EXPERIMENTAL STUDIES ON COLUMN STRENGTH OF EUROPEAN HEAVY SHAPES

Negussie Tebedge
Postdoctoral
Research Associate
of Civil Engineering

Wai-Fah Chen
Associate Professor
of Civil Engineering

Lambert Tall
Professor
of Civil Engineering
and Director, Division
for Fatigue and Fracture

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania
U.S.A.

ABSTRACT

The European Convention for Constructional Steelwork (ECCS) has performed extensive column tests on specimens of small dimensions and light weights. The work reported herein is essentially an extension of the ECCS program to columns of heavy shape. The test program consists of full-size column tests (slenderness ratio of 50 and 95) and supplementary tests, namely, tension tests (full-size and ASTM standards), residual stress measurements, and stub column tests. The test specimens include shapes from four countries: Belgium, Britain, Germany and Italy. The tests have been conducted at Fritz Engineering Laboratory, Bethlehem, Pennsylvania.

This report presents the experimental results of the column tests as well as the supplementary tests. The column test results are compared with the latest proposed European Convention Column Curves (B3-24 and C3-24). A good correlation for L/r = 95 and slightly unconservative prediction for L/r = 50 are observed.

1. INTRODUCTION

Commission 8 of the European Convention for Constructional Steelwork (ECCS) instituted Subcommittee 8.1, chaired by D. Sfintesco, to conduct "experimental studies of buckling". The Subcommittee realized that column test data were essential for accurate determination of column strength curves. The Subcommittee proposed that the maximum strength of pinned-end steel columns of prismatic cross section should be studied based on the statistical and probabilistic concept of safety applied to buckling [1,2,3].

The basic idea in the statistical approach to the column strength problem is to collect a sufficiently large number of test data, and then to obtain mean maximum loads and standard deviations possessing statistical validity. The aim of this approach is to determine, for each group of shapes of a given steel grade, a column strength curve representing a constant probability of failure for all slenderness ratios. The ultimate goal is to obtain a consistent degree of safety of factor for all members of a structure, whatever may be their shape and level of stress [4].

The statistical test program has performed well over 1000 column tests [5]. The columns were taken at random from various stockyards of steelwork fabricators in several European countries in an effort to furnish representative samples of columns normally used in actual structures. Findings from earlier experimental investigations have served to form the basis for the column curve adopted by the European Convention in June 1964 [6]. However, this test program had been limited to light shapes only of mild steel. The extension of the application of the curve to columns of larger sizes and to other types of steels was left to be performed by theoretical means and eventually to be confirmed by experimental means.

This paper presents an experimental study on heavy columns fabricated in Europe to determine the conditions by which the results from the previous program on column strengths of small dimensions and light weights can be extended to such heavy columns. Prior to testing the European heavy columns, a preliminary experimental study on different column testing methods was conducted using seven heavy columns fabricated in the United States. The specimens were prepared from a single unstraightened rolled piece and had a size comparable to the shape considered in the European heavy column test program. As a result of this study, the testing method required by ECCS [3] has been clarified and a new procedure for testing of medium and heavy columns has been proposed [7].

To obtain conclusive experimental evidence on the strength of heavy columns with minimum cost, the test program included the specimens from four countries: Belgium, Britain, Germany and Italy. The tests have been conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The test program consisted of pinned-end column tests (slenderness ratio of 50 and 95) and supplementary tests, namely, tension tests (full-size and ASTM standard), residual stress measurement, and stub column test. The results of the column tests are also compared with the recently proposed European Convention Column Curves [8].

2. THE TEST PROGRAM

Scope

The test program was limited to testing specimens from four European manufacturers found in Belgium, Britain, Germany and Italy. This choice was considered to be sufficient to furnish a good representation of population of "columns obtainable in Europe". From each manufacturer two specimens were selected as a typical representative of the production of the respective manufacturer. This results in a total of eight heavy column shapes. A typical schematic layout for the preparation of the test specimens is shown in Fig. 1. A summary of the test program is presented in Table 1.

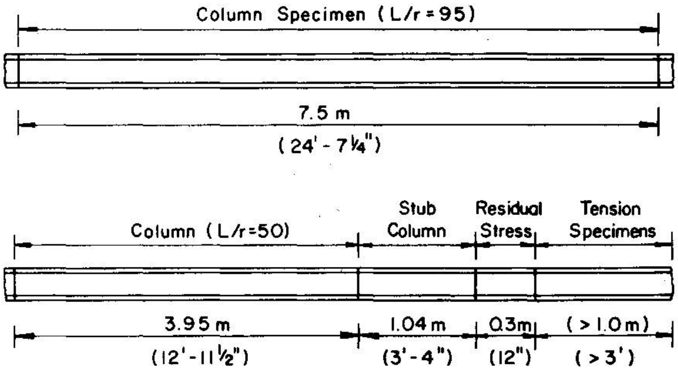


Fig. 1 Schematic Layout of Test Specimen from Each Rolling

TABLE 1: OUTLINE OF TEST PROGRAM

Source Country and Shape	Tension Tests ASTM	Tension Tests ECCS	Residual Stress Measurements	Stub Column Tests	Column Tests L/r=50	Column Tests L/r=95
BELGIUM (HEM 340)	8	8	2	2	2	2
BRITAIN (W12x161)	8	8	2	2	2	2
GERMANY (HEM 340)	8	-	2	2	2	2
ITALY (HEM 340)	8	8	2	2	2	2
Total Number of Tests	32	24	8	8	8	8

For the full-size column tests two slenderness ratios were chosen in the critical range; this was governed by practical and theoretical considerations. According to the ECCS [3] it was suggested to test columns of:
i) a slenderness ratio of 95 on the basis of theoretical considerations for which the variation of experimental results would be the greatest, and
ii) a slenderness ratio of 50 which would be of the same order of magnitude as the slenderness ratio normally used in multi-story building structures, yet, still in the critical range of slenderness ratio.

Through the applications of statistical analysis two experimental points may be established to represent the column buckling curve for heavy rolled shapes. These test points should enable a decision whether or not the experimental curve resulting from the basic program could be applied safely to heavy columns.

The Test Specimen

Two conditions governed the choice of the column size: i) the column must be a "heavy shape", and, ii) the specimen must be rolled by all of the four manufacturers.

According to ECCS definition a shape larger than HE 280 and having a thickness greater than 30 mm (1-1/8 in) is designated as a "heavy shape" [3]. To meet the above requirements, the shape HEM 340 was chosen for the specimens from the continental countries using the metric system, and the shape W12x161 from Britain (this shape is also rolled in the United States). (At the time, the HEM 340 was the heaviest shape then available in the continental countries.) The two shapes are very similar in cross sectional dimensions; the shapes are compared in Fig. 2.

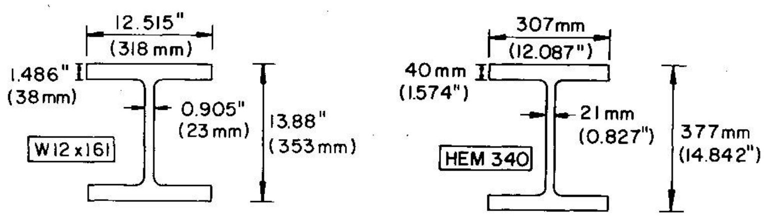


Fig. 2 Comparison of Cross Sectional Dimensions

3. SUPPLEMENTARY TESTS

The purpose of conducting supplementary tests is to determine the basic properties of specimens which are required to evaluate the theoretical column strengths. The following supplementary tests were performed:

Tension Tests

The tension tests were carried out in two ways: i) according to ASTM specifications [9] for standard 8-inch gage length specimen, and ii) following the ECCS recommendations [3] for full-size tension tests where the complete section is tested in tension using four plate specimens (the two flanges and two plates from the web). The gage length for the full-size tension tests is determined according to the formula $L = 5.65 \sqrt{A}$, where A is the cross sectional area of the tension specimen [3].

A total of thirty-two 8-inch gage length (ASTM A570) specimens were tested in tension. From each column, two coupons were prepared from the flanges and two from the web as shown in Fig. 3. The static yield strength was defined by the stress at 0.005 in/in strain. The recorded static yield strength varies between 28.7 ksi (198 N/mm^2) and 36.2 ksi (250 N/mm^2) for the flanges, and between 29.0 ksi (200 N/mm^2) and 36.7 ksi (253 N/mm^2) for the webs. Table 2 gives the test results. For most of the specimens tested, it was observed that the flange specimens had a lower yield strength and a gradual transition from the elastic to the strain hardening range, while the web specimens exhibited a higher yield strength, a "flat" yield plateau and a marked onset of strain hardening. Figure 4 shows a typical stress-strain relationship of tension specimens taken from the flange and the web.

A total of twenty-four full-size tension tests were conducted following ECCS recommendations. The static yield strength was also defined by

TABLE 2: TENSION COUPON TEST RESULTS (ASTM)

Specimen	Location	Upper Yield Stress σ_{yu} ksi (N/mm²)	Dynamic Yield Stress σ_{yd} ksi (N/mm²)	Static Yield Stress σ_{ys} ksi (N/mm²)	Ultimate Stress σ_u ksi (N/mm²)	Fracture Stress σ_f ksi (N/mm²)	Percent Elongation %	Reduction of Area %
B-1-B-1-5	1	-- --	33.4 (230)	30.8 (212)	55.2 (381)	40.7 (281)	36.9	59.5
	2	35.6 (245)	33.4 (230)	31.4 (216)	57.9 (399)	44.1 (304)	36.0	41.6
	3	-- --	34.4 (237)	33.0 (228)	58.0 (340)	45.0 (310)	34.1	59.2
	4	32.3 (223)	32.3 (223)	30.1 (208)	55.7 (384)	43.0 (296)	36.4	58.9
B-1-B-2-4	1	32.2 (222)	30.8 (212)	29.4 (203)	54.0 (572)	41.3 (285)	38.1	62.2
	2	31.1 (214)	33.9 (234)	31.7 (219)	57.0 (393)	43.4 (299)	32.3	60.0
	3	34.4 (237)	34.4 (237)	32.4 (223)	57.7 (398)	43.5 (300)	32.8	60.1
	4	32.2 (222)	32.2 (222)	30.7 (212)	52.1 (359)	41.4 (285)	36.5	59.8
B-1-GB-1-5	1	-- --	32.7 (225)	31.3 (216)	65.6 (452)	44.8 (309)	50.1	67.6
	2	36.1 (249)	35.8 (247)	34.2 (236)	66.9 (461)	47.5 (328)	31.3	61.7
	3	34.9 (246)	34.5 (238)	33.1 (228)	61.7 (425)	45.6 (314)	30.0	64.8
	4	34.0 (234)	34.9 (241)	33.5 (231)	65.3 (450)	42.9 (296)	50.4	68.7
B-1-GB-2-4	1	29.5 (203)	29.9 (206)	28.8 (199)	60.5 (417)	40.6 (280)	53.3	70.1
	2	-- --	35.1 (242)	33.2 (229)	65.4 (451)	45.5 (314)	34.0	66.3
	3	33.4 (230)	32.4 (223)	31.0 (214)	61.8 (426)	42.4 (292)	33.8	65.6
	4	31.6 (218)	31.8 (219)	30.8 (212)	60.5 (417)	39.9 (275)	62.6	69.5
B-1-D-3-5	1	34.8 (240)	33.7 (232)	32.7 (225)	58.0 (340)	44.0 (303)	38.1	62.0
	2	36.6 (252)	36.3 (250)	35.2 (243)	60.9 (420)	47.9 (330)	33.5	57.3
	3	36.9 (254)	36.9 (254)	35.0 (241)	60.9 (420)	47.0 (324)	33.1	54.6
	4	37.2 (256)	34.5 (238)	31.9 (220)	57.4 (396)	43.8 (302)	38.6	63.2
B-1-D-4-4	1	40.0 (276)	37.1 (256)	36.2 (250)	61.0 (421)	46.9 (323)	40.6	59.6
	2	40.5 (279)	39.3 (271)	36.7 (253)	65.1 (449)	53.6 (370)	35.3	54.8
	3	37.3 (257)	36.7 (253)	34.1 (235)	60.2 (415)	63.8 (440)	36.0	54.5
	4	36.6 (252)	35.6 (245)	33.4 (230)	61.0 (421)	46.9 (323)	38.5	59.2
B-1-I-1-5	1	30.4 (210)	30.4 (210)	29.3 (202)	60.4 (416)	43.0 (296)	37.5	64.2
	2	-- --	31.3 (216)	29.6 (204)	58.8 (405)	41.4 (285)	31.8	63.4
	3	35.1 (242)	33.5 (231)	31.1 (214)	59.4 (410)	43.4 (299)	34.3	61.6
	4	31.9 (220)	31.9 (220)	29.8 (205)	60.7 (419)	43.5 (300)	36.3	62.7
B-1-I-2-4	1	30.2 (208)	30.7 (212)	28.7 (198)	60.5 (417)	43.1 (297)	37.5	64.0
	2	-- --	32.7 (225)	30.8 (212)	60.7 (419)	43.6 (301)	33.8	64.8
	3	33.5 (231)	30.9 (213)	29.0 (200)	54.5 (376)	41.4 (285)	32.6	66.6
	4	-- --	30.2 (208)	29.3 (202)	59.4 (410)	43.5 (300)	37.5	66.5

Fig. 3 Location of Tension Test Specimens

the stress at 0.005 in/in strain. The test results are given in Table 3. The full-size tension test results were seen to correspond very closely to those obtained from the ASTM tests. Since the full-size tension tests did not seem to yield any additional or different information on material properties than those given by ASTM tests the full-size tension tests were not carried out for all columns.

Residual Stress Measurements

The procedure used for the residual stress measurements was the sectioning method, involving longitudinal saw cuts across the thickness of width of the component plates. A detailed discription of the sectioning method is given in Ref. 10.

Residual stress measurements were made on a total of eight specimens. To obtain a more accurate and smoother variation of the measured values, each specimen was cut into seventy longitudinal strips. Figure 5 shows the measured residual stress distributions for all specimens. A close agreement was observed for the magnitude and distribution of residual stresses in the flanges of all the specimens. The edges have compressive residual stresses with an average value of 9.5 ksi (65 N/mm²) or 0.28 σ_y.

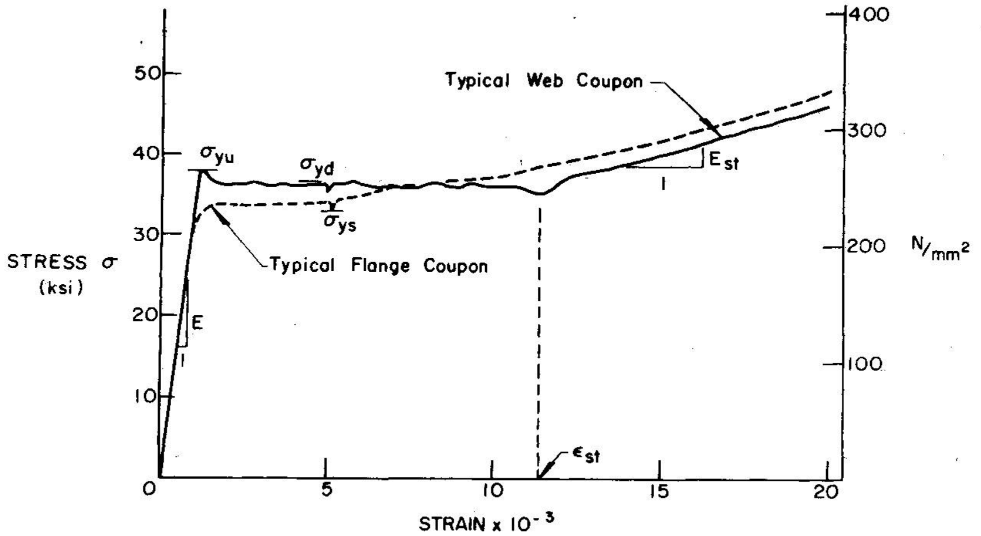


Fig. 4 Stress-Strain Curves

Stub Column Test

Prior to the testing of any column, a stub column test was prepared on a section from the same piece from which the actual column was prepared. The purpose of stub column test is to determine the average stress-strain relationship for the entire cross section which takes into account the effects of residual stress and yield strength variation over the cross section. The proportional limit, the yield strength, the elastic modulus, and the tangent modulus are the most important data furnished by the curve.

The length of the stub column was selected such that it is sufficiently long to retain the original residual stress in the column but short enough to prevent any premature failure occurring before the yield of the section is obtained. The stub column length used in this test program was 40 inches (1.02 m). The procedure used in testing the stub column is described in detail in Ref. 11

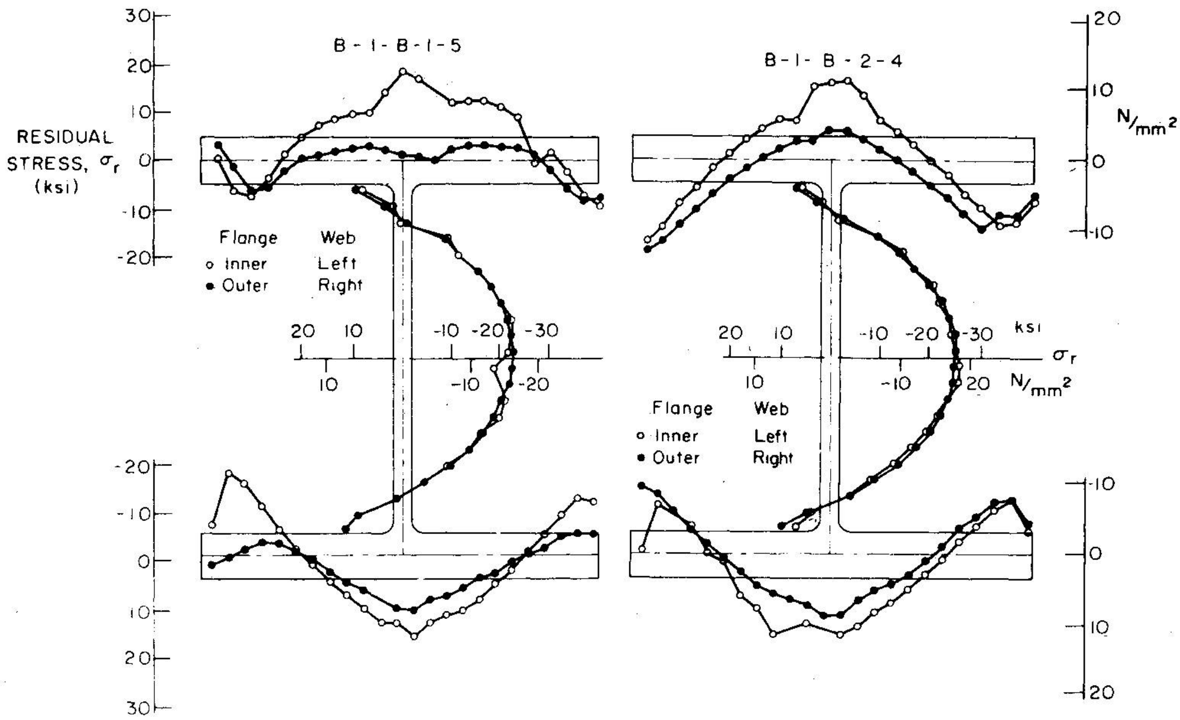


Fig. 5a Residual Stresses in HEM (Belgium)

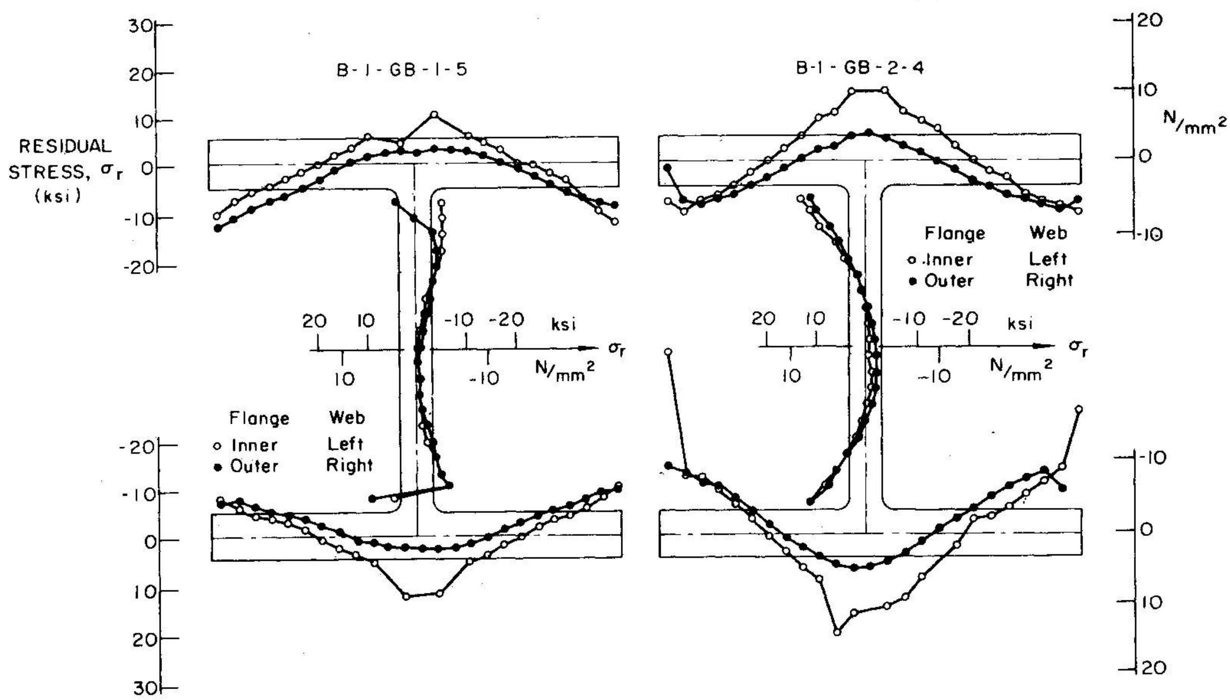


Fig. 5b Residual Stresses in W12x161 (Britain)

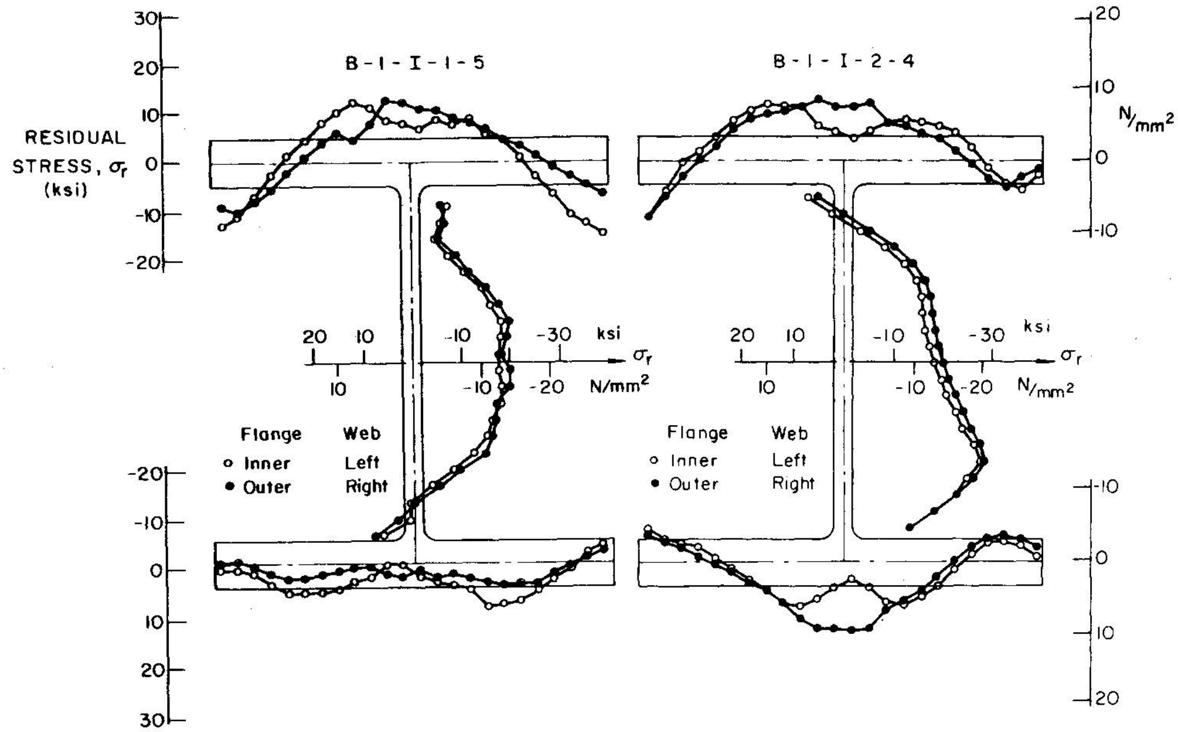


Fig. 5c Residual Stresses in HEM 340 (Italy)

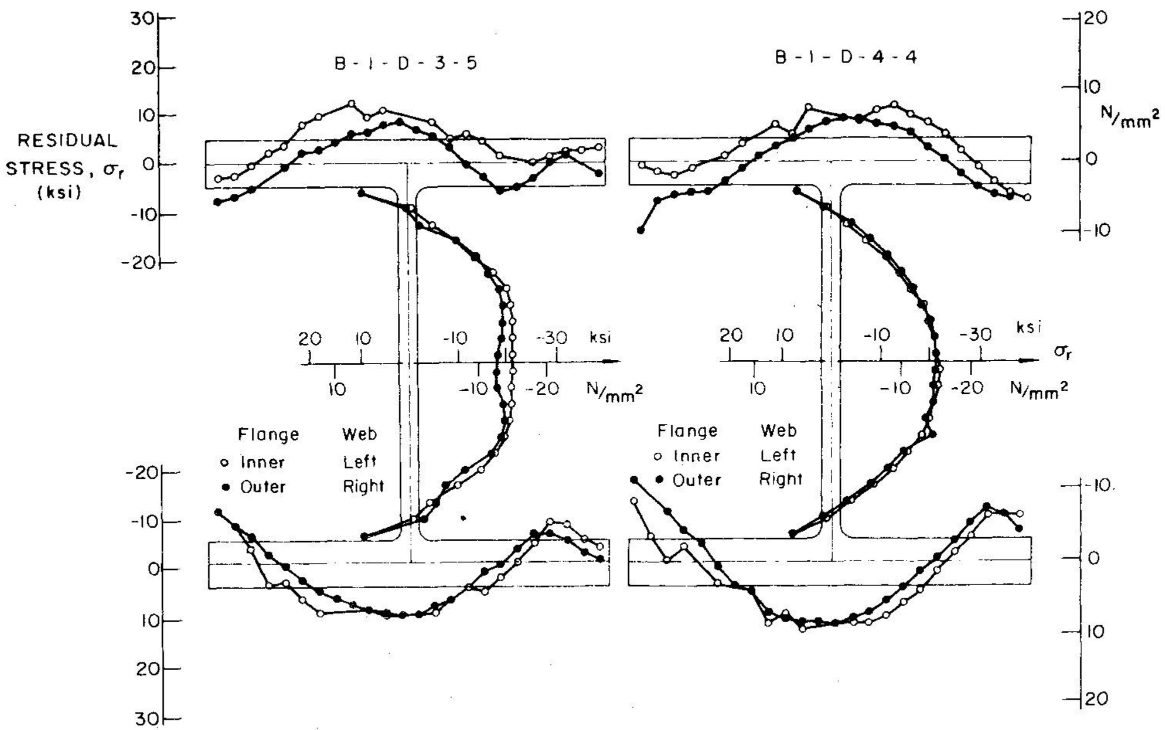


Fig. 5d Residual Stresses in HEM 340 (Germany)

TABLE 3: FULL-SIZE TENSION TEST RESULTS (ECCS)

Specimen	Location	Upper Yield Stress σ_{yu} ksi (N/mm^2)	Dynamic Yield Stress σ_{ud} ksi (N/mm^2)	Static Yield Stress σ_{ys} ksi (N/mm^2)	Ultimate Stress σ_u ksi (N/mm^2)	Fracture Stress σ_t ksi (N/mm^2)	Percent Elongation %	Reduction of Area %
B-1-B-1-5	1	33.2 (229)	32.5 (224)	30.1 (208)	56.9 (392)	49.8 (336)	41.6	42.0
	2	34.7 (239)	34.2 (236)	32.5 (224)	57.4 (396)	50.1 (345)	28.8	40.1
	3	34.8 (240)	33.5 (231)	31.7 (219)	57.3 (395)	49.1 (339)	31.9	45.0
	4	32.8 (226)	31.8 (219)	29.6 (204)	56.2 (387)	50.5 (348)	41.3	41.5
B-1-B-2-4	1	32.6 (225)	32.1 (221)	30.2 (208)	57.2 (394)	50.6 (349)	44.4	43.7
	2	35.1 (242)	34.4 (237)	32.7 (225)	57.8 (399)	49.4 (341)	29.4	38.5
	3	35.2 (243)	34.2 (236)	32.4 (223)	57.3 (395)	49.1 (339)	28.8	41.4
	4	32.6 (225)	32.3 (223)	29.4 (203)	56.2 (387)	50.4 (348)	43.1	43.5
B-1-GB-1-5	1	-- --	31.0 (214)	29.3 (202)	61.0 (421)	51.0 (352)	42.5	43.1
	2	35.2 (243)	34.5 (238)	32.2 (222)	62.8 (433)	51.7 (356)	28.1	41.9
	3	33.9 (234)	33.3 (230)	32.0 (229)	61.4 (423)	48.5 (334)	28.1	48.7
	4	-- --	31.4 (217)	29.5 (203)	60.9 (420)	40.9 (282)	40.6	41.4
B-1-CB-2-4	1	34.5 (238)	34.3 (236)	32.5 (224)	66.1 (456)	55.8 (385)	41.8	58.8
	2	37.0 (255)	35.7 (246)	33.8 (233)	64.5 (445)	53.5 (369)	24.7	59.4
	3	36.7 (253)	36.0 (248)	34.5 (238)	64.2 (443)	35.7 (246)	28.1	46.6
	4	35.0 (241)	34.9 (241)	32.9 (227)	66.7 (460)	56.0 (386)	40.6	59.6
B-1-I-1-5	1	-- --	30.7 (212)	28.2 (194)	58.9 (406)	50.5 (348)	37.5	52.3
	2	-- --	32.2 (222)	30.5 (210)	58.7 (405)	46.5 (321)	27.5	42.4
	3	32.1 (221)	31.5 (217)	30.2 (208)	59.3 (409)	49.9 (344)	28.75	40.8
	4	-- --	31.1 (214)	28.9 (199)	60.0 (414)	51.4 (354)	40.0	47.0
B-1-I-2-4	1	-- --	30.0 (207)	28.2 (194)	59.0 (407)	50.6 (349)	41.6	44.2
	2	-- --	31.9 (220)	30.2 (208)	60.3 (416)	48.3 (333)	28.8	42.3
	3	32.3 (223)	32.0 (221)	30.4 (210)	59.9 (413)	47.9 (330)	29.4	44.2
	4	-- --	31.2 (215)	29.8 (205)	61.4 (423)	52.4 (361)	42.2	42.9

TABLE 4: STUB COLUMN TEST RESULTS

Specimen	P_{yd} kips (MN)	P_{ys} kips (MN)	σ_{yd} ksi (N/mm^2)	σ_{ys} ksi (N/mm^2)
B-1-B-1-5	1550 (6895)	1450 (6450)	32.78 (226)	30.37 (209)
B-1-B-2-4	1524 (6779)	1436 (6388)	32.41 (223)	30.84 (213)
B-1-GB-1-5	1450 (6450)	1374 (6112)	31.17 (215)	29.54 (204)
B-1-GB-2-4	1552 (6904)	1470 (6539)	33.87 (234)	32.08 (221)
B-1-D-3-5	1746 (7767)	1676 (7455)	35.44 (244)	34.02 (235)
B-1-D-4-4	1744 (7758)	1670 (7428)	32.25 (222)	33.75 (233)
B-1-I-1-5	1438 (6397)	1356 (6032)	29.61 (204)	27.79 (192)
	1498 (6663)	1390 (6183)	31.56 (218)	29.29 (202)

The stub column specimens were tested in the 5-million pound capacity capacity universal hydraulic testing machine in Fritz Engineering Laboratory. Figure 6 shows the column set-up and instrumentations. Each specimen was aligned such that the deviation in strain field did not exceed 5 percent of the average value, the specimen was loaded continuously with only one stop made at the yield plateau to determine the static yield strength level. The static yield strength was found using a yield stress level criterion defined by the stress at 0.005 in/in strain [11]. A strain rate corresponding to a stress rate of $1/kp/mm^2/min$ was used throughout the test after it was established in the elastic range. The results from these tests are given in Fig. 7. The elastic modulus, the proportional limit, the tangent modulus, and the average yield strength are the important data furnished by these curves. A summary of the stub column test results is given in Table 4.

4. COLUMN TESTS

A total of sixteen full-size column tests were conducted: four from each of the four source countries at the slenderness ratios of 50 and 95. These slenderness ratios were chosen on the basis that thet cover the critical range according to theoretical and practical considerations. All column tests were conducted in the same 5 million pound capacity universal hydraulic testing machine. Pinned-end support conditions were used in the minor axis direction and fixed in the direction of the major axis.

The end fixtures used in this test program were developed at Fritz Engineering Laboratory [12] and have been used extensively and with success in previous tests. A detailed description of the instrumentation and the procedure followed in testing the columns may be found in Ref. 13.

Initial Measurements

Initial measurements of the geometric characteristics of the columns were taken since variations in cross-sectional area and shape and the initial out-of-straightness will affect the column strength. Cross-sectional measurements were taken at five locations: at the ends and at the two quarter points of the column length. The initial out-of-straightness of each specimen was measured at nine levels, each spaced at one-eighth of the column length. Measurements were taken in the direction of the two principal axes. A summary of the measured geometric characteristics of the column specimens is given in Table 5.

Testing Procedure

The testing of heavy columns requires a well-developed testing procedure, more complete in instrumentation and supplementary tests, than is the case for light columns.

The alignment of the columns, which is regarded as the most important step in column testing, was performed in accordance with the ECCS recommendations: geometrical alignment with reference to the center of the web. The end plates were first matched to the web centers at each support and were finally centered with respect to the centerline of the testing machine.

The instrumentation for each column test consisted of potentiometers attached at quarter points to measure lateral displacements in the two principal axes and the angles of twist, electric resistant strain gages at the ends and at midheight, electrical rotation gages at the supports, and dial gage to measure the overall shortening. A typical column test set-up is shown in Fig. 8.

In each test the column was loaded continuously at a constant axial strain rate corresponding to a stress rate of 1 kp/mm^2/min (1.42 ksi/min) established during the elastic stage. All measurements were instantly recorded at fixed time intervals until the maximum load was almost reached immediately after which the loading was stopped to determine the maximum "static" load. This load is determined by maintaining the cross-head movement until the applied load was stabilized. The maximum "dynamic" load, the load recognized as the "maximum" load by ECCS, was obtained as the reading indicated by the stopping of the follower of the dial in the testing machine. After the static load was recorded the loading was resumed, using the originally established rate of cross head movement, until the end of test.

The measured load versus midheight deflection curves for all column tests are shown in Fig. 9. The values shown at the zero-load level correspond to the midheight initial out-of-straightness of the columns. Table 6 summarizes the results of the column tests.

Evaluation of Column Test Results

The ECCS has proposed three column strength curves for various types of shapes [8]. The appropriate curve to a particular shape is selected on the basis of: i) steel grade, ii) thickness of component plate, and iii) the depth-width ratio of the cross section.

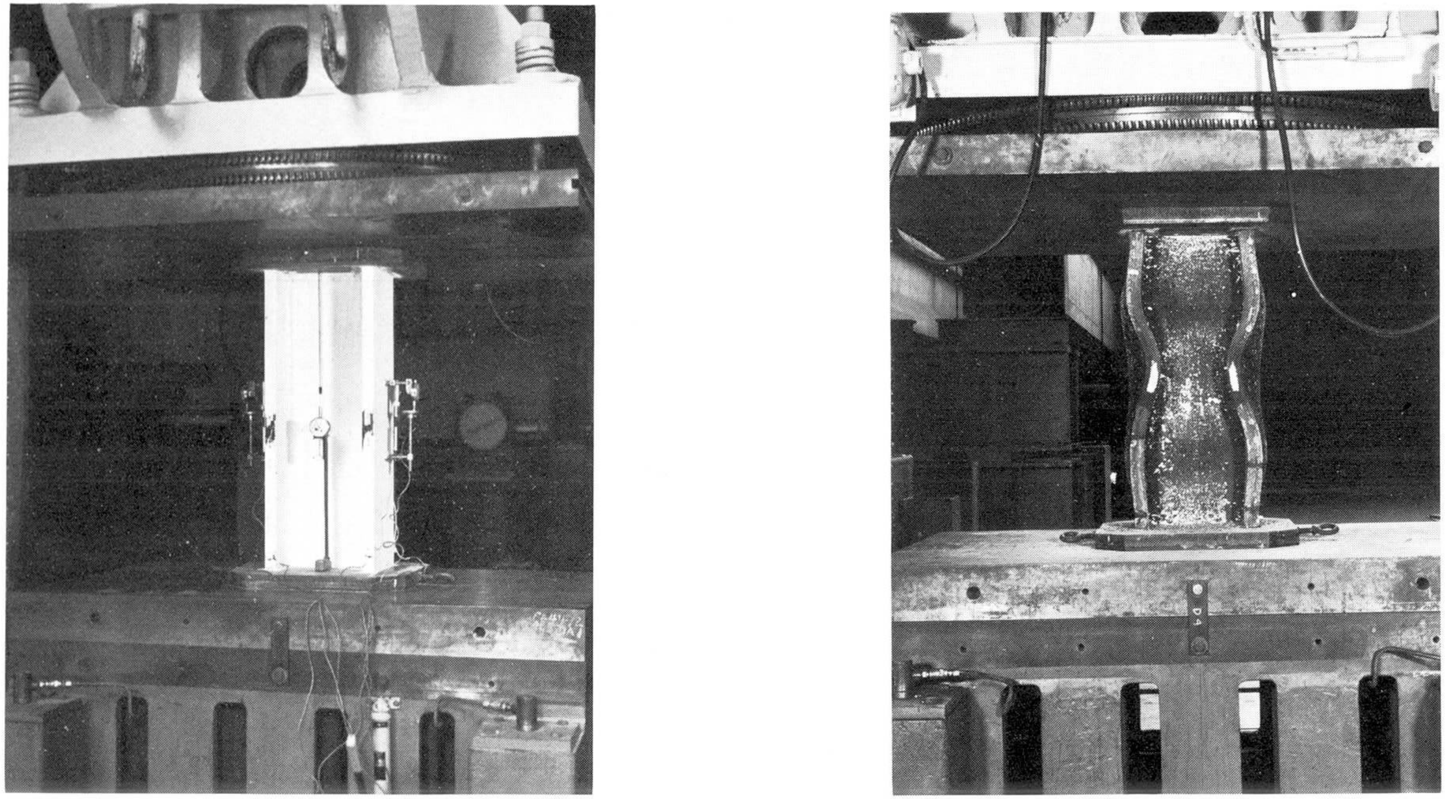
From conditions i) and ii) all specimens used in the test program belong in one group--all specimens have steel grades that relate closely to E 24 (St 37) and the thicknesses are greater than 30 mm (1-1/8 in.). Item iii), however, divides the specimens into two groups and requires use of different curves. According to the selection table given in Ref. 8, Curve B3-24 (the middle curve of the three) corresponds to the specimens from the continental countries (HEM 340) since h/b (= 1.23) > 1.20; for the British shapes (W12x161) Curve C3-24 (the lowest curve of the three) must be used since h/b (= 1.11) < 1.20. However, the assignment of different curves for these specimens does not seem justified since the shapes are essentially identical in cross-sectional properties, yield strength and residual stresses. Moreover, as shown in Fig. 10 and Table 6, comparison of the results of the British column tests discloses the same evidence. It is, therefore, recommended that a critical review be made of the depth-width ratio as a criterion for selecting the proper column strength curves.

In Fig. 10 the European Convention Curves are compared with the experimental points located at two Standard Deviations below the mean values. A good correlation for the columns with L/r = 95, but an unconservative prediction for L/r = 50, are observed.

5. SUMMARY AND CONCLUSIONS

This report presents the results of an experimental investigation into the behavior and strength of heavy European columns. The study is essentially an extension to heavy columns of the ECCS program, which has completed extensive tests of columns of small dimensions and light weights. The program was restricted to test specimens from four countries: Belgium, Britain, Germany and Italy. The experiments consisted of: i) tension tests (standard and full-size tests), ii) residual stress measurements, iii) stub column tests, and iv) full-size column tests (slenderness ratio of 50 and 95). The shapes used were HEM 340 from the continental countries and W12x161 from Britain.

Based on this investigation, the following conclusions may be made:



a) Instrumentation

b) End of Test

Fig. 6 Stub Column Test Set-Up

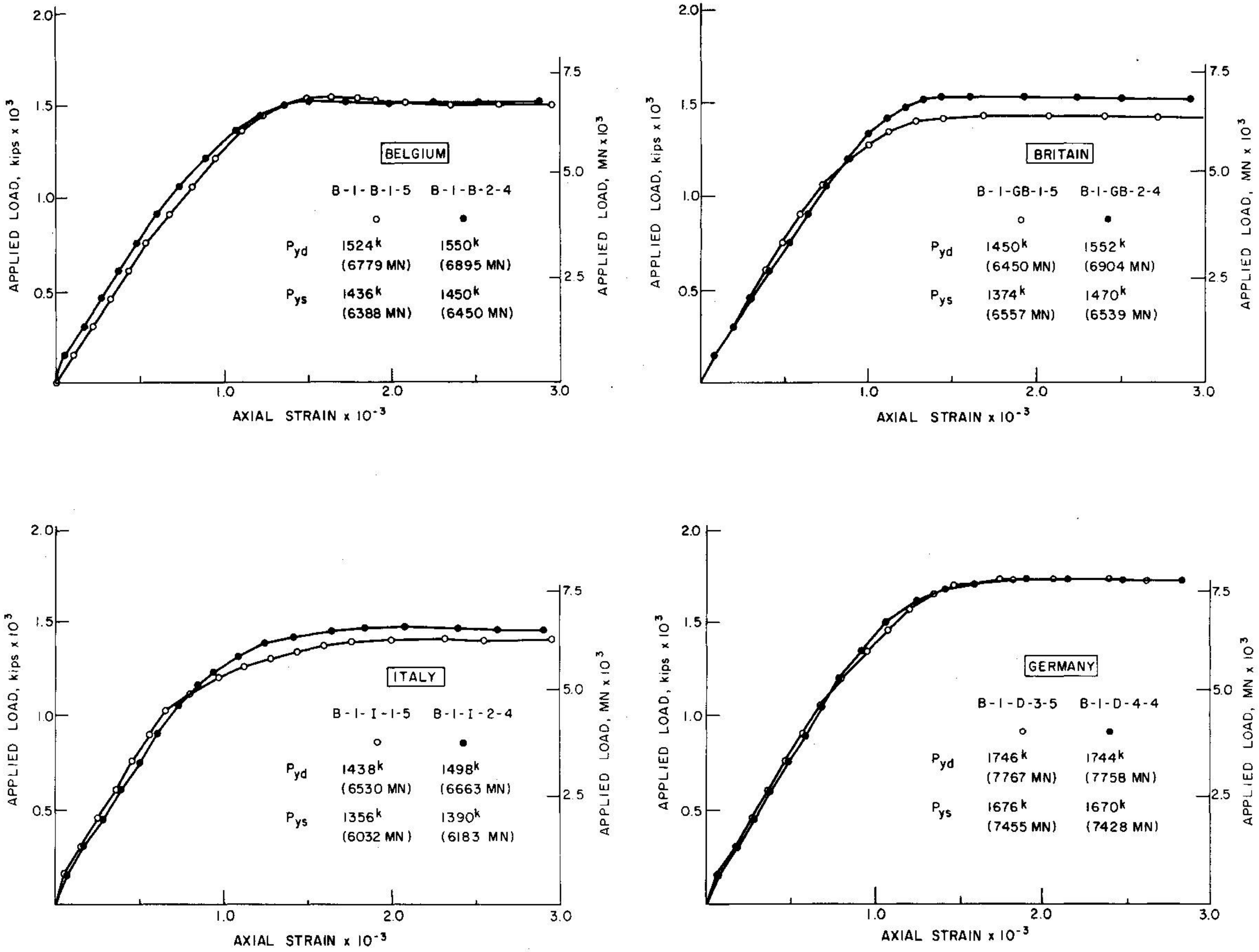
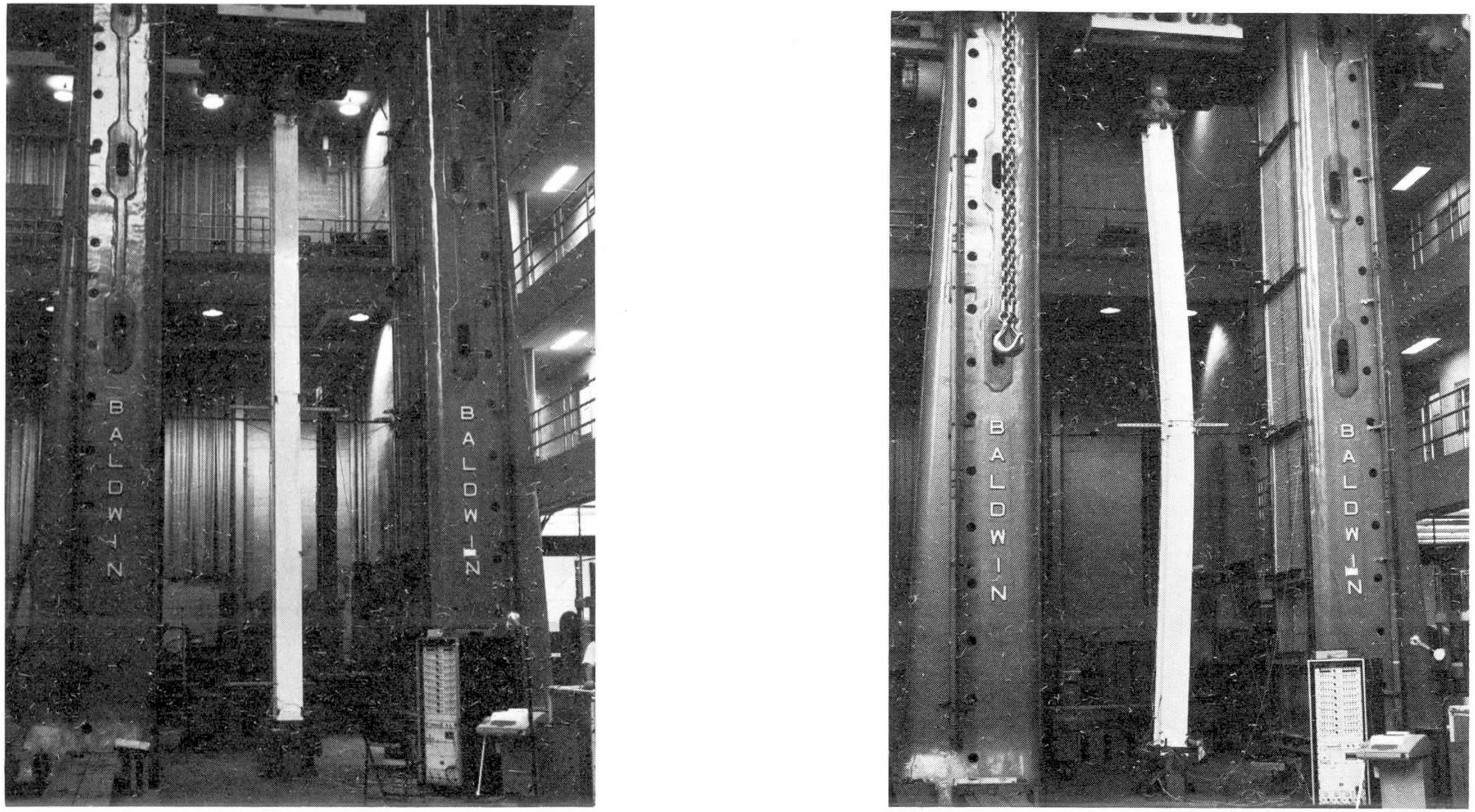


Fig. 7 Stub Column Test Results



a) Beginning of Test

b) End of Test

Fig. 8 Set-Up for Column Testing

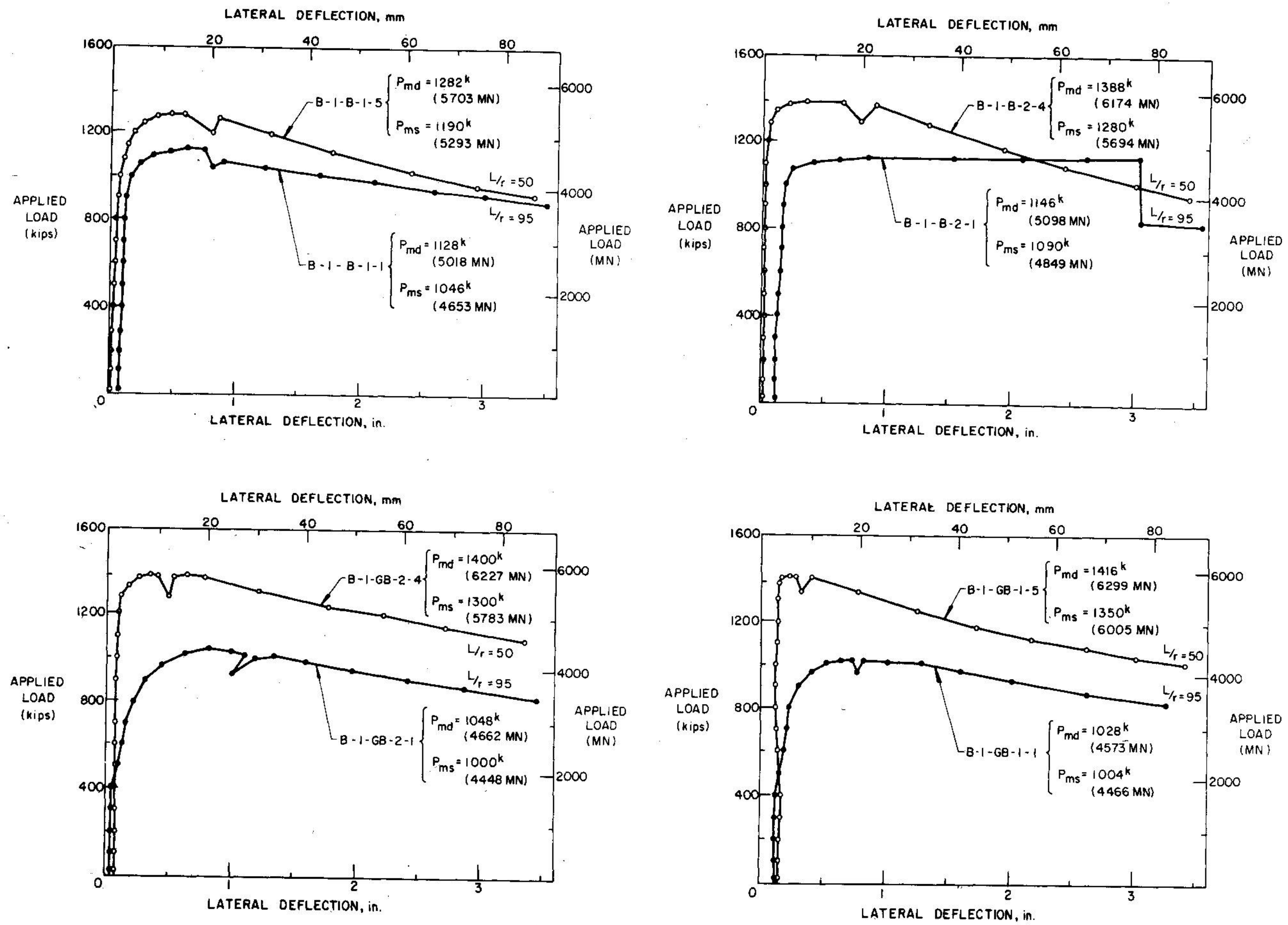


Fig. 9a Pinned-End Column Test Results

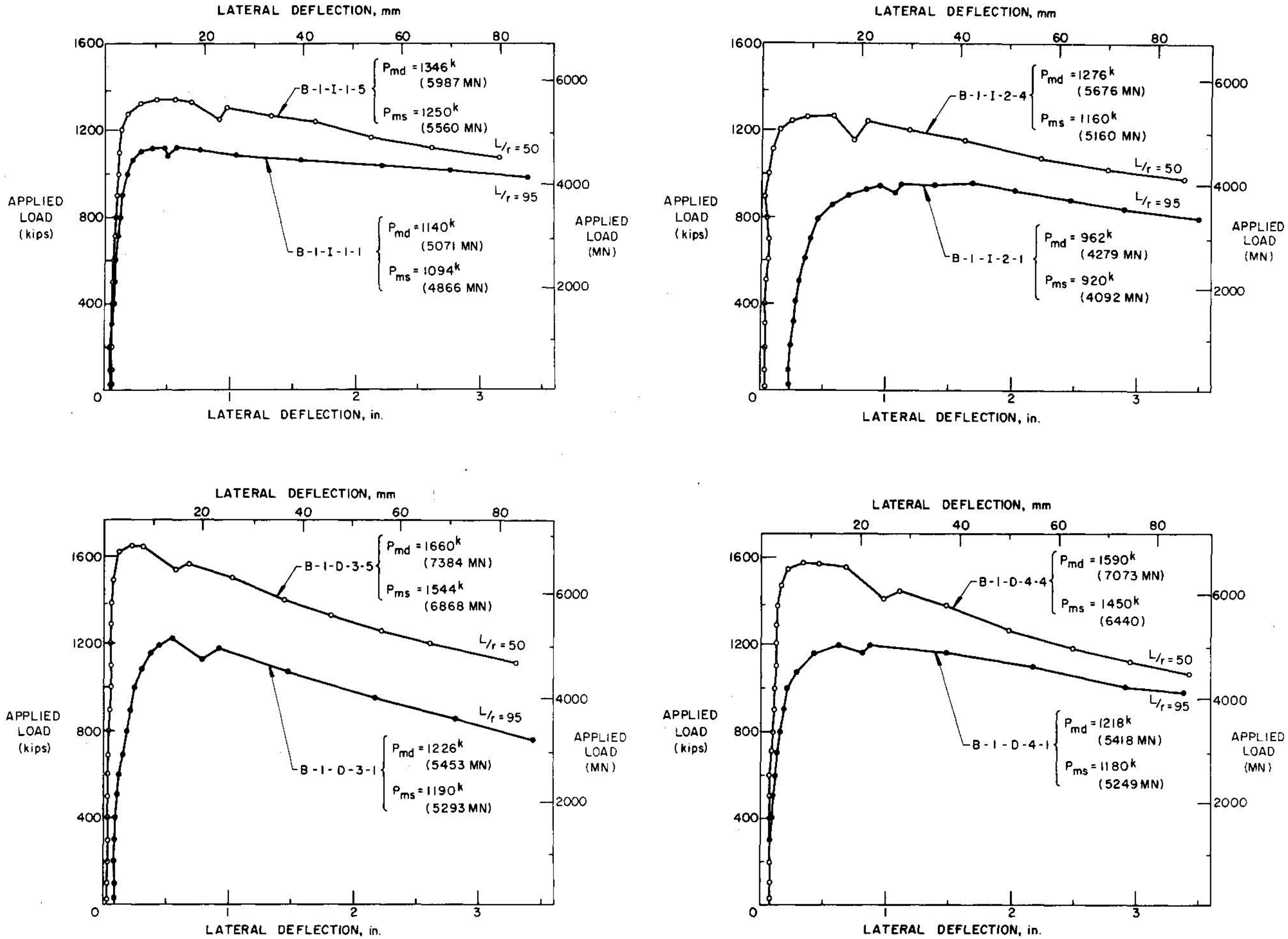


Fig. 9b Pinned-End Column Test Results

TABLE 5: MEASUREMENTS OF GEOMETRIC CHARACTERISTICS OF THE TEST SPECIMENS

Specimen	CROSS SECTION MEASUREMENTS B in. (mm)	D in. (mm)	T in. (mm)	W in. (mm)	Initial Out-of-Straightness Area in. (mm)	Minor e_x in. (mm)	Major e_y in. (mm)	Length in. (M)
B-1-B-1-5	12.17 (309)	14.90 (378)	1.54 (39)	0.82 (21)	47.28 (30503)	0.01 (0.3)	0.01 (0.3)	155.5 (3.94)
B-1-B-2-4	12.15 (309)	14.89 (378)	1.53 (39)	0.82 (21)	47.02 (30335)	0.02 (0.5)	0.01 (0.3)	155.5 (3.94)
B-1-B-1-1	12.19 (310)	14.93 (379)	1.55 (39)	0.82 (21)	47.45 (30612)	0.08 (2.0)	0.07 (1.8)	295.25 (7.50)
B-1-B-2-1	12.14 (308)	14.91 (379)	1.54 (39)	0.80 (20)	46.94 (30284)	0.11 (2.8)	0.09 (2.3)	295.25 (7.50)
B-1-GB-1-5	12.51 (318)	13.87 (352)	1.48 (38)	0.89 (23)	46.53 (30019)	0.15 (3.8)	0.02 (0.5)	160.0 (4.06)
B-1-GB-2-4	12.43 (316)	12.82 (351)	1.47 (38)	0.87 (22)	45.82 (29561)	0.08 (2.0)	0.01 (0.3)	160.0 (4.06)
B-1-GB-1-1	12.42 (315)	12.87 (352)	1.48 (38)	0.92 (23)	46.90 (30270)	0.13 (3.3)	0.04 (1.0)	304.0 (7.72)
B-1-GB-2-1	12.43 (316)	12.86 (352)	1.46 (37)	0.86 (22)	45.84 (29580)	0.04 (1.0)	0.08 (2.0)	304.0 (7.72)
B-1-D-3-5	12.26 (311)	14.89 (378)	1.59 (40)	0.89 (23)	49.27 (31787)	0.02 (0.5)	0.02 (0.5)	155.5 (3.94)
B-1-D-4-4	12.21 (310)	14.87 (378)	1.59 (40)	0.92 (23)	49.48 (31923)	0.06 (1.5)	0.92 (23.4)	155.5 (3.94)
B-1-D-3-1	12.25 (311)	14.87 (378)	1.58 (40)	0.88 (22)	49.07 (31658)	0.08 (2.0)	0.04 (1.0)	295.25 (7.50)
B-1-D-4-1	12.20 (310)	14.89 (378)	1.59 (40)	0.89 (23)	50.51 (32587)	0.06 (1.5)	0.06 (1.5)	295.25 (7.50)
B-1-I-1-5	12.11 (308)	14.93 (379)	1.60 (41)	0.84 (21)	48.57 (31335)	0.06 (1.5)	0.04 (1.0)	155.5 (3.94)
B-1-I-2-4	12.06 (306)	14.88 (378)	1.58 (30)	0.81 (21)	47.46 (30619)	0.03 (0.8)	0.02 (0.5)	155.5 (3.94)
B-1-I-1-1	12.09 (307)	14.93 (379)	1.60 (41)	0.85 (22)	48.57 (31335)	0.05 (1.3)	0.10 (2.5)	295.25 (7.50)
B-1-I-2-1	12.06 (306)	14.87 (378)	1.57 (40)	0.85 (22)	48.18 (31084)	0.22 (5.6)	0.01 (0.3)	295.25 (7.50)

TABLE 6: RESULTS OF COLUMN TESTS

Specimen	Slenderness Ratio L/r	Initial Out-of-Straightness $(\delta_o/L)x10^3$	Maximum Dynamic Load P_{md}	Maximum Static Load P_{sd}	P_{md}/P_{sd}
B-1-B-1-5	50	0.06	1282 (5703)	1190 (5293)	0.83
B-1-B-2-4	50	0.13	1388 (6174)	1280 (5694)	0.91
B-1-B-1-1	95	0.26	1128 (5018)	1046 (4653)	0.73
B-1-B-2-1	95	0.36	1146 (5098)	1090 (4849)	0.75
B-1-GB-1-5	50	0.97	1416 (6299)	1350 (6005)	0.97
B-1-GB-2-4	50	0.52	1400 (6227)	1330 (5783)	0.90
B-1-GB-1-1	95	0.43	1028 (4573)	1004 (4466)	0.71
B-1-GB-2-1	95	0.13	1048 (4662)	1000 (4448)	0.68
B-1-D-3-5	50	0.13	1660 (7384)	1544 (6868)	0.95
B-1-D-4-4	50	0.39	1590 (7073)	1450 (6440)	0.90
B-1-D-3-1	95	0.26	1226 (5453)	1190 (5292)	0.70
B-1-D-4-1	95	0.20	1218 (5418)	1180 (5249)	0.70
B-1-I-1-5	50	0.39	1346 (5987)	1250 (5560)	0.94
B-1-I-2-4	50	0.19	1276 (5676)	1160 (5160)	0.85
B-1-I-1-1	95	0.16	1140 (5071)	1094 (4806)	0.79
B-1-I-2-1	95	0.73	962 (4279)	920 (4092)	0.65

1. The testing of heavy columns requires a well-developed testing procedure, more complete in instrumentation and supplementary tests, than that for light-sized columns.

2. Full-size tension tests of heavy shapes, as recommended by ECCS, do not seem to provide additional information to that given by small specimens when taken at several locations over the cross section.

3. The measured residual stresses in the flanges of all of the eight specimens were seen to be closely consistent in pattern and in magnitude. The variations of residual stresses through the thicknesses were not significant.

4. The depth-width ratio criterion, which is one of the determining factors in selecting the column curves, is seen to be marginal for the specimens used in this investigation and consequently assigns different column curves to what are essentially identical shapes. It is recommended that this criterion be reviewed, as it could also be marginal for other heavy shapes.

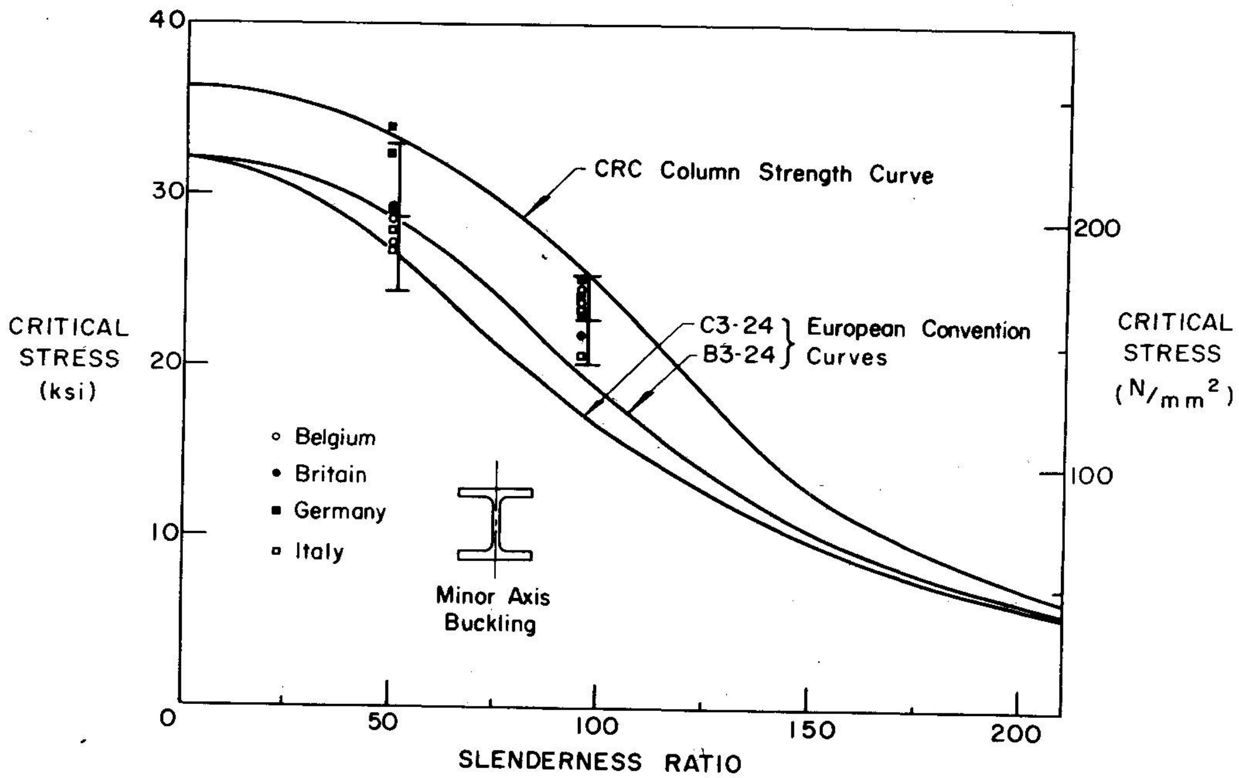


Fig. 10 Comparison of Column Test Results and the Proposed European Convention Column Curves

5. The European Convention Column Curve (B2-24) is compared with the test results. A good correlation for the columns of L/r = 95, and an unconservative prediction for L/r = 50, are observed.

6. ACKNOWLEDGMENTS

This investigation was conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The European Convention of Constructional Steelworks, the Welding Research Council, and the National Science Foundation (Grant No. GK 27302) jointly sponsored the study.

The guidance of Task Group 11 of the Column Research Council, under the chairmanship of Duililu Sfintesco, is gratefully acknowledged. Lynn S. Beedle, Director of the Column Research Council, made a number of helpful suggestions throughout the study.

Thanks are due to Kenneth R. Harpel, Laboratory Superintendent, and his staff for preparation of the specimens. The assistance of Paul Marek at the initial stages of the testing is sincerely appreciated.

Thanks are due to John Gera and Mrs. Sharon Balogh for the preparation of the drawings and to Miss Shirley Matlock for her care in typing the manuscript.

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THE PROBABILISTIC CHARACTERISTICS OF MAXIMUM COLUMN STRENGTH

Reidar Bjorhovde

Regional Engineer
American Institute of
Steel Construction, Inc.
Boston, Mass., U.S.A.
formerly with

Lambert Tall

Professor
of Civil Engineering
and Director, Division
for Fatigue and Fracture

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania
U.S.A.

ABSTRACT

This paper presents a novel method for computing the maximum strength of centrally loaded columns, whereby the probabilistic nature of the column strength factors has been taken into account. A quasi-steady probabilistic solution procedure has been used to reformulate the deterministic, incremental relationships that express the behavior of the inelastic column, so as to account for the random nature of the column strength parameters.

The contents of the paper may be outlined briefly as:

1) Evaluation of the statistical characteristics of the column strength parameters (geometric properties, yield stress, residual stresses, and initial out-of-straightness).
2) The method of probabilistic evaluation of the maximum column strength.
3) Analysis of the random variation of the maximum strength, with particular reference to probability density functions, confidence intervals, and the influence of each of the random column strength factors on the variation of the strength.

1. INTRODUCTION

Numerous theories and attempts at presenting the most rational solution to the problem of defining the strength and behavior of the centrally loaded column have been formulated throughout the years, but until fairly recently none of these studies have taken into account the stochastic characteristics of the column strength parameters. As is common to almost all physical phenomena, the factors that influence the strength and the behavior of the column exhibit unpredictable - random - variations, and an added fraction of realism therefore is introduced when the problem is treated within the realm of probability theory.

Whereas conceptually obvious, it is inconceivably complex to incorporate all factors and their variability into a practical solution, and some simplifications therefore have to be made in order to arrive at a practicable column model. This does not imply that the study and its results will be of lesser value, but rather that it represents a step in the direction of improving the method of column strength determination.

The investigation that is presented in this paper constituted a major phase of a research program that was conducted at Lehigh University. Of major concern in the study was the problem of defining the variation of the column strength, and how best it could be accounted for. In the probabilistic study that will be presented here, the variation of the relevant strength parameters has been considered explicitly in the calculation of the maximum strength. A direct analysis of the separate and joint effects of the variables therefore is made possible. To the best of the authors' knowledge, this represents the first time that the concept of probability theory has been applied towards the solution of the problem of an inelastic, initially curved column, for which the basic relationships are given in the form of incremental, iterative equations. The mathematical method of solution therefore may provide a theory that can be used in other areas of similar nature.

A complete evaluation and discussion of the theory and the results of the probabilistic study of the maximum column strength is provided by Reference 1.

2. PROBABILISTIC ANALYSIS OF COLUMN STRENGTH

The deterministic modeling of any problem implies the use of the fundamental concept of a one-to-one correspondence between the dependent and the independent variables (i.e. in a deterministic sense), and any variation of the pertinent variables is omitted from consideration. A probabilistic model, on the other hand, takes the variability explicitly into account, and the resulting solution thereby becomes expressed as a number of values, of which some are more likely to occur than others. The concept of the probability of the occurrence of an event thus is naturally introduced, whereby the multi-valued solution of the problem may be expressed either as a probability density function or a distribution function.

One of the first attempts at solving the inelastic column problem was provided by Chung and Lee[2], who presented a tangent modulus based approach. A similar technique was utilized by Rokach[3] in an effort to compare his data with the results provided by a number of European column test results. Whereas the analysis made by Augusti and Baratta[4] did incorporate the initial out-of-straightness as a random variable and therefore gave a maximum strength solution, their omission of the residual stresses in the column seriously reduced the usefulness of the study.

2.1 The Random Variation of the Column Strength Parameters

The probabilistic treatment of the maximum strength of a column is essentially a study of a structure which exhibits a random non-linear behavior. It therefore is necessary to establish the mathematical laws that reflect the random nature of the pertinent factors, prior to the formulation of the equations that govern the maximum strength. This has been done in the present study by expressing the column strength parameters in terms of probability density functions or distribution functions, and their characteristic quantities. The form of the functions have been assumed, but the available data from several experimental investigations have confirmed the validity of the assumptions made.

The detailed evaluation and analysis that have led to the determination of the probability density functions that illustrate the random nature of the column strength factors will not be presented here, due to the limitations on the length of the paper. Exhaustive developments of these relationships are given in Ref. 1, however, and therefore only the type of functions used will be outlined. These are the following:

1. For the cross-sectional properties of wide-flange and box shapes: Normal (Gaussian) distribution.
2. For the mechanical properties of the steel (only the yield stress is considered as a random variable): Type I asymptotic extreme value distribution for largest values.

3. For the residual stress in any element in the cross section: Normal (Gaussian) distribution.
4. For the initial out-of-straightness of the column: Type I asymptotic extreme value distribution for the smallest value.

It should be noted that although the residual stress in an element in the cross section is assumed to vary normally and independently of all of the other elements in the shape, the overall residual stress distribution has to satisfy force and moment equilibrium in the shape, prior to the application of any external load. The random nature of the overall residual stress distribution, as evidenced by the different patterns in rolled, welded, universal mill, and flame-cut shapes, has not been studied. This distribution is greatly influenced by the manufacturing method, and therefore has a most significant effect on the column strength. The random variations of the residual stresses that are considered are thus indicative only of deviations about the mean residual stress pattern.

Figures 1 and 2 give examples of the above described developments. Figure 1 shows the probability density function for the initial out-of-straightness, and Fig. 2 shows the derived probability density function (three-dimensional response surface) for the yield load of the rolled wide-flange shape W8x31 of steel grade ASTM A36. It should be noted that the specification[5] maximum allowable out-of-straightness, L/1000, has been assumed representative of the $97\frac{1}{2}$ percent probability level, such that values larger than this occur with a probability of 2.5 percent. The minimum out-of-straightness is 0 (zero), which is assumed to occur with a probability of 1 percent.

2.2 Probabilistic Evaluation of Maximum Column Strength

The column maximum strength may in principle be expressed by the following equation, where P_{max} denotes the maximum strength:

$$P_{max} = f(\sigma_y, \sigma_r, B, b, t, d, w, e_L, L) \qquad (1)$$

where σ_r denotes the residual stress; b, t, d, and w are geometric descriptors of the cross section of the column; and e_L is the initial out-of-straightness. The other factors have been defined previously. The function given by Eq. (1) represents a multidimensional probability density function, or a response surface, since the parameters involved may be treated as random variables.

The probabilistic characteristics of the pertinent factors have already been established. The modulus of elasticity, E, is treated as a constant, and the column length, L, is also a deterministic quantity. The column is thought of as being subjected to a deterministic load, P, which remains as such from the onset of the loading and until the maximum capacity is reached. The load-deflection analysis of the column therefore will result in the determination of a semi-probabilistic load-deflection curve with the load as a random variable if the deflection is the input-value, and vice versa. The concept of semi-probabilistic load-deflection curves is schematically illustrated in Fig. 3.

The maximum strength that is found by the solution of the incremental, iterative expressions becomes a fixed value for every given set of values of the strength parameters for a given column. The random variation of the column strength factors provides for a random variation of the strength, thus leading to the determination of the probabilistic characteristics of the strength (see Fig. 3b) for a given column and length. Solving the same problem with different values of the length eventually leads to a set of column curves that illustrates the total variation of the strength. Such a set of column curves is defined as the column curve spectrum.

The complete evaluation of the probabilistic, incremental/iterative equations will not be presented, but a detailed development is given in Ref. 1. As an illustration, however, the total stress in an element i of the cross section is given by:

$$\frac{\tilde{\sigma}_i}{\tilde{\sigma}_y} = \frac{\tilde{\epsilon}_i}{\tilde{\epsilon}_y} = \frac{1}{\tilde{\epsilon}_y}\left[\tilde{\epsilon}_{ri} + \tilde{\epsilon}_p + \tilde{\theta}.\tilde{\epsilon}_{si}\right] = \Phi\ (\epsilon) \tag{2}$$

where the tilde (~) denotes a random variable. For example, with the distribution characteristics for $\tilde{\sigma}_y$ given, those of $\tilde{\epsilon}_y$ can easily be found, since

$$\tilde{\sigma}_y = \tilde{\epsilon}_y \ .\ E \tag{3}$$

where E is a deterministic quantity. Similar principles apply for the solution for $\tilde{\epsilon}_{ri}$, $\tilde{\theta}$, and $\tilde{\theta}.\tilde{\epsilon}_{si}$. For $\tilde{\epsilon}_p$ a probability density function is used, as opposed to a specific value in the deterministic approach. This is a very significant computational advantage, since the range of ϵ_p -values is taken into account at the same time. The time-consuming and error-prone repetition of the calculations that result from incorrectly assumed ϵ_p -values is thereby eliminated.

Equation (2) forms the basis for the development of the probabilistic characteristics of the elemental stress and strain, and can be extended to incorporate all elements in the cross section. This in turn is used to determine the properties of the (random) internal force and moment, which leads to the solution for the equilibrium external load.

Various numerical methods may be used to determine the random variation of the maximum strength. The use of a Monte Carlo approach was investigated, but was discarded as an inefficient and expensive solution procedure. It may prove advantageous if purely theoretical values are used for the column strength parameters. For all practical purposes, however, the complete distribution of the maximum strength is not needed, since basically the upper and lower bounds, and a central distribution parameter such as the mean, will provide the information necessary.

3. THE VARIATION OF MAXIMUM COLUMN STRENGTH

Large amounts of data have been produced in this investigation, and only a few representative examples are shown and discussed here. The information presented is thus but a small part of what is available, but it nevertheless illustrates and emphasizes all of the important findings of the study.

Figure 4 shows the column curve spectra for the major and minor axis bending of a typical light rolled wide-flange shape (W8x31, steel grade ASTM A36). Each spectrum reflects the variation of the strength of the shape, when all of the column strength parameters vary between their respective extreme values. The spectra therefore illustrate the 95 percent confidence intervals for the maximum strength of the shape, such that there is only a probability of 5 percent that the strength of a randomly chosen W8x31 (A36) column will fall outside the interval. The upper limit of each spectrum is indicative of columns with an initial out-of-straightness of L/10,000, and the lower limit of columns with e_L = L/1000.

In order to detect and analyze the effects of the variability of the other column strength parameters, column curve spectra were prepared, for which the initial out-of-straightness was kept constant. Figure 5 shows the resulting spectra for the W8x31 (A36) shape, with e_L maintained at its mean value of L/1470.

Within the limitations and assumptions imposed by the study, the data presented in Fig. 5 show that the influence of the variability of the yield stress and the cross-sectional properties, and of the $\pm$ - variations of the residual stresses in any particular shape with a specific manufacturing method, is relatively small for the variation of the maximum strength. The two column curve spectra both indicate maximum strengths that lie within a range of 3 to 7 percent (from the upper to the lower limit), depending on the magnitude of the slenderness ratio. Extended analysis of the data, furthermore, show that this variation almost in its entirety may be attributed to the variation of the yield stress. In these analyses, means and coefficients of variation of the maximum strength were computed for various slenderness ratios, maintaining the yield stress at its minimum, mean, and maximum values. For each value of the yield stress, the corresponding mean values of the maximum strength were clearly different, although the differences were very small; and the coefficients of variation were extremely small (between 0 and 0.6 percent). No systematic influence of the varying residual stresses and cross-sectional properties was found. These statements are true for all slenderness ratios, and also for other values of the out-of-straightness.

The reason for the lack of influence of the residual stress variation about the mean residual stress distribution in the shape, is partly due to the over-riding influence of the initial out-of-straightness which strongly governs the behavior and strength of the column. It is also due to the fact that any residual stress distribution has to be in equilibrium. The effects of the geometric properties are probably almost completely over-ridden by the variation of the yield stress.

The conclusions arrived at above are basically true for all the rolled and welded wide-flange and box shapes that have been studied. However, the yield stress becomes more important as the range between its maximum and minimum values increases. For heavy rolled shapes also, the variability of the cross-sectional dimensions has a certain effect.

Figure 6 shows the major axis column curve spectrum for the W8x31 (A36) shape, together with the curves depicting its dispersion characteristics. Due to the influence of the initial out-of-straightness, which is distributed according to an extreme value density function (see Fig. 1), the maximum column strength for any given slenderness ratio also will be distributed as such. This is indicated in Fig. 6, and Fig. 7 illustrates the probability density function for the maximum strength of the W8x31 (A36) column with a non-dimensional slenderness ratio of 0.9, bent about the major axis. It was found that a Type I (Gumbel, largest value) asymptotic extreme value distribution fits the data very well; and the results for all slenderness ratios and for the other types of columns investigated also confirm this finding.

The data presented in Fig. 8 are analogous to those of Fig. 6, but represent the column curve spectrum for the minor axis bending of the W8x31 shape. The skew distribution of the maximum stength prevails, although it may be noted that it is significantly more pronounced for the intermediate and high slenderness ratios, when compared to the data in Fig. 6. This is a commom property for many column curve spectra for minor axis bending of wide-flange shapes.

The results that have been given here are indicative of the most important findings of the probabilistic study of the maximum strength of centrally loaded steel columns. Detailed and extensive data for a number of column types and shapes, in different steel grades, are provided in Ref. 1. The findings have furthermore been utilized in the development of a set of multiple column curves, which, it is believed, will provide significant improvements in the method of assessing the design strength of real columns.

4. SUMMARY AND CONCLUSIONS

Some of the most significant findings of the study presented here may be summarized briefly:

1. A probabilistic method for the solution of the problem of defining the maximum strength of centrally loaded, initially curved, pinned-end, prismatic steel columns has been developed. This represents the first time that a structure exhibiting a random non-linear behavior, for which the basic relationships are expressed as incremental, iterative equations, has been treated within the context of probability theory.
2. The random variation of the strength of a particular column, given its manufacturing method, almost entirely may be attributed to the random variation of the initial out-of-straightness. The random variation of the yield stress has a small effect, but this increases with the increasing yield stress and its range of variation.
3. The random variation of the residual stresses about their mean, and of the cross-sectional properties, do not contribute significantly to the random variation of the maximum column

strength. The probabilistic nature of the overall residual stress distribution has not been studied, and the pattern of residual stress in the shape therefore remains one of the most significant column strength parameters.

4. Due to the overall importance of the initial out-of-straightness, the maximum strength of a specific column will be distributed in a skew fashion. It has been found that a Type I asymptotic extreme value distribution is a good representation of the random column strength variation.

5. ACKNOWLEDGEMENTS

This study formed part of a research project at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, and it was sponsored by the National Science Foundation and the Column Research Council. The support of these organizations is sincerely appreciated.

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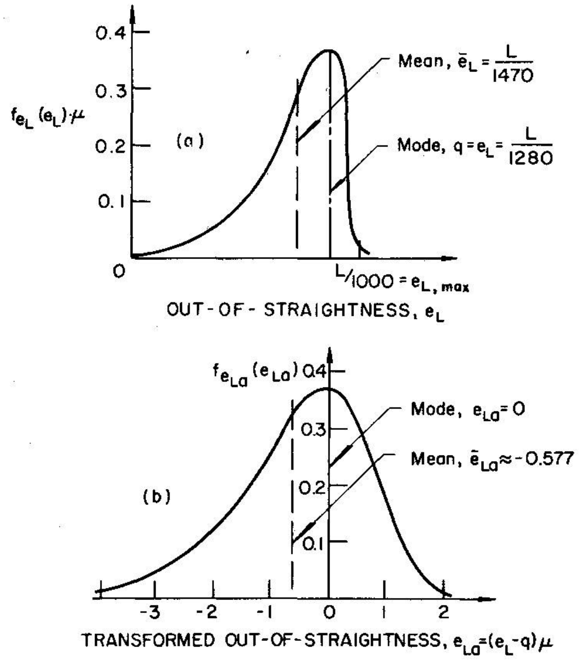


Fig. 1 The Type I Asymptotic Extreme Value Distribution Representing the Probability Density Function for the Initial Out-of-Straightness of the Column

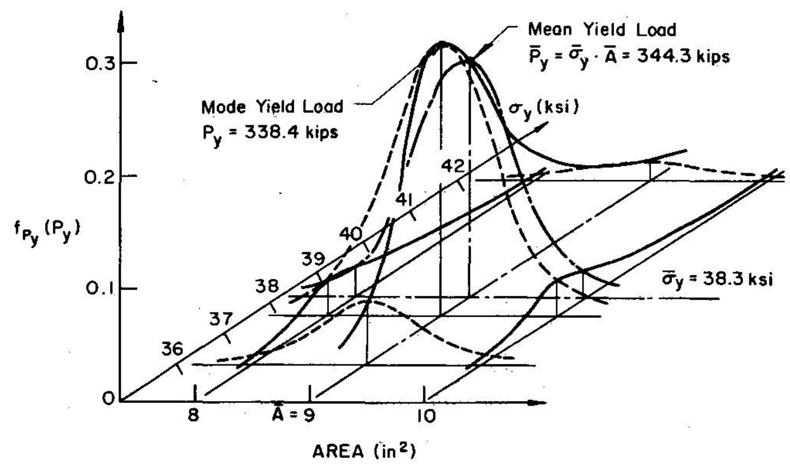
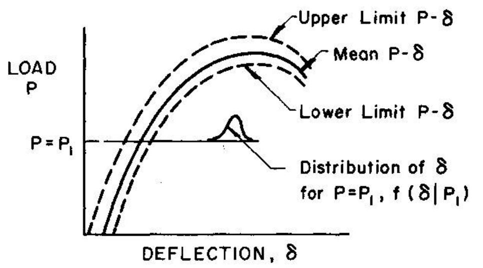
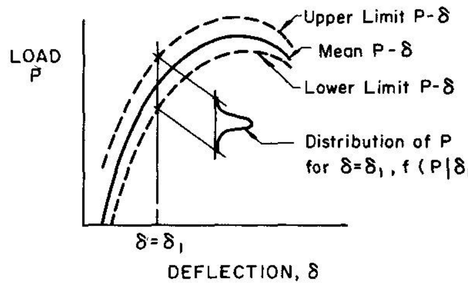


Fig. 2 The Derived Probability Density Function for the Yield Load of the Rolled Wide-Flange Shape W8x31 of Steel Grade ASTM A36



(a) Deterministic Load, Probabilistic Deflection



(b) Deterministic Deflection, Probabilistic Load

Fig. 3 A Schematic Illustration of the Concept of Semi-Probabilistic Load-Deflection Curves

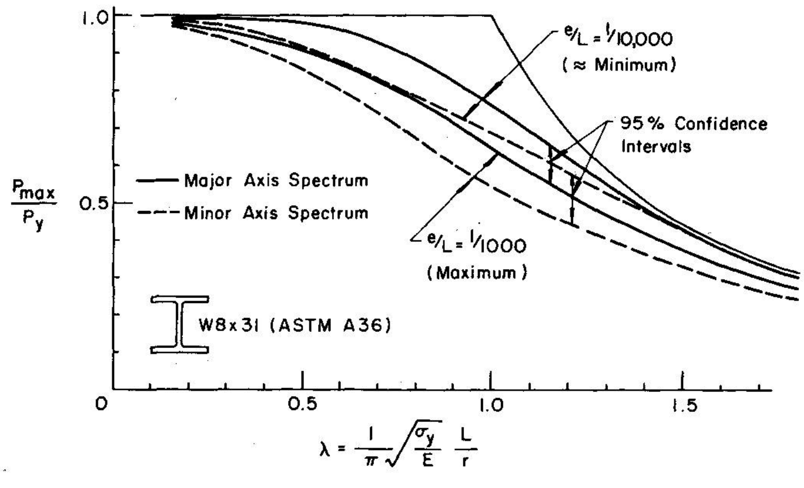


Fig. 4 The Column Curve Spectra for Major and Minor Axis Bending of the Rolled Wide-Flange Shape W8x31 (A36)

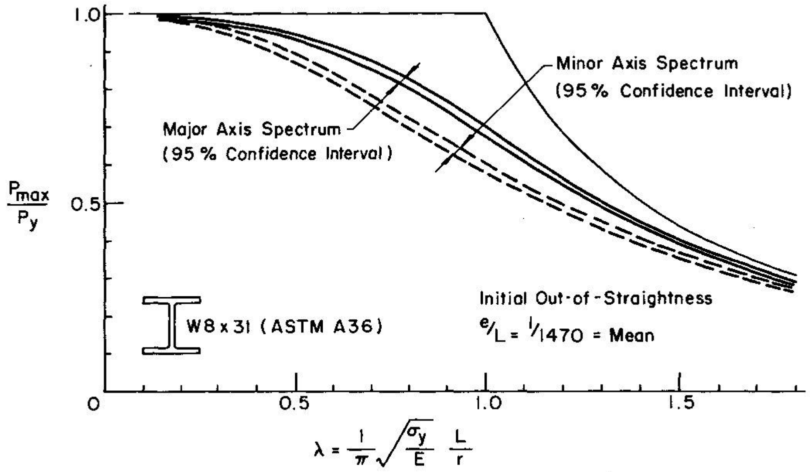


Fig. 5 The Column Curve Spectra for Major and Minor Axis Bending of the Rolled Wide-Flange Shape W8x31 (A36), with the Initial Out-of-Straightness Kept Constant (=L/1470)

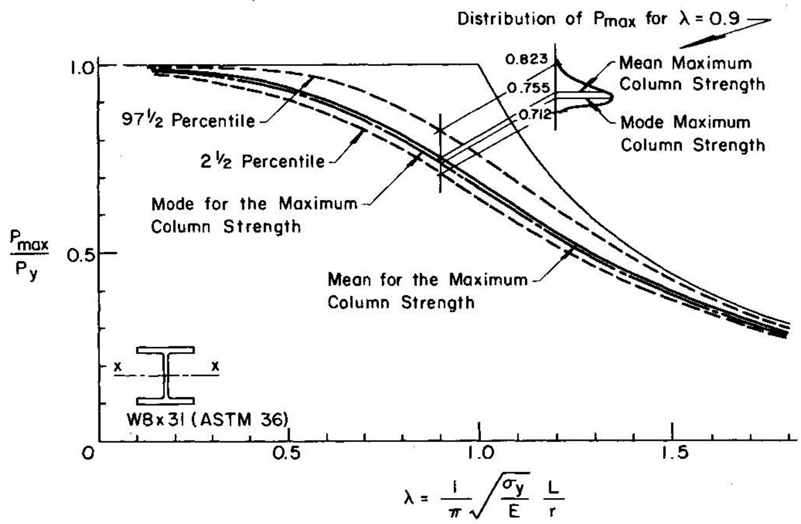


Fig. 6 The Dispersion Characteristics of the Major Axis Column Curve Spectrum for the Rolled Wide-Flange Shape W8x31 (A36)

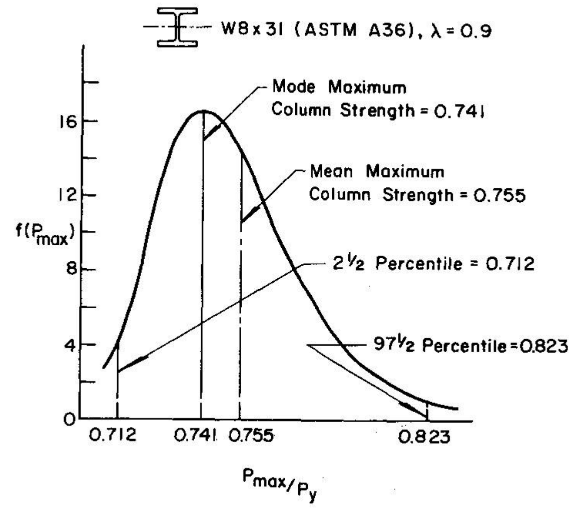


Fig. 7 The Probability Density Function (Type I Asymptotic Extreme Value Distribution) for the Maximum Strength of a Column W8x31 of Steel Grade ASTM A36, with Non-Dimensional Slenderness Ration of 0.9 (L/r = 90)

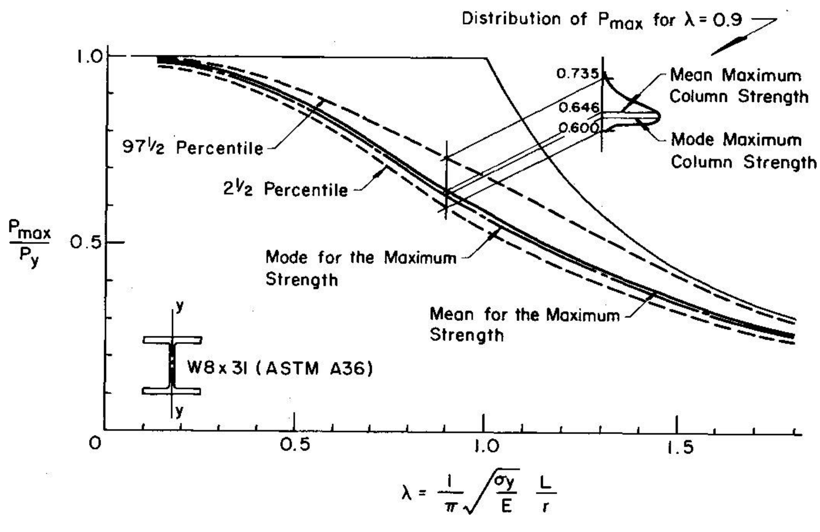


Fig. 8 The Dispersion Characteristics of the Minor Axis Column Curve Spectrum for the Rolled Wide-Flange Shape W8x31 (A36)

COMPUTER SIMULATION OF THE E.C.C.S. BUCKLING CURVE USING A MONTE-CARLO METHOD

JOHN STRATING and HAN VOS
Stevin Laboratory, Delft University of Technology
Delft, The Netherlands

ABSTRACT

The application of a Monte-Carlo simulation procedure to obtain the distribution function of the maximum load of a hinged column with imperfections is discussed. Buckling tests carried out by the E.C.C.S. on IPE 160 sections have been simulated. Information concerning the column variables is obtained from the data-sheet of the E.C.C.S. tests. The probability density function of each variable is derived or estimated. A good agreement is found between the simulated buckling curve and the experimental buckling curve.

COMPUTER SIMULATION OF THE E.C.C.S. BUCKLING CURVE USING A MONTE-CARLO METHOD.

1. INTRODUCTION

This paper describes a procedure for computer simulation of buckling tests, using a Monte-Carlo method. The variation of the parameters which determine the load-carrying capacity of a column is taken into account and the probability density-function of the buckling load is derived.

In the past years, the European Convention for Constructional Steelwork (E.C.C.S.) has carried out an extensive experimental programme on buckling of concentrically loaded, hinged columns with imperfections. The results of these tests are discussed in [1]. Most specimens tested were light-weight sections with flange thicknesses $t \leq 20$ (mm). The test series has been designed in such a way that a buckling curve with a certain probability of failure could be derived. The buckling curve is defined by means of characteristic stresses. According to the philosophy of the E.C.C.S., the characteristic buckling stress σ^{*}_{CR} is equal to

$$\sigma^{*}_{CR} = m - k.s$$

where m is the mean value and s is the standard deviation of the buckling stresses; k is a constant which depends on the type of probability density function (p.d.f.) of σ_{CR}.
The value of k must be chosen so that: prob. $[\ \sigma_{CR} \leq \sigma^{*}_{CR}\ ]$ is equal to 2.3%. If σ_{CR} follows a Gaussian p.d.f. the value of k=2.

Information concerning the type of p.d.f. of buckling stresses could be obtained only through experiments at the time the E.C.C.S. tests were started.
The number of tests involved is large, however. The p.d.f. is estimated from the results.
As shown in [1], the shape of the experimental buckling curve is determined mainly by the test results on IPE 160 sections. The buckling curve is shown in (fig. 1) together with the significant test results. A statistical analysis of the buckling stresses proved that the buckling stresses are Gaussian distributed and therefore

$$\sigma^{*}_{CR} = m - 2.s$$

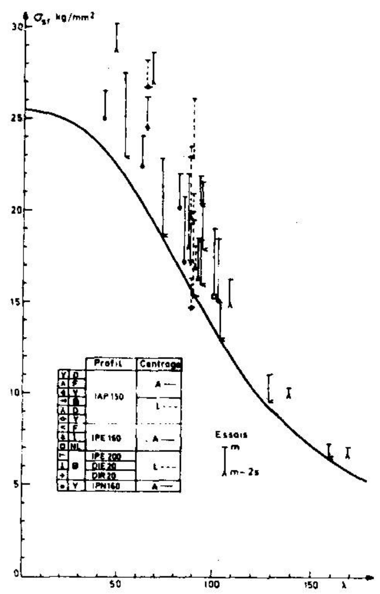


Fig. 1

Due to the great number of tests involved in the above-mentioned approach, it cannot be extended easily to all the various sectional shapes and dimensions. Neither time nor means are available to carry out these tests.
Theoretical solutions are sought, therefore which are able to predict the behaviour of an imperfect column with sufficient accuracy and which also take into account the random nature of the imperfections and the mechanical properties. Two problems can be recognized which must be solved.

1. To compute the buckling load of a concentrically loaded column, given certain imperfections and mechanical properties.

2. To compute the probability density-function of the buckling loads or stresses, given the imperfections and mechanical properties are random variables.

It is obvious that the first problem must be solved before the second problem cab be tackled. Batterman and Johnston [2]. Stüssi |3 , as well as Beer and Schulz [4] have discussed numerical methods for solving the case of a concentrically loaded column with certain imperfections. These methods are used to carry out the computations involved in the outlined procedure and they will be discussed briefly in chapter 3. This paper, is concerned primarily with the solution of the second problem, however.
A Monte-Carlo simulation procedure is applied to derive the p.d.f. of the buckling stresses. The results of the E.C.C.S.-tests on IPE specimens are analysed and used to check the validity and accuracy of this kind of approach. Information concerning the imperfections and mechanical properties of these sections has been obtained from the data sheets which were established for each test specimen. The p.d.f.'s of the column variables can be derived from this information. These functions are used as input-sources for the Monte-Carlo simulation procedure.
Finally, a buckling curve is computed with known probability of the failure. This curve compares well with the experimental E.C.C.S. buckling curve derived from the same specimens.

2. COMPUTER SIMULATION OF BUCKLING CURVES.

The buckling load of a hinged column with imperfections can be described by the following relation

$$P_{CR} = f(\sigma_{yt}, \sigma_{yc}, \sigma_r, e_o, f_o, A, E, \lambda)$$

where σ_{yt} = yield stress in tension

σ_{yc} = yield stress in compression

σ_r = residual stress

e_o = eccentricity

f_o = amplitude of the initial curvature

A = area

E = Young's modulus

λ = slenderness-ratio

It should be emphasized that the variables which appear in this relation are random variables. The number of variables can be reduced if σ_{yt} is assumed to be equal to σ_{yc}, and that E is constant; the relation can then be written as

$$P_{CR} = f\ (\sigma_y, \sigma_r, e_o, f_o, A, \lambda).$$

Proof of the influence of each variable on the scatter of the buckling load P_{CR} can be obtained through correlation analysis of tests results, as shown by Loof for the E.C.C.S. tests [5].
According to the criteria of the E.C.C.S., the characteristic buckling load is equal to

$$P^*_{CR} = \bar{P}_{CR} - k.s$$

P^*_{CR} = characteristic value of the buckling load

$\bar{P}_{CR}$ = mean value of the buckling load

s = standard deviation of the buckling load
k = constant such that prob $[P_{CR} \leq P^*_{CR}] = 2.3\ \%$

It is obvious that the value of k depends on the type of p.d.f. of P_{CR}. A value for P^*_{CR} can be determined, without much difficulties, from experiments. A theoretical solution for P^*_{CR}, is much more difficult to obtain, however. P_{CR} is a function of a number of random variables, consequently P_{CR} follows a multi-dimensional probability density-function. This function is not known generally nor can this function be derived from information concerning the p.d.f.s' of the random variables, except in a few special cases. A purely theoretical solution of the problem in question is not feasible therefore in most cases. Two approximate solutions, however, have been suggested; they are discussed below and a new approach is described.

2.1 Method I.

Various combinations of the variables are introduced into the formula for P_{CR}. Each combination leads to another buckling curve (varying λ). By comparing the computed buckling curve with the experimental E.C.C.S. buckling curve, a combination of variables can be estimated which fits the experimental curve most closely over the whole range of slenderness ratios. This method has been adopted and developed by Beer and Schulz [4] From a probabilistic point of view, this method is questionable because a lower bound curve is approximated. There is no reason to assume that the obtained solution is unique.
Extrapolation to other shapes and dimensions is realised by modifying the combination of the variables. No information concerning the scatter in the buckling loads is obtained, however . This method is therefore not truly probabilistic.

2.2 Method II.

Schor [6] and Carpena [1] assume that all variables are uncorrelated, and furthermore that the function $f(\sigma_y, \sigma_r, e_o, f_o, A, \lambda)$, can be linearized. A linear function is obtained through a Taylor expansion of f

$$f(\sigma_y, \sigma_r, e_o, f_o, A, \lambda) = f(\bar{\sigma}_y, \bar{\sigma}_r, \bar{e}_o, \bar{f}_o, \bar{A}, \lambda) + \frac{\partial f}{\partial \sigma_y}(\sigma_y - \bar{\sigma}_y) +$$

$$+ \frac{\partial f}{\partial \sigma_r}(\sigma_r - \bar{\sigma}_r) + \frac{\partial f}{\partial e_o}(e_o - \bar{e}_o) + \frac{\partial f}{\partial f_o}(f_o - \bar{f}_o) + \frac{\partial f}{\partial A}(A - \bar{A}) +$$

$$+ \frac{\partial^2 f}{\partial \sigma_y^2} \frac{(\sigma_y - \sigma_y)^2}{2\,!} + \frac{\partial^2 f}{\partial \sigma_r^2} \frac{(\sigma_r - \sigma_r)^2}{2\,!} + \ldots\ldots\ldots\ldots$$

Disregarding all terms of the second order and higher, the expansion reduces to

$$f(\sigma_y, \sigma_r, e_o, f_o, A, \lambda) \approx f(\bar{\sigma}_y, \bar{\sigma}_r, \bar{e}_o, \bar{f}_o, \bar{A}, \lambda) + \frac{\partial f}{\partial \sigma_y}(\sigma_y - \bar{\sigma}_y) +$$

$$+ \frac{\partial f}{\partial \sigma_r}(\sigma_r - \bar{\sigma}_r) + \frac{\partial f}{\partial e_o}(e_o - \bar{e}_o) + \frac{\partial f}{\partial f_o}(f_o - \bar{f}_o) + \frac{\partial f}{\partial A}(A - \bar{A}).$$

The mean value of P_{CR} can be found by substituting $(\bar{\sigma}_y, \bar{\sigma}_r, \bar{e}_o, \bar{f}_o, \bar{A})$ into this formula

$$\bar{P}_{CR} \approx f(\bar{\sigma}_y, \bar{\sigma}_r, \bar{e}_o, \bar{f}_o, \bar{A}, \lambda).$$

The variance of P_{CR}, after squaring and summing, is equal to

$$S_p^2 \approx \left(\frac{\partial f}{\partial \sigma_y} S_y\right)^2 + \left(\frac{\partial f}{\partial \sigma_r} S_r\right)^2 + \left(\frac{\partial f}{\partial e_o} S_e\right)^2 + \left(\frac{\partial f}{\partial f_o} S_f\right)^2 +$$

$$+ \left(\frac{\partial f}{\partial A} S_A\right)^2$$

where S_p = standard deviation of P_{CR}
S_y = standard deviation of σ_y
S_r = standard deviation of σ_r
S_e = standard deviation of e_o
S_f = standard deviation of f_o
S_A = standard deviation of A

It is now possible to compute the mean value of P_{CR} and the variance at each slenderness-ratio λ, provided function $f(\sigma_y, \sigma_r, e_o, f_o, A, \lambda)$ can be solved. The mean values and variances of each variable must also be known. The first derivatives of f can be obtained analytically, by partial differentiation of f or graphically from curves showing the dependance of f upon each variable. If furthermore is assumed that P_{CR} follows a Gaussian p.d.f., the desired buckling curve can be derived by computing for each λ the value $(\bar{P}_{CR} - 2S_p)$.
Essential in the abovementioned approach are the assumptions that the variables are uncorrelated and that function f can be linearized. The latter assumption must be viewed with reserve and may lead to significant errors.
The described approach can be checked against the E.C.C.S. buckling curve. The mean values and the variances of the variables can be obtained from the data-sheets available for each test specimens. Comparison of the computed buckling curve and the experimental buckling curve will show whether the linearization of f is allowed.
This method itself is basically a probabilistic approach and therefore in agreement with the criteria of the E.C.C.S.

2.3 Method III.

Carrying out a buckling test simply means loading a column, with a certain combination of imperfections and mechanical properties, until failure occurs. The values of the imperfections and the mechanical properties of a particular column cannot be predicted in advance.
Once a column has been selected for a test, however, these values can be measured. If the mathematical model of such a column is sufficiently

accurate, the buckling load of this column can be computed instead of actually carrying out a buckling test. This can be repeated for any number of columns. None of the columns are actually tested, all buckling loads are computed, the tests are "simulated". The simulation method can be further generalized if it is recognized and acknowledged that the values of the imperfections and the mechanical properties present in a column are primarily due to chance. It is sufficient to know the distribution function of each variable and the correlations between these variables, to carry out the simulation procedure. One drawing from the population of each variable, giving proper attention to the correlations between them, results in a combination of variables which can be assigned to a hypothetical column; the buckling load P_{CR} of this hypothetical column can then be computed. If this procedure is repeated a number of times, an equal number of P_{CR} values is obtained. The mean value as well as the variance of P_{CR} can be determined and a p.d.f. can be fitted to the histogram of P_{CR}-values. By doing this, the E.C.C.S. testing procedure is exactly simulated. It is very important of course, to select proper values for each variable. This can be done correctly by deriving the p.d.f.'s of the variables from representative data. A simulation procedure as described above is called a "Monte-Carlo" method. This method is particularly suited for a digital computer because numerous repeated computations are involved.
Drawing values from a particular p.d.f. can be done by generating random numbers which follow the same distribution law as the variable in question. This method allows for correlation of any kind to be introduced between the variables.

The validity of a Monte-Carlo simulation procedure will be tested by applying it to the E.C.C.S. tests carried out on IPE 160 sections. The data-sheets of these tests allow the derivation of most p.d.f.'s involved. The computed buckling curve can be compared directly with the experimental buckling curve because the shape of the latter curve is determined completely by the test results obtained on the IPE 160 specimens. Application of the discussed method to other sections simply means modifying the p.d.f.'s of the variables so that they correspond to these sections.
No buckling tests have to be carried out, only simple measurements are necessary to determine the representative values of the imperfections and the mechanical properties. These measurements are less expensive, however.
The application of the Monte-Carlo simulation method to the E.C.C.S. buckling tests on IPE 160 specimens is discussed in chapters 4, 5 and 6.

3. NUMERICAL SOLUTIONS FOR THE BUCKLING LOAD OF A COLUMN WITH IMPERFECTIONS.

Most solutions for the buckling load of a column with imperfections are based on numerically solving the equation which describes the state where in each point of a column the moment M_{ex} is equal to the internal moment M_i (fig. 2)

$$P.y = -EI_x \frac{d^2y}{dx^2}$$

For a given value of P, the deflected shape of the column is assumed: y = f(x). The external moments are computed and are assumed to be equal to the internal moments. Next the shape of the column corresponding to these internal moments is determined. P is equal to the buckling load of the column if and only if the computed shape of the deflection curve is identical to the assumed one. This is generally not the case and therefore the computation of the deflected shape is repeated starting, however, with the shape obtained in the first computation. It has been shown by various authors that this procedure is rapidly converging and that a sufficiently accurate value of P will be obtained after only a few iteration steps. [3,4].

Next consider the column shown in (fig. 2). This column is identical to a column with hinged ends and twice its length. A load P is applied to this column with an eccentricity e_o; the column is assumed to have an initial curvature which is part of a sinewave, the amplitude is f_o.

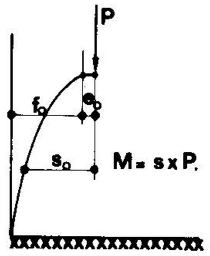


Fig. 2

As a first approximation the deflected shape of this column is also assumed to be a sinewave, the end-deflection of the column is equal to "a". The column is divided into a number of segments. The external bending moments are determined at the ends of each segment.The deflections of the column are computed numerically, by means of the reduced moment-area method and applying Simpson's rule.

For each segment the angle of rotation is computed; the deflection at the top of the column is equal to the sum of the products of the angles of rotation and the segment lengths. The computations are repeated until the computed shape is identical to the assumed shape.

In this iteration process the computed column shape of each previous step is used for the next step. The iteration is stopped if a certain degree of accuracy is obtained between two successive shapes. It is not yet necessary, however, that the computed end-deflection of the column is equal to the assumed end-deflection "a". There are two methods which can be used to bring those two deflections into agreement. In the first method, the value of P is kept constant; the length of the column, however, is varied until both deflections are equal. Next other values of "a" are adopted and for each "a" a corresponding column length (or slenderness-ratio λ) is computed. From these pairs of values (λ,a), the maximum column length is determined for which the given column will be in equilibrium under the load P. (fig. 3) Then the value of P is varied and the computations are repeated. To each value of P there corresponds a maximum column length l (or $\lambda_{max.}$).

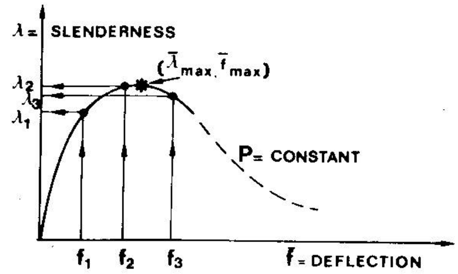


Fig. 3

In the second method the length of the column is kept constant, the value of P is varied until a value is found for which the assumed deflection is equal to the computed deflection. Next "a" is varied and other values of P are found. From the pairs of values (P,a) the collapse load of a column of given length is determined (fig. 4).

The first method has been used by Beer and Schulz for their computations [4]. They were interested in determining complete buckling curves for each combination of variables. The maximum length of a column, for any given value of P, is less interesting for the Monte-Carlo simulation procedure because a column is never tested by increasing the column length during the test until failure

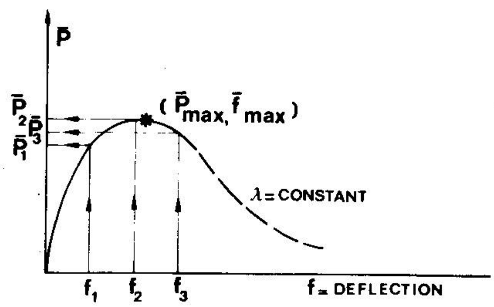


Fig. 4

occurs. Therefore the second method has been applied. For each column of given length and given imperfections, the critical load P_{CR} is computed.
The computation of the deflected shape of a column is rather complicated because the bending stiffness "EI_x" of the column is not a constant but appears to be a function of the bending moment M and the load P. The column will yield over part of the cross-section, if P is large or if the deflections are large. The bending stiffness "EI_x" will be reduced, due to this yielding. Residual stresses present in the column cause premature yielding. The value of the yield stress and the dimensions of the section will also affect the relations between M,P and EI. The bending moment is not constant over the length of the column, and consequently the bending stiffness EI varies over the column length. The relations between M,P and EI can be determined for each particular section if the stress-strain diagram, the distribution of σ_y over the cross-section and the residal-stress distribution are known. For a constant value of P, an increasing part of the cross-section is assumed to yield, the corresponding stress and strain distributions allow the values of the bending moment M and the curvature to be computed. For an IPE 160 section these relations are shown in (fig.5). The dimensions of this section are nominal, the stress-strain diagram is assumed to be bi-linear and σ_y = 24.0 kgf/mm^2. The residual stress is assumed to be parabolically distributed in the flanges and constant in the web; the maximum compressive residual-stress is equal to 0.3 σ_y. On the vertical axis of figure 5 the ratio $\bar{B}$ between the actual bending stiffness and the elastic bending stiffness is plotted; on the horizontal axis the ratio $\bar{M}$ between the actual bending moment and the plastic bending-moment is plotted. These curves provide the information necessary for the computation of the buckling loads.

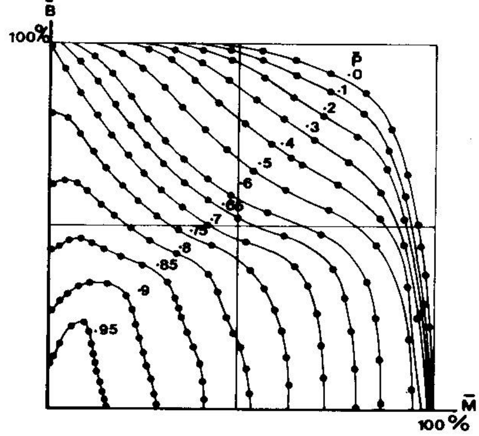


Fig. 5

From the remarks above it can be observed that the column parameters can be divided into two groups. The yield stress, residual stress and the dimensions affect the shape of the $\bar{M}$ - P - $\bar{B}$ relations while the eccentricity and the initial curvature affect the deflected shape through the external bending moment.
All the column computations which will be discussed in a later chapter, have been carried out under the following assumptions: the stress-strain diagram is bi-linear; the yield stress is constant over the cross-section; the residual stress distribution is parabolic in the flanges and constant in the web, the distribution is symmetric; the initial curvature is half a sinewave and the eccentricity is constant over the length of the column. Only weak-axis buckling is considerd. It should be mentioned that the computations involved in the Monte-Carlo simulation procedure are rather tedious because for each column a new set of $\bar{M}$ - P - $\bar{B}$ relations must be determined.
The accuracy of the computer programme is checked by comparing the output with results obtained by Beer and Schulz on a similar column. This comparison is shown in the table, below. The column is HEA 200; the initial curvature f_o = 1/1000, the maximum compressive residual stress is 0 or 0.5 σ_y, the column dimensions are nominal. The slenderness-ratio and the critical stress are given as dimensionless parameters $\bar{\lambda} = \lambda / \pi\sqrt{\frac{E}{\sigma_y}}$ and $\bar{\sigma} = \frac{\sigma_{CR}}{\sigma_y}$

$\bar{\lambda}$	α	$\bar{\sigma}$	
		This program	Beer and Schulz 1)
0.594	0	0.87	0,89
	0.5	0.77	0.78
0.810	0	0.78	0.79
	0.5	0.64	0.65
1.025	0	0.65	0.65
	0.5	0.53	0.53
1.132	0	0.58	0.59
	0.5	0.47	0.47
1.400	0	0.42	0.43
	0.5	0.36	0.35
1.725	0	0.30	0.29
	0.5	0.26	0.25

1) These values are obtained from [4] p. 40, fig. 5 and [12] p. 115, fig. 5.6.

4. COLUMN DATA.

A considerable number of the E.C.C.S. buckling tests has been carried out on IPE sections. These sections are responsible for the shape of the experimental buckling curve as derived by the E.C.C.S.. It is for this reason that these sections are chosen for the Monte-Carlo simulation procedure.
The testing procedure, established by committee 8.1* of the E.C.C.S., demanded that the following measurement be carried out on each test specimen

1. The dimensions of the specimen at 0 - 1/4 1 - 1/2 1 - 3/4 1-1
2. The initial curvature at 0 - 1/4 1 - 1/2 1 - 3/4 1-1
3. Weighting of the specimen

The mechanical properties of each bar from which specimens were cut had to be determined

4. Tensile tests
5. Stub-column test

These data had to be recorded on a standard data sheet.
In the next paragraphs the relations between the column variables and the measurements are discussed.

4.1 Eccentricity.

The dimensions of the sections are used to compute the eccentricity which is introduced because the testing procedure requires that the load must be applied at the center of the web of the specimen. The center of the web, however, does not necessarily coincide with the center of gravity of the whole section.
The center of the web lies a distance $(c + \frac{1}{2} a)$ from the right. The center of gravity of the flange lies $\frac{1}{2}$ b from the right. The difference between the two distances is equal to: $(c + \frac{1}{2} a) - \frac{1}{2} b$.
The center of gravity of the complete section is determined for the nominal area.

The eccentricity of the web is computed from the following relation

$$e_o = \frac{2A_F}{A_n} \left[c + \tfrac{1}{2} a - \tfrac{1}{2} b \right]$$

where A_F = area of a flange

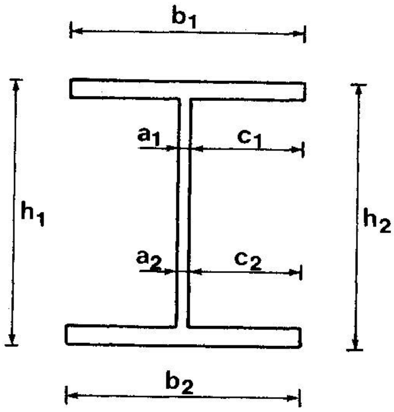
A_n = nominal area

* Committee 8.1 on "Buckling tests".

If both flanges are considered separately e_o is equal to

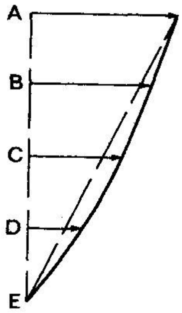
$$e_o = \frac{A_{F1} + A_{F2}}{2A_n} \left[(c_1 + \tfrac{1}{2} a_1 - \tfrac{1}{2} b_1) + + (c_2 + \tfrac{1}{2} a_2 - \tfrac{1}{2} b_2) \right]$$

The mean values of the dimensions, determined over the length of the column, are introduced into this formula.



4.2 Initial curvature.

The initial out-of straightness has been measured at five points along the length of a specimen. A digital computer is used to find the best fit of a sinewave through the points A, B, C, D and E. The amplitude f_o of the sinewave is considered as the parameter of the initial curvature. The mean value of f_o for both flanges is determined.



4.3 Area.

The weight G of a specimen is used to compute the real area of the section. The specific weight of steel is assumed to be

$$\rho = 7.85 \times 10^{-6} \text{ kgf/mm}^3$$

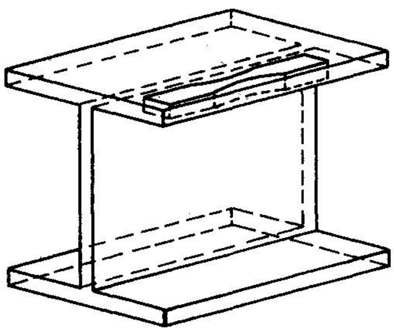
The area is equal to

$$A = \frac{G.}{\rho . l}$$

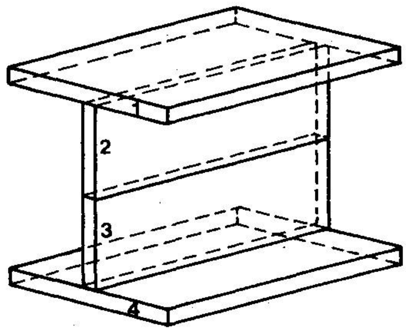
l = length
ρ = specific weight
G = weight of the specimen
A = area

4.4 Tensile tests.

Tensile tests were executed on specimens taken from the flanges, according to Euronorm 2 - 57. The yield stress obtained from these tests is denoted σ_{ye}.
Additional tensile tests were carried out on strips taken from the flanges and the web. This yield stress is denoted σ_{ys}. The figures below show how the specimens are taken from the bar.



EURONORM



STRIPS

4.5 Stub column test.

Stub column tests were carried out on specimens with slenderness-ratios $\lambda = 12$, 15 and 20. The specimens were taken from the same length of bar from which specimens were cut for the buckling tests. The yield stress obtained from these tests is called σ_y.

The individual column data are not reproduced in this paper because they are too numerous. In the next chapter histograms of these data are given, however. The data have been reduced according to the relations given in the previous paragraphs
The IPE 160 sections studied in this investigation are coded 17, 18, 19 20, 21 and 22 in Table A-1, page 30 of ref. [1]. The eccentricity and initial curvature parameters are obtained from 150 columns; the yield stresses and areas are obtained from 189 columns.

5. PROBABILITY DENSITY FUNCTIONS OF THE COLUMN VARIABLES;

The experimental data described in chapter 4 have been used to derive histograms and cumulative histograms. Cumulative distribution functions are fitted to the cumulative histograms. Throughout this chapter, the Kolmogorov-Smirnov test of significance is applied to find the best fit [7], except for the initial curvature. The Kolmogorov-Smirnov test concentrates on the deviations between the hypothesized cumulative distribution function F(x) (C.D.F.) and the observed cumulative histogram $F^*_i(x_i)$ (C.H.).
$F^*_i(x_i) = \frac{i}{n}$ where x_i is the i th largest observed value in a random sample of size n.
The following statistic is considered

$$D = \max_{i=1}^{n} \left[F^*(x_i) - F(x) \right]$$

D is, according to this formula, the largest of the absolute values of the differences between the hypothesized C.D.F. and the observed C.H., evaluated at the observed values in the sample. Critical values of D can be given at various levels of significance which will result in either accepting or rejecting the hypothesized C.D.F. Let α be the level of significance, then for large n, the critical statistic is equal to

$\alpha = 0.10$ $\bar{D} = 1.22 / \sqrt{n}$
$\alpha = 0.05$ $\bar{D} = 1.36 / \sqrt{n}$
$\alpha = 0.01$ $\bar{D} = 1.63 / \sqrt{n}$

5.1 Eccentricity.

The histogram of e_o is shown in (fig. 6). The eccentricity varies between 0 and 2.0 mm. The shape of the histogram suggests an asymmetrical p.d.f. Three C.D.F.'s are hypothesized

a Gaussian C.D.F.
a Log-normal C.D.F.
a Gamma C.D.F.

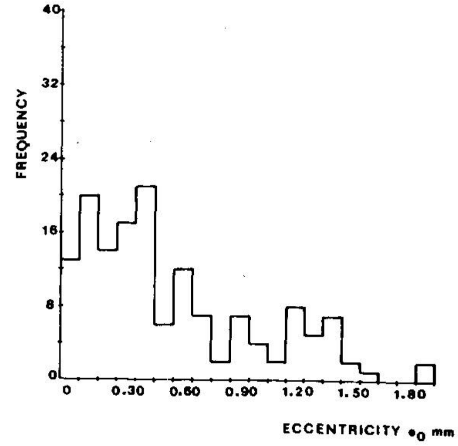


Fig. 6

In fig. 7 the observed C.H. is shown together with the hypothesized C.D.F.'s The maximum values of D which can be derived from this figure are

Gaussian C.D.F. $D = \max_{n=i}^{n} \left[F^{*}(x_i) - F(x) \right] = 0.566 - 0.420 = 0.146$

Log-normal C.D.F. D = " $= 0.915 - 0.830 = 0.085$

Gamma C.D.F. " $= 0.900 - 0.835 = 0.065$

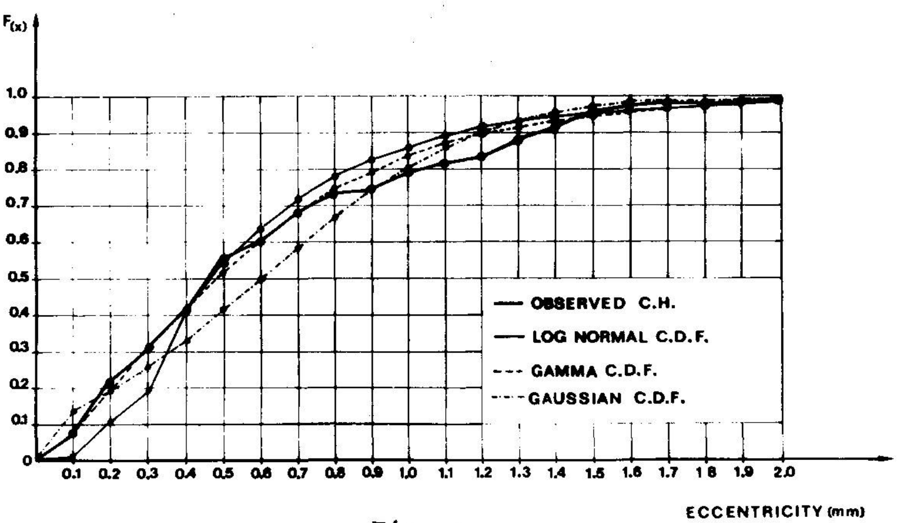


Fig. 7

The critical values of $\bar{D}$ are

$\alpha = 0.10$ $\bar{D} = 1.22 / \sqrt{150} = 0.100$

$\alpha = 0.05$ $\bar{D} = 1.36 / \sqrt{150} = 0.111$

$\alpha = 0.01$ $\bar{D} = 1.63 / \sqrt{150} = 0.133$

The log-normal and the Gamma C.D.F. cannot be rejected at the 10% level of significance. The Gamma-model is chosen for the eccentricity. The parameters of this model are

$$\left.\begin{array}{l} m = 0.5949 \text{ mm} \\ s = 0.4609 \text{ mm} \end{array}\right) \rightarrow \begin{array}{l} \lambda = 2.798 \\ k = 1.663 \end{array}$$

5.2 Initial curvature.

The initial curvature parameter f_o has been determined for each column length l involved in the simulation. It is assumed that f_o follows a Gaussian distribution function. In this case the Kolmogorov-Smirnov test is not used to check the validity of this assumption but the more refined method of "the moments" is used instead. This method is described in some detail in chapter 7. The following values are obtained for the critical parameters of this test.

	l = 1012	l = 1380	l = 1748	l = 1932	l = 2392	l = 2944
m	0.68	1.13	1.47	1.65	1.95	2.78
s	0.29	0.30	0.50	0.25	0.35	0.49
v_1	-1.40	-2.16	-4.91	-1.18	-0.60	1.84
v_2	-0.98	0.65	4.81	0.38	-0.92	1.09

The hytophesized Gaussian distribution function should be rejected if $v_1 > 3$ and $v_2 > 3$. This is only the case for l = 1748 mm ($\lambda = 95$). The hypothesized p.d.f. is accepted therefore for initial curvature.
Fig. 8 shows the computed values of m. Also plotted are the values m + 2s. It can be seen in this figure that the relations (l,m) and (l,m + 2s) can

be approximated by straight lines. This indicates that the initial curvature parameter can be described independent of the column length through the value f_o/l. This parameter is consider d in this paper; (fig. 9) shows the his togram of f_o/l.

From (fig. 8) the following values are determined for the Gaussian model

$m = 0.00085\ l$ (mm)
$s = 0.00020\ l$ (mm)

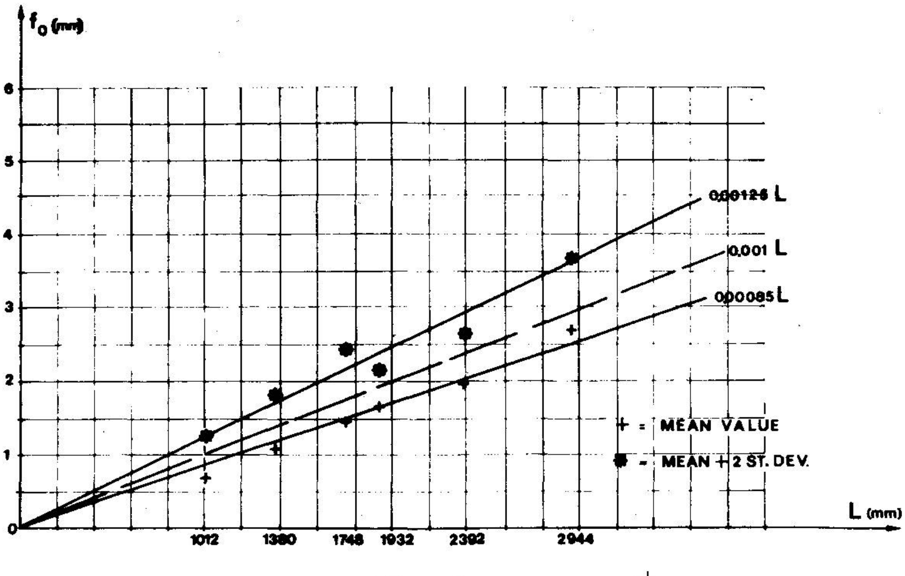


Fig. 8

5.3 Area.

The histogram of the area is given in (fig. 10). The observed C.H. and the hypothesized Gaussian C.D.F. are shown in (fig. 11). Preliminary computations indicate that hypothesizing an asymmetrical C.D.F. is not justified.
The mean area is equal to $m = 2047.33\ mm^2$
The standard deviation is equal to
$s = 81.15\ mm^2$

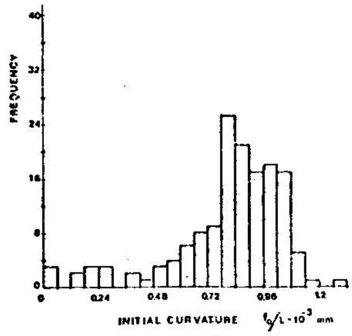


Fig. 9

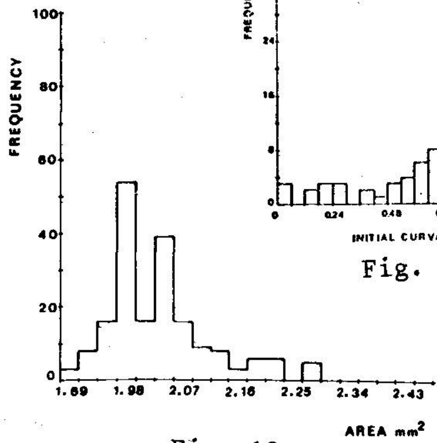


Fig. 10

The parameters k and λ of a Gamma C.D.F. are a function of m and s.

$$\frac{k.}{\lambda} = m. \; ; \; \frac{\sqrt{k}}{\lambda} = s$$

Substitution of the measured values of m and s into these formula gives

$k = 636.51$
$\lambda = 0.3109$

For large k-values the Gamma C.D.F. approaches a Gaussian C.D.F.

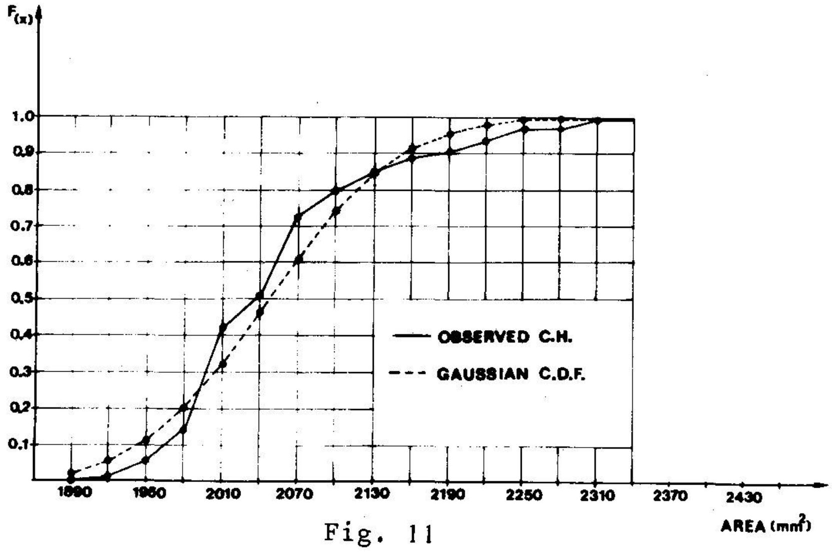


Fig. 11

The latter is the only function, therefore, which has been investigated.
The Kolmogorov-Smirnov test gives the following results.

$$D = \max_{i=1}^{n} \left[F^{*}(x_1) - F(x) \right] = 0.730 - 0.610 = 0.120$$

The critical values of $\bar{D}$ are

$$\alpha = 0.10 \qquad \bar{D} = 1.22 \,/\, \sqrt{189} = 0.089$$
$$\alpha = 0.05 \qquad \bar{D} = 1.22 \,/\, \sqrt{189} = 0.099$$
$$\alpha = 0.01 \qquad \bar{D} = 1.22 \,/\, \sqrt{189} = 0.118$$

The Gaussian model cannot be rejected at the 1 % level of significance, which is a rather questionable result. The Gaussian model is accepted, however, for reasons of convenience.The parameters of this model are

$$m = 2047.33 \text{ mm}^2$$
$$s = 81.15 \text{ mm}^2$$

For the simulation procedure, the variation in the area is assumed to be a result of the variation in the flange thickness alone. The height, width and web thickness are assumed to be equal to the nominal values. The mean value and the standard deviation of the flange thickness is obtained from the following formulae

$$A = (h - 2e)\, a + 2be = (160 - 2e)\, 5 + 2 \times 8.2\, e = 800 + 154\, e$$

mean value

$$m_e = \frac{\bar{A} - 800}{154} = 8.1 \text{ mm}$$

standard deviation

$$s_e = \frac{s_A}{154} = 0.527 \text{ mm.}$$

The parameters of the Gaussian model for the flange thickness, are

$$m = 8.1 \text{ mm}$$
$$s = 0.527 \text{ mm}$$

5.4 Yield stress.

The yield stress has been determined from three different tests.

Euronorm	$m=29.12$ kgf/mm^2	$s=2.04$ kgf/mm^2
Strips	$m=27.85$ kgf/mm^2	$s=3.17$ kgf/mm^2
Stub-column	$m=31.48$ kgf/mm^2	$s=2.65$ kgf/mm^2

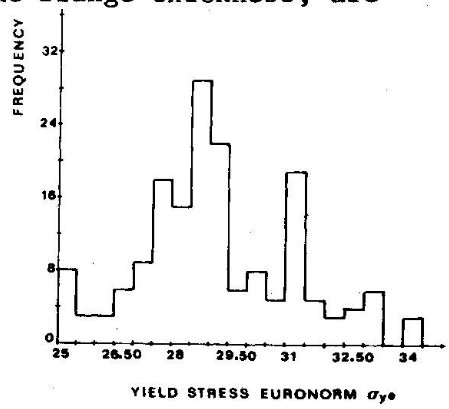


Fig. 12

The values obtained from the stub-column tests have been used in the simulation procedure because these values are the best measure for the yield stress in compression. This yield stress also determines the buckling load of a column.
The histograms of the three yield stresses are shown in (fig. 12,13 and 14). The shape of the histograms suggests a symmetrical p.d.f. Fig. 15 shows the observed CH. of the stub-column yield stress together with the hypothesized Gaussian C.D.F.
The Kolmogorov-Smirnov value D is equal to

$$D = \max_{i=1}^{n} \left[F^{*}(x_i) - F(x) \right] = 0.840 - 0.750 = 0.090$$

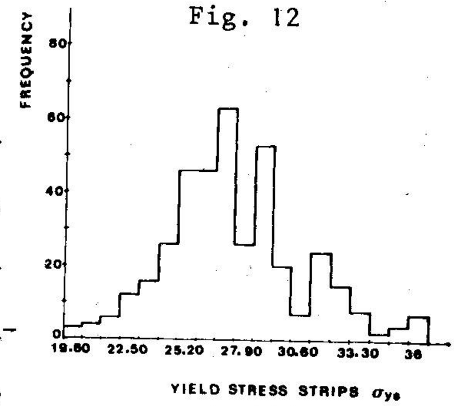


Fig. 13

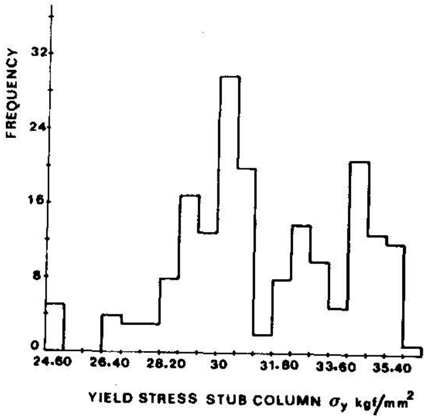


Fig. 14

The critical value $\bar{D}$ is

$\alpha = 0.10 \quad \bar{D} = 1.22 / \sqrt{189} = 0.089$
$\alpha = 0.05 \quad \bar{D} = 1.36 / \sqrt{189} = 0.099$
$\alpha = 0.01 \quad \bar{D} = 1.63 / \sqrt{189} = 0.118$

The Gaussian model for the yield stress cannot be rejected at the 10 % level of significance.
The parameters of this model are

$m = 31.48$ kgf/mm^2
$s = 2.65$ kgf/mm^2

5.5 Residual stress.

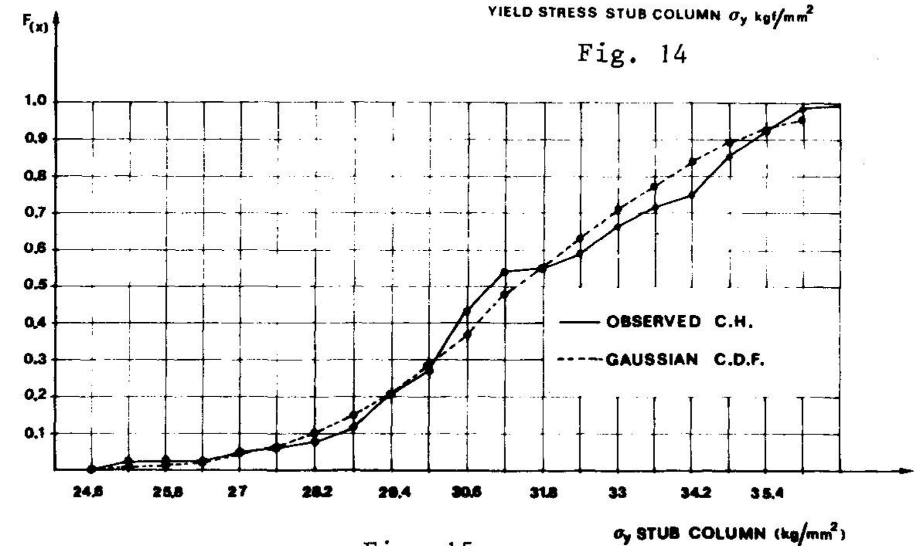


Fig. 15

The residual stresses provided most difficulties because no extensive residual stress-measurements have been done on IPE 160 sections. The distribution of the residual stresses is assumed to be parabolic in the flanges and constant in the web. As the parameter of this type of distribution the maximum compressive σ_R at the tip of the flange is chosen. Some stub-column tests were carried out in Belgium for which load-deformation diagrams were recorded [8]. From these diagrams the maximum residual stress can be estimated. Ten such diagrams are given. The maximum compressive residual stress is determined as a fraction of the yield stress.

$$\sigma_r = \alpha\, \sigma_y \rightarrow \alpha = \frac{\sigma_r}{\sigma_y}$$

A mean value $\alpha = 0.204$ and a standard deviation $s = 0.07$ are computed from the Belgian tests.
A value of $\alpha = 0.61$ is derived by Rokach. He performed a correlation analysis on the IPE 160 test results, [9]. This value of α, however, must also account for the effect of the initial curvature. For the same sections Lenz arrives at a value of $\alpha = 0.06$ [10].
Young suggests a general formula for the maximum compressive residual stress in I sections [11].

$$\sigma_R = 16.5 \left[1 - \frac{A_w}{1.2 A_F} \right] \qquad \begin{array}{l} A_w = \text{web area} \\ A_F = \text{flange area} \end{array}$$

For an IPE 160 a value of $\alpha = 0.238$ is computed. Schulz proposes a value $\alpha = 0.2$ for this type of section. [12].
The residual stress parameter α is assumed to be Gaussian distributed [13]. The validity of this assumption cannot be tested due to lack of information. For the simulation procedure, a mean value $m = 0.20$ and a standard deviation $s = 0.05$ are adopted.

5.6 Slenderness-ratio.

No variation is assumed in the slenderness-ratio λ. The length of each column has been determined with sufficient accuracy and no variation is assumed in the width of the column flanges.
For weak-axis bending, therefore, the radius of gyration is constant. The slenderness ratio λ can thus not be treated as a random variable.

5.7 Summary of the model parameters.

Random variable	Gamma C.D.F.		Gaussian C.D.F.	
	λ	k	m	s
Eccentricity (mm)	2.798	1.663	-	-
Initial curvature	-	-	0.00085 1	0.00020 1
Area (mm^2)	-	-	2047.33	81.15
Flange thickness	-	-	8.1	0.527
Yield stress kgf/mm^2	-	-	31.48	2.65
Residual stress kgf/mm^2	-	-	$0.20\ \sigma_y$	$0.05\ \sigma_y$

6. GENERATING RANDOM NUMBERS.

Random numbers with a Gaussian or uniform probability density function can be generated directly on a digital computer. Standard procedures are generally available. Values of the variables for which a Gaussian model is assumed, have been obtained on a I.B.M. 1130 computer using the procedures RANDU AND GAUSS. Generating random numbers with a Gamma p.d.f. proved more difficult. No standard procedure is available for the inversion of the incomplete Gamma function; therefore, a graphical method is used. First the Gamma C.D.F. is computed and intervals of equal probability (2.5 %) are determined.
Next random numbers with a uniform p.d.f. are generated and they are assigned to these intervals. In this particular case, the random numbers lie between 0 and 10^5; they are assigned to each interval according to the following scheme

0	-	2.500	interval 1	representative value x_1
2501	-	5.000	interval 2	representative value x_2
5001	-	7.500	interval 3	representative value x_3
	-			
97501	-	100.000	interval40	representative value x_{40}

Each interval i is represented by a single value x_i; x_i is defined as the mean value of the two boundary values of interval i. This is not correct. Theoretically x_i should be defined as the center of gravity of the area under the C.D.F. between the two boundary values. The relatively large number of intervals, however , assure that the error will be very small if the mean is considered instead of the center of gravity. The last interval must be treated with special care, because $x \to \infty$. The largest observed value of the eccentricity is chosen as the representative value of this interval. As an example of the above-mentioned procedure., let a random number 11533 be generated. This value corresponds to interval 5 and therefore to x_5. This value of x is assigned to the eccentricity. It is obvious that a Gamma p.d.f. can be approximated with increasing accuracy by raising the number of intervals.

For each variable considered in the column simulation, a series of 1000 random numbers has been generated. There is no need for a sophisticated

procedure to combine the variables because the variables are assumed to be uncorrelated. One must beware, however, of sequential effects in the random numbers. A digital computer generates random numbers according to a numerical procedure, very often the Fibonacci-method is used. Consequently, each time the random number generator is started, the same sequence of number appears. If the variables are combined according to their rank-number, they will be strongly correlated; a large value of the yield stress will be combined with a large value of the initial curvature, eccentricity etc. For this reason more than the required random numbers have been generated and each column variable has been selected at random from these numbers.
The combinations of variables obtained in this way are used as input for the computer programme described briefly in chapter 3.

7. RESULTS.

Columns of various lengths have been examined. The corresponding slenderness-ratios are λ = 55, 75, 95, 105, 130 and 160. At each slenderness-ratio experimental results are available which can be compared with the simulated buckling stresses. Each group of experimental buckling stresses had a significant influence on the shape and position of the experimental buckling curve.
A total number of 120 columns has been simulated on an I.B.M. 360/65 digital computer; 20 columns at each slenderness-ratio.
The results of the computations are given in tables I through VI.
The combinations of variables which are assigned to each column are also given in these tables. Buckling stresses are computed for the nominal area as well as for the real area. For each section the real area is determined from the value of the flange thickness e. These buckling stresses are also given in tables I through VI. Columns with a yield stress less than the guaranteed value of 24 kg/mm^2, have not been included in the computations.
The probability density-function of the buckling stress is estimated at each slenderness-ratio λ. Jaquet has shown that the experimental buckling stresses are Gaussian distributed [14]. He arrived at this conclusion by applying the method of the central-moments to the test results. This method has been described in detail by Fisher [15] .
The same method is applied to check whether the simulated buckling stresses are Gaussian distributed. A brief discussion of this method is given below. Consider a variate x and a random sample of size n, drawn from the population of x. The sums of powers of deviations from the mean are computed.

$$m = \frac{\Sigma x}{n}$$

$$s_2 = \Sigma(x-m)^2 \rightarrow k_2 = s_2/(n-1)$$

$$s_3 = \Sigma(x-m)^3 \rightarrow k_3 = n\, s_3/(n-1)(n-2)$$

$$s_4 = \Sigma(x-m)^4 \rightarrow k_4 = n\left[(n+1)s_4-3(n-1)s_2{}^2/n\right]/\left[(n-1).(n-2)(n-3)\right].$$

The two simplest measures of departure from normality are those dependent from the statistics of the 3rd and 4th degree, defined as

$$g_1 = k_3/k_2{}^{3/2} \qquad g_2 = k_4/k_2{}^2$$

If the variate x is Gaussian distributed then g_1 and g_2 are also Gaussian distributed. The sampling variances of g_1 and g_2 are

$$\hat{s}_1^{\,2} = 6n(n-1)/(n-2)(n+1)(n+3)$$

$$\hat{s}_2^{\,2} = 24\, n(n-1)^2/(n-3)(n-2)(n+3)(n+5)$$

Finally $V_1 = \frac{g_1}{\hat{s}_1}$ and $V_2 = \frac{g_2}{\hat{s}_2}$ are computed. For a perfectly Gaussian distributed variate x, the values of V_1 and V_2 are equal to zero. For each symmetrical p.d.f. $V_1 = 0$. A positive value of V_1 indicates a positive skewness whereas a negative value of V_1 indicates a negative skewness. V_2 is a coefficient of kurtosis (flatness).
A positive value of V_2 means that the p.d.f. is more filled out than a Gaussian p.d.f. whereas a negative value of V_2 means that the p.d.f. is more pointed that a Gaussian p.d.f.
The observed values of V_1 and V_2 determine whether the hypothesized Gaussian p.d.f. is to be rejected. Jaquet suggests to reject the hypothesis if V_1 and V_2 are greater than 3. For values greater than 2, the hypothesis should be reconsidered carefully.

The computed value of V_1 and V_2 are given in the tables below. The values have been determined for the nominal area as well as for the real area.

NOMINAL AREA

	55	75	95	105	130	160
m	26.60	22.09	16.58	14.74	11.22	7.73
s	2.60	1.96	1.71	1.65	0.92	0.62
V_1	- 1.44	- 0.91	- 0.22	0.88	0.08	- 0.19
V_2	0.40	0.12	- 0.56	- 0.11	- 0.14	- 1.13

REAL AREA

	55	75	95	105	130	160
m	26.04	21.39	16.60	14.59	10.86	7.63
s	1.92	1.37	1.40	1.19	0.59	0.33
V_1	- 1.80	0.43	- 0.05	0.29	- 0.14	-0.23
V_2	0.45	1.07	0.14	0.05	1.77	-0.50

All values are shown to be less than 1.8, most of them being less than 1.0 There is no reason to reject the hypothesis that the buckling stresses are Gaussian distributed. Consequently the charactaristic buckling stress σ^{*}_{CR} can be computed as

$$\sigma^{*}_{CR} = m - 2\,s$$

The values of σ^{*}_{CR} at each slenderness-ratio are given in the next tables. The simulated values and the corresponding experimental values of σ^{*}_{CR} are given.

NOMINAL AREA

		55	75	95	105	130	160
SIMULATION	m	26.60	22.09	16.58	14.74	11.22	7.73
	s	2.60	1.96	1.71	1.65	0.92	0.62
	m-2s	21.40	18.17	13.16	11.44	9.38	6.59
EXPERIMENT	m	27.90	23.15	18.70	15.27	11.35	7.44
	s	2.73	2.45	1.46	1.23	1.00	0.56
	m-2s	22.40	18.29	15.78	12.81	9.35	6.32

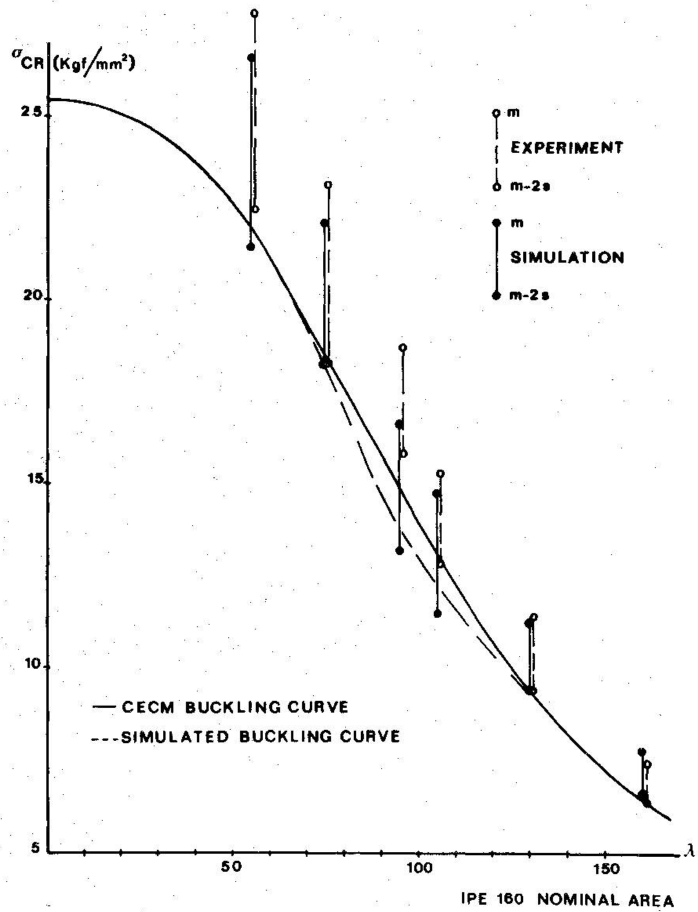


Fig. 16

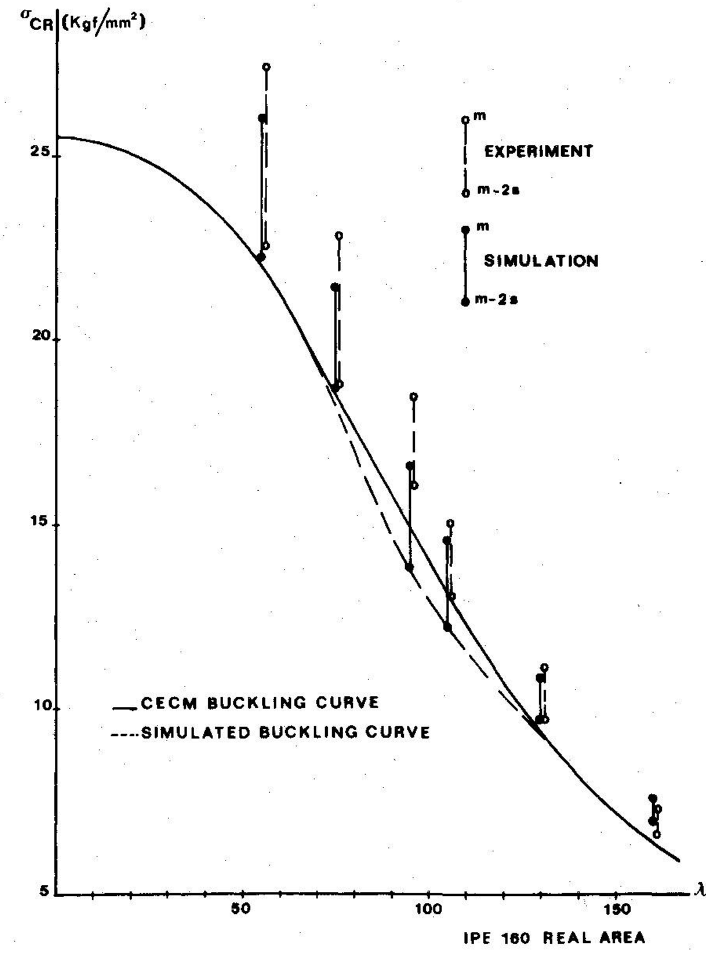


Fig. 17

REAL AREA

		55	75	95	105	130	160
	m	26.04	21.39	16.60	14.59	10.86	7.66
SIMULATION	s	1.92	1.37	1.40	1.19	0.59	0.33
	m-2s	22.20	18.65	13.80	12.21	9.68	7.00
	m	27.48	22.81	18.45	15.06	11.14	7.34
EXPERIMENT	s	2.48	2.05	1.21	1.00	0.73	0.36
	m-2s	22.52	18.71	16.03	13.06	9.68	6.62

These results are also shown graphically in (figs. 16 and 17.) A good agreement is found between the simulated buckling stresses and the experimental buckling stresses at slenderness-ratios λ = 55, 75, 130 and 160. At slenderness ratios λ = 95 and 105 the simulated buckling stresses deviate significantly from the experimental buckling stresses. The maximum deviation is 17 % (λ = 95, nom. area).

The dotted lines, in (figs. 16 and 17), correspond to a buckling curve fitted to the simulated buckling stresses (real area).
The discrepancies between both curves at λ = 95 and λ = 105 cannot be traced to exceptionally large imperfections or unfavourable mechanical properties. Confidence intervals have been determined for the means and the standard deviations. These intervals are important because the means and the standard deviations are computed from samples of limited size.

Let m and s be the sample estimates, based on a random sample of size n. the confidence interval of the mean is

$$m'' = m - t \cdot \frac{s}{\sqrt{n}} < \bar{m} < m' = m + t \cdot \frac{s}{\sqrt{n}}$$

where $\bar{m}$ is the population mean and t possesses a Student's t distribution with n-1 degrees of freedom.
The value of t **is** chosen to correspond to a 98 % confidence interval. The bounds of this interval are given in the table below.

CONFIDENCE LIMITS OF THE MEAN 98 %

		55	75	9	105	130	160
	m	26.60	22.09	16.58	14.74	11.22	7.73
NOMINAL AREA	m'	28.12	23.20	17.55	15.68	11.74	8.08
	m"	25.08	20.98	15.61	13.80	10.70	7.38
	m	26.04	21.39	16.60	14.59	10.86	7.66
REAL AREA	m'	27.16	22.17	17.39	15.26	11.19	7.84
	m"	24.92	20.61	15.81	13.92	10.53	7.48

The confidence intervals of the simulated mean stress and the experimental mean stress are shown in (figs. 18 and 19).
It can be seen that the experimental mean stresses are almost systematically greater than the simulated stresses, except at λ = 160. The confidence intervals, however, overlap slightly. The confidence interval of the mean stress obtained from the nominal area is somewhat wider than the confidence interval which corresponds to the real area. The reason is that dividing the buckling loads by the real area eliminates to some extend the influence of the flange thickness. A small flange thickness corresponds to a smaller buckling load but also to a smaller area, and vice-versa. Consequently, the scatter, in the buckling stresses will be reduced.

The confidence interval of the standard deviation s has been computed by observing that the quantity $\Sigma(x_i - m)^2/s^2$ possesses a Ψ^2 distribution with n-1 degrees of freedom.
The confidence interval is given by

$$s'' = \sqrt{\frac{\Sigma(x_i-m)^2}{\Psi_2^2}} < s < s' = \sqrt{\frac{\Sigma(x_i-m)^2}{\Psi_1^2}}$$

Ψ_1 and Ψ_2 are chosen such that they correspond to 5% and 95% confidence limits. The computed values are given in the table below.

CONFIDENCE LIMITS OF THE STANDARD DEVIATION 90 %.

		55	75	95	105	130	160
	s	2.60	1.96	1.71	1.65	0.92	0.62
NOMINAL AREA	s'	3.60	2.69	2.34	2.26	1.26	0.85
	s"	2.05	1.56	1.36	1.31	0.73	0.49
	s	1.92	1.37	1.40	1.19	0.59	0.33
REAL AREA	s'	2.66	1.88	1.92	1.63	0.81	0.45
	s"	1.52	1.09	1.11	0.95	0.47	0.26

8. CONCLUSIONS.

It has been demonstrated in this paper, that the distribution function of buckling stresses can be derived theoretically. A buckling curve which corresponds to a constant probability of failure can be determined from the distribution functions at the various slenderness ratios. The computed buckling curve is in reasonable agreement with the experimental buckling curve. Deviations between the two curves are observed at

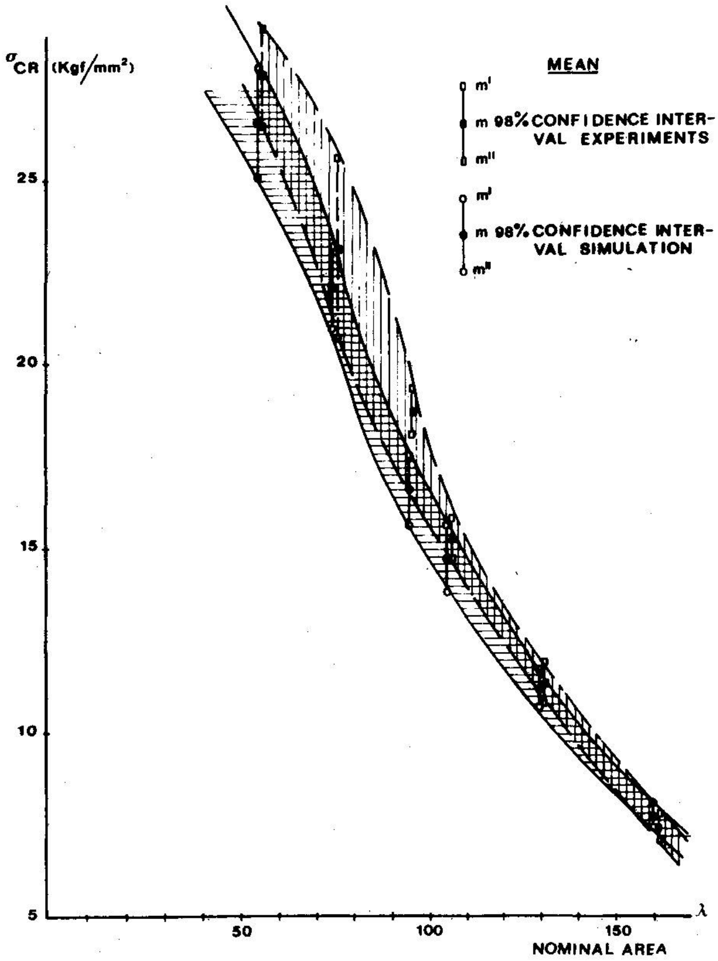


Fig. 18

σCR (Kgf/mm²)

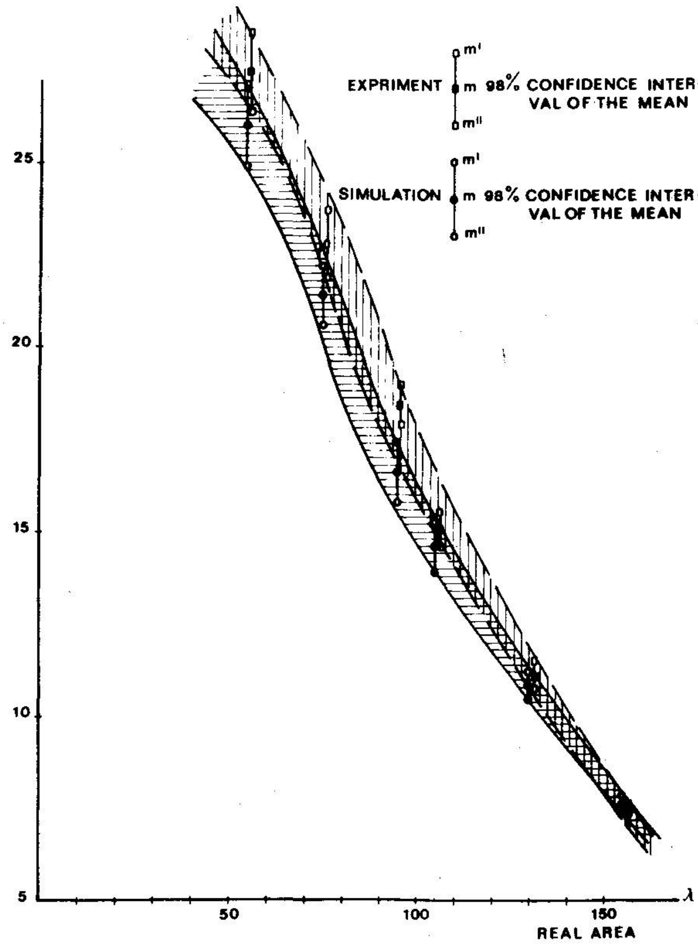


Fig. 19

slenderness-ratios $\lambda = 95$ and $\lambda = 105$. It has been pointed out by other investigations that the effect of imperfections and/or mechanical properties is most pronounced at slenderness-ratios $\lambda = 90 - 100$ [2]. One of the assumptions in the discussed simulation procedure is that all variables are uncorrelated. There is no reason to reject this assumption except for the initial curvature and the residual stresses. It is believed that some correlation exists between those two variables; consequently, the buckling stresses may be affected unfavourably.

Application of the described procedure to sections other than the IPE 160 is a rather simple matter. The distribution functions of the variables are not expected to change in character; the parameters of these functions will vary. These values can be determined by relatively simple and inexpensive measurements. Once buckling curves have been obtained for various sections, the usefullness of multiple column-curves can be decided upon. Adoption of multiple column-curves can only be justified if significant differences are shown to exist between probabilistic column curves. The buckling curves which are derived by means of the discussed procedure, are in the right format to be used as a "strength function" in load factor design. This is generally not true for most theoretically derived buckling curves.

ACKNOWLEDGEMENT.

The work described in this paper was carried out at the STEVIN LABORATORY of the DELFT UNIVERSITY OF TECHNOLOGY. It has been sponsored partly by the STICHTING CENTRUM BOUWEN IN STAAL.

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NR.	L (mm)	σ_y kgf/mm^2	e_o (mm)	f_o (mm)	a (mm)	b (mm)	e (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CRR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
1	506	34.16	0.83	1.04	5	82	8.42	160	0.1260	56586	26.99	28.15	1
2	506	23.36	0.55	0.88	5	82	8.39	160	0.2233	41971	---	---	2
3	506	27.85	1.28	1.07	5	82	7.18	160	0.2581	40659	21.34	20.23	3
4	506	31.67	0.44	0.81	5	82	8.74	160	0.1303	57751	26.91	28.73	4
5	506	34.85	0.50	0.90	5	82	8.04	160	0.2401	55752	27.35	27.74	5
6	506	34.70	1.28	0.73	5	82	8.02	160	0.2374	52886	25.99	26.31	6
7	506	31.24	0.23	0.72	5	82	7.85	160	0.1221	54492	27.13	27.11	7
8	506	32.24	0.52	0.86	5	82	8.09	160	0.0944	55397	27.08	27.56	8
9	506	30.29	0.64	0.95	5	82	7.67	160	0.1541	48787	24.63	24.27	9
10	506	30.15	1.42	0.79	5	82	8.16	160	0.2014	47209	22.95	23.49	10
11	506	29.34	0.42	1.19	5	82	8.00	160	0.1862	50294	24.75	25.02	11
12	506	30.52	0.83	0.92	5	82	8.24	160	0.1584	51741	25.00	25.74	12
13	506	31.04	0.21	0.81	5	82	7.81	160	0.2682	57418	28.67	28.57	13
14	506	36.19	0.61	0.65	5	82	8.31	160	0.2155	57917	27.85	28.81	14
15	506	35.39	0.52	0.44	5	82	7.10	160	0.2002	53026	28.00	26.38	15
16	506	33.92	0.16	0.97	5	82	8.91	160	0.2225	57686	26.56	28.70	16
17	506	31.78	0.16	1.06	5	82	9.01	160	0.2323	60876	27.83	30.29	17
18	506	34.29	1.42	0.70	5	82	8.46	160	0.2295	50175	23.86	24.96	18
19	506	30.83	0.58	0.69	5	82	9.35	160	0.1643	60453	26.99	30.07	19
20	506	32.01	0.69	0.63	5	82	6.95	160	0.1366	46680	24.96	23.22	20

$\lambda = 55$

TABLE I

NR.	L (mm)	σ_y kgf/mm^2	e_o (mm)	f_o (mm)	a (mm)	b (mm)	c (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CRR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
21	690	31.54	0.71	0.81	5	82	7.76	160	0.2020	42930	21.52	21.36	21
22	690	31.31	0.79	1.41	5	82	8.06	160	0.1430	42377	20.76	21.83	22
23	690	30.50	0.25	1.39	5	82	7.71	160	0.1715	42151	21.21	20.97	23
24	690	31.68	0.19	0.76	5	82	8.27	160	0.2375	47276	22.80	23.52	24
25	690	29.55	0.25	0.88	5	82	8.17	160	0.1411	45540	22.13	22.66	25
26	690	32.05	0.21	1.21	5	82	9.00	160	0.2321	49174	22.49	24.46	26
27	690	31.24	0.55	0.93	5	82	9.17	160	0.2106	49090	22.19	24.42	27
28	690	32.42	0.79	1.12	5	82	8.15	160	0.1766	44073	21.45	21.93	28
29	690	32.94	0.07	1.52	5	82	9.11	160	0.2801	48933	22.21	24.34	29
30	690	30.15	0.64	1.02	5	82	8.36	160	0.2211	43319	20.75	21.55	30
31	690	31.99	0.55	1.56	5	82	7.47	160	0.2497	39357	20.18	19.58	31
32	690	30.52	0.88	1.46	5	82	6.98	160	0.2657	35125	18.73	17.48	32
33	690	28.39	0.07	1.61	5	82	9.00	160	0.2192	44460	20.34	22.12	33
34	690	30.89	0.07	1.64	5	82	8.77	160	0.1602	47407	22.04	23.59	34
35	690	29.43	0.31	1.08	5	82	7.89	160	0.1887	40141	19.92	19.97	35
36	690	31.27	0.29	1.55	5	82	8.45	160	0.1047	46621	22.19	23.19	30
37	690	29.13	0.27	0.84	5	82	7.83	160	0.1582	43848	21.86	21.81	37
38	690	36.94	0.40	0.90	5	82	8.13	160	0.1993	50870	24.79	25.31	38
39	690	28.18	0.83	1.16	5	82	8.30	160	0.2278	39861	19.80	19.83	39
40	690	29.36	0.27	1.08	5	82	8.86	160	0.2438	45613	21.07	22.69	40

$\lambda = 75$

TABLE II

NR.	L (mm)	σ_y (kgf/mm^2)	e_o (mm)	f_o (mm)	a (mm)	b (mm)	c (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CRR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
41	874	34.54	0.58	1.28	5	82	7.82	160	0.1491	36172	18.05	18.00	41
42	874	39.03	1.28	1.75	5	82	7.64	160	0.2088	33020	16.71	16.43	42
43	874	34.91	1.07	1.07	5	82	7.35	160	0.1561	33186	17.18	16.51	43
44	874	30.13	0.25	1.69	5	82	7.45	160	0.1909	31729	16.29	15.79	44
45	874	29.98	0.42	1.55	5	82	7.95	160	0.2631	32727	16.17	16.28	45
46	874	29.17	0.42	1.31	5	82	7.79	160	0.1729	33545	16.78	16.69	46
47	874	27.70	1.28	2.04	5	82	7.56	160	0.2701	26730	13.67	13.30	47
48	874	28.22	0.14	1.63	5	82	8.65	160	0.2549	35072	16.45	17.45	48
49	874	30.73	1.28	1.82	5	82	7.04	160	0.1772	27868	14.79	13.86	49
50	874	29.92	1.07	1.94	5	82	7.94	160	0.2870	29699	14.68	14.78	50
51	874	28.45	0.16	1.61	5	82	7.12	160	0.1842	30554	16.11	15.20	51
52	874	34.27	0.36	1.21	5	82	8.28	160	0.2690	37220	17.94	18.52	52
53	874	26.17	0.09	1.40	5	82	8.77	160	0.1413	36645	17.04	18.23	53
54	874	30.66	0.23	0.80	5	82	8.09	160	0.1510	38397	18.77	19.10	54
55	874	26.54	0.42	1.19	5	82	8.32	160	0.1983	33961	16.32	16.90	55
56	874	32.36	0.07	1.47	5	82	7.90	160	0.1831	36275	17.99	18.05	56
57	874	34.87	1.07	1.60	5	82	7.34	160	0.2053	31289	16.21	15.57	57
58	874	31.41	0.44	1.78	5	82	8.23	160	0.2651	33596	16.25	16.71	58
59	874	32.59	1.64	1.45	5	82	7.41	160	0.2124	29532	15.23	14.69	59
60	874	30.46	0.14	0.70	5	82	8.04	160	0.1471	39469	19.37	19.64	60

$\lambda = 95$

TABLE III

NR.	L (mm)	σ_y (kgf/mm^2)	e_o (mm)	f_o (mm)	a (mm)	b (mm)	c (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CRR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
61	966	34.35	2.00	1.28	5	82	7.70	160	0.1532	27861	14.03	13.86	61
62	966	31.55	0.38	1.30	5	82	6.94	160	0.1567	27964	14.96	13.91	62
63	966	33.40	0.46	1.24	5	82	8.69	160	0.1477	35256	16.49	17.54	63
64	966	31.93	0.58	1.83	5	82	8.73	160	0.2262	31934	14.89	15.89	64
65	966	27.14	1.07	1.95	5	82	8.64	160	0.2422	28005	13.14	13.93	65
66	966	29.65	0.36	1.99	5	82	7.88	160	0.2458	28225	14.02	14.04	66
67	966	31.49	2.00	1.91	5	82	7.52	160	0.2368	24807	12.97	12.34	67
68	966	30.02	0.52	1.76	5	82	7.03	160	0.2153	25920	13.77	12.90	68
69	966	33.19	0.75	1.49	5	82	7.99	160	0.1813	30607	15.07	15.23	69
70	966	35.70	0.07	1.51	5	82	8.29	160	0.1848	34468	16.60	17.15	70
71	966	29.59	1.42	2.22	5	82	7.40	160	0.2758	24003	12.38	11.94	71
72	966	30.77	0.75	1.28	5	82	8.49	160	0.1543	32212	15.28	16.03	72
73	966	33.94	0.50	2.18	5	82	8.41	160	0.2704	30570	14.59	15.21	73
74	966	33.79	0.42	1.82	5	82	8.18	160	0.2239	31199	15.15	15.52	74
75	966	32.98	0.36	2.12	5	82	7.82	160	0.2649	28934	14.44	14.40	75
76	966	31.51	0.16	1.20	5	82	8.91	160	0.1434	36597	16.85	18.21	76
77	966	32.03	0.29	2.09	5	82	8.29	160	0.2594	30308	14.59	15.08	77
78	966	29.24	0.52	1.35	5	82	7.54	160	0.1629	28758	14.66	14.31	78
79	966	31.08	0.25	1.66	5	82	7.18	160	0.2040	27797	14.59	13.83	79
80	966	32.26	0.70	2.26	5	82	7.74	160	0.2825	29292	13.70	13.58	80

$\lambda = 105$

TABLE IV

NR.	L (mm)	σ_y (kgf/mm^2)	e_o (mm)	f_o (mm)	a (mm)	b (mm)	c (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
81	1196	34.67	0.09	1.71	5	82	8.65	160	0.2334	24992	11.72	12.43	81
82	1196	33.53	0.71	2.75	5	82	7.30	160	0.1932	19399	10.08	9.65	82
83	1196	31.73	0.64	1.29	5	82	7.93	160	0.2405	22183	10.98	11.04	83
84	1196	29.27	1.07	1.63	5	82	8.43	160	0.1752	22378	10.67	11.33	84
85	1196	31.44	0.94	1.84	5	82	7.74	160	0.0975	21288	10.69	10.59	85
86	1196	32.95	0.71	2.90	5	82	8.50	160	0.2073	22198	10.53	11.04	86
87	1196	31.15	0.23	2.37	5	82	8.08	160	0.2045	21985	10.75	10.94	87
88	1196	33.99	0.94	2.23	5	82	8.58	160	0.1393	23315	10.99	11.60	88
89	1196	33.51	0.44	2.44	5	82	8.42	160	0.1116	23390	11.16	11.64	89
90	1196	32.37	1.00	1.10	5	82	8.65	160	0.2213	24011	11.26	11.95	90
91	1196	33.22	0.23	1.08	5	82	8.75	160	0.1686	26259	12.23	13.06	91
92	1196	36.06	0.40	2.37	5	82	8.20	160	0.2034	22833	11.07	11.36	92
93	1196	27.63	0.07	3.06	5	82	8.04	160	0.1756	20820	10.22	10.36	93
94	1196	34.44	0.19	2.20	5	82	7.22	160	0.1854	20545	10.75	10.22	94
95	1196	29.99	0.58	1.70	5	82	7.85	160	0.2327	21292	10.60	10.59	95
96	1196	35.48	0.71	2.51	5	82	8.35	160	0.2174	22511	10.79	11.20	96
97	1196	32.35	0.46	2.25	5	82	9.24	160	0.1897	25153	11.36	12.56	97
98	1196	31.21	0.64	1.37	5	82	8.42	160	0.2495	23322	11.12	11.60	98
99	1196	29.41	0.38	2.78	5	82	9.05	160	0.1467	23773	10.84	11.83	99
100	1196	26.95	0.61	3.11	5	82	7.97	160	0.2815	19144	9.44	9.52	100

$\lambda = 130$

TABLE V

NR.	L (mm)	σ_y (kgf/mm^2)	e_o (mm)	f_o (mm)	a (mm)	b (mm)	c (mm)	h (mm)	α	P_{CR} (kgf)	σ_{CRR} (kgf/mm^2)	σ_{CRN} (kgf/mm^2)	NR.
101	1472	35.37	0.03	2.22	5	82	8.26	160	0.1567	16683	8.05	8.30	101
102	1472	30.26	0.09	1.20	5	82	6.98	160	0.1977	14276	7.61	7.10	102
103	1472	32.43	1.28	1.86	5	82	8.73	160	0.1762	16641	7.76	8.28	103
104	1472	33.95	0.27	1.74	5	82	8.76	160	0.2422	17512	8.15	8.71	104
105	1472	34.80	0.61	2.67	5	82	7.09	160	0.1957	13707	7.25	6.82	105
106	1472	32.33	0.42	2.28	5	82	8.44	160	0.1368	16573	7.89	8.25	106
107	1472	29.21	0.03	1.96	5	82	7.56	160	0.2153	14980	7.63	7.45	107
108	1472	33.37	0.50	1.82	5	82	8.65	160	0.2313	17055	8.00	8.49	108
109	1472	31.57	1.64	0.75	5	82	8.03	160	0.1848	15609	7.66	7.77	109
110	1472	29.10	0.31	2.13	5	82	8.33	160	0.1758	16221	7.79	8.07	110
111	1472	36.58	0.33	2.19	5	82	7.97	160	0.2043	15886	7.84	7.90	111
112	1472	32.79	0.75	2.07	5	82	7.48	160	0.2704	14326	7.34	7.13	112
113	1472	33.64	0.03	1.23	5	82	8.44	160	0.2239	17297	8.24	8.61	113
114	1472	33.82	0.64	3.21	5	82	7.16	160	0.1649	13626	7.16	6.78	114
115	1472	33.35	1.07	3.24	5	82	8.38	160	0.1934	15519	7.42	7.72	115
116	1472	32.21	1.28	3.14	5	82	7.36	160	0.1594	13589	7.03	6.76	116
117	1472	33.06	0.88	2.28	5	82	7.80	160	0.1129	15139	7.56	7.53	117
118	1472	27.94	0.42	1.89	5	82	7.57	160	0.2040	14696	7.48	7.31	118
119	1472	27.47	0.64	1.32	5	82	8.27	160	0.1825	16205	7.81	8.06	119
120	1472	31.63	0.09	2.40	5	82	7.77	160	0.2985	15029	7.53	7.48	120

$\lambda = 160$

TABLE VI

STATISTICAL STRENGTH ANALYSIS AND STEEL COLUMNS

C. Allin Cornell
Associate Professor
Department od Civil Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, U.S.A.

A.J. Rokach
Structural Engineer
Weiskopf and Pickworth
New York, N. Y.
U. S. A.

ABSTRACT

Strength theories and test data represent the two most important sources of information available to the designer of structural members. A method for combining any particular theory, available member tests, and auxiliary data on material and geometrical properties is discussed in the paper. Its application is illustrated using the tangent modulus theory of inelastic buckling, European column test data, and associated material information. The procedure is consistent with the type of information needed to implement second-moment code formats. For the theory and data considered in the illustration, the column strength uncertainty (as measured by the variance) due to imperfect theories and due to imperfect information about the internal residual stress distribution outweighs that column strength uncertainty due to the yield strength of the material.

THE FULL TEXT OF THIS REPORT HAS NOT BEEN AVAILABLE FOR PUBLICATION

LOAD FACTOR DESIGN OF COLUMNS USING SECOND MOMENT PROBABILISTIC METHOD

Theodore V. Galambos
Professor of Civil Engineering, Washington University
St. Louis, Mo. USA
Chairman, Column Research Council, USA

ABSTRACT

A method of steel column design is presented which is based on first order probabilistic theory, utilizing only the mean values and the coefficients of variation of the relevant parameters. A reliability factor, called the "safety index" is defined and a value for it is obtained by calibration to an existing design code. Subsequently a design format

$$\phi R_n \geq \gamma Q_n$$

is developed, where ϕ is a strength factor, R_n is the nominal resistance, γ is a load factor and Q_n is the nominal load effect.

1. INTRODUCTION

This report will outline a simplified method of steel column design based on the "first order" or "second moment" probabilistic theory (1,2,3). In the interest of simplicity it will be assumed that the resistance R of the column is independent of the load effect Q. Both R and Q are random functions, and thus the probability of failure can be expressed by either of the following expressions.

$$P_F = P[(R-Q) < 0] \quad (1)$$

$$P_F = P[R/Q < 1] \quad (2)$$

$$P_F = P[\ell n\ (R/Q) < 0] \quad (3)$$

If we consider the "standardized variate"

$$U = \frac{\ell n\ (R/Q) - [\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}} \quad (4)$$

in which $[\ell n\ (R/Q)]_m$ and $\sigma_{\ell n\ (R/Q)}$ are the mean and standard deviation of the natural logarithm of the ratio R/Q, then

$$P_F = P\left[U < \frac{-[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}}\right] = F_U\left\{\frac{-[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}}\right\} \quad (5)$$

in which F_U is the cumulative distribution function of this standardized variate. The quantity $[\ell n\ (R/Q)]_m / \sigma_{\ell n\ (R/Q)}$ defines the reliability of the element; hence it is called the "safety index," denoted by β. For example, if the random variable R/Q is lognormally distributed, then the area under the tail $R/Q < 1$ i.e., the probability of failure, is 3.2×10^{-5} if $\beta = 4$. Similarly, the failure probabilities are 2.3×10^{-2}, 1.4×10^{-3} and 2.9×10^{-6} for $\beta = 2$, 3 and 5, respectively. The values of β can be quite different if the shape of the distribution of R/Q in the tail is different. In practice, the probability distribution of R/Q is unknown and only R_m, Q_m, σ_R and σ_Q are estimated. However, β still indicates, in an approximate way, the failure probability, and an increase or a decrease of β by unity roughly decreases or increases the probability of failure by an order of magnitude (i.e., 10^{-1}). If the distribution of R/Q were lognormal or any of a number of other commonly used distributions (e.g. Extreme Value Type I), β would directly indicate a value of the probability of failure. In the first order probabilistic design method used here, β is only a relative measure of reliability, and it is hence called the "safety index." Within the context of the information available, i.e., just R_m, Q_m, σ_R and σ_Q, a constant value of β effectively approximates constant reliability for all similar structural elements.

The expression for the safety index β, i.e.,

$$\beta = \frac{[\ell n\ (R/Q)]_m}{\sigma_{\ell n\ (R/Q)}} \quad (6)$$

can be simplified by using first order probability theory as follows:

$$[\ell n\ (R/Q)]_m \simeq \ell n\ (R/Q)_m \simeq \ell n\ \frac{R_m}{Q_m} \tag{7}$$

and

$$\sigma^2_{\ell n\ (R/Q)} \simeq \left[\frac{\partial \ell n\ (R/Q)}{\partial R}\right]^2_m \sigma_R^2 + \left[\frac{\partial \ell n\ (R/Q)}{\partial Q}\right]^2_m \sigma_Q^2 = \frac{\sigma_R^2}{R_m^2} + \frac{\sigma_Q^2}{Q_m^2} \tag{8}$$

Since $\sigma_R/R_m = V_R$ and $\sigma_Q/Q_m = V_Q$, where V_R and V_Q are the coefficients of variation of R and Q, respectively,

$$\beta \simeq \frac{\ell n\ (R_m/Q_m)}{\sqrt{V_R^2 + V_Q^2}} \tag{9}$$

or,

$$\frac{R_m}{Q_m} = \theta = \exp\ (\ \beta \sqrt{V_R^2 + V_Q^2}\ ) \tag{10}$$

In Eq. 10, θ is the "central safety factor."

ESTIMATION OF THE SAFETY INDEX β

The "safety index" β is related to the probability of failure. In order to develop a design criterion, β must be specified. There are several ways in which β can be determined: it can be a value agreed upon by the profession to give the desired degree of reliability, or it can be obtained by adjusting β such that the same degree of reliability is attained for the new criterion as in the existing design method for a given standard situation. This procedure is called "calibration," and it will be used here. The "standard situation" selected here is the design of an interior column in a braced frame with simple beam-to-column connections according to Part 2 of the "Specification for The Design, Fabrication and Erection of Structural Steel For Buildings," American Institute of Steel Construction (AISC), 1969. According to this specification the columns are designed to resist the axial load as axially loaded elements with an effective length equal to the center-to-center story height. The factored axial load on an interior column in a braced frame in the n^{th} floor below the top of the frame (counting the roof as level n = 1) is approximately equal to

$$P_n = [D_c\ An + L_c'\ (1\text{-}RF)\ An\ ]LF \tag{11}$$

where

LF = Load Factor; LF = 1.70
D_c = dead load intensity
L_c = live load intensity specified in the code for the occupancy type
A = area on any floor level contributing to the load on the column
RF = live load reduction factor specified in the code

Equation 11 assumes that the weight of the columns is included in the dead load, that the loads on the top level (roof) are the same as for the other levels, and that L_c, D_c and A are the same at every level. Thus a regularly loaded regular structure is assumed. It is stipulated that columns in such a structure are satisfactory when designed by the present code.

The live load reduction factor RF is, according to A58.1 (1972) of the American National Standards Institute (ANSI) Code, a function of the total tributary area and the ratio D_c/L_c. The maximum reduction is RF = 0.6 if the total tributary area is more than 750 sq. ft. (70 sq. in.), or RF = 0.23 (1 + D_c/L_c), whichever is smaller.

The column capacity is equal to (AISC 1969, Part 2) 1.7 $A_c F_a$, where A_c is the cross-sectional area of the column and F_a is the allowable stress

$$F_a = \frac{F_y (1 - 0.25 \lambda^2)}{5/3 + 3/8 (\lambda/\sqrt{2}) - 1/8 (\lambda/\sqrt{2})^3} \tag{12}$$

Equation 12 is the Column Research Council Basic Column Curve Equation in the numerator, divided by a factor of safety. It is valid for $\lambda \leq \sqrt{2}$; F_y is the yield stress and

$$\lambda = \frac{h}{r} \sqrt{\frac{F_y}{\pi^2 E}} \tag{13}$$

where h/r is the column slenderness ratio and E is the modulus of elasticity.

By setting 1.7 $A_c F_a = P_n$, the required column area according to the AISC Specification is:

$$A_c = \frac{nA [D_c + L_c (1-RF)][5/3 + 3/8 (\lambda/\sqrt{2}) - 1/8 (\lambda/\sqrt{2})^3]}{F_y (1 - 0.25 \lambda^2)} \tag{14}$$

In the following derivation β will be determined such that the column area A_c from Eq. 14 serves as the basis of the calibration for the new format.

In order to evaluate β from Eq. 9 it is necessary to estimate the mean and the coefficient of variation of the resistance, R_m and V_R, and the corresponding values of the load effect, Q_m and V_Q. The mean strength of the column is equal to

$$R_m = A_c F_m \tag{15}$$

where A_c is the column area required according to the present code and F_m is the mean stress at failure. This stress is a function of a number of variables, such as the yield stress, the residual stress, the shape, the initial crookedness, the unintentional eccentricity of the axial load, and the end restraints. Each of these variables is random, and an analysis could be made if the relevant statistical parameters of each were known. This would be a formidable task if all these effects were included, although analyses with some of the variables have been made (4,5,6,7). In order to circumvent this problem, the mean failure stress was expressed in the following way:

$$F_m = [\text{Bias Factor}] [\text{Nominal Formula}] \tag{16}$$

where

$$[\text{Bias Factor}] = \left[\frac{\text{Test capacity}}{\text{Theoretical prediction}}\right]_m \left[\frac{\text{Theoretical prediction}}{\text{Nominal strength}}\right]_m \tag{17}$$

and the nominal formula is the column strength equation which is to be used in the new code for the particular type of column section. For the sake of demonstration and because the formula fits fairly well for medium size rolled columns, the CRC Basic Column Curve was chosen, i.e.,

$$F_n = F_y (1 - 0.25 \lambda^2) \tag{18}$$

Since the theoretical prediction of column strength, including all effects, is fairly complicated and all the necessary data were not available to make the analysis, it was decided to determine the bias factor by directly comparing test results to predictions from the nominal formula.

In order to assess the mean and standard deviation of the test-to-prediction ratio, test data from reports of the Fritz Engineering Laboratory of Lehigh University were analyzed (8,9). These samples are not truly random because the tests were not designed statistically, and so a better basis, involving the omitted step of the theoretical prediction, or statistically designed tests, will eventually have to be used. The sample used here includes about 50 US rolled medium size column shapes and the bias factor for these was found to be equal to 1.03 and the corresponding coefficient of variation was 0.14 (9). The test-to-prediction ratio was determined for the nominal yield stress and so the numbers above account also for the variability of the yield stress.

The mean column resistance is thus equal to

$$R_m = 1.03\, A_c F_y (1 - 0.25 \lambda^2) \tag{19}$$

where A_c is determined from Eq. 14 and F_y is the specified yield stress. The coefficient of variation is equal to

$$V_R = \sqrt{V_{Bias}^2 + V_{Fabrication}^2} = \sqrt{0.14^2 + 0.05^2} = 0.15 \tag{20}$$

The coefficient of variation due to fabrication represents an estimate of dimensional variations of the column cross sections.

The mean load effect is the mean load on the column, and it is equal to

$$Q_m = E_m\, An\, [D_m + L_m] \tag{21}$$

where A and n are the tributary area and the story number, as defined earlier, E is a random variable which accounts for the structural analysis by which the idealized loads are translated into axial forces ($E_m = 1.0$ and $V_E = 0.1$ will be assumed in this analysis), and D_m L_m are the mean dead and the mean lifetime maximum live loads, respectively. In the ensuing derivations it will be assumed that

$$D_m = D_c \quad \text{and} \quad V_D = 0.04 \tag{22}$$

$$L_m = \frac{L_c\,(1\text{-}RF)}{1 + K_L \sqrt{V_E^2 + V_L^2}} \tag{23}$$

and

$$V_L = \frac{C}{\sqrt{n}} \tag{24}$$

The coefficient of variation of the load effect is equal to

$$V_Q^2 = V_E^2 + \frac{(nA\ D_m\ V_D)^2 + (nA\ L_m\ V_L)^2}{[nA\ (D_m + L_m)]^2} \tag{25}$$

Substitution and non-dimensionalization permits the determination of β from Eq. 9, and it is a function of A, n, λ, V_R, V_E, C, K_L, V_D, V_L and D_c/L_c. A numerical study was performed by varying these parameters as follows:

$0.98 \le [\text{Test/Prediction}]_m \le 1.10$

$0.15 \le V_R \le 0.20$

$0.1 \le D_c/L_c \le 10$

$0 \le RF \le 0.6$

$0.25 \le \lambda \le 1.25$

$1 \le K_L \le 3$

$0.05 \le V_E \le 0.15$

$0.2 \le C \le 0.4$

$2 \le n \le 40$

$0.04 \le V_D \le 0.1$

These variations in the pertinent parameters defining β are thought to be larger than what one would expect for the structure for which calibration is being performed. The graphs in Figs. 1 and 2 give the variation of β with almost every one of the parameters, except for the effects of V_R and $[\text{test/prediction}]_m$. Since three values were changed at once, the results for these variations are best shown in tabular form:

$[\text{test/prediction}]_m^*$	λ	V_R^*	β
1.10	0.25	0.20	3.19
1.03	0.50	0.15	3.86
1.03	0.75	0.15	4.01
1.00	1.00	0.16	3.82
0.98	1.25	0.18	3.37

The values of β in this table were computed with $D_c/L_c = 2$; $K_L = 2$; $V_E = 0.1$; $C = 0.25$; $n = 10$; $V_D = 0.04$. This table, as well as the curves in Figs. 1 and 2 show that β varies from about 3.2 to 4.5, depending on the values of the variables affecting the results. The coefficient of variation of the live load, $V_L = c/\sqrt{n}$, the number of stories, and the code dead-to-live load ratio does not appear to result in much change in β, while the changes in the other variables have pronounced effects.

Based on this study a value of $\beta = 4$ is arbitrarily chosen as a reasonable and representative value of the reliability of medium size rolled wide-flange columns in braced simple multi-story frames as designed by the 1969 AISC Specification. Arguments could, of course, be advanced that in some

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* Based on reasonable estimates, not test results.

cases the profession permits a lower reliability (say $\beta = 3.2$), or that in other cases it demands a higher reliability (say $\beta = 4.5$), but the choice of $\beta = 4$ is one which appears neither on the low nor on the high side, and it will be used hereafter in this report.

THE LOAD FACTOR DESIGN EQUATION

Once β is selected from the calibration process, the design equation can be written from Eq. 10 as:

$$\theta = R_m/Q_m \geq \exp \beta \sqrt{V_R^2 + V_Q^2} \qquad (26)$$

Unfortunately the resistance and the load effects are not separated in this equation. Separation is achieved by using an approach suggested by Lind (3), where an approximation

$$\theta_a = \exp \alpha_R \beta V_R \exp \alpha_Q \beta V_Q \qquad (27)$$

is introduced such that the error $(\theta_a - \theta)/\theta$ is a minimum. If the extreme ranges $2 \leq \beta \leq 5$, $0.1 \leq V_R \leq 0.2$ and $0.1 \leq V_Q \leq 0.5$ are used, the values of the α's become equal to

$$\alpha_R = 0.52$$
$$\alpha_Q = 0.90$$

For the most unlikely combinations the error in θ becomes approximately 16%; for most of the prevalent combinations the error is less than $\pm$ 5%.

By the introduction of α_R and α_Q the separation between resistance and load effect is achieved very simply. Furthermore, the α's are independent of the other variables. Thus a design equation

$$\frac{R_m}{\exp \alpha_R \beta V_R} \geq Q_m \exp \alpha_Q \beta V_Q \qquad (28)$$

can be written. This equation can now be still further modified into the form

$$\theta R_n \geq \gamma Q_n \qquad (29)$$

where R_n and Q_n are nominal load effects, and

$$\phi = \frac{R_m}{R_n} \exp (- \alpha_R \beta V_R) \qquad (30)$$

and

$$\gamma = \frac{Q_m}{Q_n} \exp \alpha_Q \beta V_Q \qquad (31)$$

For example, if $\beta = 4$, $\alpha_R = 0.52$,

$$R_m = 1.03 F_y (1 - 0.25 \lambda^2) \qquad (32)$$

$$R_n = F_y (1 - 0.25 \lambda^2) \qquad (33)$$

$$V_R = \sqrt{0.14^2 + 0.05^2} = 0.15 \tag{34}$$

$$\phi = 1.03 \exp(-0.52 \times 4 \times 0.15) \simeq 0.75 \tag{35}$$

The nominal load effect is

$$Q_n = An\,[D_c + L_c\,(1\text{-}RF)] \tag{36}$$

and the mean load effect is assumed to be

$$Q_m = An\,[D_m + L_m] \tag{37}$$

for an axially loaded column in a braced simple multi-story frame, then

$$\frac{Q_n}{Q_m} = \frac{D_c + L_c\,(1\text{-}RF)}{D_m + L_m} \tag{38}$$

$$V_Q^2 = V_E^2 + \frac{(D_m V_D)^2 + (L_m V_L)^2}{(D_m + L_m)^2} \tag{39}$$

If it is assumed that

$$D_m = D_c\ ,\quad V_D = 0.04 \tag{40}$$

$$L_m = \frac{L_c\,(1\text{-}RF)}{1 + K_L \sqrt{V_E^2 + V_L^2}} \tag{41}$$

$$K_L = 2 \quad \text{and} \quad V_L = \frac{0.25}{\sqrt{n}} \tag{42}$$

then γ can be computed from Eq. 31. Since γ appears not to vary a great deal with n and D_c/L_c, (see Fig. 3), a single value of γ can be selected which is $\gamma = 1.30$.

The new design equation can then be expressed as follows:

$$0.75\,F_y\,(1 - 0.25\,\lambda^2) \geq 1.30\,An\,[D_c + L_c\,(1\text{-}RF)] \tag{43}$$

In case it is not desirable to use the same γ value, the load factor can always be determined from Eqs. 38 and 39 directly. This approach becomes necessary if dead and live load plus wind load is present in the load effect term. In this case

$$Q_m = c_1 D_m + c_2 L_m + c_3 W_m \tag{44}$$

$$V_Q^2 = V_E^2 + \frac{(c_1 D_m V_D)^2 + (c_2 L_m V_L)^2 + (c_3 W_m V_W)^3}{(c_1 D_m + c_2 L_m + c_3 W_m)^2} \tag{45}$$

where c_1, c_2, c_3 are the deterministic coefficients from structural analysis, and W_m and V_W are the mean wind load intensity and the coefficient of variation of the wind load.

SUMMARY AND CONCLUSIONS

A simplified method of column design, based on the second moment or first-order probabilistic approach, has been presented. While the method lacks in the elegance of the more sophisticated probabilistic approaches, it is far advanced of the traditional approach of selecting a factor safety by concensus based on experience. The essential statistical and probabilistic elements are all present, and their relationship is simple enough to permit a rapid study of the outcome should one or several parameters change. The approach uses the data in about as sophisticated a form in which they are presently available. In fact, there are still many elements about which educated guesses must be made. The formulation is open-ended, permitting improvement as new or better data becomes available, and it allows an analysis of the consequences if one, two, or three different column curves are to be used. Furthermore, the level of reliability can also be adjusted by changing β.

ACKNOWLEDGEMENTS

The work reported in this paper is part of a larger project dealing with load factor design criteria of steel building structures. This research is sponsored by the American Iron and Steel Institute. The work described was developed by a colleague of the author, Dr. M. K. Ravindra of Washington University. The manuscript was typed by Mrs. Alice Bletch.

NOMENCLATURE:

A	:	Tributary area in one story
A_c	:	Cross-sectional area of column
C_c	:	Coefficient in Eq. 24
c_1, c_2, c_3	:	Coefficients from structural analysis (Eqs. 44 and 45)
D_c	:	Code-specified dead load intensity
D_m	:	Mean dead load intensity
E	:	Modulus of elasticity
E	:	Random variable accounting for uncertainties in structural analysis
E_m	:	Mean of E
F_a	:	Allowable column stress
F_m	:	Mean column failure stress
F_n	:	Nominal column failure stress
F_y	:	Specified yield stress
K_L	:	Coefficient in Eq. 23
L_c	:	Code-specified live load intensity
L_m	:	Mean lifetime maximum live load intensity
LF	:	Load factor in current design code
P_n	:	Load on column n-stories below roof level
Q	:	Load effect
Q_m	:	Mean load effect
Q_n	:	Nominal load effect
R	:	Resistance
RF	:	Live load reduction factor
R_m	:	Mean resistance

R_n : Nominal resistance

V_D, V_E, V_L, V_Q, V_R, V_W : Coefficients of variation of D, E, L, Q, R and W, respectively

W_m : Mean wind load intensity

h : Story height

n : Number of stories below roof level

r : Radius of gyration

α_R, α_Q : Coefficients in Eq. 28

β : Safety index

γ : Load factor

ϕ : Resistance factor

θ : Central safety factor

λ : Non-dimensional slenderness parameter

σ_Q, σ_R : Standard Deviation of Q and R, respectively.

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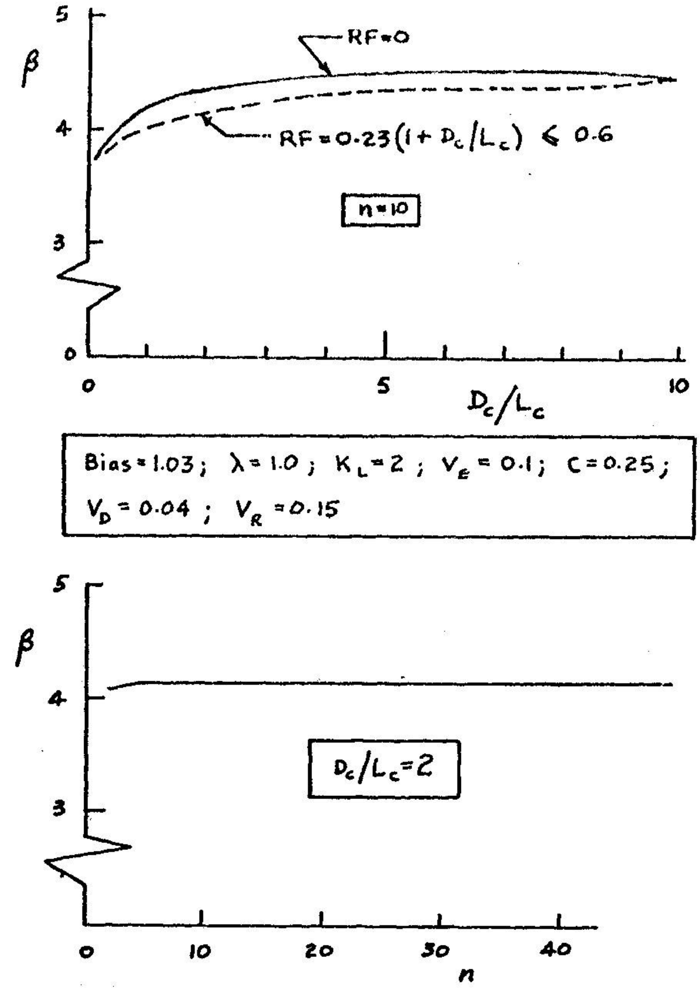

Fig.1 Variation of β with D_c/L_c and n

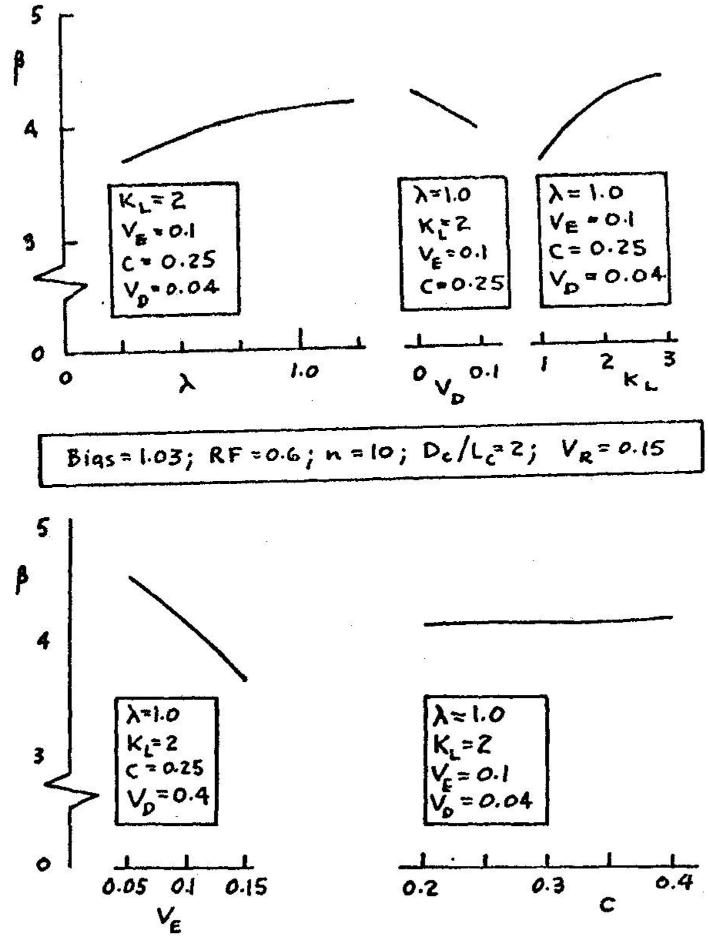

Fig.2 Variation of β with λ, V_D, K_L, V_E and C

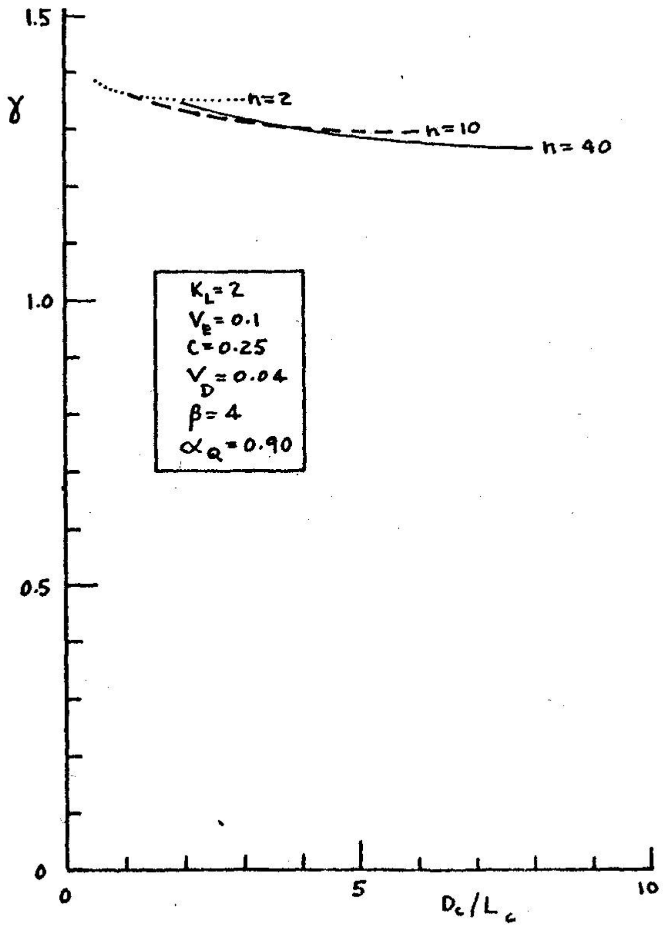

Fig.3 Variation of γ with D_c/L_c and n