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USE OF PERRY FORMULA TO REPRESENT THE NEW EUROPEAN STRUT CURVES

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ABSTRACT

The derivation of the Perry-Robertson strut formula is described, along with the other variants which have been used in codes of practice. The limitations in the derivation of the formula are noted.

The evolution of the new European strut curves is summarised, and the report shows how these curves may be represented by a modified Perry formula. The advantages of this representation are noted. Modifications to cater for welded struts and Jumbo rolled sections are described.

1. INTRODUCTION

This report is concerned with the basic design formula for pin-ended steel columns, relating failure stress to slenderness ratio.

Many such formulae are used. The resulting "strut-curves" show a remarkable variation from country to country. Some of the formulae are purely empirical, while others have a degree of theoretical justification. One of those with a more rational background is our own Perry formula, presented in 1886 by Ayrton and Perry[1]. In the form proposed by Robertson[2] (the "Perry-Robertson" formula), following his classic research in the early 1920's, this has formed the basis of British column design for over 40 years, although a variant was introduced by Godfrey[4] in 1962. A slightly different version of the Perry formula is used in France, based on the work of Dutheil[3].

Extensive column testing at Lehigh has shown that different classes of section have significantly different strut-curves[5]. The causes are partly geometrical and partly differences in the locked-in stresses. Members which suffer from severe locked-in compression in their extreme fibres undergo premature yield in these regions when loaded as struts, resulting in reduced flexural stiffness and impaired strength.

A clear indication has emerged that a better approach in codes would be to have several strut-curves for any given steel, with different classes of section allotted to different curves. Thus a Universal Column section, buckling about its minor axis, has a clearly inferior strut performance to a tube, and should be treated accordingly.

Theoretical studies have reinforced this conclusion and multiple strut-curves are likely to be adopted in new European codes. Independent studies at Graz[7] and at Cambridge[10] came up with remarkably similar proposals for such curves. The minor differences have since been ironed out. It seems likely that a Euro-Brittanic strut treatment will emerge.

Three curves are currently proposed. Their derivation has been complex and they cannot be precisely defined by simple formulae. Empirical polynomials have been devised to represent them. This report puts forward a simpler and more rational formula, based on the Perry formula, which has various advantages. The curves are not significantly altered.

Even for one class of section it would be impossible to produce <u>the</u> true strut-curve, which accurately represented the performance of any test specimen in that class. Strut performance is governed by imperfections, which vary between specimens and cause considerable scatter. The proposed new design curves aim to provide a reasonable lower bound on this scatter for each group.

2. FACTORS AFFECTING STRUT STRENGTH

2.1 Material properties

The most important material properties for strut performance are yield stress σ_y and Young's modulus E.

For design purposes the yield stress must be taken as the specification value, depending mainly on the grade of steel and also on product thickness. The range of values specified in BS.4360: Part 2: 1969 for each overall grade is as follows:

	N/mm^2
Grade 43	220 to 280
Grade 50	325 to 355
Grade 55	400 to 450

These values refer to the tensile yield stress obtained from mill-tests at a rather high strain rate. A slower and more appropriate rate of straining would give lower values. However, the compressive yield stress, which is what matters for struts, runs higher than the tensile figure for a given sample. Although these two effects tend to cancel out, we would expect to find that a significant amount of actual production would not reach the specified yield values in slowly conducted compression tests. Despite this, it is unlikely to be practical politics to do anything but adopt the BS.4360 yield values as a basis for strut design.

In I-sections the values specified effectively refer to the flange material, which always has a lower yield than the web material. This is all right, because flange properties are what matter.

Turning to E, Baker[12] records a variation from about 203 to 207 kN/mm^2 for structural steel and has come up with strong recommendation for accepting a figure* of 205 kN/mm^2. This value is likely to be used in the new British Codes. Previous codes have used values of 201 and 210 kN/mm^2. On the Continent there is some support for a high value of 214 kN/mm^2 (= 21.0 kgf/cm^2).

A further material property affecting stocky struts is the strain-hardening modulus E_s. It is seldom quoted, but is believed to be in the range E/40 to E/30. Conventional strut theories ignore the beneficial effect of strain-hardening, but its importance has been clearly demonstrated by workers in the truss field[13,14].

2.2 Imperfections

In the practical range of slenderness, where yield and instability are of comparable importance, the performance of a strut is critically affected by its imperfections, both crookedness and locked-in stress. They obviously vary a lot, and design rules must assume pessimistic values.

BS.4 specifies a straightness tolerance on rolled steel sections of L/960. On the strength of this Young[10] assumed a crookedness of L/1000 in deriving his proposed strut-curves. The Graz workers[7] adopted an identical figure. L/1000 may be a reasonable tolerance for rolled sections, but welded sections could well be more crooked than this, especially if unsymmetrical. The original Merrison appraisal rules for box-girder bridges[15] took a figure as high as L/600 to L/400 for plate stiffener combinations.

In view of the importance of initial crookedness in strut behaviour it is surprising that so little is known statistically about the values which actually occur.

Residual stresses are harder to tie down. For rolled sections the level and the pattern of the locked-in stresses inevitably varies between specimens, even from the same mill. The best documented class is the I-section[9], for which a reasonably well defined trend is apparent provided the rolling practice is taken into account. It is found that column sections tend to carry appreciable compression in the flange toes, typically approaching 100 N/mm^2, which impairs their strut performance. Beam sections do better because their flanges only contain low compressive stresses; webs may carry very high compression, sometimes approaching yield, but this is less important for column buckling.

Young[9] analysed many residual stress measurements for rolled I-sections and produced a pattern for design purposes, representing the most adverse condition

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* 205 kN/mm^2 = 20.8×10^3 kgf/mm^2 = 13.3×10^3 ton/in^2 = 29.7×10^6 lb/in^2.

likely to arise. His pattern depended on the flange-web area ratio, but was independent of the yield stress, suggesting that as the yield stress goes up, the relative importance of the residual stresses goes down. For any given shape of section his assumed residual stresses were independent of the absolute size or thickness. This is believed to be reasonable for the main run of I-sections, but not for the very thick "Jumbo" sections made in America, which can have very severe locked-in stresses.

Little is known about residual stresses in other types of rolled section (channel, angle, tee, bulb-flat, hollow), for which regrettably few determinations have been made. It is supposed that hot rolled hollow sections contain very low residual stresses but this fact has yet to be established.

In sections fabricated from plate, the residual stresses are more predictable in that the shrinkage forces in the welds can be estimated from the size of the weld[9]. This is of limited help, because one cannot reasonably require a designer to perform such calculations. For code purposes one must base design rules on the most adverse residual stresses for a given class of section, bearing in mind likely extremes of weld size. One difficulty is that the shape of the "tension block" in the region of the weld is not properly known. Some workers have assumed a rectangular pattern while others have taken a triangular one, leading to appreciably different strut predictions. The true pattern is probably an intermediate trapezoidal shape.

3. PERRY-ROBERTSON FORMULA

3.1 Basic derivation of Perry formula

The Perry strut formula[1] is based on the following assumptions:

(a) The strut is pinned and centrally loaded at its ends.
(b) It has an initial sinusoidal bow.
(c) There are no locked-in stresses.
(d) It behaves elastically up to failure.
(e) Failure occurs when the stress in the inner extreme fibre reaches yield*.

The resulting equation for the average applied stress σ at failure is:

$$(\sigma_E - \sigma)(\sigma_y - \sigma) = \eta\,\sigma_E\sigma \qquad (1)$$

where σ_E is the Euler stress and σ_y the yield stress.

The non-dimensional quantity η (the "Perry constant") measures the initial out-of-straightness Δ at mid-depth, and is defined thus:

$$\eta = \frac{\Delta}{c} \qquad (2)$$

in which c is the "semi-core" of the section, given by:

$$c = \frac{r^2}{y} \qquad (3)$$

where r is the radius of gyration and y is the distance from the centroid to the yielding extreme fibre.

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*Note that for very unsymmetrical cross-sections it is possible, at high slenderness ratios, for tensile yield to occur at the outer extreme fibres before compressive yield is reached at the inner extreme fibres. Equation (1) does not cover this possibility.

3.2 Putting a value to η

The shape of strut-curve which results from Equation (1) is critically affected by the value taken for η, which depends of the assumed initial crookedness and which must obviously vary with the length of the strut. Robertson[2] suggested that η should be assumed proportional to the slenderness ratio, that is:

$$\eta = \alpha \frac{L}{r} \quad \dots \dots \dots \dots \dots \dots \dots \dots \quad (4)$$

This is equivalent to assuming that the initial out-of-straightness is proportional to the length, a reasonable assumption.

Rather than determine α from measurements of Δ, Robertson adjusted the value to give good agreement with actual strut tests. He found that the curve corresponding to $\alpha = 0.003$ formed a reasonable lower bound to the scatter of results from his own carefully conducted tests and from those of other workers.

The Perry formula, with $\eta = 0.003\ (L/r)$, became adopted for the British Standards covering bridges (BS.153) and buildings (BS.449).

In 1962 Godfrey[4] suggested a new expression $\eta = 0.3\ (L/100r)^2$ .... (5)
BS.449 adopted this change but BS.153 retains the earlier expression. Fig. 1 shows that the BS.449 version gives appreciably higher stresses at $L/r < 100$, and lower ones at $L/r > 100$ than does the BS.153 version.

Godfrey's expression (5) for η is equivalent to assuming that the initial bow Δ is preparational to L^2. The same applies to the version of the Perry formula, due to Dutheil[3], employed in the French code, which in effect uses the following expression for η:

$$\eta = 0.38 \left(\frac{\sigma_y}{250}\right)\left(\frac{L/r}{100}\right)^2 \quad \dots \dots \dots \dots \dots \dots \quad (6)$$

where the yield stress σ_y is in N/mm^2. This implies that the initial crookedness gets worse as the yield stress increases. It leads to stresses below the Godfrey curve, even for mild steel.

4. FAULTS OF THE PERRY-ROBERTSON FORMULA

The Perry-Robertson formula is the right kind of formula in that it talks about stress magnification, and it has the virtue of simplicity. It should not be regarded as a precise treatment, as it contains a certain degree of empiricism. In making the following criticisms so many years later, the author in no way wishes to belittle the great achievement of Professor Robertson in producing a strut treatment that has been widely used for nearly 50 years.

The assumption that failure occurs when yield is first reached in the extreme fibres is slightly pessimistic. The error depends of the shape factor of the section.

By using this criteria, strain-hardening is ignored, as indeed it is in most strut treatments. This makes the strut-curve dive as soon as it leaves the stess-axis, suggesting that it is never possible to achieve yield in compression, which is not true.

Robertson's adoption of a constant α in equation (4) leads to inconsistency between sections of different geometry.

From equations (2), (3) and (4):

$$\Delta = \eta c = \alpha . \frac{L}{r} . \frac{r^2}{y} \quad \text{which gives:} \quad \frac{\Delta}{L} = \alpha . \frac{r}{y} \quad \dots \dots \dots \dots \quad (7)$$

Thus with α held constant at 0.003 Robertson's assumed crookedness Δ, as a proportion of the length, depends on r/y and so alters with the shape of the section. The variation is $2\frac{1}{2}$:1 between various sections. In reality one would expect the amount of initial bow to be independent of the section geometry for rolled sections and symmetrical welded ones.

The Perry equation (1) takes no account of locked-in stresses. Using the theoretical value for η, the predicted strength of a strut containing appreciable residual stresses will be too high.

Robertson partly covered this since the design value of $\eta = 0.003\ (L/r)$ was based on tests and corresponds to a fictitious initial bow, greater than L/1000. In effect he made some allowances for residaul stress effect by exaggerating the crookedness, but did not take into account the variation from one class of section to another.

5. NON-DIMENSIONAL PRESENTATION

A popular way of presenting strut data in academic work employs the non-dimensional quantities N and λ, where:

$$N = \frac{\sigma}{\sigma_y} \quad \text{and} \quad \lambda = \frac{L/r}{\pi\sqrt{E/\sigma_y}}$$

so that the Euler curve reduces to:

$$N = \frac{1}{\lambda^2}$$

This supposedly facilitates comparison between results obtained for steels of differing yield stress. There is, however, no logical reason why results obtained for widely varying steels should give identical N-λ curves.

It is perhaps not generally appreciated that the Perry-Robertson formula presented on such an N-λ plot, gives a curve which rises with the yield stress. The Dutheil version, because it makes η increases with σ_y, leads to the same N-λ curve for all strengths of steel.

6. RIGOROUS STRUT TREATMENTS

The Perry-Robertson formula is a simple approach which takes crookedness into account, but ignores locked-in stress. An equally simple type of approach is possible which allows for residual stress, but ignores crookedness[17]. What is required is a treatment which properly considers both.

Such treatments do exist, but are laborious. They are often called after Newmark[16], whose numerical integration procedure they generally employ. First one must compute moment-curvature curves for the section at various levels of axial load, taking the residual stresses into account. These M-ϕ-P curves are then used to obtain the behaviour of any given unstraight strut made of the section concerned. An iterative procedure is used to obtain the correct deflected shape corresponding to a given load P. In this manner a load-deflection curve can be generated.

Young[9,10] used a comparable finite-difference procedure.

Even these painstaking methods are not entirely rigorous, as they consider no reversal of stress, and ignore strain hardening. However, the Newmark (or Young) type of method is a valuable research tool which enables accurate (except at low L/r) strut-curves to be generated for a given section, taking into account initial bow and residual stresses.

7. THE EUROPEAN STRUT-CURVES

7.1 Evolution

The proposed new European strut-curves stem mainly from the work carried out under the late Professor Beer at Graz[7], for Commission 8 of the European Convention for Constructional Steelwork. More recently Cambridge University, supported by the Construction Industry Research and Information Association became involved[10]. The final curves have resulted from interaction between the two teams.

The procedures adopted by Schulz (at Graz) and Young (at Cambridge) were similar. A number of specific sections were chosen for study. A suitable pattern of residual stress was assumed for each, and a rigorous method (Newmark or equivalent) then employed to obtain the strut-curve. This was mostly done for mild steel with an assumed initial crookedness (Δ) of L/1000.

From the range of curves thus obtained each worker then selected a limited number for use as design curves. At Graz three such curves were eventually settled on, and at Cambridge four. In conjunction with each set a selection table or chart was provided, showing which curve to use with any given section.

The theoretical work at Graz was backed up by a massive programme of column testing, carried out in various countries[8]. Because of the variation in the imperfections from one specimen to another there was a good deal of scatter in the results, as is inevitable in strut testing. It was found that for each class of section considered, the proposed design curve formed a good lower bound to the spread of results obtained. The programme therefore provided valuable support in favour of the theoretical curves.

7.2 Plateau at low L/r

The curves as calculated did not allow for strain-hardening, and therefore started to descend immediately on leaving the stress axis. This is not in accord with the fact that a stocky member can reach its squash load, and may well exceed it. To overcome this discrepancy the Cambridge team decided to make an arbitrary adjustment in the region of low L/r, such that the curves would have an initial horizontal portion before starting to descend. The calculated strut-curves then had to be raised to join the end of this horizontal portion.

The extent of the flat part, defined by L/r < S was determined from:

$$\lambda_o = \frac{S_o}{\pi\sqrt{E/\sigma_y}} = 0.2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

giving the following typical values for S_o:

	σ_y	S_o
Grade 43	250 N/mm²	18
Grade 50	350 N/mm²	15
Grade 55	450 N/mm²	13

In arriving at the arbitrary figure of $\lambda_o = 0.2$ in equation (8) some credence was given to the notion that the plateau should extend to point where the Euler curve drawn with E replaced by E_s cuts the line $\sigma = \sigma_y$. In fact the value 0.2 corresponds to $E_s = E/25$, which may be thought a rather high value for E_s, but account must be taken of the pronounced plateaus observed in research on trussels[13,14].

7.3 The Curves

The Graz and Cambridge curves were in good general agreement except at low L/r. After a meeting in Graz to attempt to bridge the differences, it was decided to promote the three European curves but with the British plateau incorportated at low L/r. These proposals are now going forward and will, it is hoped, be adopted in the countries concerned including Britain.

The proposed curves (a,b,c) are shown in Fig.3. They result from actual computations made for mild steel, from which were then determined the non-dimensional N-λ curves given in the figure. The intention is that these non-dimensional curves, although based on mild steel, will still remain applicable as σ_y changes and be used as the basis of design for any grade of steel. The validity of so doing is discussed later.

7.4 Curve selection

Table 1 gives the agreed allocation of rolled sections to the three curves. Welded sections fabricated from plate are discussed later.

The appropriate curve for a given member depends both on the shape of section, and on the axis about which buckling occurs. The factors which push a section onto a low curve are: (a) residual compression in the extreme fibres, and (b) low value of r/y.

In some cases the curve allocation is somewhat tentative, because not enough is known about locked-in stresses. As more information comes to light, there may be some changes in the curve selection table. Even as it is now predictions will generally be an improvement over those obtained from current codes having a single strut-curve for a given steel.

7.5 Empirical formula

Young[10] has proposed the following polynominal formula to represent the non-dimensional strut-curves:

$$\lambda = \sqrt{\frac{C_o}{N} + C_1 + C_2N + C_3N^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (9)$$

Values of the four coefficients, which provide a close fit to the computed European N-λ curves, are listed in Table 2.

This note puts forward an alternative to equation (9).

8. APPLICATION OF PERRY FORMULA TO THE EUROPEAN CURVES

8.1 Required changes

It is proposed that the Perry formula (1) should continue as a basis for strut design. Its method of application will need to be modified to provide a range of curves to cover different section groups and to incorporate a horizontal plateau at low L/r.

8.2 Adjustment of α

A range of curves can be provided by varying α in expression (4). A section which performs well as a strut (low residual stresses, high r/y) should be accorded a low value of α, while a poor performer (unfavourable residual stress, low r/y) should be given a high α.

If there were no residual stresses and if first yield caused immediate failure, the appropriate α would be, from (7): $\alpha = (\Delta/L).(y/r)$. Taking an initial crookedness of L/1000, this becomes $\alpha = 0.001\ (y/r)$ .. .. .. (10).

Ideal values of α range from 0.001 to 0.003. They do not form a true basis for design, because residual stresses have been ignored. For nearly all sections the adverse effect of locked-in stress will more than cancel out the conservative nature of the first yield failure criterion, and increased values of α will be needed.

8.3 Formation of plateau

The need for a plateau up to $L/r = S_o$ (see section 7.2), can be conveniently provided by replacing equation (4) with the following:

$$L/r < S_o \text{ .... } \eta = 0$$

$$L/r > S_o \text{ .... } \eta = \alpha\left(\frac{L}{r} - S_o\right) \quad \text{.. .. ..} \quad (11)$$

The strut is fictitiously taken as being initially straight if its length is less than S_o; it it is longer than this, its initial bow is taken as proportional to $(L - rS_o)$. S_o is given by (8).

Fig. 1 shows the resulting curves for mild steel struts with various values of α, compared with the present British Standards.

8.4 Representation of the European curves

Using the method just described, the Perry strut formula (1) can be readily employed to represent the new European strut-curves. Summarizing:

(i) The basic formula remains as given by equation (1).
(ii) η is now obtained from (11). S_o is assigned the value given by (8).
(iii) For each curve a value of α is selected, to obtain a good fit.

It is suggested that the following values of α should be adopted:

	α
Curve a	0.0020
Curve b	0.0035
Curve c	0.0055

The resulting modified Perry curves for mild steel are shown in Fig. 2, where they may be compared with the European curves plotted from the polynominal expressions (9). The agreement is acceptable.

Some might argue that in the important range $L/r = 40$ to 100 the accuracy of representation of the two upper curves (a,b) could be improved by a slight increase in the values adopted for α. Bearing in mind the many uncertainties in strut prediction and the doubts about the assumed imperfections, the author considers that this would suggest a degree of accuracy that does not really exist and that it would be more sensible to adopt the "round-number" values listed above.

9. TREATMENT OF HIGH YIELD STRESS STEELS

The European proposals consist of a set of three non-dimensional N-λ curves. They result from computations performed on mild steel struts, but are intended to give the necessary σ-L/r curves for design in any steel.

In fact there is no reason why, for a given section, the strut-curves for different grades of steel should all lie on top of each other when shown on an N-λ plot. Even when residual stresses are ignored, the true N-λ curve tends to become raised as the steel gets stronger. This is apparent from Fig. 3.

Use of the same N-λ curve for all yield stresses, would imply increasing initial crookedness as the yield stress goes up, for which there is no justification.

When residual stresses are introduced, the effect becomes more pronounced. It is believed that the absolute level of locked-in compressive stress in a memeber of given section is largely independent of the yield stress of the steel. As the yield stress goes up, the relative importance of the residual stresses therefore goes down. This suggests that the N-λ curve appropriate to a certain section in mild steel will be even further over-safe when employed for design in high yield. This contention is supported by computations performed by Young[10], which show that a Universal Column section (buckling about yy) in Grade 55 steel has a significantly higher N-λ curve than the same section in Grade 43, eqivalent to a rise from his curve C to curve B.

Young, in the Cambridge proposals[10], envisaged that this effect of yield stress would be taken into account in the curve selection procedure; his selection chart enabled a higher curve to be used, when the yield stress was sufficiently high. This suggestion did not find favour with the Europeans, as it was thought to only bring benefit for very strong steels. With Grade 50 steel the improvement, in N-λ terms, was not enough to permit a rise to the next design curve up.

The European N-λ curves (Fig. 3) have therefore gone forward as a proposed basis for design, without any allowance being made for yield stress in the selection table (see Table 1). This arrangement, if finally adopted, will penalise members made of higher yield steels.

The proposed adaption of the Perry formula, summarized in 8.4, attempts to overcome this difficulty and does not penalise the stronger steels to much. Taking a given value for α automatically makes the N-λ curve move up with increasing yield stress - as it should. The proposed values for α (0.0020, 0.0035, 0.0055) have been chosen so as to fit the European curves (a,b,c), when these are applied to mild steel members. For higher yield steels the European treatment becomes increasingly over-safe, whereas the procedure of 8.4 preserves reasonable accuracy. This is apparent from Fig. 3 which shows N-λ plots for three grades of steel, based on the proposed Perry treatment with $\alpha = 0.0020$, compared with the (unvarying) European curve a. Even so, the theoretical results obtained by Young[10] suggest that for members containing unfavourable residual stresses, as for example a Universal Column buckling about yy, the Perry treatment will still tend to penalise the stronger steels a little.

10. SECTIONS FABRICATED FROM PLATE

The proposed new strut-curves, used in conjunction with Table 1, directly cover rolled sections including I-sections reinforced with flange cover-plates. Members built up from plate, such as welded I- and box-sections are less straightforward. The locked-in stresses caused by welding are not properly understood, the exact pattern in the vicinity of the weld being uncertain. The appropriate strut-curves are therefore not yet clearly determined, but it is apparent that the curves developed for rolled sections do not quite have the right shape.

Theoretical results by Young[10], although based on an over-idealized pattern of residual stress, indicate a characteristic difference between rolled and welded strut-curves. This is shown in Fig. 4, which compares the strut-curve for rolled Universal Column section buckling about yy (i.e. curve c), with those which are believed to be typical for welded sections of similar shape. The essential point is the depression of the welded curves in the earlier part of their range. This is governed by the severity of the residual compressive stress at the toes of the flanges, which will depend on the weld heat input relative to the area of the section.

The European proposals cope with this situation by utilizing for the welded sections a lowered curve based on a fictitiously reduced yield stress. The reduction in σ_y ought strictly to be related to the size of the welds, but in view of the various uncertainties a uniform reduction is made. Thus lightly welded sections tend to be penalized and heavily welded ones favoured. The proposed yield reductions translated into British terms would be as follows, the figure for grade 55 being the author's own extrapolation:

	assumed reduction in σ_y
Grade 43	15 N/mm^2
Grade 50	20 "
Grade 55	25 "

Although the compressive residual stress is largely independent of the yield stress for a given size of weld, it is still reasonable to have more reduction with the stronger steels, because the welds will tend to be bigger.

The class of curve to be used, with suitably reduced yield stress, is as follows:

	Curve
Welded I-sections (buckling about xx)	b
Welded I-sections (buckling about yy)	c
Welded box sections	b

In the case of "heavily welded" boxes a further modification is proposed, but it is thought that this will not affect practically designed columns.

It is interesting to note, that when the flanges of a welded I-section are known to be flame-cut from plate instead of being rolled flats, higher stresses are permissible because of favourable tension induced in the flange toes. In this case curve b may be used for xx or yy buckling without any reduction to σ_y.

11. JUMBO SECTIONS

Work at Lehigh[6] has clearly shown that very massive "Jumbo" I-sections (with say 60 mm flanges) can contain far worse residual stresses than do sections of normal thickness. The locked-in compression at the toes can approach yield. Column tests have shown these sections to have an impaired column capacity.

It has been suggested that when the flange thickness exceeds 40 mm, the next lower strut curve should be used. For a Jumbo column buckling about yy a new curve below c would become necessary. In anticipation of Jumbo rolling on this side of the Atlantic, an appropriate curve has been included in the relevent figures of this report, computed for $\alpha = 0.0080$. At present only two section in the British book quality for this curve, if 40 mm is in fact to be the change-over thickness.

A sudden jump to a lower curve could sometimes lead to anomalies, and it might be thought preferable to have a sliding scale for α when the thickness passes to 40 mm.

12. ADVANTAGES OF THE PERRY FORMULA

The advantages of the modified Perry formula are:

(a) It is simpler than the polynomial expression currently proposed.
(b) σ may be expressed in terms of L/r, as well as L/r in terms of σ.
(c) With suitable α values, it fits the agreed curves.
(d) High yield steels are not penalised as much as in the current proposals.
(e) Extra curves may be added by selecting suitable values of α.

13. DESIGN DATA

Design curves and tables are given in the author's full report[18].

Fig. 1 compares existing British Standards with the present proposals. The spread of the proposed curves embraces the present B.S. curves. One hopes that any loss in economy for sections allocated to a low curve will be offset by revisions of load factors.

14. CONCLUSIONS

(1) Several strut curves are necessary for diffining types of section.
(2) The European curves provide a suitable basis for strut design.
(3) The modified Perry formula represents these curves simply and conveniently.
(4) It does not penalise high strength steels as heavily as does the N-λ form.
(5) The selection table (table 1) may be revised as knowledge improves.
(6) More information is needed on crookedness and locked-in stresses.
(7) Further work is needed on welded fabricated struts.

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The author would like to pay tribute to the late Professor Beer for his work towards a unified basis of strut design in Europe. He would also like to thank Mr. P.K. Clarkson for plotting the strut-curves in this reoprt. This paper was shortened for publication here by Mr. C.D. Bradfield.

Table 1

CURVE SELECTION TABLE FOR ROLLED SECTIONS

Axis	x-x		y-y	
SECTION	European curve	α	European curve	α
Universal column	b	.0035	c	.0055
Universal beam	a	.0020	b	.0035
UC or UB with cover-plates	b	.0035	a	.0020
Channel or tee	c	.0055	c	.0055
Angle (any axis)	c	.0055		
Round tube	a	.0020	a	.0020
Rectangular Hollow Section	a	.0020	a	.0020

Notes: 1. The curve allocation is generally in accordance with the European proposals.
2. The allocation for angles is the author's own suggestion, pending more information.
3. Universal columns with flanges thicker than 40 mm to have $\alpha = 0.0080$ for yy buckling.
4. For welded I- and box-sections refer to Section 10.

Table 2

COEFFICIENTS FOR USE IN POLYNOMIAL EXPRESSION (9)

$$\lambda = \sqrt{C_0/N + C_1 + C_2N + C_3N^2}$$

Curve	C_0	C_1	C_2	C_3
a	+1.0	-0.61	+1.29	-1.64
b	+0.92	-0.51	+0.43	-0.80
c	+0.92	-0.39	-0.74	+0.25

Table 3

ASSUMED YIELD STRESSES FOR STRUT DESIGN

Hot Rolled Sections

BS 4360 Grade	assumed σ_y (N/mm^2)	thickness range (mm)
43	225 240 255	40 to 63 16 to 40 < 16
50	340 355	16 to 63 < 16
55	410 430 450	40 to 63 16 to 40 < 16

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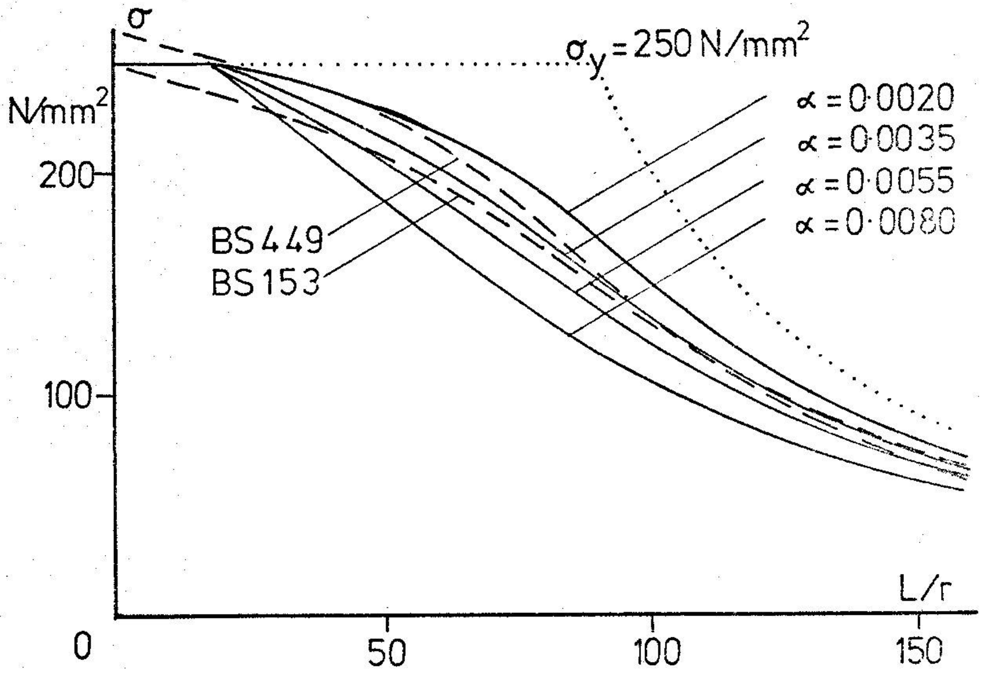

Fig. 1 Comparison of current British design curves with the new proposals.

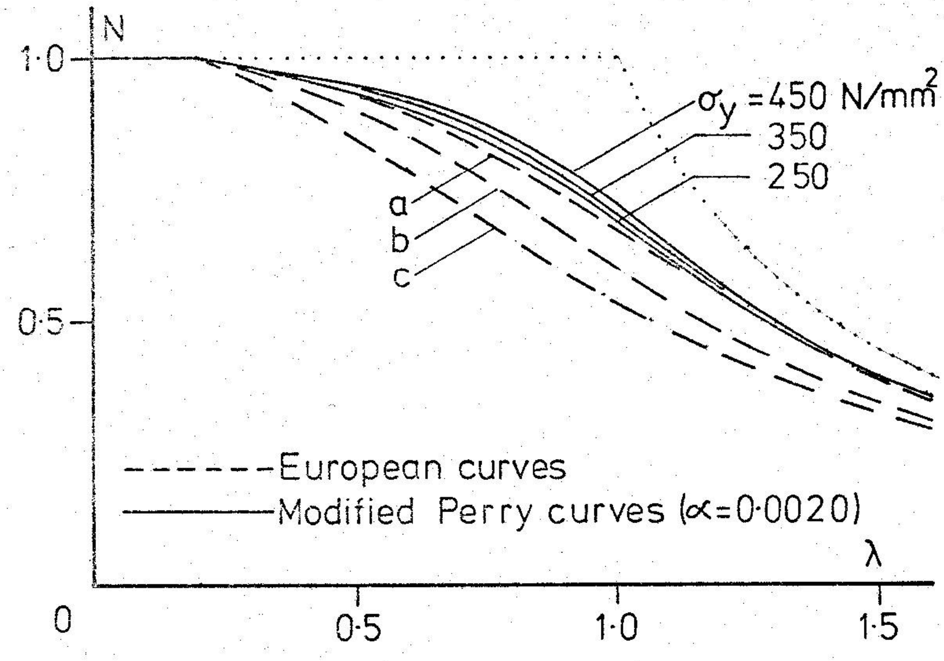

Fig. 3 Non-dimensional plot showing the increasing difference between the appropriate Perry curves and European curve a as the yield stress increases. The comparison would be similar for curves b and c.

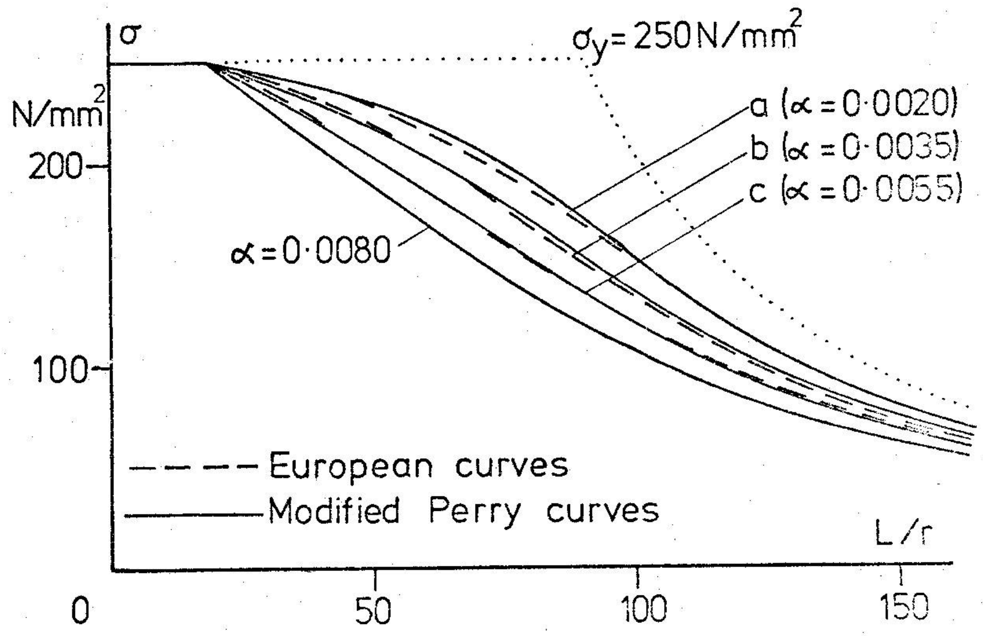

Fig. 2 Comparison of the modified Perry curves, plotted for mild steel, with the corresponding European curves. The lowest curve is

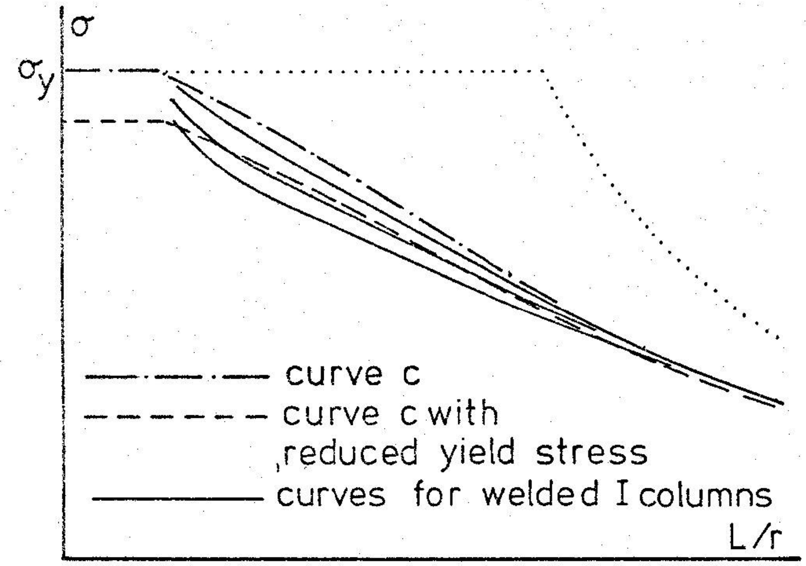

Fig. 4 Treatment of welded columns.