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Then the stress states are subdivided into four fundamental groups in relation to the ensemble of the physical and mechanical phenomena that they can induce in the material. Therefore both general and particular safety criteria can be established, on each occasion if you want, by demanding that suitable stress states belong to one or more predetermined groups depending on the efficiency and the safety requirements of the whole structure or of its special parts.

We have introduced the correspondence between domains and groups because the domains can vary, while the groups are to be considered "fixed" in the sense that each group is a representation of a certain well-determined condition of the material to be approximately ascribed to all stress states belonging to the same group. In our opinion safety criteria have to be established along the lines outlined before as regards these last groups.

As a matter of fact to consider the influence of time and of the stress path means to consider that the failure surface and the other limiting surfaces can change; and so the domains can change too. Lastly a consideration with reference to some papers of Prof. Zienkiewicz and Dr. Argyris and others, which I have read with care even if I am not a "finite element man". I would like to suggest that our "limiting surfaces", the failure surface too, behave very well as "loading surfaces": in this case it is possible to incorporate every safety criteria established as outlined before in the constitutive relations themselves, as it should be. Thank you.

### III-6 Prof. O. DE DONATO

Ladies and gentlemen, I will briefly discuss one point regarding the way in which the path-dependency of the material constitutive laws has been taken into account in the underground opening problem considered in the paper. The assumed plasticity condition for the soil was the extended Mohr Coulomb criterion, proposed by Drucker and Prager; the same criterion was also adopted for the concrete but assuming a small tensile strength and elastic perfectly plastic behaviour.

As everybody knows, the path-dependency is a very important aspect in soil problems because it is related both to the path-dependency of the material constitutive laws and to the history of the loads. The latter depends greatly on the assumed sequence of the phases of construction, such as, in the problem in hand, the consolidation, the opening, the centring of the tunnel, the concrete casting, etc.

Among the different non linear analysis procedures so far proposed for these kinds of problems, the incremental method has been very frequently used and, in fact, it is attractive. But there are well known circumstances for which the procedure becomes very laborious involving increments of the external loads at each step which are too small and then requiring a high number of steps to reach the given final loads. It would be more advantageous instead to choose a procedure which subdivides "a priori" the loading history in finite loading steps. This is operated in a recently proposed procedure named "multi-stage method" explicitly divided for general non proportional loadings and here applied to the problem in hand.

The main aspects of the multistage method are: (i) piecewise linearization of the yield surface, (ii) subdivision of the loading history in a sequence of families of loads (loadings stages) each governed by only one parameter monotonically increasing,

(iii) solution of the analysis problem for a given loading stage (says  $k^{\text{th}}$ ) assuming holonomic flow laws, i. e., that no sequence of loading-unloading takes place in the stress path of any point of the structure during the application of the load, (iv) solution of the analysis problem for the following  $(K + 1)^{\text{th}}$  loading stage assuming again holonomy, but with a suitable modification of the constitutive laws (in practice of the yield limit) for those stress points that were in the plastic range at the end of the  $k^{\text{th}}$  stage.

It is possible to show that for both phases (iii) and (iv) the analysis problem can be formulated in terms of various quadratic programming problems involving as unknowns the plastic multipliers, or the plastic multipliers and displacements, or the plastic multipliers and stresses (see references for more details).

I would like to conclude with some remarks on the adopted procedure. An important advantage is the possibility to subdivide "a priori" the final loads in to proportional loading steps. However there are some sources of error in applying the multistage procedure. First is the piecewise linearization, but this source of error can be reduced in its importance by increasing the number of planes describing the linearized yield surface. Second is the possibility of loading-unloading sequences during a step; but this source of error too can be reduced in its importance by decreasing the amplitude of the loading step.

An important point to be mentioned is the possibility of reducing the number of variables in the problem by forecasting the yield planes which will be activated at the solution under the final loads (on the basis e. g. of the elastic solution). This sometimes reduces so drastically the number of variables, that large size problems can often be solved very easily.

Finally, another point is about the algorithm to adopt to solve the quadratic programming problem. There are different algorithms which all have the common important feature of guaranteeing convergence to the solution of the problem in every case (of course in the presence of overall stability). A recent algorithm (by Cottee) can even say whether, during the loading step, there is some load-unloading sequence, i. e., whether the source of error already mentioned is present; this information is an important tool for suitable ajustement of the length of the loading step. Thank you.

## SUR LE COMPORTEMENT MECANIQUE DES BETONS A LA LIMITE DU DOMAINE REVERSIBLE.

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L'étude des lois de comportement des mortiers et bétons et de leurs mécanismes de fissuration et de rupture fait l'objet d'un programme de recherche développé depuis plusieurs années au Département de Génie Civil de l'I. N. S. A. de Toulouse.

Nous présentons, dans cette communication, les conclusions de ces travaux en ce qui concerne la définition d'un critère de réversibilité pour les mortiers et bétons.