A probabilistic contribution to the safety analysis of thick arch-gravity concrete dams

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A probabilistic contribution to the safety analysis of thick arch-gravity concrete dams

Une contribution probabilistique à l'analyse de la sécurité des barrages poids-voûte en béton

Wahrscheinlichkeitsbeitrag zur Sicherheitsanalyse von Schwerkraft-Bogen-Staudaemmen aus Beton

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1. INTRODUCTION

In conventional design, the safety of concrete dams is usually measured in terms of deterministic safety factors [1, 2, 3, 4, 5].

Nevertheless, because of various design uncertainties (such as ones, for example, on the strength and deformation of rock foundations, on the true distribution of uplift, on the temperature variations,......) and also because of the fact that concrete exhibits quite often a considerable degree of spacewise randomness in its mechanical properties (elastic moduli and local yield strength), a probabilistic quantity, such as the probability of failure, seems to be an alternative preferable measure (or more precisely countermeasure) of the dam safety [6, 7].

In the present study, as a preliminary approach to the question concerning the randomness of concrete properties, an attempt is made to apply to arch-gravity dams recent concepts of probabilistic safety analysis and stochastic continuum mechanics.

The study consists of two main parts :

1. Evaluation of the probability of functional dam failure corresponding to a conventional limit state of serviceability, in which elastically constrained plastic deformations arise in some selected points of the structure. As it will be explained in that follows, the main idea developed in this part lies in

the interpretation that, as well as the concrete local yield strength [8, 9] , also the three-dimensional elastic stresses in the dam are random functions of space variables.

2. Description of a method for obtaining a lower bound, as large as possible, of the probability of structural dam failure, corresponding to a limit state of collapse in which the maximum load that the dam can carry is reached. This method constitutes a probabilistic modified version, adapted for the case of arch-gravity dams, of a previous deterministic one, established in [10] for the case of pure arch dams with one or double curvature.

2. PROBABILISTIC MODEL FOR YIELD STRENGTH OF DAM CONCRETE

It is assumed herein that a thick arch-gravity concrete dam can be regarded as a three-dimensional random continuum, whose elastic moduli and local yield strength have to be described by means of stochastic functions of space coordinates.

Being unnecessary, in view of the purpose of the following analysis, a specific probabilistic characterization of elastic moduli (whose random effects on the stress state will be implicitly taken into account by later assumptions concerning the stochastic distribution of elastic stresses), we now deal with the only probabilistic specification of the concrete yield strength.

Following earlier works [8, 9, 11, 12] , in that follows we assume the compressive local yield strength Y of dam concrete to be a homogeneous Gaussian process of the spatial cylindrical coordinates r , θ , z , with mean value m_Y , standard deviation D_Y , and normalized auto-correlation function

$$\rho_Y = \exp(- d^2/ d_0^2) \tag{2.1}$$

In (2.1), $d^2 = r^2 - 2\, r\, r'\cos(\theta - \theta') + r'^2 + (z-z')^2$ is the square of the distance between two typical points P and P' of the dam (having coordinates r , θ , z and r', θ', z', respectively), and d_0 is a real constant. As it is shown in Fig.2.1, ρ_Y becomes negligible when $d > 2\, d_0$.

Thus, denoting by y and y' the permissible values of Y at the points P and P' , respectively, the two-dimensional probability density of the concrete yield strength can be expressed in the form :

$$p_{YY'}(y,y') = \frac{1}{2\,\pi\, q\, D_Y^2} \exp(- p^2/2\, q^2\, D_Y^2) \tag{2.2}$$

where $p^2 = (y-m_Y)^2 - 2\,\rho_Y(y-m_Y)(y'-m_Y) + (y'-m_Y)^2$, and $q^2 = 1 - \rho_Y^2$.

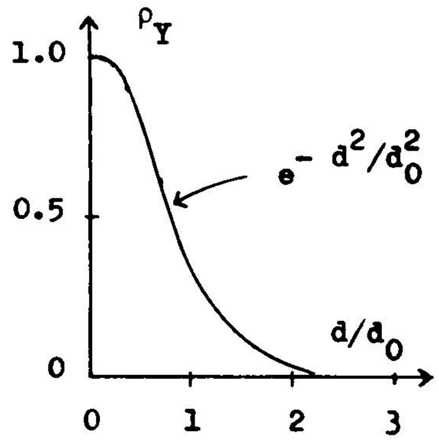


Fig.2.1

With regard to the expression (2.2), firstly it has to be observed that both m_Y and D_Y are supposed herein to be independent of space variables.

Secondly, it has to be noted that D_Y must be sufficiently small by comparison with m_Y ; this is a necessary condition to avoid inaccuracies once the normal probability density for the compressive (non-negative) yield strength has been introduced. As it is clear, being $D_Y = 0.2\ m_Y$ a reasonable value for the standard deviation of concrete yield strength [9, 13] , such a condition can be considered herein as sufficiently satisfied.

In that follows, it will be usefull to remember that, as it is easy to deduce from (2.2), the one-dimensional probability density of Y is

$$p_Y(y) = \int_{-\infty}^{\infty} p_{YY'}(y,y')\ dy' = \frac{\exp\left[-(y-m_Y)^2/2\ D_Y^2\right]}{\sqrt{2\pi}\ D_Y} \tag{2.3}$$

3. ASSUMPTIONS ON THE PROBABILISTIC DISTRIBUTION OF PRINCIPAL STRESSES

As a consequence of the randomness of the mechanical properties characterizing either elastic or inelastic response of dam concrete, also the stress state really exhibits in the dam a peculiar stochastic nature. Thus, also its components have to be described as random functions of spatial coordinates.

In the present analysis, it is assumed that, as a general consequence of all possible causes of material randomness, the principal stresses Σ_i $(i = 1,2,3)$ can be considered as uncorrelated normally distributed random functions with mean values m_{0i} and standard deviations D_{0i} .

Then, their joint probability density can be expressed as

$$p_{\Sigma_1\Sigma_2\Sigma_3}(\sigma_1,\sigma_2,\sigma_3) = p_{\Sigma_1}(\sigma_1)\ p_{\Sigma_2}(\sigma_2)\ p_{\Sigma_3}(\sigma_3) \tag{3.1}$$

where

$$p_{\Sigma_i}(\sigma_i) = \frac{\exp\left[-(\sigma_i-m_{0i})^2/2\ D_{0i}^2\right]}{\sqrt{2\pi}\ D_{0i}} \tag{3.2}$$

is the marginal probability density of Σ_i, and σ_i is its permissible value.

In view of the purpose of defining an adequate probabilistic criterion for the local failure of concrete, it is now convenient to introduce the three so-called cylindrical stress invariants X_1, X_2, X_3, whose permissible values x_1, x_2, x_3 can be expressed in terms of principal stresses as

$$x_1 = \left[\frac{1}{3} \sum_{i>j} (\sigma_i - \sigma_j)^2\right]^{\frac{1}{2}}$$

$$x_2 = \text{arc tan} \frac{\sqrt{3}\,(\sigma_2 - \sigma_3)}{2\,\sigma_1 - \sigma_2 - \sigma_3}$$

$$x_3 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}}$$

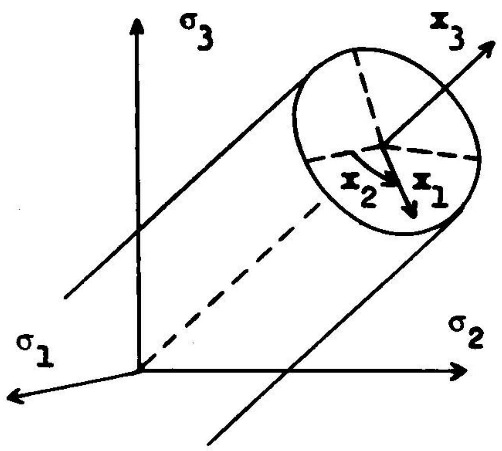


Fig.3.1

As it is well known, such invariants are the cylindrical coordinates of the point having, in the stress space (Fig.3.1), the principal stresses as Cartesian ones.

It has been indicated previously [11, 12] that, by assuming for the sake of simplicity $D_{01} = D_{02} = D_{03} = D$ and setting

$$m_1 = \left[\frac{1}{3} \sum_{i>j} (m_{0i} - m_{0j})^2\right]^{\frac{1}{2}} \qquad (3.3)$$

$$m_2 = \text{arc tan} \sqrt{3}\,(m_{02} - m_{03})(2\,m_{01} - m_{02} - m_{03})^{-1} \qquad (3.4)$$

$$m_3 = (m_{01} + m_{02} + m_{03}) \,/\, \sqrt{3} \quad , \qquad (3.5)$$

the joint probability density of the cylindrical invariants can be written as

$$p_{X_1X_2X_3}(x_1,x_2,x_3) = p_{X_1X_2}(x_1,x_2)\; p_{X_3}(x_3) \qquad (3.6)$$

where

$$p_{X_1X_2}(x_1,x_2) = \frac{1}{2\,\pi\,D^2} \exp\left(-\frac{x_1^2 - 2\,x_1 m_1 \cos(x_2 - m_2) + m_1^2}{2\,D^2}\right) \qquad (3.7)$$

$$p_{X_3}(x_3) = \frac{1}{\sqrt{2\,\pi}\;D} \exp\left(-\frac{(x_3 - m_3)^2}{2\,D^2}\right) \qquad (3.8)$$

With regard to equations from (3.2) to (3.8), it has to be pointed out that, in the general case, m_{0i} and D_{0i}, as well as m_1 and D, are functions depending on the coordinates of space points and also on time (or on a load parameter depending on time).

Furthermore, it is interesting to note that, as it is easy to deduce (i), the marginal probability density of X_1 can be rigorously expressed in the form :

$$p_{X_1}(x_1) = \int_0^{2\pi} p_{X_1X_2}(x_1,x_2)\, dx_2 = \frac{x_1}{D^2} \exp\left(-\frac{x_1^2+m_1^2}{2\,D^2}\right) I_0\left(\frac{x_1 m_1}{D^2}\right) \qquad \ldots (3.9)$$

where

$$I_0(\rho) = \frac{1}{2\pi}\int_0^{2\pi} e^{\rho\cos\mu}\, d\mu$$

is the modified Bessel function of order zero.

In order to express the probability density of X_1 in a simplified manner, more suitable for further elaborations, we now observe that, if D is sufficiently small by comparison with m_1 (and this is that we suppose in the present case), then we can approximately write (ii) :

$$I_0\left(\frac{x_1 m_1}{D^2}\right) \cong \frac{D}{\sqrt{2\pi\, x_1 m_1}} \exp\left(\frac{x_1 m_1}{D^2}\right)$$

Consequently, from (3.9) it is easy to deduce :

$$p_{X_1}(x_1) \cong \sqrt{x_1/m_1}\, \frac{1}{\sqrt{2\pi}\, D} \exp\left(-\frac{(x_1-m_1)^2}{2\,D^2}\right) = \sqrt{x_1/m_1}\; g(x_1,m_1,D) \qquad \ldots (3.10)$$

where $g(x_1,m_1,D)$ is a Gaussian curve.

Therefore, because of the factor $\sqrt{x_1/m_1}$, the approximate expression (3.10) of the probability density of X_1 differs from a Gaussian curve in such a way that, as shown in Fig.3.2, we have $p_{X_1}(x_1) \lessgtr g(x_1,m_1,D)$ in accordance with the value of $x_1 \lessgtr m_1$.

In this connection, it has to be pointed out that, in the range $x_1 >> m_1$ (which is the most interesting from the point of view of the safety analysis), the effect produced by the factor $\sqrt{x_1/m_1}$ is similar to that caused by an increase of the standard deviation of $g(x_1,m_1,D)$.

(i) See, for instance, A. Papoulis, "Probability, Random Variables, and Stochastic Processes", MacGraw-Hill Book Company, New York, 1965, pp.195-196.

(ii) See, for instance, M. Boll, "Tables numériques universelles", Dunod, Paris, 1957, p.739.

Thus, taking into account the considerable uncertainty of the true value of D and in view of several practical advantages, it seems reasonable to adopt, as a possible alternative approximate expression of the probability density of X_1 , the Gaussian form :

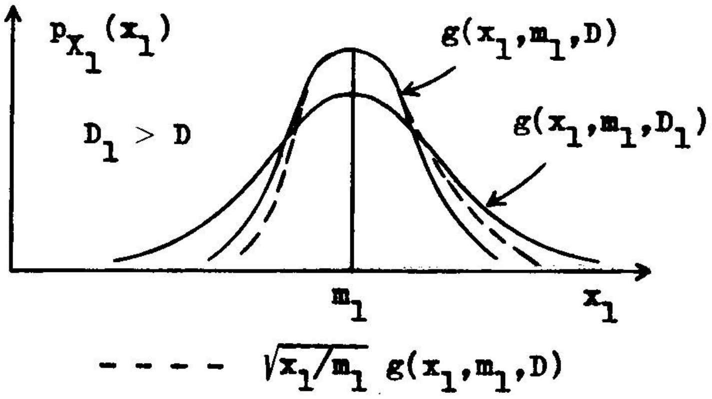


Fig.3.2

$$p_{X_1}(x_1) \cong \frac{1}{\sqrt{2\pi}\, D_1} \exp\left(-\frac{(x_1-m_1)^2}{2\, D_1^2}\right) \qquad (3.11)$$

in which a conveniently selected value $D_1 > D$ is introduced for the standard deviation.

4. PROBABILITY OF FUNCTIONAL DAM FAILURE

For simplicity, as failure surface for plain concrete under complex states of stress, we adopt in the present note a circular cone having the axis coinciding, in the space of principal stresses, with the x_3-axis (Fig.4.1). Then, such a failure surface can be expressed by the relation

$$X_1 + \sqrt{2}\,(1-\alpha)\,X_3 = \sqrt{2/3}\,\alpha\,Y \qquad (4.1)$$

in which α is a deterministic constant coefficient depending upon the material.

In connection with (4.1), it is convenient to observe that such an equation can be regarded either as a particular case (in probabilistic version) of the more general (deterministic) yield surface described in [14, 15] or as a consistent generalization of the well known experimental formula (obtained by Richart, Brantzaeg and Brown, and recently discussed in [16]) for predicting ultimate load capacity of triaxially loaded concrete

$$\sigma' = m_Y + 4.1\,\sigma'' \qquad (4.2)$$

in which σ' is the axial (compressive) stress at failure and σ'' the lateral pressure.

In order to obtain that (4.1) can coincide with (4.2) for the case of the triaxial state of stress, it is easy to deduce that α must be approximately equal to 1/2 .

Let us now denote by $B = B_x \cup B_y$ the union of the region B_x of the stress space, containing all the points with coordinates x_1 , x_2 , x_3 such that

$$x_1 + \sqrt{2}(1-\alpha)\, x_3 < \sqrt{2/3}\; \alpha\, y \qquad \text{and} \qquad 0 \leqq x_2 \leqq 2\pi \quad ,$$

with the set B_y containing all permissible values $0 \leqq y \leqq \infty$ of Y .

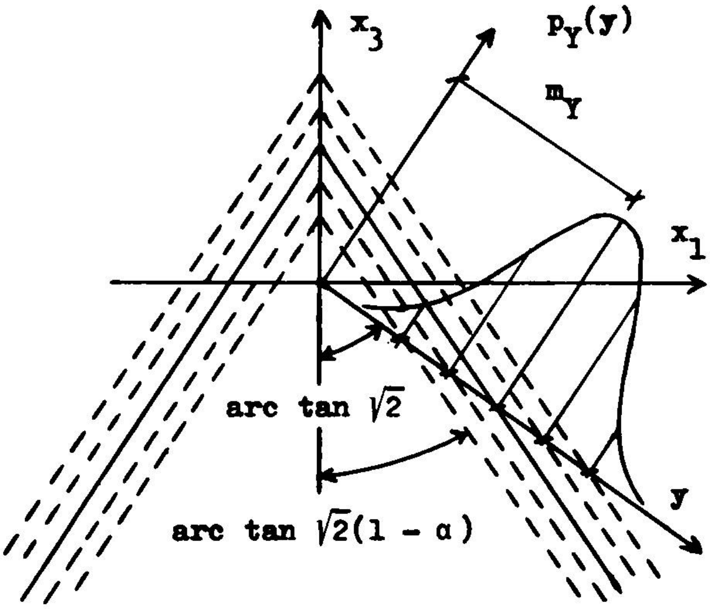


Fig.4.1

Therefore, the probability P_E of the elastic dam survival, without reaching in any material point the conventional limit state (4.1) of functional (or local) failure, is evidently the probability of the event that the random point with coordinates X_1, X_2, X_3 and Y occurs in the set B . As well known, P_E can be then expressed as the integral of the joint probability density $p_{X_1X_2X_3Y}(x_1,x_2,x_3,y)$ of X_1 , X_2 , X_3 , Y taken over B :

$$P_E = \Pr\Big[(X_1,X_2,X_3,Y) \in B\Big] = \int_B p_{X_1X_2X_3Y}\, dx_1 dx_2 dx_3 dy \qquad (4.3)$$

and the functional failure probability $P_F = 1 - P_E$ can be explicitly written as

$$P_F = 1 - \int_0^{\infty} dy \int_{-\infty}^{h_0 y} dx_3 \int_0^{2\pi} dx_2 \int_0^{h(x_3,y)} p_{X_1X_2X_3Y}\, dx_1 \qquad (4.4)$$

where $h_0 = \alpha/\sqrt{3}(1-\alpha)$ and $h(x_3,y) = \sqrt{2/3}\,\alpha y - \sqrt{2}(1-\alpha)\, x_3$.

Assuming for simplicity the stress state to be stochastically independent of the local yield strength Y (an analogous hypothesis, even if somewhat debatable, is habitually employed in structural safety analysis), namely

$$p_{X_1X_2X_3Y}(x_1,x_2,x_3,y) = p_{X_1X_2X_3}(x_1,x_2,x_3)\; p_Y(y) \quad , \qquad (4.5)$$

and taking into account equations (3.6) and (3.9), it is easy to recognize that expression (4.4) can be rewritten as

$$P_F = 1 - \int_0^{\infty} p_Y(y)\, dy \int_{-\infty}^{h_0 y} p_{X_3}(x_3)\, dx_3 \int_0^{h(x_3,y)} p_{X_1}(x_1)\, dx_1 \qquad (4.6)$$

Equation (4.4) unrestrictedly, and equation (4.6) subordinately to assumption (4.5), provide a rigorous expression of the functional failure probability P_F .

In order to obtain an alternative simplified expression of such a probability, more suitable for rapid calculations, we now consider a new random variable Z defined as

$$Z = X_1 + \sqrt{2}(1-\alpha)\, X_3 - \sqrt{2/3}\ \alpha\, Y \qquad (4.7)$$

Assuming as probability densities of X_1 , X_3 and Y the Gaussian ones expressed by relations (3.11), (3.8) and (2.3), respectively, and supposing such relations to be valid for every value of x_1 , x_3 and y included between $-\infty$ and $+\infty$ (this requires, to avoid inaccuracies, $D_1 \ll m_1$ and $D_Y \ll m_Y$, being X_1 and Y both essentially non-negative), also the probability density of Z is Gaussian :

$$p_Z(z) = \frac{1}{\sqrt{2\pi}\, D_Z} \exp\left(-\frac{(z-m_Z)^2}{2\, D_Z^2}\right) \qquad (4.8)$$

Moreover, the mean value m_Z and dispersion D_Z^2 of Z can be expressed, respectively, in terms of m_1 , m_3 , m_Y and D_1 , $D_3 = D$, D_Y as

$$m_Z = m_1 + \sqrt{2}(1-\alpha)\ m_3 - \sqrt{2/3}\ \alpha\ m_Y \qquad (4.9)$$

$$D_Z^2 = D_1^2 + 2(1-\alpha)^2 D_3^2 + \frac{2}{3}\alpha^2 D_Y^2 \qquad (4.10)$$

Being in virtue of (4.1) and (4.7)

$$P_E = 1 - P_F = \Pr(Z < 0) = \int_{-\infty}^{0} p_Z(z)\, dz \quad ,$$

it is then immediate to deduce :

$$P_E = \frac{1}{2} - \Phi(m_Z/D_Z) \quad , \quad P_F = \frac{1}{2} + \Phi(m_Z/D_Z) \qquad (4.11)$$

where

$$\Phi(\rho) = \frac{1}{\sqrt{2\pi}} \int_0^{\rho} e^{-\mu^2/2}\, d\mu$$

is a well known error-type function, for which exhaustive numerical tables are available.

5. APPLICATION TO A DAM WITH A SCHEMATIC STRESS DISTRIBUTION

For the purpose of exemplification, let us consider a typical cylindrical arch-gravity dam, having a cross-section such as that shown in Fig.5.1.

Denoting by H the height of the dam and by R the radius of its upstream face, the vertical variation of the thickness t, which has been considered herein, is such that

$$\frac{t}{R} = 1 - \sqrt{1 - a\,\frac{z}{H}} \qquad (5.1)$$

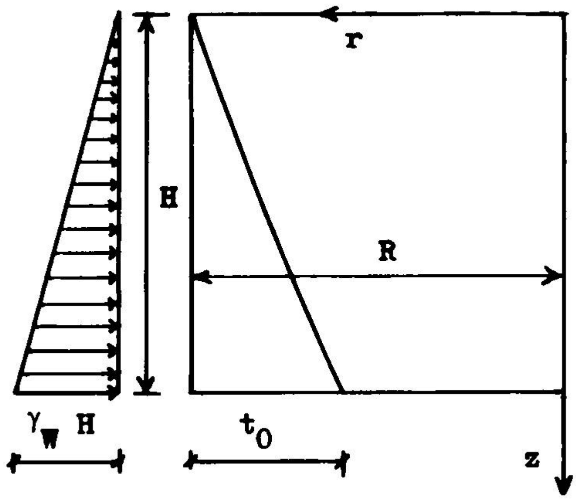


Fig.5.1

where $a = (2 - t_0/R)t_0/R$ and t_0 is the thickness at $z = H$ (Fig.5.1).

It has to be pointed out that if, as this is frequently the case, t_0/R is small enough by comparison with the unit, then (5.1) can be linearized and t can be approximately expressed as

$$t \cong t_0\,\frac{z}{H} = b\,z \qquad (5.2)$$

For the simplicity of exemplification, we assume in this section that in the deterministic average elastic case (i.e., in the case in which the elastic moduli of dam concrete are defined as the average values of the actual ones) the elastic three-dimensional stress state of the dam can be schematically represented by means of a stress tensor having the following non-zero physical components

$$\begin{aligned}
\sigma_r &= -\,\sigma_A\,\frac{1-\varphi}{a} - \sigma_B\,\frac{z}{H}\\
\sigma_\theta &= -\,\sigma_A\,\frac{1+\varphi}{a}\\
\sigma_z &= \sigma_B\left(\frac{z}{H} - 2\psi\right)b^{-2} - \sigma_C\left(\frac{z}{H} - \psi\right)\\
\tau_{rz} &= \sigma_B\,\frac{\psi}{b}
\end{aligned} \qquad (5.3)$$

in which we have introduced the notations

$$\sigma_A = \gamma_W\,H\,(1-\omega) \quad , \quad \sigma_B = \gamma_W\,H\,\omega \quad , \quad \sigma_C = \gamma_C\,H$$

$$\varphi = (1 - a\,z/H)(R/r)^2 \quad , \quad \psi = (R - r)/t_0$$

where γ_W is the specific weight of the water, γ_C is the one of the concrete, and ω is a repartition coefficient for the load taken by the arch action and by the cantilever one.

It is worthwhile to mention, in connection with (5.3), that the case $\omega = \gamma_C = 0$ (for which we have $\sigma_A = \gamma_W H$ and $\sigma_B = \sigma_C = 0$) corresponds to an exact three-dimensional axially symmetric solution, obtained in a previous paper [17] , for a cylindrical tank having thickness variation similar to one expressed by (5.1) and subjected to only hydrostatic pressure.

On the contrary, the case $\omega = 1$ (which corresponds to $\sigma_A = 0$, $\sigma_B = \gamma_W H$ and $\sigma_C = \gamma_C H$) represents the well known Lèvy's solution for gravity dams, having thickness variation similar to one expressed by (5.2) and subjected both to hydrostatic pressure and to dead load.

Moreover, it has to be observed that, as it is well known, the Lèvy's solution for dead load can be regarded as an almost exact solution also for a cylindrical arch-gravity dam, in consequence of the fact that such a dam is normally built of blocks separated by vertical joints, which are usually open during the course of construction.

For the purpose of demonstration, and in order to avoid further complications, we suppose at this point that, in evaluating m_1 and m_3 by means of (3.3) and (3.5), respectively, the mean values m_{01} , m_{02} and m_{03} can be calculated as principal values of the stresses (5.3) :

$$
\begin{aligned}
m_{01} &= \frac{1}{2}(\sigma_r + \sigma_z) + \frac{1}{2}\left[(\sigma_r - \sigma_z)^2 + 4\tau_{rz}^2\right]^{1/2} \\
m_{02} &= \sigma_\theta \\
m_{03} &= \frac{1}{2}(\sigma_r + \sigma_z) - \frac{1}{2}\left[(\sigma_r - \sigma_z)^2 + 4\tau_{rz}^2\right]^{1/2}
\end{aligned}
\qquad (5.4)
$$

Then, if, in view of simplifying numerical computations, we assume $a = b = 1/2$ and $\gamma_C/\gamma_W = 2.4$, from equations (5.3), (5.4), (3.3) and (3.5) we can deduce what follows.

1. <u>At upstream face</u> (for which we have $r = R$, $\varphi = 1 - a\frac{z}{H}$ and $\psi = 0$), the mean values m_1 and m_3 are

$$
\begin{aligned}
m_1 &= \sqrt{2/3}\,\gamma_W H\left[C_0 + C_1(z/H) + C_2(z/H)^2\right]^{1/2} \\
m_3 &= -\sqrt{1/3}\,\gamma_W H\left[C_3 + C_4(z/H)\right]
\end{aligned}
\qquad (5.5)
$$

where

$$C_0 = 16(1-\omega)^2 \quad , \quad C_1 = -2.4(1-\omega)(9 - 10\omega) \quad ,$$

$$C_2 = 0.12(73 - 205\omega + 175\omega^2) \quad ,$$

$$C_3 = 4(1-\omega) \quad , \quad C_4 = 2.4 - 3\omega$$

2. <u>At downstream face</u> (for which we have $r = R - t$, $\varphi = 1$ and $\psi = z/H$), m_1 and m_3 can be written as

$$
\begin{aligned}
m_1 &= \sqrt{2/3}\,\gamma_W H\left[D_0 + D_1(z/H) + D_2(z/H)^2\right]^{1/2} \\
m_3 &= -\sqrt{1/3}\,\gamma_W H\left[D_3 + D_4(z/H)\right]
\end{aligned}
\qquad (5.6)
$$

where

$D_0 = 16\ (1-\omega)^2 = C_0\ ,\quad D_1 = -\ 20\,\omega\,(1-\omega)\ ,\quad D_2 = 25\,\omega^2\ ,$

$D_3 = 4\ (1-\omega) = C_3\ ,\quad D_4 = 5\,\omega$

If, in addition, we suppose for simplicity $\alpha = 1/2$ and $\beta = m_1/D_1 = m_3/D_3 = m_Y/D_Y$, then, taking into account relations (5.5), (5.6), (4.9) and (4.10) and setting $\eta = \gamma_W H/m_Y$, we can write the failure probability P_F , previously expressed by the second of relations (4.11), in the form :

$$P_F = \frac{1}{2} - \Phi\left(\beta\,\frac{1-\eta\ (2\,M-N)}{\sqrt{1+\eta^2(4\,M^2+N^2)}}\right) \tag{5.7}$$

where, in the case of the upstream face, one must assume

$M^2 = C_0 + C_1\ z/H + C_2(z/H)^2$ and $N = C_3 + C_4\ z/H$,

while, in the case of the downstream one, it has to be assumed

$M^2 = D_0 + D_1\ z/H + D_2(z/H)^2$ and $N = D_3 + D_4\ z/H$.

In Fig.5.2, to illustrate the foregoing results, curves $P_F = P_F(z/H)$, deduced by (5.7) for $\beta = 4$ and $\eta = 1/40$, are plotted for $r = R$ (upstream face) and for $\omega = 0$, $1/4$, $1/2$, $3/4$, 1 .

In Fig.5.3, analogous curves are plotted for the case of the downstream face.

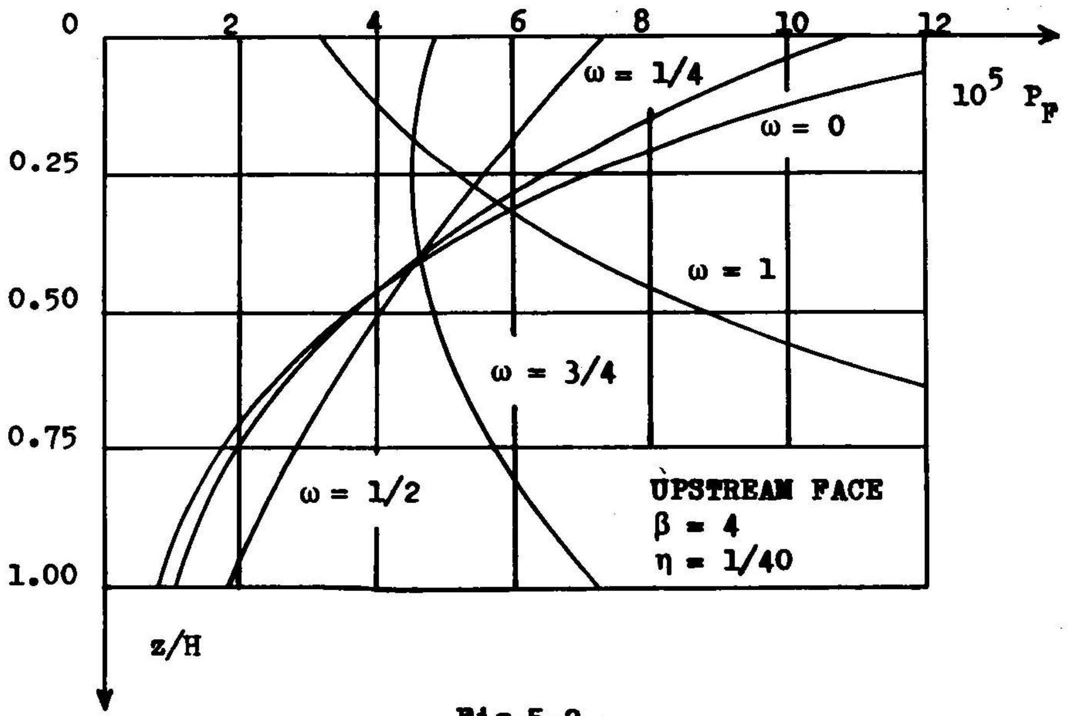


Fig.5.2

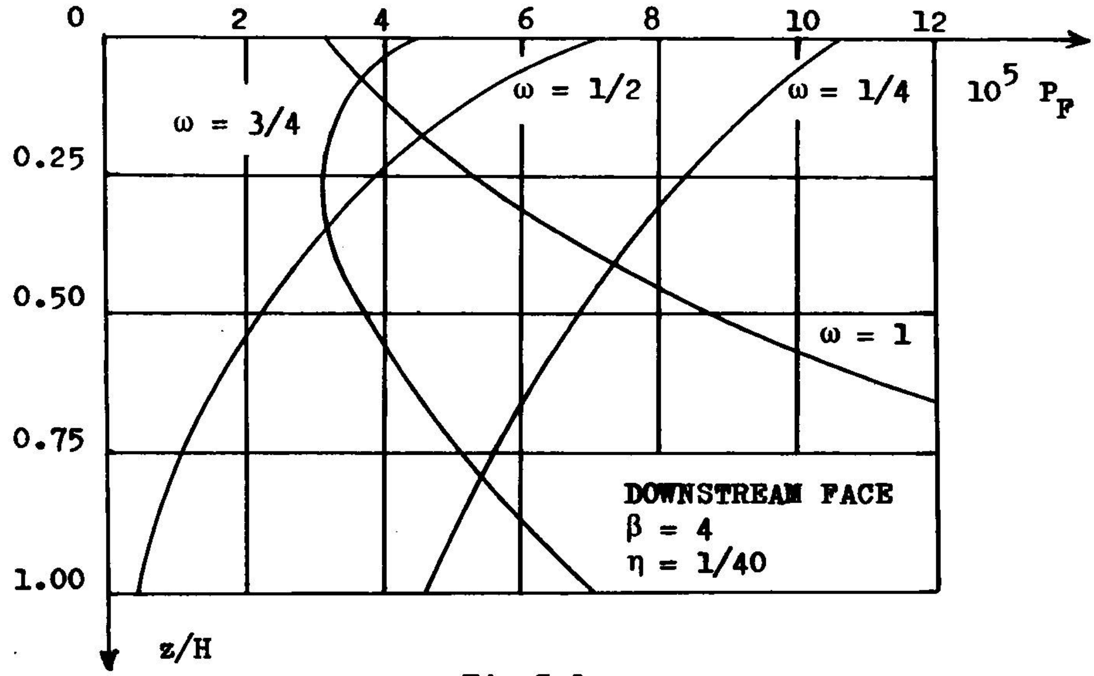


Fig.5.3

6. PROBABILISTIC FORMULATION OF A METHOD FOR OBTAINING A LOWER BOUND FOR THE DAM COLLAPSE LOAD

As it has already been discussed in several previous papers, lower and upper bounds for the average value and dispersion of ultimate load [18, 19, 20] of a structure with random yield strength, as well as for the probability of its collapse failure [21, 22, 23, 24, 25, 26] , can be found by employing the fundamental theorems of the limit analysis theory.

In the present section, defering to a later paper the study concerning the upper bound (it has to be noted that a research on such a bound, from the deterministic point of view, has already been developed in [27] for the case of arch dams), we deal only with a probabilistic formulation of a method for evaluating a lower bound, as large as possible, of the maximum hydrostatic-type load that an arch-gravity dam can sustain.

Such a method, whose deterministic formulation has previously been established in [10] with regard to the case of pure arch dams, is founded on the limit analysis theorem which states that a statically admissible field of stresses and forces in equilibrium defines a lower bound for the ultimate load that a structure can carry.

From the mathematical point of view, as it will be explained in what follows, a stochastic linear programming problem (with deterministic objective function and random constraints) arises from this procedure.

For the solution of such a problem, appropriate computations by means of a recent technique of digital simulation (with random generation of normal variables) are now in progress and will be presented as soon as possible.

Discretization of the structure. Let us consider the typical dam portion included between diametral planes $\theta = \theta_1$ and $\theta = \theta_2$, shown in Fig.6.1. Any horizontal cross-section of such a portion (cantilever) exhibits, at the upstream side, constant breadth

$$c = R\,(\theta_2 - \theta_1) \quad (6.1)$$

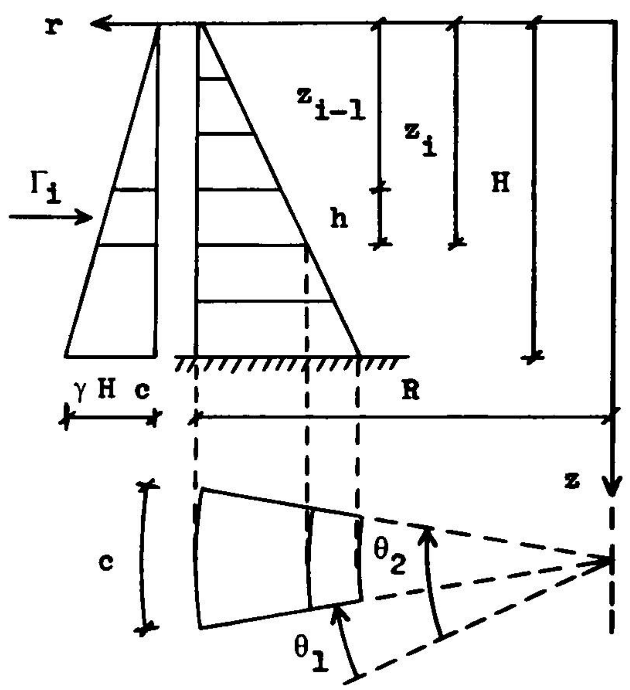


Fig.6.1

As indicated in Fig.6.1, let us divide the whole cantilever into n finite elements having the same height $h = H/n$, and denote by $z_i = i\,h$ $(i = 1,2,..,n)$ the value of the coordinate z at elevation i.

Thus, the element i is included between the horizontal cross-sections $z = z_{i-1}$ and $z = z_i$.

If the upstream face of the dam is subjected in every point to the hydrostatic pressure γz, the horizontal force applied on the i-th element is then the following (Fig.6.2) :

$$\Gamma_i = 2 \int_0^{(\theta_2-\theta_1)/2} \gamma\,(z_i - h/2)\, h\, R \cos\mu \; d\mu$$

and, if the angle $\theta_2 - \theta_1$ is assumed to be small enough, we can suppose approximately

$$\Gamma_i \cong \gamma\,(2\, z_i - h)\, \frac{h\, c}{2} \qquad (6.2)$$

Disregarding for simplicity torsional action of the dam, we now denote by p_i the part of Γ_i taken by arch action, and by q_i the one taken by cantilever action. As a consequence of this, we can rewrite (6.2) as

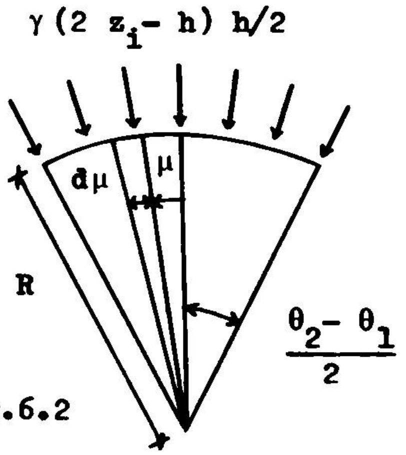


Fig.6.2

$$\gamma = \frac{2\,(p_i + q_i)}{h\,c(2\,z_i - h)}$$

Taking into account the fact that, for each possible value of i, we have

$$\frac{p_i + q_i}{2\,z_i - h} = \frac{p_n + q_n}{2\,z_n - h}\,, \qquad (6.3)$$

we can express γ also in the form

$$\gamma = \frac{2}{h\,c\,n} \sum_{i=1}^{n} \frac{p_i + q_i}{2\,z_i - h} \qquad (6.4)$$

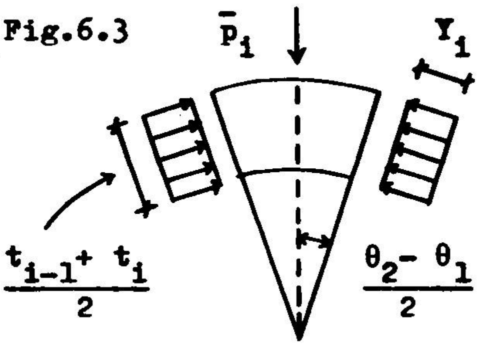
<u>Ultimate load capacity of arches</u>. Indicating by t_i the thickness of the dam wall at horizontal section i and by

$$A_i = (t_{i-1} + t_i)\,h/2$$

the area of the cross-section of the i-th arch, we now assume that i-th arch failure occurs when in the whole area A_i the mean nominal stress $\sigma_{\theta i}$ reaches the yield value (of random nature) :

$$Y_i = \frac{1}{A_i} \int_{A_i} Y\,dA_i \qquad (6.5)$$

Fig.6.3



Neglecting, for the sake of simplicity, the tensile strength of dam concrete and denoting by $\bar{p}_i$ the maximum part of Γ_i which can be carried by the i-th arch, we can write

$$0 \leq p_i \leq \bar{p}_i \qquad (6.6)$$

where (Fig.6.3) :

$$\bar{p}_i = 2\,Y_i\,A_i\,\sin\,(\theta_2 - \theta_1)/2 \cong Y_i\,A_i\,c/R \qquad (6.7)$$

<u>Ultimate bending capacity of cantilevers</u>. Indicating by a_i' the depth and by

$$A_i' = a_i'\,(R - t_i + a_i'/2)\,c/R$$

the area of the compressive zone at the downstream side of the i-th horizontal section (Fig.6.4), it is assumed that the cantilever failure at section i, due to the positive limit bending moment Q_i', occurs when in all point of the area A_i' the mean nominal stress σ_{zi} reaches the random yield value

$$Y_i' = \frac{1}{A_i'} \int_{A_i'} Y\,dA_i' \qquad (6.8)$$

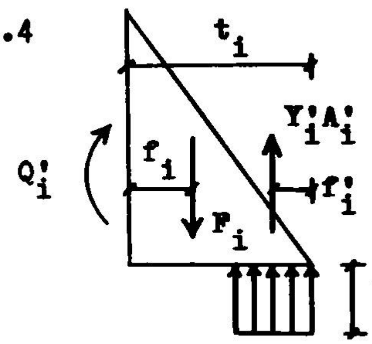
In a perfectly analogous way, denoting by a''_i and

$$A''_i = a''_i\,(R - a''_i/2)\;c/R$$

the depth and the area, respectively, of the compressive zone at the upstream side of the same section i (Fig.6.5), it is also assumed that the cantilever failure, due to the negative limit moment Q''_i, occurs when in all points of the area A''_i the mean nominal stress σ_{zi} reaches the yield value

$$Y''_i = \frac{1}{A''_i}\int_{A''_i} Y\,dA''_i \qquad (6.9)$$

Fig.6.4



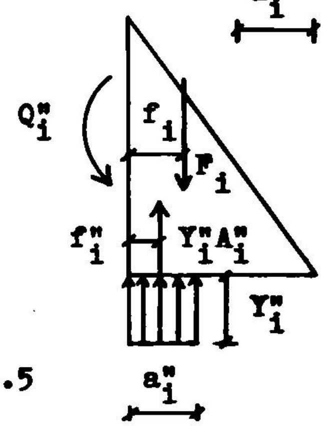


Fig.6.5

If we now denote by

$$F_i = Y'_i\,A'_i = Y''_i\,A''_i \qquad (6.10)$$

the (deterministic) vertical force applied on the section i (due to dead weight), by

$$f_i = t_i/3$$

its distance from the upstream face, and by

$$f'_i = \frac{a'_i}{2}\left(1 + \frac{1}{6}\,\frac{a'_i}{R - t_i + a'_i/2}\right)$$

$$f''_i = \frac{a''_i}{2}\left(1 - \frac{1}{6}\,\frac{a''_i}{R - a''_i/2}\right)$$

the distances of the forces $Y'_i\,A'_i$ and $Y''_i\,A''_i$ from downstream and upstream face, respectively, it is easy to obtain, from figures 6.4 and 6.5, the expressions :

$$Q'_i = F_i\,(t_i - f_i - f'_i) \cong F_i\left(\frac{2}{3}\,t_i - \frac{1}{2}\,a'_i\right) \qquad (6.11)$$

$$Q''_i = -F_i\,(f_i - f''_i) \cong -F_i\left(\frac{1}{3}\,t_i - \frac{1}{2}\,a''_i\right) \qquad (6.12)$$

in which Q'_i and Q''_i, as a consequence of the randomness of a'_i and a''_i produced by (6.10), are also random quantities.

In view to determine the maximum part $\bar{q}'_i$ and the minimum one $\bar{q}''_i$ of Γ_i which can be carried by the cantilever, it is now convenient to remember that the bending moment Q_i applied on the i-th horizontal section can be expressed in the form (Fig.6.6)

$$Q_i = \sum_{j=1}^{i} (z_i - z_j + h/2)\,q_j \qquad (6.13)$$

Consequently, denoting by $\{Q_i\}$ and $\{q_j\}$ two column vectors having as components the bending moments Q_i and the cantilever loads q_j, respectively, and by $[Q_{ij}]$ the low triangular matrix whose elements are

$$Q_{ij} = \begin{cases} 0 & \text{if} \quad i < j \\ z_i - z_j + h/2 & \text{if} \quad i \geq j \end{cases} ,$$

it is easy to recognize that (6.13) can be rewritten in matrix notation as

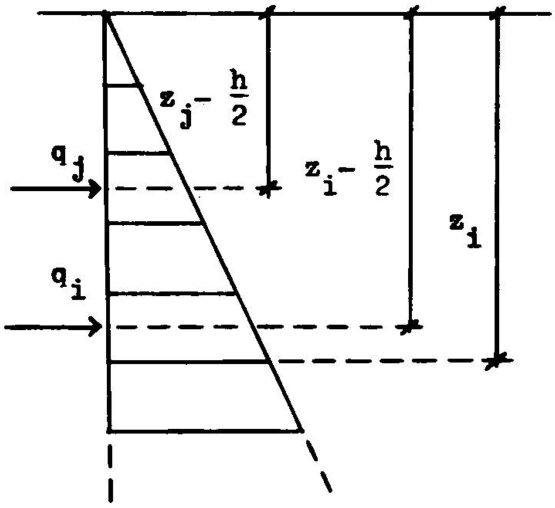


Fig.6.6

$$\{Q_i\} = [Q_{ij}]\{q_j\} \qquad (6.14)$$

or also in the alternative form

$$\{q_i\} = [Q_{ij}]^{-1}\{Q_j\} \qquad (6.15)$$

in which $[Q_{ij}]^{-1}$ indicates the inverse of $[Q_{ij}]$.

If, in addition, we denote by $\{Q_i'\}$ and $\{Q_i''\}$ the column vectors of the limit bending moments, we have

$$\{Q_i''\} \leq \{Q_i\} \leq \{Q_i'\} \qquad (6.16)$$

or also

$$[Q_{ij}]^{-1}\{Q_j''\} \leq \{q_i\} \leq [Q_{ij}]^{-1}\{Q_j'\} \qquad (6.17)$$

By setting for the sake of convenience

$$\{\bar{q}_i'\} = [Q_{ij}]^{-1}\{Q_j'\} \qquad \text{and} \qquad \{\bar{q}_i''\} = -[Q_{ij}]^{-1}\{Q_j''\} ,$$

inequality (6.17) can be apparently rewritten as

$$-\bar{q}_i'' \leq q_i \leq \bar{q}_i' \qquad (6.18)$$

Lower bound of dam collapse load. In virtue of foregoing considerations, the problem of finding a lower bound for the dam collapse load can be reduced to determining the vectors

$$\{p_i\} \text{ and } \{q_i\}$$

which maximize the specific weight γ, expressed by the (deterministic) objective function (6.4), subjected to the random constraints (6.6) and (6.18), and to the additional (deterministic) restriction (6.3).

It has to be noted that, in order to obtain that all the variables are positive, it is evidently sufficient to assume, instead of q_i, the new variable $q_i^* = q_i + \bar{q}_i''$ and to rewrite (6.18) as

$$0 \leq q_i^* \leq \bar{q}_i' + \bar{q}_i'' .$$

<u>Mean value and dispersion of</u> Y_i . Because of space limitations, which preclude adequate discussion upon the random yield resistances Y_i , Y_i' , Y_i'' previously introduced, we are now concerned with only two short considerations on the arch resistances Y_i , and we defer to a later work a more extensive treatment of the matter.

In connection with Y_i , it has firstly to be pointed out that, being by assumption m_Y independent of space coordinates, it is immediate to obtain (by commutating the operations of mathematical expectation and integration) that

$$m_{Y_i} = E\left[\frac{1}{A_i}\int_{A_i} Y\, dA_i\right] = \frac{1}{A_i}\int_{A_i} E\left[Y\right] dA_i = m_Y$$

Namely, the mean value of Y_i is independent of A_i .

Secondly, remembering that, as well as m_Y , also deviation D_Y is independent of space points, we observe that the dispersion of Y_i can be written in the form :

$$D^2_{Y_i} = \left(\frac{D_Y}{A_i}\right)^2 \int_{A_i}\int_{A_i} \rho_Y\, dA_i\, dA_i \tag{6.19}$$

in which, by virtue of (2.1), the normalized auto-correlation function ρ_Y can be expressed (being $\theta = \theta'$ for all points of A_i) as

$$\rho_Y = \exp\left(-\frac{(r - r')^2 + (z - z')^2}{d_0^2}\right) \tag{6.20}$$

Inserting (6.20) into (6.19) leads to

$$D^2_{Y_i} = \left(\frac{D_Y}{A_i}\right)^2 \int_{z_{i-1}}^{z_i}\int_{z_{i-1}}^{z_i} \exp\left(-\frac{(z-z')^2}{d_0^2}\right) K(z,d_0)\, dz\, dz'$$

where

$$K(z,d_0) = \int_{R-bz}^{R}\int_{R-bz}^{R} \exp\left(-\frac{(r-r')^2}{d_0^2}\right) dr\, dr'$$

can be easily expressed in terms of the error function.

7. CONCLUSIONS

Probability of local dam failure was rigorously investigated in the general case. An approximate formula for determining such a probability was obtained and numerically applied to the particular case of a typical arch-gravity dam with a schematic three-dimensional stress distribution.

In connection with the structural dam failure, a probabilistic formulation of a previous deterministic method for obtaining a lower bound of the ultimate load that the dam can carry was presented.

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ZUSAMMENFASSUNG

Man hat versucht die kürzlich erreichten Begriffe der Wahrscheinlichkeitsanalyse für die Sicherheit einer Struktur bei einer Gewölbe-Gewichtsperre aus Beton anzuwenden, idem die Vermutung gemacht wurde, dass diese als "Random"-Mittel betrachtet werden könnte, dessen mechanische Eingenschaften (elastische Module und ötliche Elastizitätsgrenze) als stokastische Funktionen der Raumkoordinaten beschrieben werden können.

Insbesonders wurde die örtliche Bruchwahrscheinlikeit des Staudammes berechnet und, angesichts der Wahrscheinlichkeit, wurde eine massgebende Methode, die schon früher für die Bewertung einer Mindesgrenze der Bruchbelastung eines Staudammes gebraucht, neu formuliert und für den Fall der Gewölbe-Gewichtsperren angeeignet.

SUMMARY

Assuming that an arch-gravity concrete dam can be regarded as a random medium, whose mechanical properties (elastic moduli and local yield strength) can be described as stochastic functions of space coordinates, an attempt is made to apply recent concepts of probabilistic safety analysis to such a structure.

In particular, the probability of local dam failure is evaluated, and a previous deterministic method for obtaining a lower bound for the collapse load of pure arch dams is reformulated from the probabilistic point of view, and adapted to arch-gravity ones.

RESUME

En l'hypothèse de considérer un barrage poids-voûte comme un milieu random dont les proprietés mécaniques (module et limite locale d'élasticité) peuvent se considérer comme fonctions stochastiques des coordonnées spatiales, on a essayé l'application à une structure de telle sorte de quelques récents concepts de l'analyse probabilistique de la sécurité.

En particulier, on a calculé la probabilité de rupture locale du barrage, et on a modifié, en l'adaptant à la considération des barrages poids-voûte sous le point de vue probabilistique, une méthode déterministe déja formulée pour l'evaluation d'une limite inférieure de la charge de ruine d'un barrage-voûte.