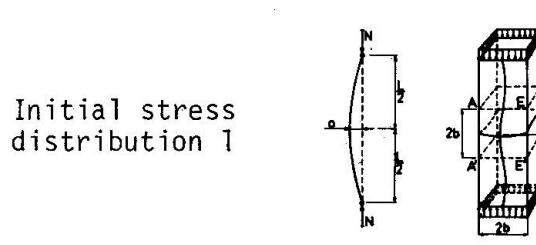


Fig. 10 Critical stress $\sigma_{kkn} = N_{kkn} / 2bd$ at column buckling divided by σ_{su} at different α_{buckl} as a function of $\alpha_{kn} = \sqrt{\sigma_{su}} / \sigma_{el}$. ($\sigma_{su} = \sigma_y$); $\sigma_{su} = 2600 \text{ kp/cm}^2$; $a/l = 1/1000$; $f_0/2b = 0$.

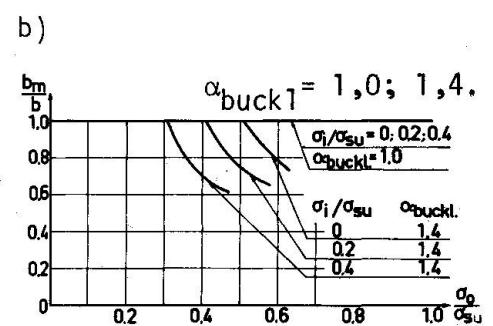
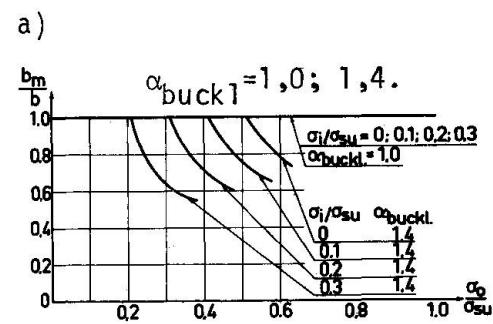
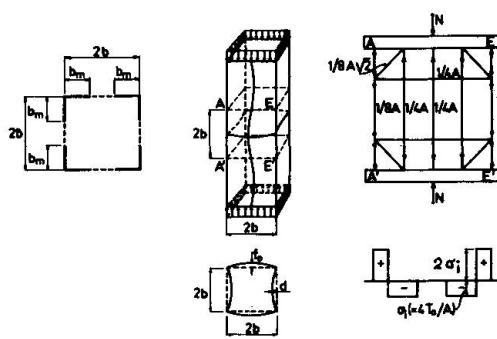
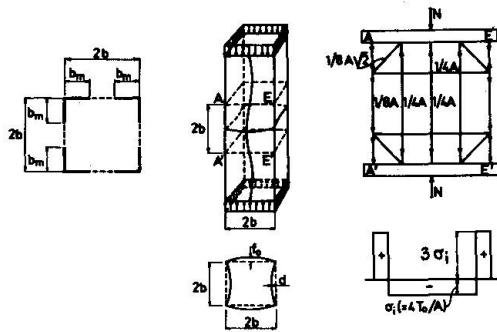


Fig. 11 Effective width b_m , divided by b at different σ_i/σ_{su} as a function of $\sigma_0 = N/2bd$, divided by σ_{su} . ($\sigma_{su} = \sigma_y$) $\sigma_{su} = 2600 \text{ kp/cm}^2$ and $\sigma_{su} = 7000 \text{ kp/cm}^2$. $f_0/2b = 0$.

- a) Initial stress distribution 1.
 b) Initial stress distribution 2.