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COLUMN BUCKLING CURVE OF WELDED STEEL TUBE

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ABSTRACT

This paper presents the results of experimental research regarding the buckling strength of centrally loaded welded steel tubular columns.

The specific contents of this paper are:

- (1) Effect of thermal residual stress and locked-in stress induced by cold forming on the tangent modulus in elasto-plastic range.
- (2) Effect of supporting fixtures.
- (3) Formulation of column buckling curve and comparison with test results.

## 1. INTRODUCTION

The column curve for steel tubular struts adopted by Commission 8 of the European Convention of Constructional Steelwork seems to be derived from the knowledge of the performance of seamless tube which is almost free from residual stress, and it was ranked with the superior class(a) of the eventually settled three curves. For the use of structural members, however, welded steel tubes produced by cold forming and high frequency induction welding are much more popular than seamless tubes because of their excellent productivity and of versatility of sizes. Mechanical properties of welded steel tube as a whole are somewhat different from that of seamless steel tube mainly by the influences of welding thermal residual stress and of locked in stress induced by cold forming. Hence the column buckling behavior of welded steel tube in inelastic region may also differ from that of seamless tube.

A series of buckling test of welded steel tubular columns were carried out under pin-ended centrally loaded condition. Test results are compared with the theoretical prediction based on the mechanical properties obtained from the stub-column tests. They are also compared with design loads allowed by the current Japanese specification.

## 2. TANGENT MODULUS AND CRITICAL STRESS OF WELDED STEEL TUBE

In a welded tube, three types of stress may be introduced during its producing process;

1) Elastic and plastic bending stress of tube wall along the circumferential direction as shown in Fig.1(a). Combined with the applied compressive axial load, this causes the biaxial state of stress and thus affects the yielding of a column. On this problem, a study on the basis of mathematical plasticity had been made (1).

2) When a steel strip is bent to tubular shape by cold forming, it will warp upward as shown in Fig.1(b). This is forced to straighten up in the course of cold rolling process. Thus bending stress of wall in longitudinal direction is induced. Several examples of measurement of the distribution of this bending stress(2) are shown in Fig.2(b).

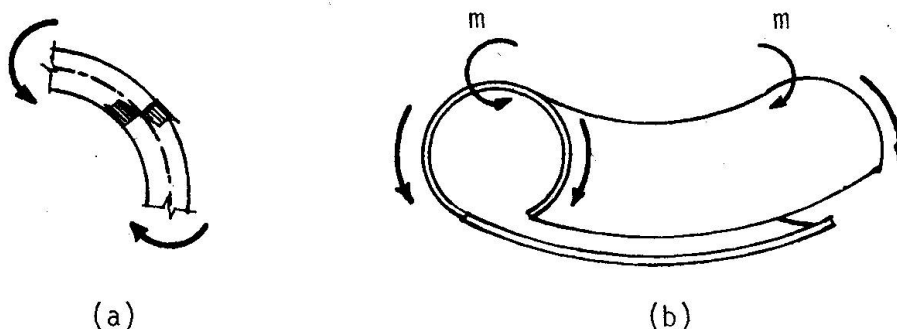
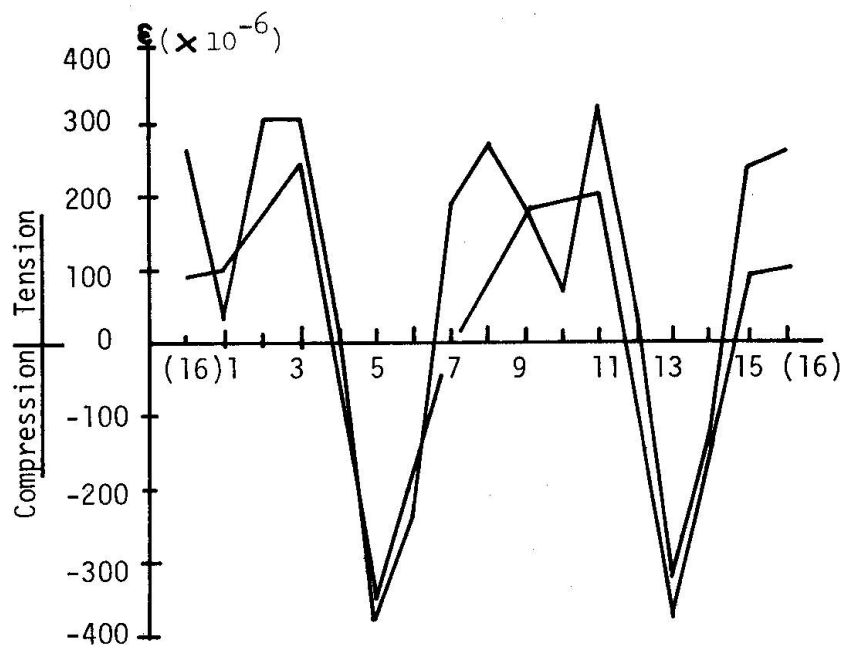
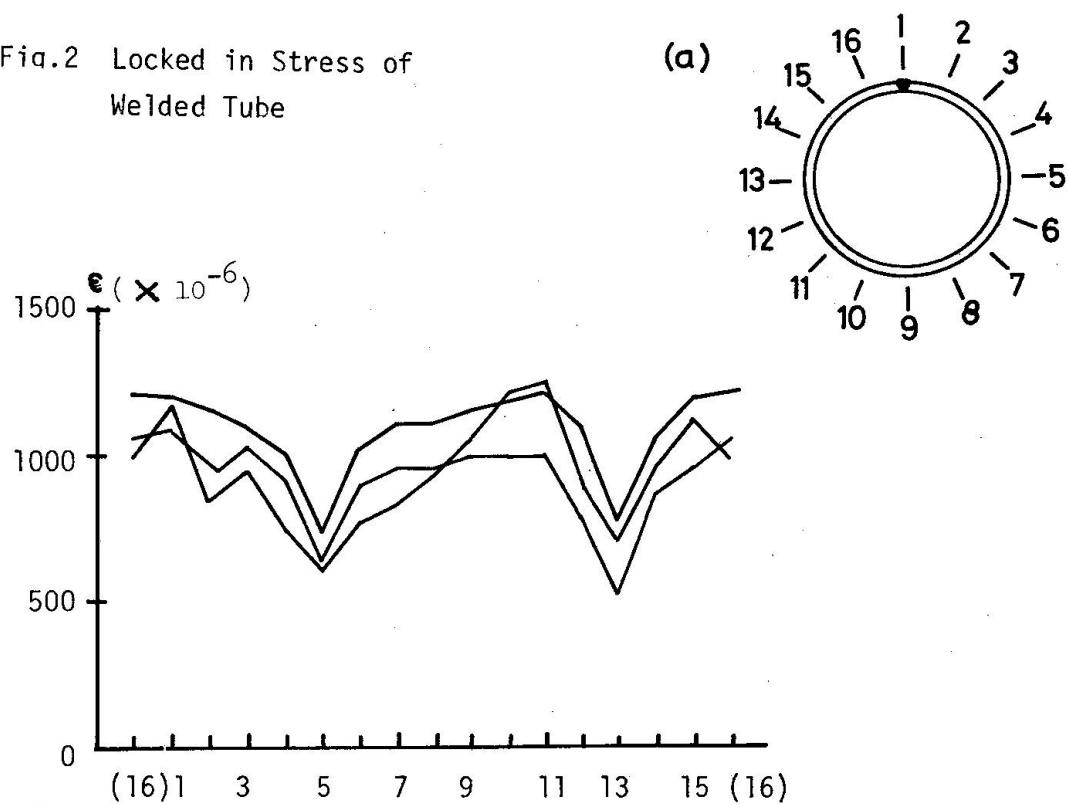


Fig.1 Cold Forming of Steel Tube

Fig.2 Locked in Stress of  
Welded Tube



3) Thermal residual stress due to welding. Examples of the distribution of this residual stress(2) are shown in Fig.2(c).

Yielding of the column subject to axial compression is affected by these locked in stresses, and the average stress-strain relationship obtained from stub-column test shows so called round house shape as is shown in Fig.3. In case of seamless tube which is almost free from residual stress, it shows rather clear yield point(Fig.3).

To obtain the general expression of stress-strain relationship of this round house type, stub-column tubes with different yield points and diameter-to-thickness ratios were tested. As shown in Fig.4,  $\sigma_p / \sigma_y$  ratios can be roughly

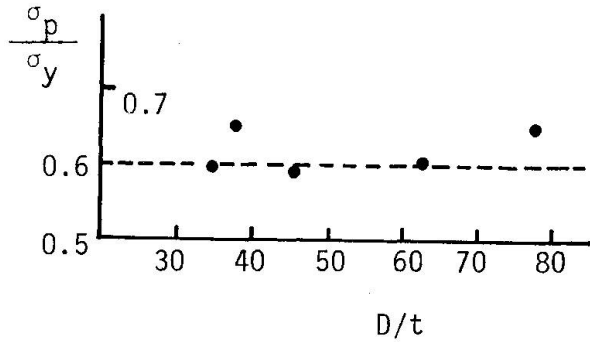


Fig.4  $\sigma_p / \sigma_y$  ratios

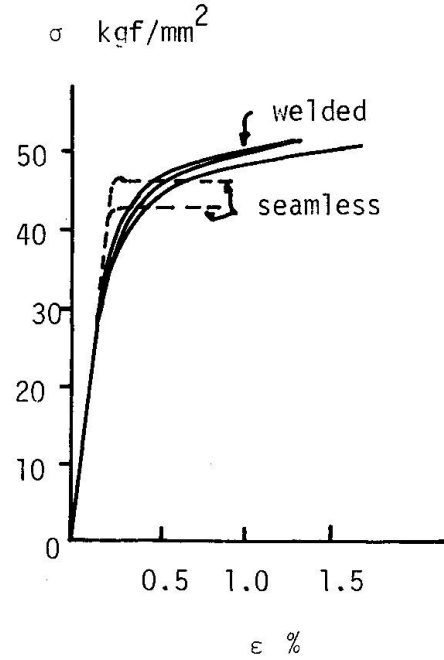


Fig.3  $\sigma$ - $\epsilon$  curve from stub-column tests

estimated as 0.6 through all tests, where  $\sigma_p$  is the proportional limit and  $\sigma_y$  is the yield strength defined by 0.2% offset basis. Assuming that  $\sigma_p = 0.6 \sigma_y$ , it was found that  $\sigma$ - $\epsilon$  relation could be well approximated as

$$\sigma = \alpha(80\alpha + 1.0)E - \frac{\alpha^2 (80\alpha + 0.4)^2 E}{\epsilon + \alpha (80\alpha - 0.2)} \quad (1)$$

where  $\alpha = \sigma_y / E$   
 $E$  = Young's modulus

From eq.(1), tangent modulus  $E_t$  can be expressed as

$$E_t = \frac{d\sigma}{d\epsilon} = \frac{[\alpha(80\alpha + 1.0)E - \sigma]^2}{\alpha^2(80\alpha + 0.4)^2 E} \quad (2)$$

Then the critical stress in inelastic region is

$$\sigma_{cr} = \frac{\pi^2 E_t}{\lambda^2} = \frac{\pi^2}{\lambda^2} \frac{[\alpha(80\alpha + 1.0)E - \sigma_{cr}]^2}{\alpha^2(80\alpha + 0.4)^2 E} \quad (3)$$

Eq. (3) can be written in nondimensional form as

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{1}{\bar{\lambda}^2} \left[ \frac{80\alpha + (1.0 - \sigma_{cr}/\sigma_y)}{80\alpha + 0.4} \right]^2$$

where

$$\bar{\lambda} = \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}}$$

$$\lambda = l/r = \text{slenderness}$$

### 3. EFFECT OF SUPPORTING FIXTURES

It has been reported that it is very difficult to realize the ideal pin-end condition in column testing(3)(4). Knife edges and conventional spherical seats were reported to be unsatisfactory because of their inevitable friction. Shown in Fig.5 are hydraulically-supported spherically seated compression testing machine platens invented by R.L.Templin(5), which seems to be one of the best devices. Two series of test results of tubular columns are shown in Fig.6. Templin type platens were used in one

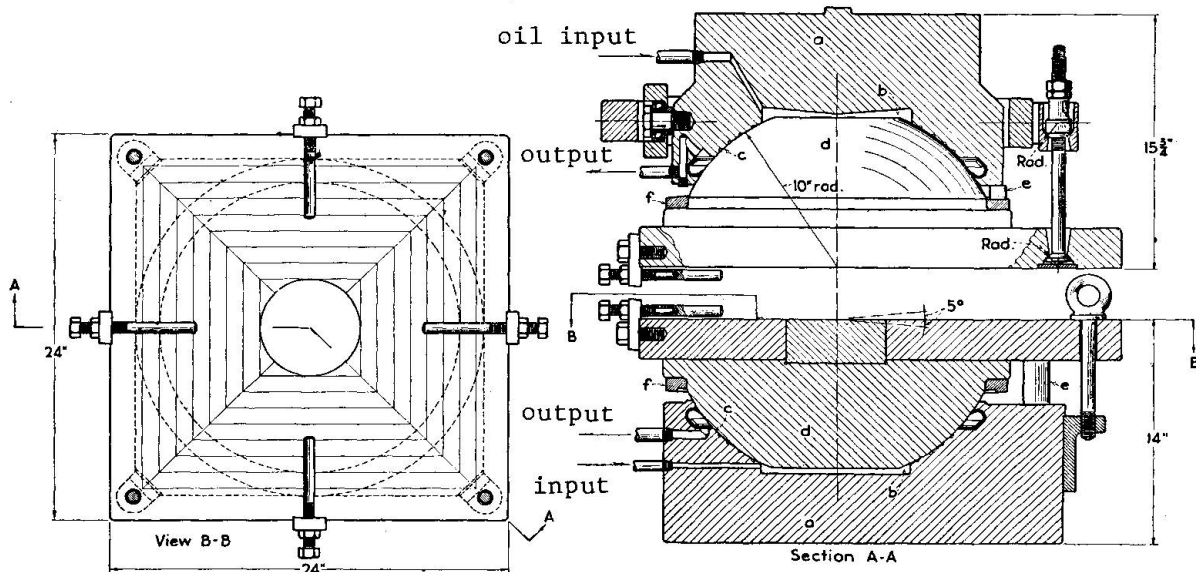


Fig.5 Templin Type End Fixture

test, while conventional spherical seats were used in the other(6). The latter test resulted in higher values up to 40% than theoretical prediction. Hence, test results of tubular columns referred in this report are limited to those obtained by using Templin type platens.

#### 4. TEST RESULTS

Available test results of welded tubular columns are shown in Fig.7 and in Table 1.(7)(8)(9). Test results are compared with eq.(4). Deviation from theoretical prediction becomes large in elastic-to-plastic transitive region. Many test results are higher than Euler value in elastic region, which means that even the Templin's device would be not ideal.

In Fig.7, test results of seamless tubes are also plotted by open circles(6)(8). The column curve after DIN4114 is shown by dashed line in the same figure, which is described as

$$\sigma_{cr} = \frac{\pi^2 E \tau}{\lambda^2}$$

$$\tau = 1 - \left( \frac{\sigma_{cr} - \sigma_p}{\sigma_y - \sigma_p} \right)^2, \quad \sigma_p = 0.6 \sigma_y \quad (5)$$

This can be written in nondimensional form as

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{1}{\bar{\lambda}^2} \left[ 1 - \left( \frac{\sigma_{cr}/\sigma_y - 0.6}{0.4} \right)^2 \right] \quad (6)$$

$$\bar{\lambda} = \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}}$$

Test results on seamless tubes which are almost free from residual stress seem to show better correlation with eq.(6) than with eq.(4). Substantial difference can be seen between the buckling strength of welded tubes and of seamless tubes.

#### 5. COMPARISON WITH JAPANESE COLUMN FORMULA

Column formulae specified by Japan Architectural Institute(A.I.J.)(10) are as follows;

$$f_c = \left[ 1 - 0.4 \left( \frac{\lambda}{\Lambda} \right)^2 \right] \sigma_y / \nu \quad \text{for } \lambda \leq \Lambda$$

$$f_c = \frac{0.6 \sigma_y}{\nu \left( \frac{\lambda}{\Lambda} \right)^2} \quad \text{for } \lambda > \Lambda \quad (7)$$

where  $f_c$  = allowable compressive stress

$$\Lambda = \sqrt{\frac{\pi^2 E}{0.6 \sigma_y}}$$

$$\nu = \frac{3}{2} + \frac{2}{3} \left( \frac{\lambda}{\Lambda} \right)^2 \quad \text{for } \lambda \leq \Lambda$$

$$= 2.17 \quad \text{for } \lambda > \Lambda \quad (8)$$

Above formulae can be written in nondimensional form as

$$\frac{f_c}{\sigma_y} = \frac{1 - 0.31 \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}}}{1.5 + 0.4 \left( \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}} \right)^2} \quad \text{for } \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}} \leq 1.29$$

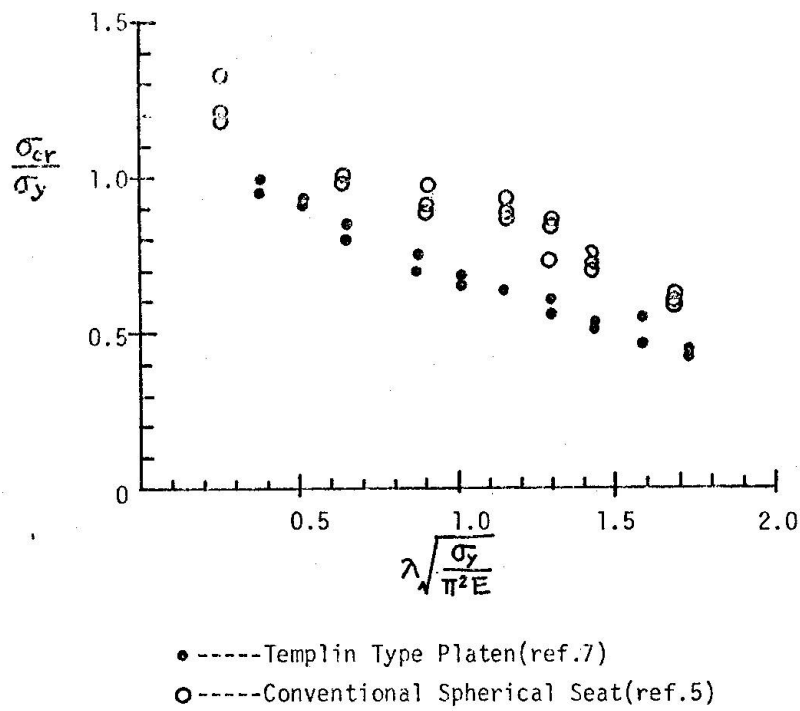


Fig.6 Effect of End Support Fixture

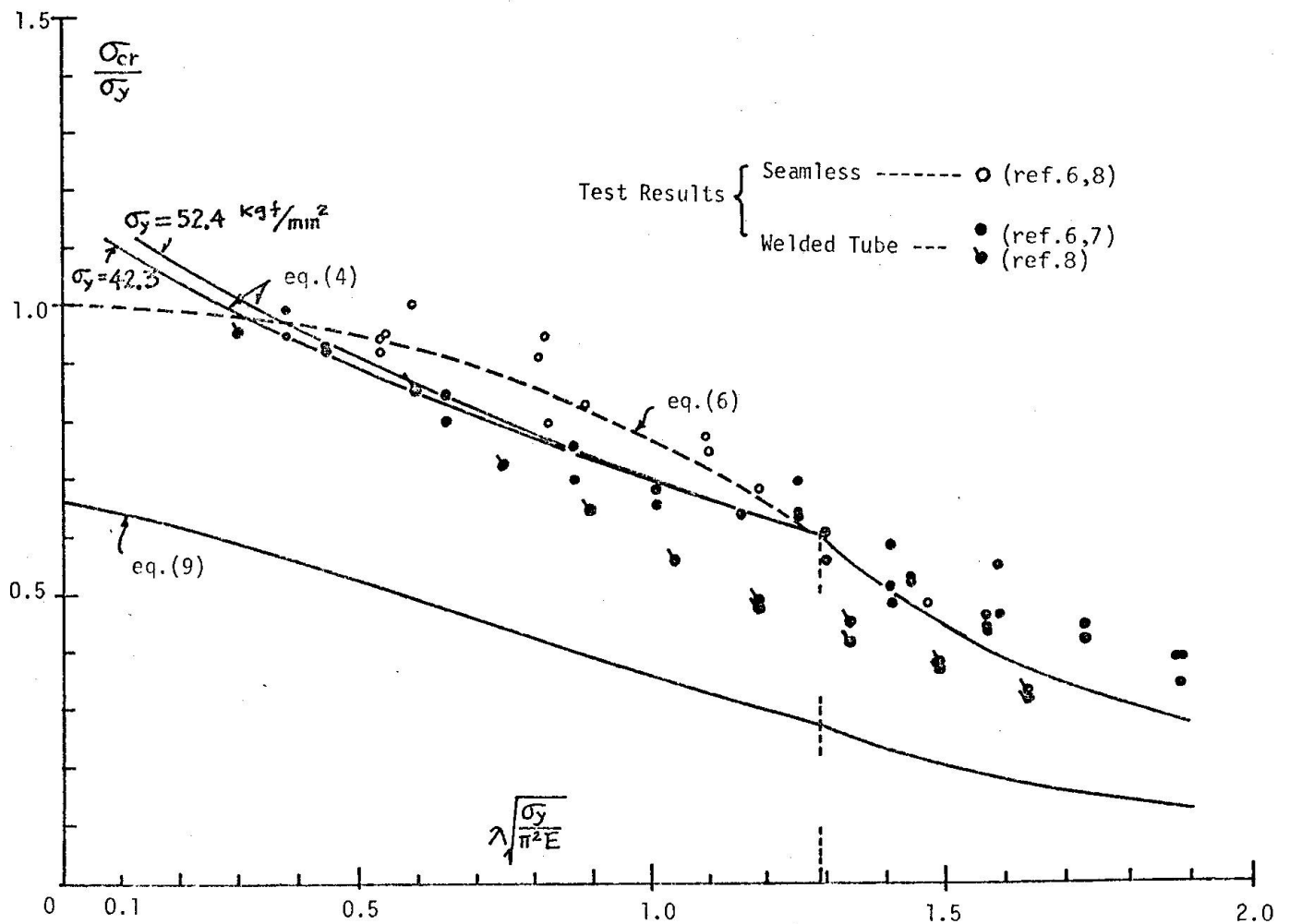


Fig.7 Test Results, Theoretical Curve & Design Formula



$$\frac{f_c}{\sigma_y} = \frac{1}{2.17} \frac{1}{\left( \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}} \right)^2} \quad \text{for} \quad \lambda \sqrt{\frac{\sigma_y}{\pi^2 E}} > 1.29$$

----- (9)

The column curve expressed by eq.(9) is depicted in Fig.7 to compare with test results.

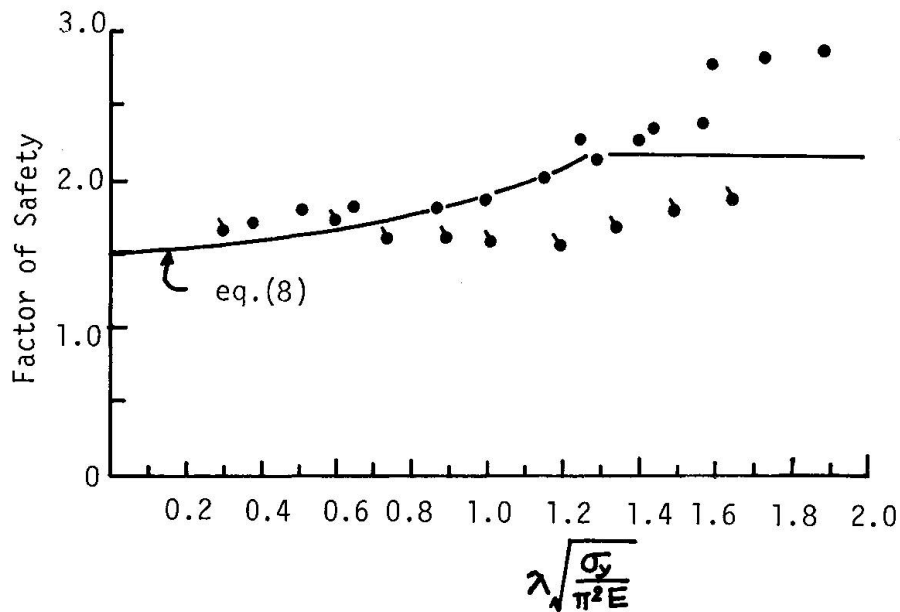


Fig.8 Factor of Safety

Each average of test results which have identical slenderness ratio are divided by corresponding nondimensional allowable stress comes from eq.(9), and is plotted in Fig.8. Nominal factor of safety specified by A.I.J. standard (eq.(8)) is also shown in the figure. In inelastic region, specified factor of safety increases parabolically starting from 1.5, while it is constantly designated as 2.17 throughout elastic region as seen in eq.(8). Safety factor of 1.5 is the one which is designated for the tension member. Test results of reference(8) show rather lower value. This might be caused by some imperfections(initial curvature, eccentricity) or unfavorable locked in stress for which no information is available from reference(8). Anyhow, as far as available test results are concerned, factor of safety of 1.5 is secured through the whole range.

Table 1. TEST RESULTS

Test A. Reference(6), Welded Tube, Grade of Steel: STK 50,  $\sigma_y = 52.4 \text{ kgf/mm}^2$

size $\phi \times t. \text{mm}$	length mm	slenderness $\lambda = l/r$	$\sigma_{\max}(\sigma_{cr})$ $\text{kgf/mm}^2$	$\sigma_{\max}/\sigma_y$	$\lambda \sqrt{\frac{\sigma_y}{2\pi E}}$
60.5×2.9	1,600	79.1	36.4	0.695	1.255
		79.1	33.4	0.638	1.255
		79.1	33.0	0.630	1.255
	1,800	88.9	25.1	0.480	1.413
		88.7	26.8	0.511	1.410
		88.9	30.6	0.585	1.413
	2,000	98.7	23.2	0.443	1.570
		98.9	23.4	0.446	1.571
		98.9	24.0	0.458	1.571
	2,400	118.5	18.1	0.346	1.885
		118.7	20.1	0.384	1.890
		118.4	20.2	0.386	1.885

Test B. Reference(7), Welded Tube, Grade of Steel: STK 50,

$\sigma_y = 42.3 \text{ kgf/mm}^2$  for 139.8 $\phi$ ×4.5mm       $\sigma_y = 43.8 \text{ kgf/mm}^2$  for 60.5 $\phi$ ×3.2mm

139.8×4.5	1,410	26.7	42.0	0.993	0.382
	1,409	26.7	40.0	0.945	0.382
	1,910	36.2	39.0	0.921	0.517
	1,910	36.2	39.2	0.926	0.517
	2,412	45.6	33.8	0.799	0.651
	2,413	45.6	35.9	0.850	0.651
60.5×3.2	1,210	59.7	33.2	0.758	0.871
	1,211	59.7	30.5	0.696	0.871
	1,411	69.5	28.5	0.651	1.010
	1,411	69.5	29.7	0.678	1.010
	1,610	79.4	—	—	1.155
	1,612	79.4	27.8	0.635	1.155
	1,812	89.2	24.6	0.561	1.299
	1,812	89.2	26.7	0.610	1.299
	2,012	99.1	23.1	0.527	1.440
	2,013	99.1	22.9	0.523	1.440
	2,213	109.1	20.3	0.464	1.590
	2,213	109.1	24.2	0.552	1.590
	2,412	118.9	18.6	0.425	1.730
	2,412	118.9	19.5	0.445	1.730

Test C. Reference(8), Welded Tube, Grade of Steel:STK 50,  $\sigma_y=46.3 \text{ kgf/mm}^2$

size $\phi \times t, \text{mm}$	length mm	slenderness $\lambda = l/r$	$\sigma_{\max}(\sigma_{cr})$ $\text{kgf/mm}^2$	$\sigma_{\max}/\sigma_y$	$\lambda \sqrt{\frac{\sigma_y}{\pi^2 E}}$
101.6×2.9	3,840	110.0	14.6	0.316	1.640
	3,840	110.0	15.1	0.326	1.640
	3,491	100.0	17.4	0.377	1.490
	3,491	100.0	16.9	0.365	1.490
	3,142	90.0	19.1	0.411	1.340
	3,142	90.0	20.8	0.448	1.340
	2,793	80.0	21.7	0.468	1.193
	2,793	80.0	22.2	0.478	1.193
	2,444	70.0	25.6	0.552	1.043
	2,095	60.0	29.7	0.643	0.895
	1,746	50.0	33.5	0.725	0.745
	1,396	40.0	39.5	0.852	0.597
	698	20.0	44.0	0.952	0.299

Test D. Reference(6), Seamless Tube,  $\sigma_y=52.0 \text{ kgf/mm}^2$  for  $60.5\phi \times 2.9\text{mm}$ .  
 $\sigma_y=37.6 \text{ kgf/mm}^2$  for  $89.1\phi \times 3.5\text{mm}$ ,  $101.6\phi \times 3.5\text{mm}$ ,  $114.3\phi \times 4.5\text{mm}$ .

60.5×2.9	1,400	68.8	40.3	0.770	1.095
	1,400	69.0	39.2	0.749	1.100
89.1×3.5	1,200	39.9	35.4	0.940	0.535
	1,800	60.0	34.2	0.910	0.807
101.6×3.5	1,400	40.8	35.8	0.950	0.550
	2,100	60.8	35.6	0.945	0.818
114.3×4.5	1,550	39.9	34.7	0.920	0.537
	2,300	59.1	31.0	0.824	0.795

Test E. Reference(8), Seamless Tube,  $\sigma_y=46.1 \text{ kgf/mm}^2$

101.6×2.9	3,442	99.0	22.26	0.482	1.470
	2,793	80.0	31.31	0.680	1.192
	2,095	60.0	38.30	0.832	0.895
	1,396	40.0	46.45	1.020	0.596

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