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LOAD FACTOR DESIGN OF COLUMNS USING SECOND MOMENT PROBABILISTIC METHOD

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ABSTRACT

A method of steel column design is presented which is based on first order probabilistic theory, utilizing only the mean values and the coefficients of variation of the relevant parameters. A reliability factor, called the "safety index" is defined and a value for it is obtained by calibration to an existing design code. Subsequently a design format

$$\phi R_n \geq \gamma Q_n$$

is developed, where ϕ is a strength factor, R_n is the nominal resistance, γ is a load factor and Q_n is the nominal load effect.

1. INTRODUCTION

This report will outline a simplified method of steel column design based on the "first order" or "second moment" probabilistic theory (1,2,3). In the interest of simplicity it will be assumed that the resistance R of the column is independent of the load effect Q. Both R and Q are random functions, and thus the probability of failure can be expressed by either of the following expressions.

$$P_F = P[(R-Q) < 0] \tag{1}$$

$$P_F = P[R/Q < 1] \tag{2}$$

$$P_F = P[\ell n\ (R/Q) < 0] \tag{3}$$

If we consider the "standardized variate"

$$U = \frac{\ell n\ (R/Q) - [\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}} \tag{4}$$

in which $[\ell n\ (R/Q)]_m$ and $\sigma_{\ell n\ (R/Q)}$ are the mean and standard deviation of the natural logarithm of the ratio R/Q, then

$$P_F = P\left[U < \frac{-[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}}\right] = F_U\left\{\frac{-[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}}\right\} \tag{5}$$

in which F_U is the cumulative distribution function of this standardized variate. The quantity $[\ell n\ (R/Q)]_m / \sigma_{\ell n\ (R/Q)}$ defines the reliability of the element; hence it is called the "safety index," denoted by β. For example, if the random variable R/Q is lognormally distributed, then the area under the tail $R/Q < 1$ i.e., the probability of failure, is 3.2×10^{-5} if $\beta = 4$. Similarly, the failure probabilities are 2.3×10^{-2}, 1.4×10^{-3} and 2.9×10^{-6} for $\beta = 2$, 3 and 5, respectively. The values of β can be quite different if the shape of the distribution of R/Q in the tail is different. In practice, the probability distribution of R/Q is unknown and only R_m, Q_m, σ_R and σ_Q are estimated. However, β still indicates, in an approximate way, the failure probability, and an increase or a decrease of β by unity roughly decreases or increases the probability of failure by an order of magnitude (i.e., 10^{-1}). If the distribution of R/Q were lognormal or any of a number of other commonly used distributions (e.g. Extreme Value Type I), β would directly indicate a value of the probability of failure. In the first order probabilistic design method used here, β is only a relative measure of reliability, and it is hence called the "safety index." Within the context of the information available, i.e., just R_m, Q_m, σ_R and σ_Q, a constant value of β effectively approximates constant reliability for all similar structural elements.

The expression for the safety index β, i.e.,

$$\beta = \frac{[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}} \tag{6}$$

can be simplified by using first order probability theory as follows:

$$[\ell n\ (R/Q)]_m \simeq \ell n\ (R/Q)_m \simeq \ell n\ \frac{R_m}{Q_m} \tag{7}$$

and

$$\sigma^2_{\ell n\ (R/Q)} \simeq [\frac{\partial \ell n\ (R/Q)}{\partial R}]^2_m\ \sigma_R^2 + [\frac{\partial \ell n\ (R/Q)}{\partial Q}]^2_m\ \sigma_Q^2 = \frac{\sigma_R^2}{R_m^2} + \frac{\sigma_Q^2}{Q_m^2} \tag{8}$$

Since $\sigma_R/R_m = V_R$ and $\sigma_Q/Q_m = V_Q$, where V_R and V_Q are the coefficients of variation of R and Q, respectively,

$$\beta \simeq \frac{\ell n\ (R_m/Q_m)}{\sqrt{V_R^2 + V_Q^2}} \tag{9}$$

or,

$$\frac{R_m}{Q_m} = \theta = \exp\ (\ \beta \sqrt{V_R^2 + V_Q^2}\ ) \tag{10}$$

In Eq. 10, θ is the "central safety factor."

ESTIMATION OF THE SAFETY INDEX β

The "safety index" β is related to the probability of failure. In order to develop a design criterion, β must be specified. There are several ways in which β can be determined: it can be a value agreed upon by the profession to give the desired degree of reliability, or it can be obtained by adjusting β such that the same degree of reliability is attained for the new criterion as in the existing design method for a given standard situation. This procedure is called "calibration," and it will be used here. The "standard situation" selected here is the design of an interior column in a braced frame with simple beam-to-column connections according to Part 2 of the "Specification for The Design, Fabrication and Erection of Structural Steel For Buildings," American Institute of Steel Construction (AISC), 1969. According to this specification the columns are designed to resist the axial load as axially loaded elements with an effective length equal to the center-to-center story height. The factored axial load on an interior column in a braced frame in the n^{th} floor below the top of the frame (counting the roof as level n = 1) is approximately equal to

$$P_n = [D_c\ An + L_c'\ (1\text{-}RF)\ An\ ]LF \tag{11}$$

where

- LF = Load Factor; LF = 1.70
- D_c = dead load intensity
- L_c = live load intensity specified in the code for the occupancy type
- A = area on any floor level contributing to the load on the column
- RF = live load reduction factor specified in the code

Equation 11 assumes that the weight of the columns is included in the dead load, that the loads on the top level (roof) are the same as for the other levels, and that L_c, D_c and A are the same at every level. Thus a regularly loaded regular structure is assumed. It is stipulated that columns in such a structure are satisfactory when designed by the present code.

The live load reduction factor RF is, according to A58.1 (1972) of the American National Standards Institute (ANSI) Code, a function of the total tributary area and the ratio D_c/L_c. The maximum reduction is RF = 0.6 if the total tributary area is more than 750 sq. ft. (70 sq. in.), or RF = 0.23 (1 + D_c/L_c), whichever is smaller.

The column capacity is equal to (AISC 1969, Part 2) 1.7 $A_c F_a$, where A_c is the cross-sectional area of the column and F_a is the allowable stress

$$F_a = \frac{F_y (1 - 0.25 \lambda^2)}{5/3 + 3/8 (\lambda/\sqrt{2}) - 1/8 (\lambda/\sqrt{2})^3} \quad (12)$$

Equation 12 is the Column Research Council Basic Column Curve Equation in the numerator, divided by a factor of safety. It is valid for $\lambda \leq \sqrt{2}$; F_y is the yield stress and

$$\lambda = \frac{h}{r} \sqrt{\frac{F_y}{\pi^2 E}} \quad (13)$$

where h/r is the column slenderness ratio and E is the modulus of elasticity.

By setting 1.7 $A_c F_a = P_n$, the required column area according to the AISC Specification is:

$$A_c = \frac{nA [D_c + L_c (1\text{-RF})][5/3 + 3/8 (\lambda/\sqrt{2}) - 1/8 (\lambda/\sqrt{2})^3]}{F_y (1 - 0.25 \lambda^2)} \quad (14)$$

In the following derivation β will be determined such that the column area A_c from Eq. 14 serves as the basis of the calibration for the new format.

In order to evaluate β from Eq. 9 it is necessary to estimate the mean and the coefficient of variation of the resistance, R_m and V_R, and the corresponding values of the load effect, Q_m and V_Q. The mean strength of the column is equal to

$$R_m = A_c F_m \quad (15)$$

where A_c is the column area required according to the present code and F_m is the mean stress at failure. This stress is a function of a number of variables, such as the yield stress, the residual stress, the shape, the initial crookedness, the unintentional eccentricity of the axial load, and the end restraints. Each of these variables is random, and an analysis could be made if the relevant statistical parameters of each were known. This would be a formidable task if all these effects were included, although analyses with some of the variables have been made (4,5,6,7). In order to circumvent this problem, the mean failure stress was expressed in the following way:

$$F_m = [\text{Bias Factor}] [\text{Nominal Formula}] \quad (16)$$

where

$$[\text{Bias Factor}] = \left[\frac{\text{Test capacity}}{\text{Theoretical prediction}}\right]_m \left[\frac{\text{Theoretical prediction}}{\text{Nominal strength}}\right]_m \quad (17)$$

and the nominal formula is the column strength equation which is to be used in the new code for the particular type of column section. For the sake of demonstration and because the formula fits fairly well for medium size rolled columns, the CRC Basic Column Curve was chosen, i.e.,

$$F_n = F_y (1 - 0.25 \lambda^2) \tag{18}$$

Since the theoretical prediction of column strength, including all effects, is fairly complicated and all the necessary data were not available to make the analysis, it was decided to determine the bias factor by directly comparing test results to predictions from the nominal formula.

In order to assess the mean and standard deviation of the test-to-prediction ratio, test data from reports of the Fritz Engineering Laboratory of Lehigh University were analyzed (8,9). These samples are not truly random because the tests were not designed statistically, and so a better basis, involving the omitted step of the theoretical prediction, or statistically designed tests, will eventually have to be used. The sample used here includes about 50 US rolled medium size column shapes and the bias factor for these was found to be equal to 1.03 and the corresponding coefficient of variation was 0.14 (9). The test-to-prediction ratio was determined for the nominal yield stress and so the numbers above account also for the variability of the yield stress.

The mean column resistance is thus equal to

$$R_m = 1.03\, A_c F_y (1 - 0.25 \lambda^2) \tag{19}$$

where A_c is determined from Eq. 14 and F_y is the specified yield stress. The coefficient of variation is equal to

$$V_R = \sqrt{V_{Bias}^2 + V_{Fabrication}^2} = \sqrt{0.14^2 + 0.05^2} = 0.15 \tag{20}$$

The coefficient of variation due to fabrication represents an estimate of dimensional variations of the column cross sections.

The mean load effect is the mean load on the column, and it is equal to

$$Q_m = E_m\, An\, [D_m + L_m] \tag{21}$$

where A and n are the tributary area and the story number, as defined earlier, E is a random variable which accounts for the structural analysis by which the idealized loads are translated into axial forces ($E_m = 1.0$ and $V_E = 0.1$ will be assumed in this analysis), and D_m L_m are the mean dead and the mean lifetime maximum live loads, respectively. In the ensuing derivations it will be assumed that

$$D_m = D_c \quad \text{and} \quad V_D = 0.04 \tag{22}$$

$$L_m = \frac{L_c\,(1\text{-}RF)}{1 + K_L \sqrt{V_E^2 + V_L^2}} \tag{23}$$

and

$$V_L = \frac{C}{\sqrt{n}} \tag{24}$$

The coefficient of variation of the load effect is equal to

$$V_Q^2 = V_E^2 + \frac{(nA\ D_m\ V_D)^2 + (nA\ L_m\ V_L)^2}{[nA\ (D_m + L_m)]^2} \qquad (25)$$

Substitution and non-dimensionalization permits the determination of β from Eq. 9, and it is a function of A, n, λ, V_R, V_E, C, K_L, V_D, V_L and D_c/L_c. A numerical study was performed by varying these parameters as follows:

$0.98 \le [\text{Test/Prediction}]_m \le 1.10$

$0.15 \le V_R \le 0.20$

$0.1 \le D_c/L_c \le 10$

$0 \le RF \le 0.6$

$0.25 \le \lambda \le 1.25$

$1 \le K_L \le 3$

$0.05 \le V_E \le 0.15$

$0.2 \le C \le 0.4$

$2 \le n \le 40$

$0.04 \le V_D \le 0.1$

These variations in the pertinent parameters defining β are thought to be larger than what one would expect for the structure for which calibration is being performed. The graphs in Figs. 1 and 2 give the variation of β with almost every one of the parameters, except for the effects of V_R and $[\text{test/prediction}]_m$. Since three values were changed at once, the results for these variations are best shown in tabular form:

$[\text{test/prediction}]_m^*$	λ	V_R^*	β
1.10	0.25	0.20	3.19
1.03	0.50	0.15	3.86
1.03	0.75	0.15	4.01
1.00	1.00	0.16	3.82
0.98	1.25	0.18	3.37

The values of β in this table were computed with $D_c/L_c = 2$; $K_L = 2$; $V_E = 0.1$; $C = 0.25$; $n = 10$; $V_D = 0.04$. This table, as well as the curves in Figs. 1 and 2 show that β varies from about 3.2 to 4.5, depending on the values of the variables affecting the results. The coefficient of variation of the live load, $V_L = c/\sqrt{n}$, the number of stories, and the code dead-to-live load ratio does not appear to result in much change in β, while the changes in the other variables have pronounced effects.

Based on this study a value of $\beta = 4$ is arbitrarily chosen as a reasonable and representative value of the reliability of medium size rolled wide-flange columns in braced simple multi-story frames as designed by the 1969 AISC Specification. Arguments could, of course, be advanced that in some

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* Based on reasonable estimates, not test results.

cases the profession permits a lower reliability (say $\beta = 3.2$), or that in other cases it demands a higher reliability (say $\beta = 4.5$), but the choice of $\beta = 4$ is one which appears neither on the low nor on the high side, and it will be used hereafter in this report.

THE LOAD FACTOR DESIGN EQUATION

Once β is selected from the calibration process, the design equation can be written from Eq. 10 as:

$$\theta = R_m/Q_m \geq \exp \beta \sqrt{V_R^2 + V_Q^2} \qquad (26)$$

Unfortunately the resistance and the load effects are not separated in this equation. Separation is achieved by using an approach suggested by Lind (3), where an approximation

$$\theta_a = \exp \alpha_R \beta V_R \exp \alpha_Q \beta V_Q \qquad (27)$$

is introduced such that the error $(\theta_a - \theta)/\theta$ is a minimum. If the extreme ranges $2 \leq \beta \leq 5$, $0.1 \leq V_R \leq 0.2$ and $0.1 \leq V_Q \leq 0.5$ are used, the values of the α's become equal to

$$\alpha_R = 0.52$$
$$\alpha_Q = 0.90$$

For the most unlikely combinations the error in θ becomes approximately 16%; for most of the prevalent combinations the error is less than $\pm$ 5%.

By the introduction of α_R and α_Q the separation between resistance and load effect is achieved very simply. Furthermore, the α's are independent of the other variables. Thus a design equation

$$\frac{R_m}{\exp \alpha_R \beta V_R} \geq Q_m \exp \alpha_Q \beta V_Q \qquad (28)$$

can be written. This equation can now be still further modified into the form

$$\theta R_n \geq \gamma Q_n \qquad (29)$$

where R_n and Q_n are nominal load effects, and

$$\phi = \frac{R_m}{R_n} \exp(-\alpha_R \beta V_R) \qquad (30)$$

and

$$\gamma = \frac{Q_m}{Q_n} \exp \alpha_Q \beta V_Q \qquad (31)$$

For example, if $\beta = 4$, $\alpha_R = 0.52$,

$$R_m = 1.03 F_y (1 - 0.25 \lambda^2) \qquad (32)$$

$$R_n = F_y (1 - 0.25 \lambda^2) \qquad (33)$$

$$V_R = \sqrt{0.14^2 + 0.05^2} = 0.15 \tag{34}$$

$$\phi = 1.03 \exp(-0.52 \times 4 \times 0.15) \simeq 0.75 \tag{35}$$

The nominal load effect is

$$Q_n = An\,[D_c + L_c\,(1\text{-}RF)] \tag{36}$$

and the mean load effect is assumed to be

$$Q_m = An\,[D_m + L_m] \tag{37}$$

for an axially loaded column in a braced simple multi-story frame, then

$$\frac{Q_n}{Q_m} = \frac{D_c + L_c\,(1\text{-}RF)}{D_m + L_m} \tag{38}$$

$$V_Q^2 = V_E^2 + \frac{(D_m V_D)^2 + (L_m V_L)^2}{(D_m + L_m)^2} \tag{39}$$

If it is assumed that

$$D_m = D_c\ , \quad V_D = 0.04 \tag{40}$$

$$L_m = \frac{L_c\,(1\text{-}RF)}{1 + K_L\sqrt{V_E^2 + V_L^2}} \tag{41}$$

$$K_L = 2 \quad \text{and} \quad V_L = \frac{0.25}{\sqrt{n}} \tag{42}$$

then γ can be computed from Eq. 31. Since γ appears not to vary a great deal with n and D_c/L_c, (see Fig. 3), a single value of γ can be selected which is $\gamma = 1.30$.

The new design equation can then be expressed as follows:

$$0.75\,F_y\,(1 - 0.25\,\lambda^2) \geq 1.30\,An\,[D_c + L_c\,(1\text{-}RF)] \tag{43}$$

In case it is not desirable to use the same γ value, the load factor can always be determined from Eqs. 38 and 39 directly. This approach becomes necessary if dead and live load plus wind load is present in the load effect term. In this case

$$Q_m = c_1 D_m + c_2 L_m + c_3 W_m \tag{44}$$

$$V_Q^2 = V_E^2 + \frac{(c_1 D_m V_D)^2 + (c_2 L_m V_L)^2 + (c_3 W_m V_W)^3}{(c_1 D_m + c_2 L_m + c_3 W_m)^2} \tag{45}$$

where c_1, c_2, c_3 are the deterministic coefficients from structural analysis, and W_m and V_W are the mean wind load intensity and the coefficient of variation of the wind load.

SUMMARY AND CONCLUSIONS

A simplified method of column design, based on the second moment or first-order probabilistic approach, has been presented. While the method lacks in the elegance of the more sophisticated probabilistic approaches, it is far advanced of the traditional approach of selecting a factor safety by concensus based on experience. The essential statistical and probabilistic elements are all present, and their relationship is simple enough to permit a rapid study of the outcome should one or several parameters change. The approach uses the data in about as sophisticated a form in which they are presently available. In fact, there are still many elements about which educated guesses must be made. The formulation is open-ended, permitting improvement as new or better data becomes available, and it allows an analysis of the consequences if one, two, or three different column curves are to be used. Furthermore, the level of reliability can also be adjusted by changing β.

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NOMENCLATURE:

A	:	Tributary area in one story
A_c	:	Cross-sectional area of column
C^c	:	Coefficient in Eq. 24
c_1, c_2, c_3	:	Coefficients from structural analysis (Eqs. 44 and 45)
D_c	:	Code-specified dead load intensity
D_m	:	Mean dead load intensity
E	:	Modulus of elasticity
E	:	Random variable accounting for uncertainties in structural analysis
E_m	:	Mean of E
F_a	:	Allowable column stress
F_m	:	Mean column failure stress
F_n	:	Nominal column failure stress
F_y	:	Specified yield stress
K_L	:	Coefficient in Eq. 23
L_c	:	Code-specified live load intensity
L_m	:	Mean lifetime maximum live load intensity
LF	:	Load factor in current design code
P_n	:	Load on column n-stories below roof level
Q	:	Load effect
Q_m	:	Mean load effect
Q_n	:	Nominal load effect
R	:	Resistance
RF	:	Live load reduction factor
R_m	:	Mean resistance

R_n : Nominal resistance
V_D, V_E, V_L, V_Q, V_R, V_W : Coefficients of variation of D, E, L, Q, R and W, respectively
W_m : Mean wind load intensity
h : Story height
n : Number of stories below roof level
r : Radius of gyration
α_R, α_Q : Coefficients in Eq. 28
β : Safety index
γ : Load factor
ϕ : Resistance factor
θ : Central safety factor
λ : Non-dimensional slenderness parameter
σ_Q, σ_R : Standard Deviation of Q and R, respectively.

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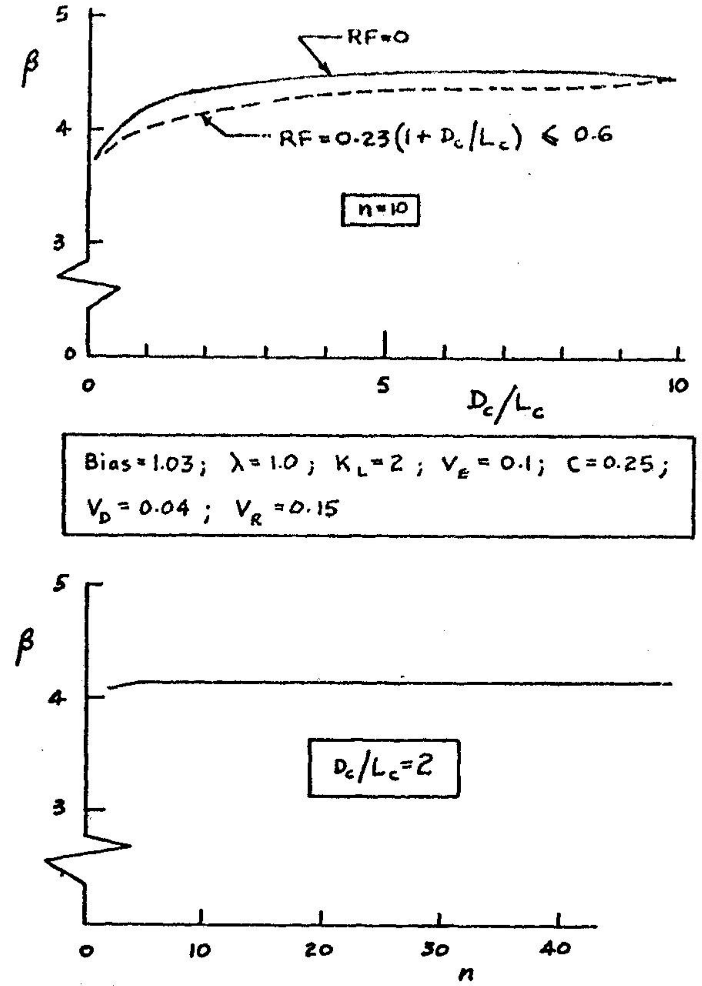

Fig.1 Variation of β with D_c/L_c and n

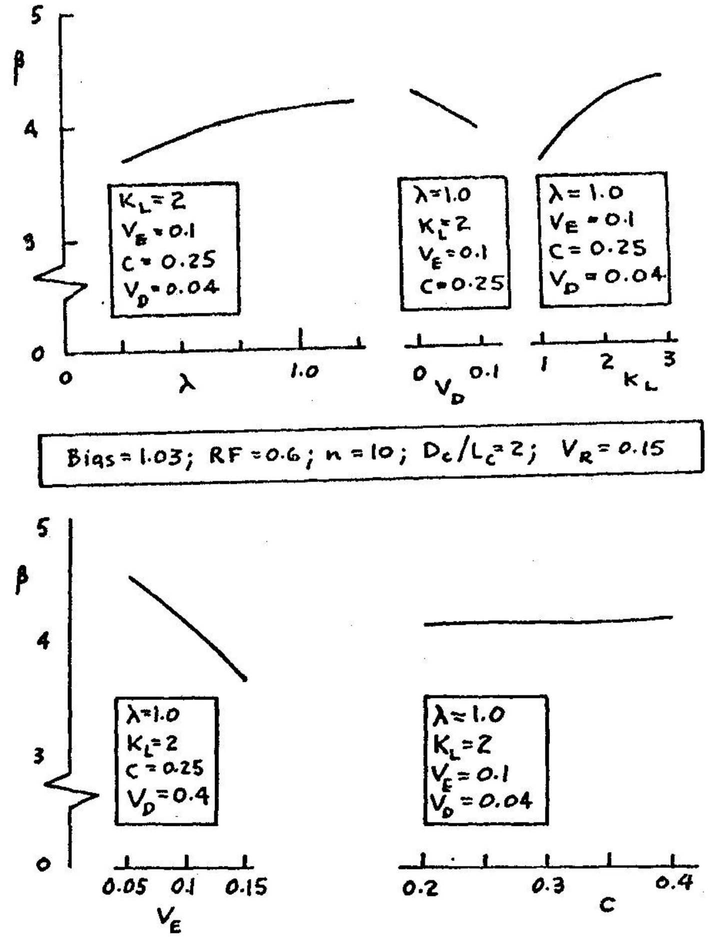

Fig.2 Variation of β with λ, V_D, K_L, V_E and C

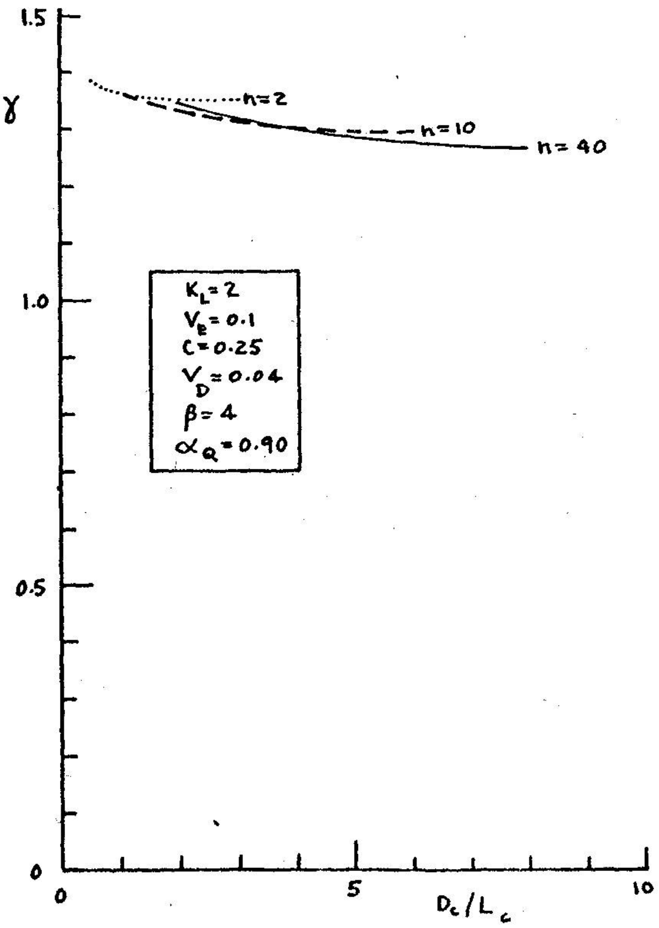

Fig.3 Variation of γ with D_c/L_c and n