Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 23 (1975)

Artikel: The analysis of column buckling behaviour

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DOI: https://doi.org/10.5169/seals-19806

THE ANALYSIS OF COLUMN BUCKLING BEHAVIOUR

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ABSTRACT

A new treatment of the axially loaded column problem is presented which has led to design proposals for the revised versions of British Structural Steel Codes. The analytical procedure takes account of initial curvature and residual stresses due to hot-rolling and welding. Column strength calculations using non-linear moment-curvature properties based on residual stress distributions are reported for a range of typical structural sections. Comparisons are made with existing British column curves and with recent European proposals.

1. INTRODUCTION

Revisions of the two major British structural codes (BS 153 "Steel Bridges" and BS 449 "Structural Steel in Buildings") at present underway, have encouraged renewed study of a number of basic design problems. One specific problem has been the prediction of the maximum strength of steel columns (1,2) and as an introduction to this work, the present paper describes the treatment of the axially loaded column with pinned ends which is free from both applied terminal moments and lateral loading. The paper deals with the effect of imperfections on overall buckling. Local buckling and its interaction with overall failure is not discussed.

The imperfections in structural members examined here are residual stresses, (which might be due to hot-rolling, welding or flame-cutting operations) and initial lack of straightness. These imperfections have been incorporated in the analysis of a range of sections by means of computer simulated column tests. The results have permitted a rational study of the separate effects of imperfections, method of manufacture, variations in section shape, and material yield stress on column strength. The need to provide a proper treatment for all these parameters has led to the recommendation of a number of column curves to suit different classes of section.

A similar investigation commissioned by the European Convention of Steelwork Associations was proceeding concurrently with the present study. This work provides the basis for a set of European column curves. The analytical method is very similar to that discussed here and there is close correspondence between the resulting curves.

2. HISTORICAL SURVEY

The history of the axially loaded column problem is well known and it is not necessary to enter into great detail here except to note the main events in order to place the present study in perspective.

The major modification of Euler's original work (3) on elastic columns occurred at the end of the nineteenth century when Engesser proposed his tangent and reduced modulus theories (4,5) for initially straight inelastic columns. Later (in 1947) Shanley (6) was able to show that the tangent and reduced modulus loads gave lower and upper bounds respectively for the collapse of such columns. The tangent modulus approach has since found favour with American researchers (7, 8) wishing to investigate the effect of residual stresses on column strength. In practice, the relation between the tangent modulus and the axial load in the member is found from the stress-strain curve obtained during a stub-column test. The concept of a constant tangent modulus for a particular axial load is only applicable to straight columns and is therefore an unsatisfactory means of dealing with real members.

An alternative method of predicting real member strength is the assumption of an initial curvature of sufficient magnitude to allow for all imperfections. Robertson (9) evaluated empirically an initial curvature factor in the Perry formula. This formula, which relies on the attainment of yield to define limiting column strength has provided a design method in British codes for many years. The similar Perry-Duthiel formula is used in France.

A much more satisfactory way of dealing with the real, initially curved column is to determine its maximum strength. As with the Perry approach, an initial curvature is assumed so that there is a continuous development of column deflexions without bifurcation of equilibrium. Unlike Perry, the criterion of failure is now the attainment of a maximum in the axial load-lateral deflexion curve for a column of given slenderness ratio. Residual stresses can be accounted for when the load-deflexion curve is determined for a particular cross-section.

Little previous work on the strength of steel columns has allowed for the simultaneous action of initial curvature and residual stress. Batterman and Johnston (10) have considered the simplified case of I-section columns in which only the flanges are assumed to be load-carrying. Hall and Stup (11) have made further developments of this work. The recent European study of the maximum strength of real columns is due to Beer and Schulz (12).

3. THE TREATMENT OF IMPERFECTIONS

The magnitude and distribution of hot-rolled and welded residual stresses assumed for the present analysis have been discussed elsewhere (14,15,16). From a knowledge of the residual stresses it is possible to compute moment-thrust-curvature (M-P-ϕ) curves for the range of column sections shown in Table I. This information is then fed into the computer when the full column analysis program is in operation.

The initial centreline of the column takes the shape of a half sine-wave with an amplitude standardised at 1/1000 of the length. In view of the general uncertainty about the allowance for initial curvature, provision is made in the calculations for varying the magnitude of the initial bow, although keeping it proportional to the column length. The effect of initial bow on column strength is discussed in Ref. 1. Beer and Schulz also choose 1/1000 of the column length as their initial bow.

4. THE MAXIMUM STRENGTH OF AXIALLY LOADED COLUMNS

4.1 The Basic Method

The criterion of failure which determines the maximum strength of a column has already been described (Section 2) as the attainment of a maximum in the load-deflexion curve for a column of fixed slenderness ratio. However, in view of the way in which the moment-curvature properties have been calculated it is much more convenient to carry out column calculations for fixed axial load so that the criterion of failure must be restated as the attainment of zero slope in the slenderness-ratio versus central deflexion curve for constant axial load.

The procedure for determining this criterion is first to select a column length of appropriate initial curvature which is stable under the given axial load. A numerical method which allows for the presence of residual stresses in then used to determine the deflexions which satisfy equilibrium requirements for this column. As the column length is increased, new deflected shapes are determined for each increment until finally a length is reached for which it is impossible to satisfy the conditions of equilibrium, a plastic hinge having formed at the middle of the column.

The increments in length are initially equal to the radius of gyration (r) of the column section about the chosen axis of bending. When the limiting length is reached, the calculation returns to the last stable length and proceeds to determine column deflexions for increments in length of 0.1r until a new (shorter) limiting length is reached.

The computation process reflects the sudden failure typical of column buckling. At the limiting, just stable length for a given axial load the maximum moment in the column is often appreciably less than the plastic moment, yet an increase in length of less than 0.1r is sufficient to allow a hinge to develop at the centre of length and deflexions to increase indefinitely to collapse.

4.2 Calculations of Column Deflexions

The basis for the calculation of column deflexions is simple. All that is necessary is the satisfaction of the condition of equilibrium between internal and external moments at every point in the column length. Column deflexions may then be obtained by a relationship between the internal moment and the curvature. The problem becomes complicated when, due to the presence of residual stresses in the cross-section, this relationship is non-linear. For this reason an iterative process to determine the deflexions must be used.

For equilibrium of moments at any point in the column,

$$M_i = M_e = P\ (w+y) \qquad (1)$$

where M_e and M_i are the external and internal column moments respectively

P is the axial load

y is the intial deflexion before axial load is applied

and w is the additional deflexion induced by the load.

In non-dimensional terms, this equilibrium equation becomes

$$\frac{M_i}{M_Y} = \frac{P}{P_Y}\ (W+Y) \qquad (2)$$

where $W = w/c_e$, $Y = y/c_e$

c_e is the section core (=Z/A)

M_Y is the first yield moment with zero axial load

and P_Y is the squash load.

The internal moment is related to the curvature by the equation

$$\frac{M_i}{M_Y} = i\frac{\phi}{\phi_Y} \qquad (3)$$

In a cross-section which remains elastic, i is a constant equal to unity and the calculation of column deflexions involves substitution for M_i/M_Y into equation (2) followed by simple integration.

We are considering here the behaviour of elasto-plastic columns for which i is a function of the internal moment and the axial load. Moment-curvature curves for partially plastic sections have been computed (16) and it is from these curves that we are able to obtain the function i. The loading path followed assumes no elastic unloading of previously yielded regions of the cross-section.

The curvature is related to small column deflexions by the familiar expression

$$\phi = -\frac{d^2w}{dx^2}$$

Due to the non-linear nature of the problem, the deflexions in the expression above are differentiated numerically to obtain the curvatures. The differential coefficient is therefore replaced by a finite difference expression for the curvature at a typical node 0, thus

$$\phi_o = -\left(\frac{w_1 + w_3 - 2w_o}{h^2}\right) \qquad (4)$$

where h is the width of the interval between nodes.

The initial, unloaded deflected shape of the column is assumed to be given by a half-sine curve, thus

$$y = Y c_e = \alpha L \sin \frac{\pi x}{L}$$

where L is the column length
and α is normally set equal to 1/1000.

At the node 0

$$Y_o = \frac{\alpha (L/r)}{(c_e/r)} \sin \pi \left(\frac{a_o}{a_n}\right) \qquad (5)$$

where a_o is the number of intervals of width h from one end of the column and a_n is the number of intervals into which the whole column length is divided (=L/h). The ratio of section core to radius of gyration (c_e/r) can be obtained (16) in terms of the ratio of web to flange area, A_W/A_F.

From equations (2), (3), (4) and (5) we now have the following non-dimensional equilibrium equation at a typical column node,

$$W_1 + W_3 - 2W_o + \frac{z_2}{i_o} \cdot \left(\frac{P}{P_Y}\right) \left[W_o + z_1 \sin \frac{\pi a_o}{a_n}\right] = 0 \qquad (6)$$

where $z_1 = \frac{\alpha (L/r)}{(c_e/r)}$

and $z_2 = \frac{(L/r)^2}{{a_n}^2} \left(\frac{\sigma Y}{E}\right)$

A computer program employing a relaxation method was used to give a set of column deflexions which satisfy equation (6). Sufficient accuracy was obtained with the column divided into twenty intervals and an out-of-balance between external and internal moments of not more than 0.1%.

5. COMPUTED COLUMN CURVES

The computer program determined critical column lengths for discrete values of axial load between 10% and 95% of the squash load. A number of typical column members were treated in order to examine the influence on column strength of cross-section shape, method of manufacture, material yield stress and distribution of residual stress. The various members (coded M1 to M12) are identified in Table I.

5.1 Universal Sections (Major Axis Bending)

The computed column curves for hot rolled I-sections bending about the major axis (members M1 to M4) appear in Figure 1 in which P/P_Y is plotted against the slenderness, λ_{cx}.

Note that the slenderness is defined by

$$\lambda = \frac{L}{r} \cdot \frac{1}{\pi}\sqrt{\frac{\sigma_Y}{E}}$$

thus λ is unity when the Euler critical load is equal to the squash load, (P_Y).

Table I

Moment-curvature properties and column buckling strength curves were computed for the following typical structural members:

a) Hot-rolled sections

Member No.	Description	Axis of Bending	$\frac{A_W}{A_F}$	σ_Y MN m^{-2}
M1	Universal Column Shape	Major	0.3	250
M2	Medium Universal Beam Shape	"	0.75	"
M3	Extreme Universal Beam Shape	"	1.2	"
M4	As M1	"	0.3	450
M5	As M1	Minor	0.3	250
M6	As M2	"	0.75	"
M7	As M3	"	1.2	"
M8	As M1	"	0.3	450

b) Welded Sections

Member No.	Description	Axis of Bending	$\frac{A_W}{A_F}$	S_f a)	S_w a)	t/T b)
M9	Moderately Welded H-Column Shape	Major	0.3	0.2	0.2	0.6
M10	Moderately Welded Square Box	-	1.0	0.2	0.2	1.0
M11	Lightly Welded Square Box	-	1.0	0.05	0.05	1.0
M12	Moderately Welded H-Column Shape	Minor	0.3	0.2	0.2	0.6

a) S_f and S_w indicate respectively the proportion of flange and web occupied by weld tension block.
b) T = flange thickness, t = web thickness

As we have seen in Ref. 14, the section geometry determines the residual stress. This effect is illustrated by the three curves for members M1, M2 and M3 in which the ratio A_W/A_F takes the values 0.3, 0.75 and 1.2. The yield stress for the three members is 250 MN m^{-2}. It is evident from these curves that the geometry does not have a significant effect for major axis buckling of Universal sections, it would therefore be reasonable to assume a single design curve for major axis buckling of all hot-rolled I-shapes in a particular grade of steel.

Member M4 has a yield stress of 450 MN m^{-2} and $A_W/A_F = 0.3$. The reduced significance of the residual stress apparent here is due to the fact that these stresses are independent of the material yield stress. Where the residual stresses in the flange toes of member M1 were 50% of the yield stress, in member M4 the same absolute value is now only 28%.

5.2 Universal Sections (Minor Axis Bending)

The variation of cross-section geometry and the resulting distributions of residual stress have a slightly more pronounced effect on the minor axis buckling of hot-rolled Universal sections. The column curves for these members appear in Figure 2. The ratio A_W/A_F takes the values 0.3, 0.75 and 1.2 for members M5, M6 and M7 which have a common yield stress of 250 MN m^{-2}. The results for member M8 which has a yield stress of 450 MN m^{-2} and $A_W/A_F = 0.3$ (compare with curve for M5) can barely be distinguished from those for member M7. The strength of these two members is therefore shown by a single curve in Figure 2. Comparison of curves for M1 and M4 (Figure 1) and for M5 and M8 (Figure 2) indicates that an increase in material yield stress has approximately the same proportional effect on column strength in minor axis bending as in major axis bending.

5.3 Welded Sections

Column curves for welded sections are shown in Figure 3. Due to the variation in strength of a particular section induced by differing amounts of welding it is convenient to plot these curves on the basis of a fictitious reduced yield stress, σ_{YR}.

For box sections and I-sections bending about the major axis,

$$\sigma_{YR} = \sigma_Y - \frac{1}{2}\sigma_r$$

where σ_r is the residual compressive stress induced by welding. For I-sections bending about the minor axis;

$$\sigma_{YR} = \sigma_Y - \sigma_r$$

A virtue of using this form of presentation for welded members is that all column curves tend to fall close to a single design curve (shown dashed in Figure 3).

It must be noted that the strength of welded members predicted in this study is likely to be pessimistic because of the severe assumption made for the distribution of compressive residual stresses.

6. BASIC COLUMN DESIGN CURVES

6.1 Very Short Columns

It is possible to dispose of all very short columns regardless of their subsequent behaviour by allowing them to reach the squash load, P_Y. This is not unreasonable since otherwise it would not be possible to perform a stub column test. In these circumstances, strain-hardening begins to have an effect and an estimate of the limiting slenderness for the development of the squash load can be obtained by replacing the elastic modulus in Euler's equation with the strain-hardening modulus E_S ($\simeq \frac{E}{30}$), then at the squash load, λ is approximately 0.20.

This figure is not adhered to exactly for all design curves but is used as a general guide for the squash length of a stocky column. The concept represents an improvement on the Perry-Robertson curves of the existing British Codes which only allow the development of the squash load at zero column length.

6.2 Buckling Curves for Hot-rolled and Welded Sections

It is clear from the discussion in Section 5 that a single column buckling curve is insufficient to specify the range of column strengths dictated by section geometry, method of manufacture, axis of bending and material yield stress. It is therefore suggested that for column design a number of basic curves be provided to cover the band of results obtained during this study.

The curves derived for the hot-rolled sections M1, 2,3, 7 and 8 fall into a group for which a single design curve would suffice. We will refer to this as Curve B. Curves A and C are chosen to coincide with those derived for members M4 and M5. As Figure 3 shows, Curve B is also satisfactory for welded members provided the appropriate reduced yield stress is used.

6.3 Allocation of Sections to Design Curves

We are now in a position to relate members to their appropriate design curve. The chart in Figure 4 permits the selection of column curves for a wide range of structural sections including some which have not been examined specifically.

To demonstrate the use of the chart in Figure 4, consider the allocation of a column design curve to a Universal Beam section bending about the minor axis. The section has a web to flange area ratio (A_W/A_F) of 0.75 and a flange thickness of 22 mm. The steel is Grade 50, thus σ_Y is 350 MN m^{-2}. From the type of section, axis of bending, area ratio and yield stress, the point 1 is found in the lower half of the chart. Projecting upwards from point 1, the intersection (point 2) with the horizontal line corresponding to the flange thickness is obtained in the upper half of the chart. Point 2 lies in the region labelled B, thus column design curve B should be used for the member.

The study of hot-rolled sections which led to the establishment of the design curves has already been discussed. This work was principally concerned with sections for which the flange thickness was less than about 25 mm. Although this restriction includes a high proportion of the total tonnage of Universal sections rolled in Great Britain, design rules must also be provided for thicker sections which suffer from more severe residual stresses (14). A further basic design curve, (Curve D) is therefore provided for these members.

Hot finished tubes and rectangular hollow sections have been placed on the right-hand side of the chart in Figure 4 in view of the generally small residual stresses expected in these sections (17). The higher stresses which might occur with increased wall thickness are automatically allowed for. Further study of the stresses in these sections is needed.

Hot-rolled channel and angle sections have received little attention in the past, but O'Connor (18) has reported large residual stess measurements in a few channel and angle sections and as a consequence these members appear on the left-hand side of the selection chart. More information on residual stress distributions in these sections is required.

Although the thickness of components is allowed for when the residual stress is calculated, welded sections are usually made up from Universal Mill flats which already contain residual stresses from the hot-rolling operation. These additional stresses have not been included in the present analysis but this is not a serious objection for thin plates since the design curves for welded members tend to be conservative. The use of thick plates which could contain considerable cooling residual stresses requires a lower design curve. The column selection chart in Figure 4 allows for this. More research on the problem is needed, together with the treatment of flame-cut flanges.

6.4 Code Presentation

To promote the use of computers in structural design, the column curves have been specified by simple formulae. The general expression for the limiting slenderness is then,

$$\lambda_c^2 = \sum_{n=1}^{n=4} C_n \left(\frac{P}{P_Y}\right)^{n-2} \qquad (7)$$

where values of the coefficients for all the basic design curves are given in Table II.

Table II

Coefficients in column formulae (equation 7)

Design Curve	C_1	C_2	C_3	C_4
A	+1.07	-1.15	+2.97	-2.83
B	+0.97	-0.46	+0.84	-1.30
C	+0.92	-0.08	-1.14	+0.34
D	+0.87	0	-1.71	+0.87

As formulae are too cumbersome for direct use in codes, it is proposed that column strength data should consist of four separate charts (derived from the above formulae) which correspond to the basic design curves. Each chart will have families of curves for different values of yield stress and will show maximum column stress against slenderness ratio.

7. EXPERIMENTAL RESULTS AND OTHER COLUMN DESIGN CURVES

7.1 Experimental Results

The interpretation of test results for nominally axially loaded pin-ended columns is generally unsatisfactory since small amounts of restraint or eccentricity of loading are inevitably present and can have an appreciable effect on the column strength.

Some measure of experimental comparison with the present work can be made with results reported by Beedle (19) on a large number of sections tested at Lehigh University. These tests confirm the wide range of column strengths due to variations in section geometry and method of manufacture. Particularly noticeable from this series are the low values of critical axial load obtained for welded members, a conclusion already reached in the present study.

A series of tests on high yield strength steel sections conducted by Strymowicz and Horsley (20) have been compared with Basic Curves A and B in Ref.1. A large number of the results in this series lay above the Euler hyperbola which suggests that some restraint may have developed in the support bearings. If the test points are moved to the left so that they all lie below the Euler curve, there is good agreement with the Basic Curves.

7.2 Comparison with Perry-Robertson Curves

Curves A, B, C and D are compared with existing BS 449 and BS 153 column curves in Figure 5. A and B lie above the code curves for all values of load, while the C and D curves are below the code curves for values of λ in the range 0.8 to 1.2. The BS 449 curve falls rapidly as the slenderness increases and lies below curve D for values of thrust less than 0.3 P_Y.

It is clear from this comparison that the single Perry-Robertson curve cannot cater adequately with all types of member. While it can be argued that these curves have been satisfactory for design hitherto, it is now possible to make a much more favourable allowance for many classes of section.

7.3 Comparison with the European Column Curves

The late Prof. Beer and Dr. Schulz (12) have recommended three column curves to Commission 8 of the European Convention of Steelwork Associations which take initial curvature and residual stresses into account. The curves are derived analytically and are supported by an extensive test programme. The design information is presented for code use in the form of tables of critical stress. The three European curves a, b, c are compared with basic curves A, B, C and D in Figure 6, where it should be noted that they fall away immediately from the squash load. The types of member to which each curve applies are shown in tabular form.

Points of difference between the two approaches to axially loaded column design have been the subject of recent discussions. It is hoped that the final European column curves will contain the best elements of both versions.

8. CONCLUSIONS

a) Four column design curves based on real member imperfections have been found necessary to specify the strength of a wide range of structural steel sections. The single Perry-Robertson curves of the British Codes (BS 153 and 449) are considered inadequate to deal efficiently with all sections.

b)Residual stresses due to hot-rolling or welding have a marked effect on column strength. The magnitudes and distributions of these stresses assumed in the calculation of moment-curvature properties are pessimistic, thus the computed column curves may be viewed as lower bounds to true collapse loads.

c) The initial central bow in the column was chosen to be L/1000. This figure is considered to be a satisfactory allowance for practical columns. Increase in size of bow above this figure begins to have a considerable effect on column strength.

d) Comparison with current European column design curves has shown a similarity in form to those presented here. Some disagreement with the allocation of members to these curves exists and the situation is at present under review by the European study group.

ACKNOWLEDGEMENTS

The author is grateful to the Construction Industry Research and Information Association (CIRIA) for providing the financial support of this work and to the Engineering Department of the University of Cambridge for the generous provision of facilities. In particular he would like to thank Mr. J. B. Dwight of the Engineering Department, Cambridge without whose initiative and unflagging enthusiasm this work might never have come to fruition.

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NOTATION

A - cross-sectional area of member
A_F - total flange area
A_W - web area
a_o - number of intervals from one end of column to node 0
a_n - total number of intervals into which the column is divided (L/h)
C_n - column formulae coefficients (equation 7)
c_e - section core (=Z/A)
E - Modulus of Elasticity
h - interval between nodes
I - second moment of area
i - effective stiffness function from moment curvature relation
L - column length
M - general moment
M_e - external moment
M_i - internal moment
M_Y - yield moment ($=Z\sigma_Y$)
P - axial load
P_Y - squash load ($=A\sigma_Y$)
P_{YR} - reduced squash load ($=A\sigma_{YR}$)
r - radius of gyration
t - plate thickness
W - non-dimensional deflexion ($=w/c_e$)
w - column deflexion induced by load
x - distance along column
Y - non-dimensional deflexion ($=y/c_e$)
y - initial column deflexions under no load
Z - elastic section modulus
α - initial column bow/column length
ϕ - curvature
ϕ_Y - yield curvature ($= M_Y/EI$)
λ - slenderness ($= \frac{L/r}{\pi} \sqrt{\frac{\sigma_Y}{E}}$)
σ_Y - yield stress
σ_{YR} - reduced yield stress
σ_r - compressive residual stress due to welding

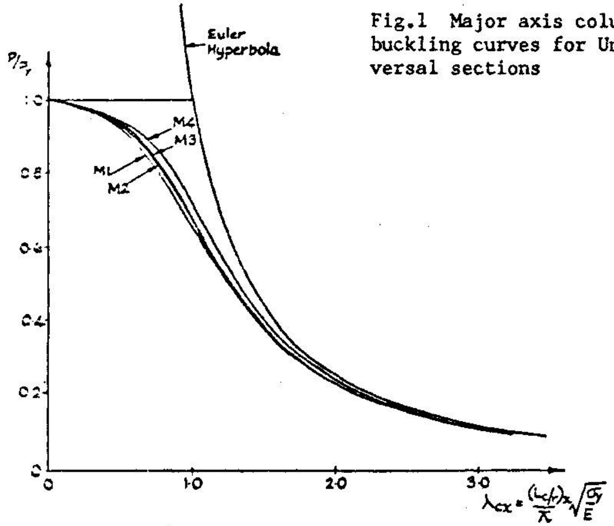

Fig.1 Major axis column buckling curves for Universal sections

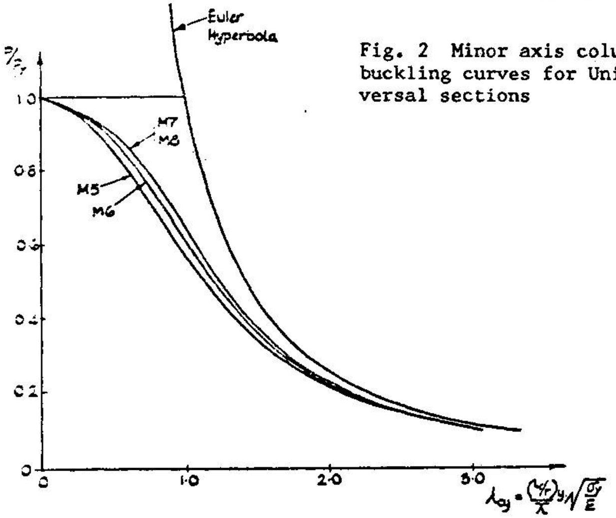

Fig. 2 Minor axis column buckling curves for Universal sections

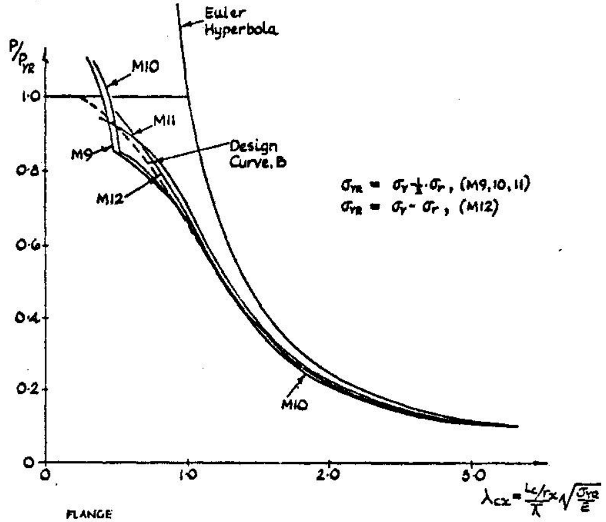

Fig. 3 Adjusted column curves for buckling of welded members

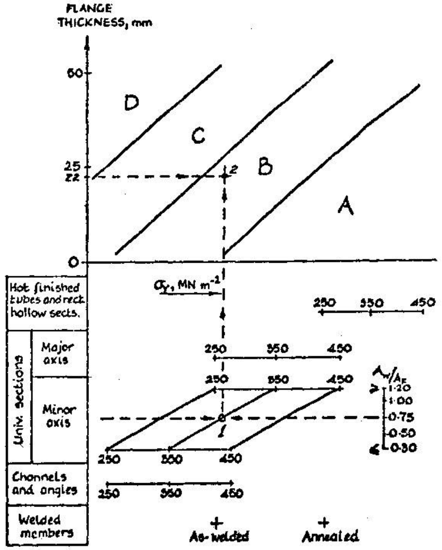

Fig. 4 Column curve selection chart

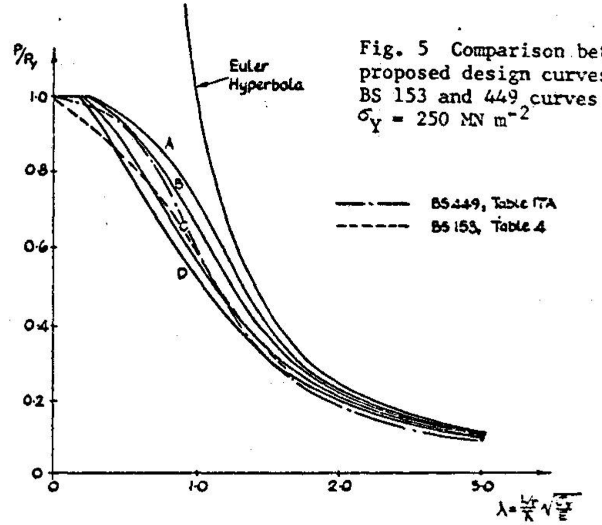

Fig. 5 Comparison between proposed design curves and the BS 153 and 449 curves for $\sigma_Y = 250$ MN m^{-2}

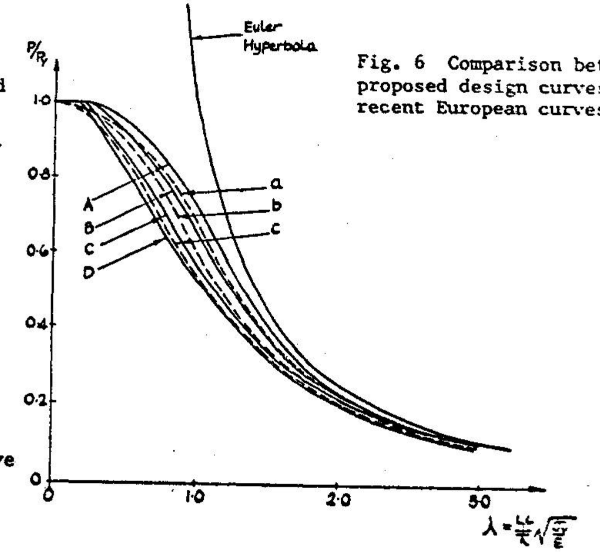

Fig. 6 Comparison between proposed design curves and recent European curves