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Elaboration of triaxial extensometer data on arch dams

*Elaboration des données d'extensomètre à trois dimensions
placées en des barrages voûte*

Verarbeitung triaxialer Dehnungsmessungen in Bogenmauern

ENEL - Direzione delle Costruzioni (*)

Roma - Italia

1) GENERAL

Only two of ENEL's arch and gravity-arch dams now in service, the Vajont and the Val Gallina dams, are equipped with a set of triaxial strain gauges in addition to ordinary instrumentation. The Place Moulin dam on the river Buthier, Val d'Aosta, is also equipped with four sets, each of which consists of seven strain gauges. But since the instruments were installed only in the radial and the tangential planes, the measurements cannot be very useful in determining stresses in three dimensions.

Both the Vajont and the Val Gallina dams are part of the Piave-Boite-Vajont hydroelectric development.

The Vajont dam, which spans a deep gorge immediately upstream of the confluence of the river Vajont and the river Piave, was completed in 1961. It is 261.5 m high, its thickness at the crown section varying from 21.50 m at the base to 3.40 m at the top.

The Val Gallina dam, which was constructed in a mountainous formation on the left bank of the river Piave, forms a weekly compensating reservoir for the regulation of the water used by the Soverzene power station, and is therefore subject to frequent, though partial, fill-ups and

(*) A. REBAUDI, former Director of «Centro Progettazione e Costruzione Idraulica ed Elettrica» - Venice

drawdowns. The structure, which was completed in 1952, is 92 m high. Its thickness at the crown section varies from 15.24 m at the base to 2.54 m at the top.

The two triaxial sets were installed along the centre lines of the crown sections of the two dams. The set in the Vaiont dam is located at one third of the total height of the structure where thickness is 14.00 m. The set in the Val Gallina dam is located in the upper part of the plug where thickness is 15.00 m.

Whereas the positions of the two sets in the structures are almost analogous, the layout and the sensitivity of the instruments vary considerably, which results in the measurements having a different degree of reliability.

2) Layout of the strain gauges in the sets

The Val Gallina set consists of six electroacoustic strain gauges, three of which lie on the edges of a cube while the other three are inclined in the direction of the diagonals of the faces.

Thus the number of instruments is the least required for the determination of stresses in three dimensions.

The Vaiont set consists of seven electroacoustic strain gauges, three of which lie on the edges of a cube while the other four are inclined in the direction of the diagonals.

This particular layout enables an immediate check to be made of the theoretical condition requiring the sum of the stresses on the axes to be equal to $3/4$ of the sum of the stresses on the diagonals.

For this reason as well as for the higher sensitivity of the instruments, the information obtained from the Vaiont strain gauge set was handled before the data from the Val Gallina strain gauge set, to which all the devices that emerged in the process were equally applied.

3) Theoretical determination of the results

The data furnished by the two strain gauge sets continuously in time were handled following the procedures suggested by the theory of elasticity for the purpose of determining:

- the principal stresses in three dimensions and their time variations;
- the secondary stresses in the three planes and their time variations.

By these procedures (1) the above results can generally be achieved only after the stresses σ_x σ_y and σ_z on the coordinate axes and the shear stresses τ_{xy} , τ_{yz} and τ_{zx} have been determined. In the present case these six values were derived from the information obtained from the strain gauges of known orientation by application of the necessary geometric properties. (2)

4) Adaptation of measurements to theory

As strain gauges are instrument for measuring lengths and not stresses, the corresponding $\frac{\sigma}{E}$ were substituted for the stresses σ and τ . These values were derived from the strains Δl taking into account lateral contraction for which Poisson's ratio was assumed to be equal to 0.2.

This substitution will be implied whenever the stresses σ or τ are mentioned hereunder.

In expressing the change in length Δl the substitution of lengths for stresses is possible if $\Delta l = l_t - l_0 - \Delta l_c$ where l_t = length measured at time t ;

l_0 = length at zero stress:

Δl_c = change in length due to anelastic phenomena.

These phenomena include shrinkage, free thermal expansion and thermal effect on instruments, which are generally comprised in the measurements made with a specimen instrument incorporated in a small concrete block "isolated" from the structure.

With regard to the values of l_0 , they are necessary for determining the absolute origin of magnitudes and consequently the meaning of non-linear functions, especially of angles.

When determining these values no direct reference can be made to the measurements carried out in the initial stage when anelastic and especially non-isotropic changes occur which practically invalidate the measurements. This initial anelastic stage, however, was considered through examination of the daily data starting from the time of installation of the instruments for the purpose of establishing when the elastic phenomenon had begun. Admittedly, if a phenomenon is elastic and its

- (1) Obert Duvall: Rock Mechanics and the Designer of Structures in Rock - "Whiley & Sons".
- (2) C.Bellucci: Determination of Strains by Means of a Triaxial Strain Gauge Rosette. Energia Elettrica, Vol. XL, 1963

anelastic components are isotropic the value $\Delta s = \sum \Delta l$ axes - $\frac{3}{4} \sum \Delta l$ inclined gauges, should equal zero or, if the values of Δl_0 are not known, should remain constant in time. (*)

The period immediately following the setting-in of the elastic phenomenon is characterized by significant load variations due to the concreting of the overlying lifts and by measurement variations that can be correlated with the load.

Based on these correlations and assuming the existence of a single force vertically directed (a reasonable assumption considering the pronounced verticality of the part of structure that was concreted during the short period under investigation) it was eventually possible to determine the values of Δl_0 .

With these values the phenomenon investigated will conform to theory if the values of Δs defined above remain equal to zero.

Yet, considering the experimental nature of the data, random and systematic errors might be encountered in time. Whereas random errors are due to measurement errors inherent in the instrumentation, systematic errors can be imputed to non-isotropic anelastic phenomena. In this respect, it should be kept in mind that, being applied equally to all instruments, compensation for anelastic phenomena (Δl_c) is valid only where the phenomena are isotropic. In any case, original data should always be modified so as to verify the condition $\Delta s = 0$.

Similarly, for the theorems to be applicable the sum of the angles measured in a triangle should be made equal 2π . If the difference is noticeable the measurement is meaningless; if it is negligible, any compensation principle may be acceptable. In the case in hand Δs was made to equal zero by assuming the sum of the modified data to be equal to the sum of the original data.

The standard deviation of the modified data from the original data was considered as "error in the data" (ϵ_v) inherent in the instrumentation and hence the same for all the individual components. As will be seen further on, this value is of primary importance for the determination of the uncertainty in the results (ϵ_R). (**)

(*) In the present case this condition occurred with good approximation a few days after installation.

(**) During the period investigated the value of ϵ_v never exceeded 8×10^{-6} .

5) Computation of the results

As the numerical elaboration of the measurements involved, among other things, the solution of a cubic equation and a number of inverse functions for each set of data, recourse was had to a computer.

The input data of the programme were the readings from the seven strain gauges and the data obtained from the isolated one, associated with the instrument constants so as to obtain the strains.

The first part of the programme determined the values of σ and τ in relation to the location of the individual instruments in the set.

The second part of the programme, of a more general nature, calculated the following results, σ and τ being known:

- principal stresses in algebraic order (maximum, intermediate, minimum);
- angles of each of the principal stresses with respect to the x, y and z axes;
- secondary stresses (maximum and minimum) in the xy, yz and zx planes;
- angles of the secondary stresses in each plane.

6) Uncertainty in the results

It is to be noted that, though the results of this procedure are always real, the resulting values are reliable to a different degree depending on the error in the data and on the way the data contribute to the results. Hence, it is essential that the uncertainty in the results be determined as an indication of the accuracy with which the phenomenon can be expressed by the instrumentation.

If we apply the theory of errors in the present case, the seven input data will be the variables V_i with uncertainty ϵ_V ; and the uncertainty in a result R will be expressed by

$$\epsilon_R^2 = \epsilon_V^2 \sum_{i=1}^7 (\delta R / \delta V_i)^2$$

To avoid conceptual errors when formulating the partial derivatives, these were determined numerically (incremental ratio) using a loop for each variable which is incremented only once.

Not to complicate computation unnecessarily, this iterative method was applied only to the determination of

6.

stresses and angle components. The error for each angle was obtained analytically as a function of the errors in the components and of the partial derivatives of their inverse function, which was easy to formulate.

It should be observed that the error in the data (ε_v) was defined and employed as a random value while it was partly systematic. It follows from this that the trend of the results in time often appears to be more regular than indicated by the band of uncertainties (ε_R).

7) Graphical presentation of the results

The output data, in a graphical form, are a function of time. All the values are denoted by crosses whose upward strokes, symmetrical with the resulting values, represent their uncertainty.

For the hydraulic load, the scale is in metres and has an arbitrary origin.

For the stresses σ , the scale is in units $\frac{\sigma}{E} = 10^{-5}$, the negative cartesian direction indicating compression.

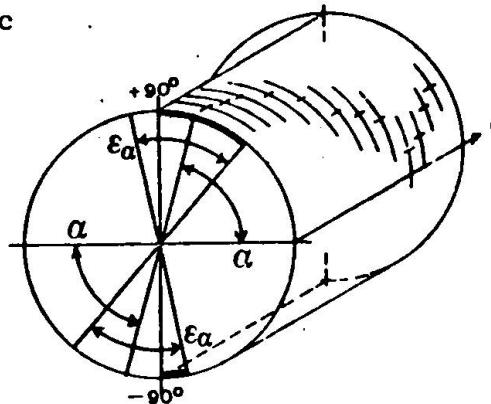
For the angles α representing the direction of a generic σ with regard to an axis, the graph should be regarded as the development of a semicylinder whose length is the time abscissa, while the extremity of a diameter of the cylinder rotated by the angle α is plotted as the ordinate.

The diameter is symmetrically bounded by two opposed circular sectors with openings equal to the uncertainty ε_α whose development is also represented by the ordinate.

With regard to the layout of the strain gauge set in the dam, the coordinated axes are oriented in the following directions:

- x : lateral (left-right)
- y : radial (downstream-upstream)
- z : vertical (base-top)

The positive cartesian direction of the axes is given by the second indication. ("Left-right" refers to the structure



seen from downstream).

For the secondary σ in the plane, the angle α represents the direction of σ_{\max} (algebraically maximum) in respect of the axis relating to the first of the two indices that define the plane. σ_{\min} is at right angles to σ_{\max} .

8) Results of the Vajont strain gauge set

The period investigated goes from the date of installation of the instruments (June 1959) to September 1963. For each month in this period a measurement was considered which was the mean of the most significant data obtained in that month and was compared with the average hydraulic load.

The results, in a graphical form obtained directly from the plotter (see par. 7), can be seen in Tables 1) and 2).

Table 1 - Vajont strain gauge set

Stresses in three dimensions

Q = hydraulic load (m)

σ_n = principal stresses in algebraic order (maximum, intermediate, minimum for $n = 1, 2, 3$)

$\alpha_{n\xi}$ = angle between σ_n and the ξ axis ($\xi = x, y, z$)

It will be convenient first to define as minimum hydraulic load the one active in March 1960, when the dam had virtually been completed, which scarcely exceeded the elevation of the strain gauge set.

The value of σ appear to be correlated with the hydraulic load. A small sinusoidal component with an annual period was probably due to thermal stress.

The variation of σ_3 (maximum compression) in the initial stage during which concreting was in progress, is notable and so is the trend of σ_2 which appears to be strictly correlated with the hydraulic load. Its values approximate those of σ_1 at minimum hydraulic load and those of σ_3 at maximum hydraulic load.

With regard to the trend of the angles α , all the show a tendency towards following the directions of the axes ($\alpha = 0^\circ$ or $\pm 90^\circ$). In particular, one at least of the σ always lies on one of the axes.

As the trend of the angles is not linear, it will be well to examine some of the most significant situations of the hydraulic load.

At minimum hydraulic load, σ_3 (maximum compression) is markedly vertical ($\alpha_{3z} \approx 90^\circ$). Hence, the plane on which σ_1 and σ_2 lie is concurrent with the horizontal xy plane to a considerable extent.

The scalar values of σ_1 and σ_2 are very near, which sometimes gives rise to uncertainties (*) as to their direction in the plane. In any case, σ_1 (minimum compression) clearly follows the lateral direction (x axis).

At a slightly higher hydraulic load σ_1 and σ_2 rotate by 90° on the horizontal plane. Hence, σ_1 lies in the radial direction while σ_2 lies in the lateral direction. The situation remains practically unchanged till the hydraulic load becomes intermediate.

When the hydraulic load rises from intermediate to maximum, σ_2 and σ_3 rotate in a plane near the vertical-lateral zx plane, which is clearly defined by the constant and precise value of the angle $\alpha_{1y} \approx 0$. On the other hand, the values of the angles in this plane are more uncertain owing to the nearness of the scalar values of σ_2 and σ_3 (*). Anyhow, at maximum hydraulic load the directions of σ_2 and of σ_3 are nearly vertical and nearly lateral respectively.

Table 2 - Vajont strain gauge set

Stresses in the planes ($\frac{\sigma}{E} 10^{-5}$)

Q = hydraulic load (m)

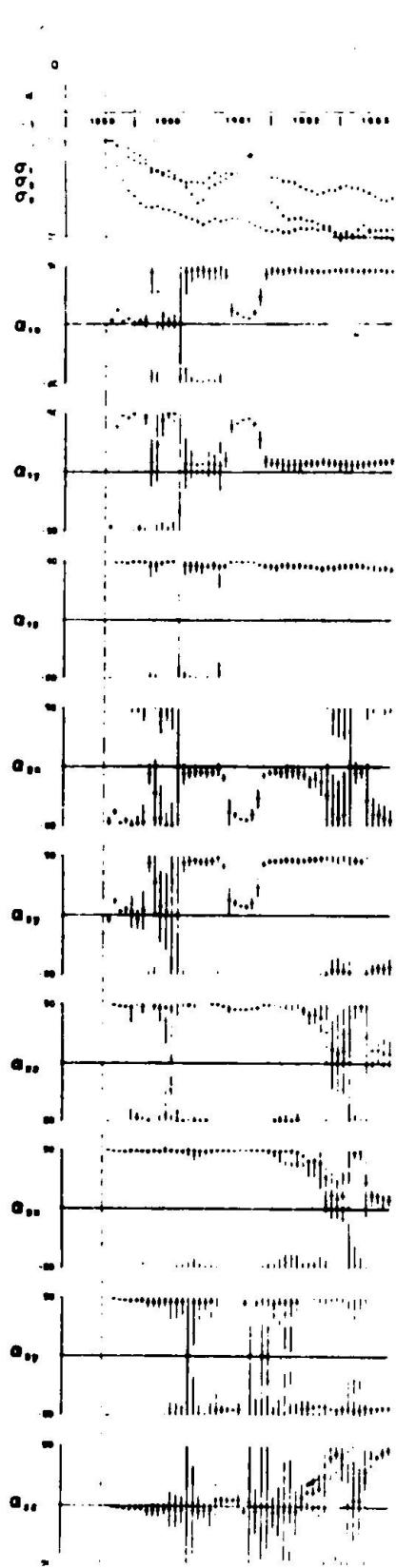
σ_ξ = stress on the ξ axis ($\xi = x, y, z$)

$\sigma_{\xi', \xi''}$ = maximum and minimum stresses in algebraic order in the ξ', ξ'' plane ($\xi', \xi'' = x, y, z$)

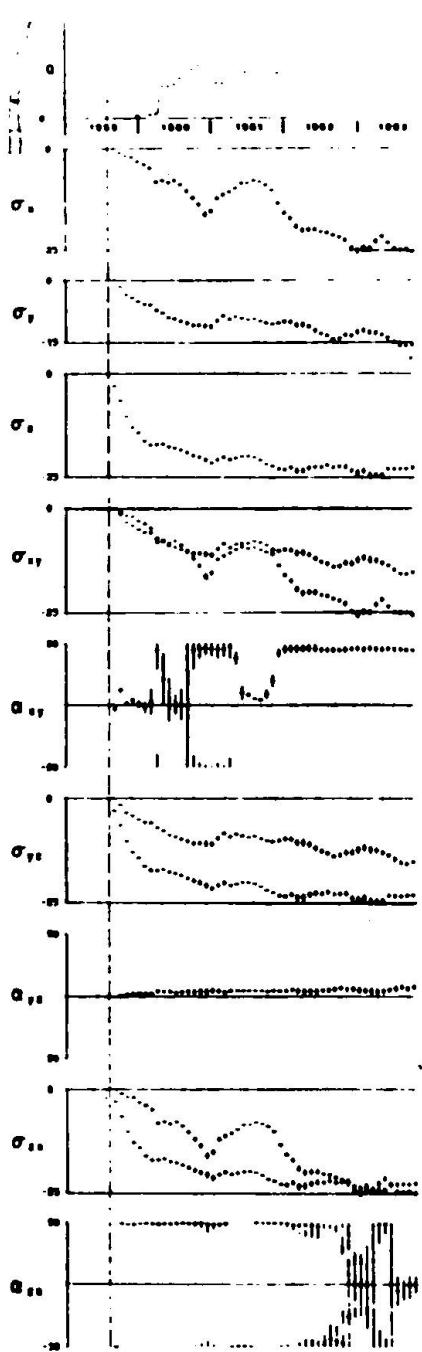
$\alpha_{\xi', \xi''}$ = angle between the maximum stress and the ξ' axis (or between the minimum stress and the ξ'' axis).

The σ on the axes are markedly concurrent with the principal stresses in three dimensions. Hence, the trend of the principal stresses in three dimensions taken by pairs is

(*) When the scalar values of a pair of σ are very near, the quadratic surface of the stresses in the plane containing the pair of σ tends towards a circle. Thus, if the two are equal their direction in the plane is indeterminate. Generally speaking, the uncertainty in the direction is a function of the probability of the two σ being equal as a consequence of their scalar uncertainty. By definition, however, they remain perpendicular to each other.



TAB.1



TAB.2

the same as the trend of the secondary stresses in the planes. Here, however, uncertainties are fewer and the interpretation is more immediate.

In the horizontal xy plane:

- at minimum hydraulic load the minimum compression (σ_{\min}) is lateral;
- at a slightly higher hydraulic load the pair σ_{\max} , σ_{\min} rotate by 90° and remain there till the maximum hydraulic load sets in.

In the radial-vertical yz plane:

- the maximum compression (σ_{\max}) is always vertical.

In the vertical-lateral zx plane:

- the maximum compression (σ_{\max}), which is vertical till intermediate hydraulic load, becomes lateral at maximum hydraulic load.

9) Results of the Val Gallina strain gauge set

As was mentioned above, for the Val Gallina strain gauge set it was impossible to ascertain whether measurements corresponded with theory. Consequently, no compensation could be introduced.

Moreover, the behaviour of the isolated strain gauge was found to be unreliable, which prevented the compensation of anelastic phenomena.

An attempt, however, was made to compensate that part of the anelastic component Δl_c that is a function of temperature. To this purpose, a coefficient was derived from the strain-temperature correlation for the other isolated strain gauges in the dam. The values of l_0 were determined almost at random, as the data obtained after the installation of the instruments do not seem to indicate any trend that could be correlated with the increase in dead weight. Most probably, this trend was concealed by other concurrent effects that were not compensated.

Thus, for l_0 conventional values were assumed that were the means of observations made from the fifteenth to the thirtieth day after installation.

Of course, the assumption of incorrect values for l_0 and Δl_c may have had some consequences on the results. Whereas an error in l_0 , which is an initial reference, does not affect the time variations of the results, which are linear functions of the data, it affects non-linear functions, espe-

ctly angles. A error in δ_{lc} , which is a value variable with time, affects the time variations of the linear results but does not affect the angles whose determination is independent of this value.

As to the error in the data (ϵ_v) an estimated value $\epsilon_v = 10^{-5}$ was assumed for it.

The results can be seen in Tables 3 and 4 which are very similar to Tables 1 and 2.

The first period investigated goes from the date of the installation of the instruments to about three years later. The second period starts about ten years later and coincides with the only total drawdown of the reservoir.

Apart from the tension values imputable to an error in the formulation of the initial values and in the subsequent thermal compensations, the interpretation of these results is far from being easy. From January to May 1951, when concreting was stationary, compression appears to have decreased.

A greater decrease in compression occurred from February to April 1953 when the reservoir level was drawn down, but the successive fill-up did not produce the contrary effect.

Ten years later a creep towards compression can be observed together with a better correlation with the hydraulic load.

It is to be noted, however, that the scalar values of σ do not differ from one another very much. In this situation the quadratic surface of the stresses tends towards a spherical from: hence, the uncertainty in the angles is considerable.

A slight tendency of σ_3 towards the z axis can also be inferred, and this is perhaps the least baffling aspect of the whole phenomenon.

At any rate, it is reasonable to believe that for this strain gauge set the elastic theory was applied to phenomena that were mostly anelastic.

10) Considerations

All the results of this investigation seem to point out how important it is that each result should be associated with its own uncertainty. Among other things, uncertainties indicate which results are not sufficiently reliable and should be discarded before interpretation.

The values of the errors in the data, on which the uncertainty in the results depends, can either be estimated or, as for the Vaiont strain gauge set, be derived from a comparison of measurements with theory.

With regard to instrumentation, the investigation has stressed the importance of a reliable isolated instrument or any other means for the compensation of anelastic components. That applies not only to the time during which the phenomenon is in progress, but also to the initial stage when these components are prevailing and the determination of the lengths at zero stress depends on the determination of the elastic behaviour of the structure. As noted, these values enable an absolute origin to be established for the scalar and angular values.

Owing to the lack of means necessary for fulfilling these conditions, the Val Gallina strain gauge set has failed to yield satisfactory results. The results achieved at the Vaiont dam, on the other hand, may be regarded not only as reliable, but also as corresponding with the actual behaviour of the structure.

Examination of these results shows that one at least of the principal stresses in three dimensions lies in the direction of one of the axes. Under these conditions the phenomenon can be represented almost as thoroughly by the secondary stresses in three dimensions.

Moreover, the results reveal that τ is sensibly equal to zero in the x y plane.

If this assumption were extended to the Place Moulin dam which, as was mentioned at par 1), is also equipped with strain gauge sets, the procedure for the determination of stresses in three dimensions could be applied also to this structure.

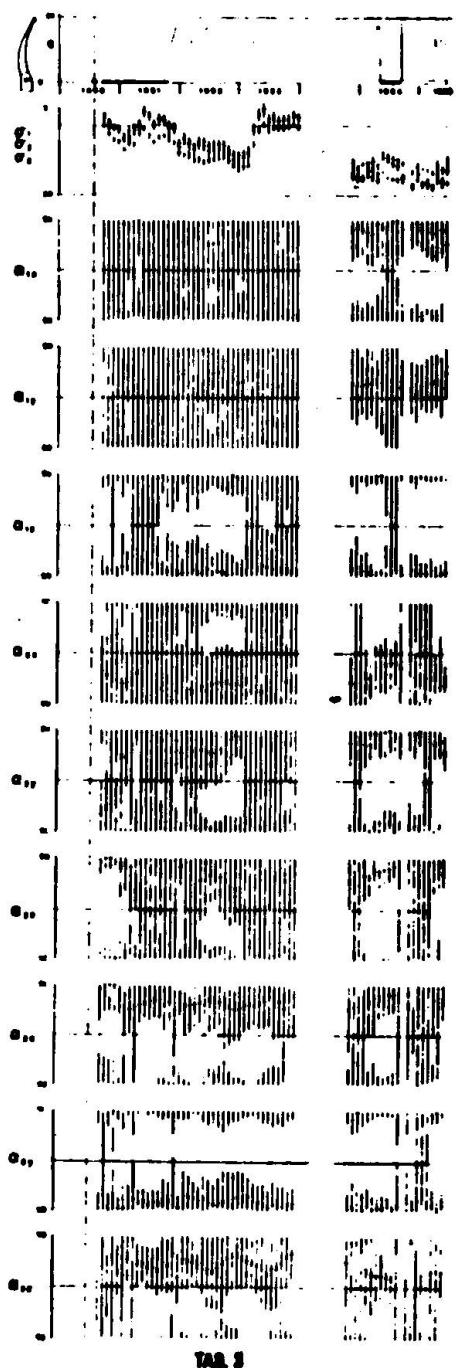


FIG. 3

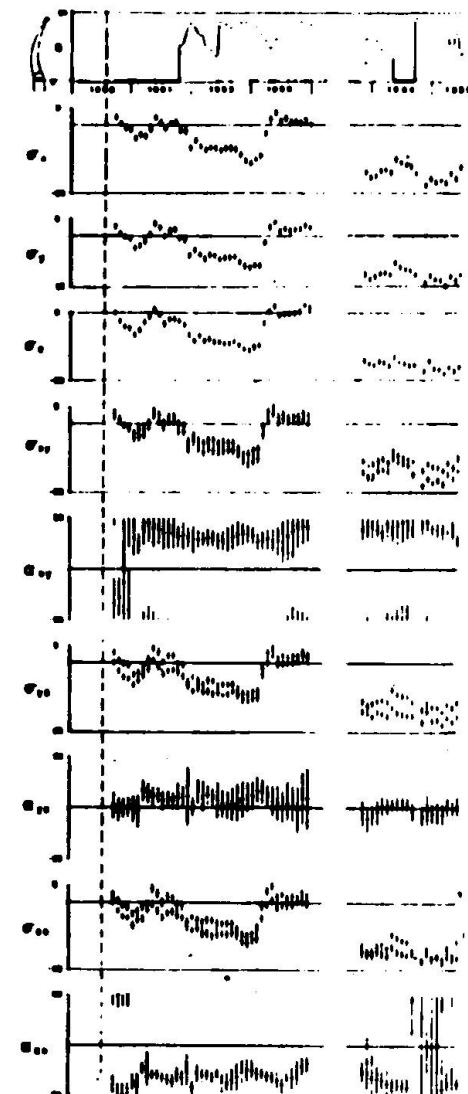


FIG. 4

Summary

As a preliminary remark, it is noted that the available experimental data are limited to the strain measurements furnished by two spacial sets respectively of 6 and 7 strain gauges, every set being installed in an arch dam.

After the correction of the experimental data, which is permitted by the 7 strain gauge set, to obtain correspondance between phenomena and theory, the value and the direction of the principal stresses are determined.

The research is also extended to the determination of the value and direction of the secondary stresses in the three planes established by the reference axes.

After stressing the importance of knowing the reliability of results, which is a function of the errors through which gauges can express physical phenomena, the uncertainty in every result is calculated.

The values obtained are represented in a series of graphs as a function of time and compared with the hydraulic load.

Considerations on the triaxial measurements and comparisons with the measurements in the planes are made.

Résumé

On relève, d'abord que la disponibilité des données expérimentales est limitée aux mesures de déformation fournies par deux groupes à trois dimensions de six et sept extensomètres placés chacun dans un barrage-vôûte.

Après avoir corrigé les données expérimentales pour obtenir la correspondance entre phénomène et théorie, (ce qui est permis par le seul groupe de sept extensomètres) on va déterminer la valeur et la direction des contraintes principales.

La recherche est aussi élargie à la détermination de la valeur et de la direction des contraintes secondaires situées dans les trois plans, indiqués par les axes coordonnées.

Après avoir relevé l'importance de connaître la précision des résultats, fonction de l'erreur par laquelle la instrumentation est à même d'exprimer le phénomène physique, on procéde à la valutation de l'erreur que chaque résultat comporte.

Les résultats obtenus sont représentés en diagrammes en fonction du temps et comparés à la charge hydrostatique.

On déduit ensuite quelque considérations sur les mesures à trois dimensions et des comparaisons de ces mesures avec celle qui sont dans les plans.

Zusammenfassung

Es wird vorausgesetzt, dass die Verfuerbarkeit experimentaler Werte auf die Dehnungsmessungen zwei räumlicher Messergruppen beschränkt ist, denen die eine aus 6 und die andere aus 7 Messern besteht; die zwei Gruppen sind in zwei verschiedenen Bogenmauern verlegt.

Nach der Korrektur der experimentalen Werte (die durch die 7 - Messer-Gruppe ermöglicht ist) werden Grösse und Richtung der Hauptspannungen ermittelt, um die Übereinstimmung zwischen Phänomen und Theorie zu erhalten.

Die Forschung ist überdies auf die Bestimmung von Grösse und Richtung der Sekundärspannungen erweitert die auf den drei durch die Bezugssachsen ermittelten Ebenen liegen.

Nachdem bemerkt wird, wie wichtig es ist, die Zuverlässigkeit der Ergebnisse zu kennen - sie ist eine Funktion des Fehlers, mit welchem die Geräte das physikalische Phänomen wiedergeben - wird die Messunsicherheit für jedes einzelne Ergebnis berechnet.

Die erhaltenen Werte werden in einer Reihe von Diagrammen als Funktion der Zeit und mit Bezug auf den hydrostatischen Druck dargestellt.

Es werden daraus Betrachtungen über die räumlichen Messungen und Vergleiche mit den Messungen auf einzelnen Ebenen gezogen.

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