Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 19 (1974)

Artikel: Analysis of creep in concrete structures under general states of stress

Autor: Jordaan, I.J.

DOI: https://doi.org/10.5169/seals-17533

IABSE
AIPC
IVBH

SEMINAR on:
«CONCRETE STRUCTURES SUBJECTED TO TRIAXIAL STRESSES»
17th-19th MAY, 1974 - ISMES - BERGAMO (ITALY)

Analysis of creep in concrete structures under general states of stress

Calcul de fluage dans les structures en béton sous l'effet de contraintes quelconques

Kriechen von Betonbauten die einem Allgemeinen Spannungszustand Ausgesetzt sind

I. J. JORDAAN - Associate Professor
Department of Civil Engineering the University of Calgary
Calgary, Alberta - Canada

INTRODUCTION

In many concrete structures, it is important to calculate time-dependent stresses or strains in addition to those at the time of application of loads. Time-varying strains and deflections are a simple matter to calculate if creep is linear and homogeneous throughout the structure. Redistribution of stresses results in cases where creep is non-linear or non-homogeneous, or both. Non-homogeneity of the creep parameters occurs where the temperature varies within the structure, where the concrete is drying, or where the structure is composed of two or more materials. Examples of the latter are reinforced and prestressed concrete structures. Stress redistribution also takes place if boundary conditions alter, for instance in a statically indeterminate beam with a settling support. A further case where redistribution of stress occurs is in those structures in which Poisson's ratio varies with time. This will be discussed in more detail later.

The first part of this paper will be devoted to an outline of a recently developed method of analysis which leads to direct calculation of creep effects, and the necessity to perform iterative calculations in time is obviated. In addition, the creep solution is obtained directly from an elastic solution. It is assumed in this analysis that the strain compliance is the same at all the points in the structure. Several investigators [1, 2, 3 for example] have studied the effect of spatial variation of temperature. An additional variation of creep compliance results from different degrees of drying at various distances from exposed surfaces; this has been recognized in earlier work, for example, by Pickett [4] and has been studied in some detail in terms of the structure of concrete by Illston [5]. In the present paper the variation of creep parameters due to drying is considered, together with the formulation of constitutive equations for the analysis of concrete structures under these conditions. The relationships are somewhat tentative but can be improved when more experimental evidence is available.

CORRESPONDENCE PRINCIPLE FOR AGEING CONCRETE

Details of the method have been presented elsewhere [6] and only a brief outline will be given here. The method may be used for sealed concrete, or drying concrete if an average axial compliance is used. Analyses for concrete structures using the elastic-viscoelastic analogy have previously been carried out on the assumption of a time-invariable creep law; for instance, a combination of Maxwell and Kelvin elements are fitted to the data

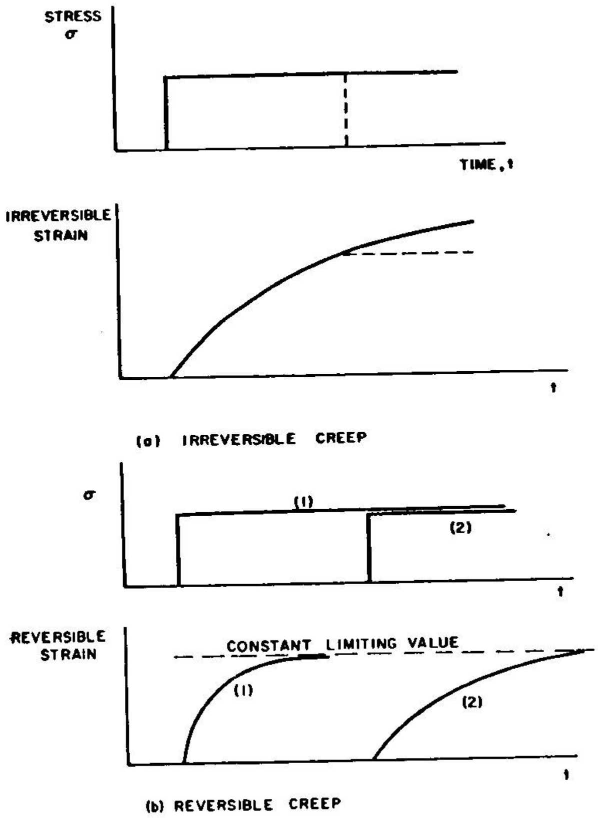


Fig. 1. Reversible and irreversible components of creep.

Fig. 1. Les composants réversibles et irréversibles du fluage.

Bild 1. Umkehrbare und nicht umkehrbare Komponenten des Kriechens.

so as to model the concrete deformation, or the Maxwell model alone is adopted. The principal disadvantage to this procedure is that both the irreversible and the reversible creep due to unit stress show the effect of ageing (Fig. 1) and cannot be represented adequately by standard rheological elements. The irreversible component is not linear with time while the reversible creep curves are also of different shape for different ages. In passing, it should be noted that the reversible creep should, in practice, be obtained by means of recovery tests [6, 7].

The essence of the method is the replacement of the true time, t, with a pseudo-time, t', equal to the irreversible creep per unit stress; Fig. 1(a) illustrates the monotonic one-to-one relationship between t and t'. The creep problem is then solved using t' throughout; the solution can subsequently be converted to real time by using the known relationship between t and t'. The reason for this subterfuge becomes clear when the components of creep are plotted against t', as in Fig. 2. A linear relationship is, of

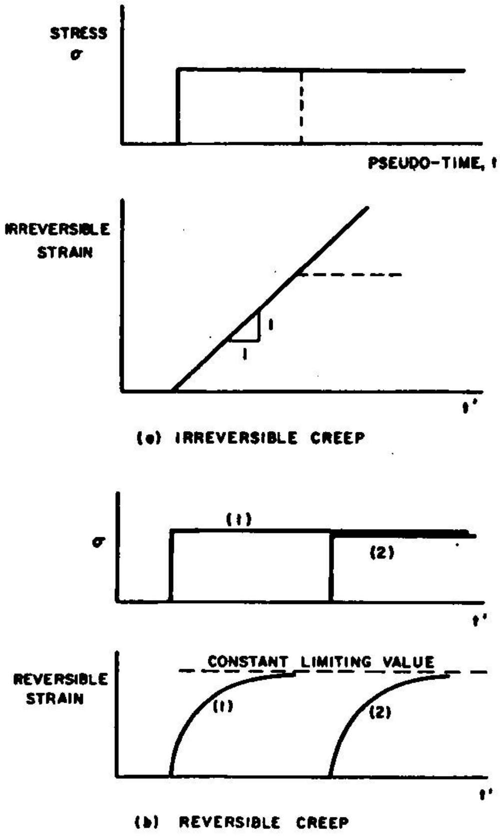


Fig. 2. Reversible and irreversible components of creep plotted against t'.

Fig. 2. Variation des composants réversibles et irréversibles du fluage avec le temps "fictif," t'.

Bild 2. Umkehrbare und nicht umkehrbare Komponenten des Kriechens als Funktion von t' dargestellt.

course, obtained between irreversible creep and t'; this can be modelled by means of a single dashpot. In addition, the curves of reversible creep are now of similar shape (Fig. 2(b)). This should be regarded as an experimentally determined property [7] and while it is not exact, it is at worst a good approximation.

The strain $\varepsilon(x,t')$ in concrete may therefore be determined from the following equation (considering uniaxial loading only):

$$\varepsilon(x,t') = \int_{0}^{t'} \frac{\partial\sigma(x,\tau')}{\partial\tau'} \left\{\frac{1}{E(\tau')} + (t'-\tau') + g(t'-\tau')\right\} d\tau' \tag{1}$$

where $\sigma(x,t')$ is the stress at t', $E(\tau')$ the elastic modulus at τ', and $g(t'-\tau')$ the reversible strain after duration $t'-\tau'$. In equation (1) quiescence is assumed for $t' < 0$; x denotes the position in the body.

The elastic modulus is considered to be a function of time in equation (1); in many problems (for ages of concrete 28 days or more) E may be treated as a constant although the reversible and irreversible creep change significantly with age. Hence, under these conditions equation (1) may be written as an integral equation of the convolution type:

$$\varepsilon(x,t') = \frac{\sigma(t')}{E} + \int_{0}^{t'} \frac{\partial\sigma(x,\tau')}{\partial(\tau')} \left\{(t'-\tau') + g(t'-\tau')\right\} d\tau' \tag{2}$$

which has the Laplace transform

$$\bar{\varepsilon}(x,s) = \bar{\sigma}(x,s) \left\{\frac{1}{E} + \frac{1}{s} + s\bar{g}(s)\right\} \tag{3}$$

in which the bar denotes the Laplace transform and s is the transform parameter. It should also be noted that the viscoelastic model corresponding to equation (2) is a combination of Maxwell and Kelvin elements (Fig. 3).

Advantage in the use of equation (3) lies in the correspondence principle of linear viscoelasticity: if the elastic solution to a problem is known, replacement of the reciprocal of the elastic modulus (1/E) by $(1/E + 1/s + s\bar{g}(s))$ will give the Laplace transform of the creep solution. This is one form of the analogy between elastic and viscoelastic solutions; Volterra [8] was apparently the first to establish a reciprocal theorem of this kind.

The assumption of a constant elastic modulus in equation (2) may be avoided and the elastic-viscoelastic analogy still used, if a step variation of 1/E with time is assumed (see reference 6 for details).

THREE-DIMENSIONAL ANALYSIS-SEALED CONCRETE

In massive structures, only the outer skin of concrete will be subjected to drying [9, 10, 11] particularly during the period just after loading. It follows that the appropriate constitutive equation to be used is that for sealed concrete, if the effect of the differences in creep and shrinkage rates in the outer layers is to be disregarded.

Tests on sealed concrete have been performed by several investigators. Correct results are difficult to obtain because of the small magnitude of the strains to be measured: in addition, if the results depend upon the calculated difference between two large strains, it is difficult to obtain results that are statistically significant. These and other experimental difficulties have been discussed elsewhere [12]. A study of the available literature and recent experimental results [13] shows that under constant stress Poisson's ratio may be taken to be a constant and equal to the static elastic value; for variable stress [14] this is also found to be a reasonable assumption.

The constitutive equation for concrete under these conditions may be written (by analogy to the elastic equation $\varepsilon_{ij} = (1 + \nu)\sigma_{ij}/E - \nu\delta_{ij}\sigma_{kk}/E$):

$$\varepsilon_{ij}(x,t') = (1 + \nu) \int_0^{t'} \frac{\partial\sigma_{ij}(x,\tau')}{\partial\tau'} J(t',\tau')d\tau' - \nu\delta_{ij} \int_0^{t'} \frac{\partial\sigma_{kk}(x,\tau')}{\partial\tau'} J(t',\tau')d\tau' \tag{4}$$

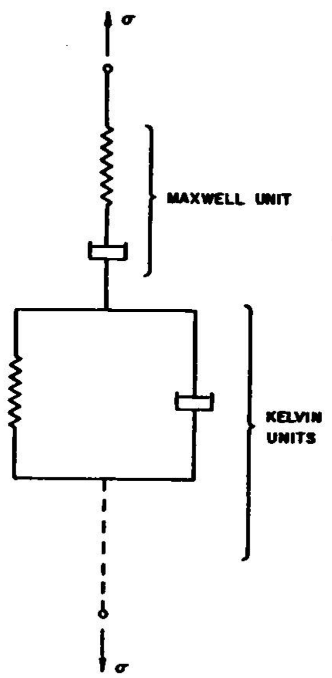


Fig. 3. Viscoelastic model for concrete; pseudo-time t' rather than the time t is used as the time variable.

Fig. 3. Le modèle visco-élastique pour le béton; le temps fictif t' est utilisé comme le variable du temps au lieu du tempts t.

Bild 3. Viskoelastisches Modell für Beton: Die Ersatzzeit t' wird anstelle der Zeit als zeitabhängige Variable benützt.

where ν, Poisson's ratio, is treated as a constant throughout, δ_{ij} is the Kronecker delta and $J(t',\tau')$, the uniaxial compliance, $= 1/E + (t'-\tau') + g(t'-\tau')$. Note that ε_v, the volumetric strain is equal to $\varepsilon_{kk}/3$. If the variation in E with time is neglected, $J(t',\tau') = J(t'-\tau')$; otherwise E is treated as a step function of time, as discussed above.

It is possible as an alternative to equation (4) to express the stress and strain in terms of their volumetric and deviatoric components, i.e.,

$$\sigma_{ij} = S_{ij} + \frac{\sigma_{kk}}{3}\delta_{ij} \tag{5}$$

$$\varepsilon_{ij} = e_{ij} + \frac{\varepsilon_{kk}}{3}\delta_{ij} \tag{6}$$

where $\sigma_{kk}/3$ and $\varepsilon_{kk}/3$ are the volumetric stresses and strains, S_{ij} and e_{ij} the stress and strain deviators, and δ_{ij} the Kronecker delta. The time-varying strains e_{ij} and ε_{kk} may then be written as

$$e_{ij}(x,t') = \int_0^{t'} \frac{\partial S_{ij}(x,\tau')}{\partial \tau'} J_d(t',\tau')d\tau' \tag{7}$$

$$\varepsilon_{kk}(x,t') = \int_0^{t'} \frac{\partial \sigma_{kk}(x,\tau')}{\partial \tau'} J_v(t',\tau')d\tau' \tag{8}$$

where $J_d(t',\tau') = (1+\nu)\ J(t',\tau')$ (9)

and $J_v(t',\tau') = (1-2\nu)\ J(t',\tau')$. (10)

The correspondence principle for the three-dimensional case is as follows: either treat (1/E) as outlined in the previous section and ν as a constant throughout, or

$$\frac{1}{2G} = \frac{1+\nu}{E} \rightarrow (1+\nu)s\,\bar{J}(s) \text{ in the Laplace domain} \tag{11}$$

and $$\frac{1}{3K} = \frac{1-2\nu}{E} \rightarrow (1-2\nu)s\,\bar{J}(s) \text{ in the Laplace domain.} \tag{12}$$

THREE-DIMENSIONAL ANALYSIS-DRYING CONCRETE

Drying at the surface of the concrete introduces several complications: firstly, shrinkage which will vary from a maximum at the surface to a minimum in the interior of the concrete, introducing stresses and subsequently stress redistribution, and secondly the rate of creep is considerably enhanced if the concrete is drying. Many investigations have been carried out on the creep of drying concrete, but in order to specify the spatial distribution of creep compliances the tests should be performed on comparatively thin specimens at different relative humidities (thus having a constant relative humidity throughout each specimen). Such tests have been performed by Parrott [15], whose tests also included measurements of Poisson's ratio. Drying at a R.H. of 65 per cent was investigated and behaviour compared to that of concrete at

95 per cent R.H.

Since Parrott obtained readings of creep recovery, he was able to determine the effect of drying on the reversible and irreversible strains. Considering first the axial strains (under uniaxial loading), the effect on reversible creep strains is minor and of little practical significance. In addition, the elastic modulus of the concrete varied only a small amount under the different drying conditions. The relatively minor effect of drying at 65 per cent R.H. on the elastic modulus is supported by other data [16], although after long durations the effect does become significant. For very low humidities, the change in elastic modulus also becomes important.

In studying the effect of drying on irreversible creep, the basic irreversible creep (i.e., from specimens maintained at 95 per cent R.H.) is taken as the pseudo-time t'. Values of irreversible creep for drying concrete are given in Fig. 4. It is readily seen that in mixes with a normal water cement ratio (mix B) at 65 per cent R.H. the irreversible creep is more than

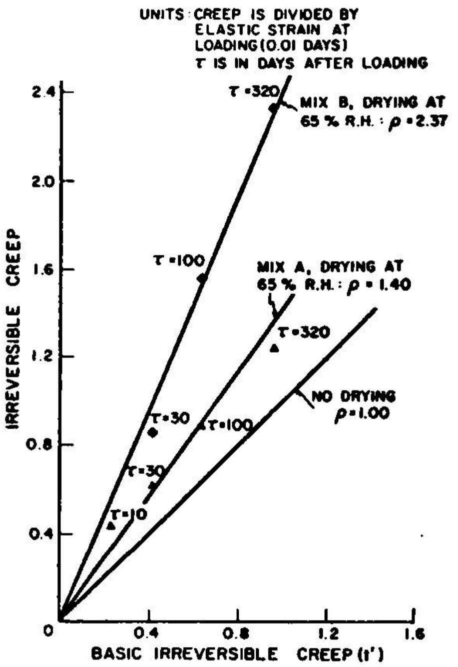


Fig. 4. Irreversible creep at relative humidities of 95% (basic creep) and 65%. Mix A has a water cement ratio of 0.28 while for mix B the ratio is 0.55. The aggregate/cement ratios were 2.0 and 5.5 respectively.

Fig. 4. Fluage irréversible à des humidités relatives de 95% (fluage fondamental) et de 65%. Ciment/eau = 0.28 pour Béton A, et 0.55 pour Béton B. Granulat/ciment = 2.0 et 5.5, respectivement.

Bild 4. Nicht umkehrbares Kriechen bei einer relativen Luftfeuchtigkeit von 95% (Grundfließen) und 65%. Die Mischung A hat einen Wasserzementwert von 0,28 und die Mischung B von 0,55. Das Verhältnis von Zuschlagstoffen zu Zement war 2,0 bzw. 5,5.

twice the basic value. In addition, it is clearly reasonable to assume a linear relationship, i.e., that the "drying" irreversible creep (at a *constant* R.H.) is a constant (ρ) times the basic value.

If the conclusions above are combined in mathematical form, we obtain the following axial compliance:

$$J(x,t',\tau') = \frac{1}{E(\tau')} + \rho(x)(t'-\tau') + g(t'-\tau') \tag{13}$$

where $\rho(x)$ is a function of spatial coordinates.

Comprehensive results on the lateral compliance are not available for drying concrete. However, sufficient results are available to formulate a tentative relationship. Firstly, there is a decrease in the elastic Poisson's ratio with moisture content in *cement pastes* [17, 12]; this effect is of moderate significance (e.g., from .25 at 100 per cent to .17 at 20 per cent). It is chiefly associated with a change in axial compliance and is not significant for *concrete*; this has been reported in test results, e.g., by Plowman [16] who found no effect of change in humidity on the static Poisson's ratio. It is possible that for extremely low humidities this conclusion might not be valid.

The results for lateral creep indicate a Poisson's ratio for creep that is lower than the elastic value, under drying conditions [12]; however on closer inspection this result can be interpreted differently. Results by Meyer [18] and by Parrott [15] show that the values of lateral creep are very similar, regardless of drying: the reduced value of Poisson's ratio for creep is due to increased axial strain. Hence the lateral compliance J_ℓ is not a function of position in space:

$$J_\ell(t',\tau') = \nu\left\{\frac{1}{E(\tau')} + (t'-\tau') + g(t'-\tau')\right\} \tag{14}$$

where ν is the static elastic Poisson's ratio.

It is interesting to note that temperature affects the creep behaviour in a similar way to drying: the relationship is the same as in equation (13) for axial creep. The spatially varying function $\rho(x)$ would be instead a constant multiplied by the temperature at that point. One difference is to be noted: Poisson's ratio is constant, both for elastic and creep strains [19].

If equations (13) and (14) are combined into the appropriate integral equations, corresponding to (4) for basic creep, we obtain

$$\begin{aligned}\varepsilon_{ij}(x,t') = {} & (1+\nu)\int_0^{t'} \frac{\partial\sigma_{ij}(x,\tau')}{\partial\tau'}\left\{\frac{1}{E(\tau')} + \frac{\rho(x)+\nu}{1+\nu}(t'-\tau') + g(t'-\tau')\right\} d\tau' \\ & - \delta_{ij}\,\nu\int_0^{t'} \frac{\partial\sigma_{kk}(x,\tau')}{\partial\tau'}\left\{\frac{1}{E(\tau')} + (t'-\tau') + g(t'-\tau')\right\} d\tau'.\end{aligned} \tag{15}$$

If decomposition into volumetric and deviatoric components is carried out as before (equations (5) and (6)), then

$$e_{ij}(x,t') = \int_0^{t'} \frac{\partial S_{ij}(x,\tau')}{\partial\tau'}\, J_d(x,t',\tau')d\tau' \tag{16}$$

and $$\varepsilon_{kk}(x,t') = \int_0^{t'} \frac{\partial\sigma_{kk}(x,\tau')}{\partial\tau'}\, J_v(x,t',\tau')d\tau' \tag{17}$$

$$\text{where}\quad J_d(x,t',\tau') = \frac{1+\nu}{E(\tau')} + \{\rho(x) + \nu\}\{t'-\tau'\} + (1+\nu)g(t'-\tau') \tag{18}$$

$$\text{and}\quad J_v(x,t',\tau') = \frac{1-2\nu}{E(\tau')} + \{\rho(x) - 2\nu\}\{t'-\tau'\} + (1-2\nu)g(t'-\tau') \tag{19}$$

The constant ν is the static elastic Poisson's ratio. If $\rho=1$, the expressions are the same as those for sealed concrete.

It is to be noted that the Laplace transformations of equations (15), (16) and (17) are easy to evaluate if $E(\tau')$ is taken as a constant. The correspondence principle may then be stated as: either

$$\frac{1}{E} \rightarrow \frac{1}{E} + \frac{\rho}{s} + s\bar{g}(s) \tag{20}$$

$$\text{and}\quad \nu \rightarrow \frac{\nu\{1/E + 1/s + s\bar{g}(s)\}}{1/E + \rho/s + s\bar{g}(s)} \tag{21}$$

$$\text{or}\quad \frac{1}{2G} = \frac{1+\nu}{E} \rightarrow \frac{1+\nu}{E} + \frac{\rho(x)+\nu}{s} + (1+\nu)s\bar{g}(s) \tag{22}$$

$$\text{and}\quad \frac{1}{3K} = \frac{1-2\nu}{E} \rightarrow \frac{1-2\nu}{E} + \frac{\rho(x)-2\nu}{s} + (1-2\nu)s\bar{g}(s) \tag{23}$$

A step variation of E may again be incorporated as outlined elsewhere [6]. Fig. 5 summarizes the assumption for lateral compliance for drying concrete in the form of curve (a), while curve (b) shows a less accurate alternative.

DISCUSSION

The development above represents a formulation of the equations for the three-dimensional creep of concrete which are amenable to direct calculation, and which represent accurately the experimentally determined behaviour in three dimensions. If iterative calculations in time are to be performed, greater flexibility is possible in the description of material behaviour but the above discussion should nevertheless provide the basis for realistic parameters for three-dimensional analysis.

It can readily be seen that the axial compliances presented are sensitive to drying whereas the lateral compliances are not. In particular, it should be noted that the relative humidities at points within the concrete change with time as the drying progresses. Ideally, therefore, the creep parameters should be determined taking into account the different rates of drying at different distances from the exposed surfaces. However, an inspection of information on the drying [9, 5, 11] shows that it would be reasonable for considerable periods of time (up to several months) to take an average value of relative humidity since the rate of change with time is slow, particularly for mild drying after the first few weeks of rapid drying. The direct methods presented can be used in these cases: it should also be noted that most of the stress redistribution occurs in a comparatively short period.

A further practical problem is the lack of data on the spatial variation of relative humidity or other related variables. This is available in numerical form but practical curves giving the variation with a reasonable approximation are needed.

In spite of the difficulties in carrying out analyses where the creep parameters are functions of position in concrete, the direct analyses summarised above can be used with confidence for sealed concrete, or where an average value of creep is used (i.e., the spatial variation ignored), as is common practice. In the following, two simple structures will be analysed, considering two extremes: firstly, very thick structures in which a constant compliance corresponding to basic creep may be used and secondly, very thin members in which uniform drying may be assumed. The examples are idealised and are chosen for their simplicity so as to illustrate the method of calculation.

EXAMPLES

Thick Reinforced Cylinder

Fig. 6 gives the geometrical details; the concrete cylinder with internal pressure p is reinforced with steel on the outer circumference. It is assumed that the reinforcement offers no resistance to deformation in the direction of the axis of the cylinder. The elastic solution to the problem is

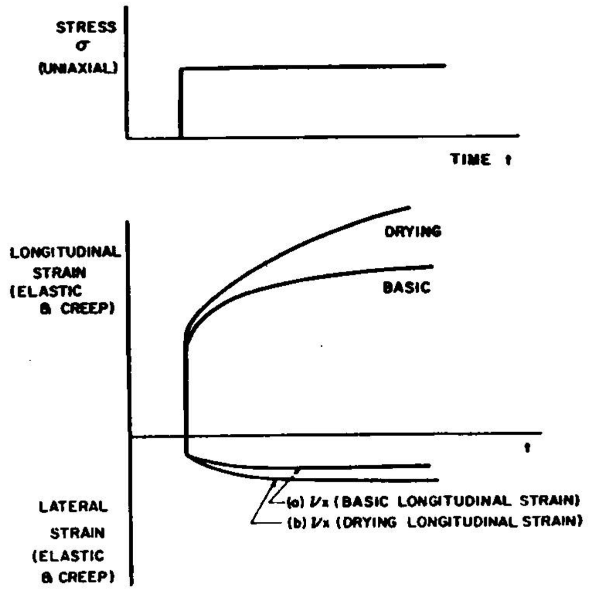


Fig. 5. Axial and lateral strain resulting from uniaxial loading; case (a) represents the assumption embodied in equations (21)-(23) while case (b) represents the assumption of a constant Poisson's ratio for drying concrete.

Fig. 5. Allongement axial et latéral produit par un chargement uni-axial. (a) représente la supposition incorporée dans les équations (21)-(23) tandis que le cas (b) représente la supposition d'une valeur de Poisson constante pour le béton en train de sécher.

Bild 5. Längs- und Querdehnung bei einachsiger Belastung. Der Fall (a) stellt die in den Gleichungen (21)-(23) enthaltenen Annahmen, der Fall (b) die Annahme einer konstanten Querdehnungszahl für austrocknenden Beton dar.

as follows, considering compression to be positive:

$$\sigma_{r,\theta} = p\left[A\left\{1 \mp \left(\frac{r_1}{r}\right)^2\right\} \pm \left(\frac{r_1}{r}\right)^2\right] \tag{24}$$

where σ_r = radial stress at distance r from the centre,

σ_θ = circumferential stress at r from the centre, and

r_1, r_2 = internal and external radii respectively.

The constant A is given by

$$A = \left\{1-\lambda(1+\nu)\right\} \Big/ \left[1-\lambda(1+\nu) - \left(\frac{r_2}{r_1}\right)^2 \left\{1+\lambda(1+\nu)(1-2\nu)\right\}\right] \tag{25}$$

in which ν = Poisson's ratio,

$\lambda = (E_s A_s)/(E_c r_2)$

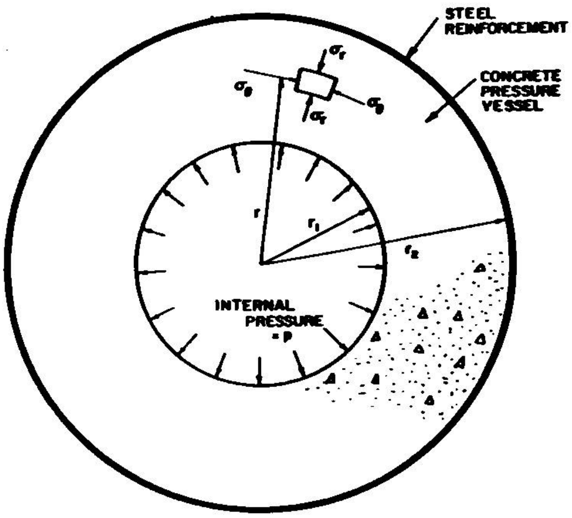


Fig. 6. Thick cylinder, reinforced on outside circumference with layer of steel.

Fig. 6. Un cylindre épais, armé d'une couche d'acier sur la circonférence extérieure.

Bild 6. Ein an der Außenfläche mit einer Lage Stahl bewehrter Zylinder.

A_s = area of steel per unit length of cylinder,

E_s = modulus of elasticity for steel, and

E_c = modulus of elasticity for the concrete.

The above solution corresponds to plane strain, i.e., $\varepsilon_z = 0$ where the z axis is along the centre line of the cylinder.

We will consider that p is applied as a step at $t' = 0$ (arbitrary origin) and that only basic creep need be considered (massive section). The concrete creep data are as follows, abstracted approximately from reference [15]:

(i) t' in terms of t is given in an experimental curve (see Fig. 7); $\rho = 1$

(ii) E_c = constant = $.024 \times 10^6$ MN/m^2; $E_s = .207 \times 10^6$ MN/m^2

(iii) $g(t') = g_o(1-e^{-\alpha t'})$, where $g_o = 11 \times 10^{-6}$ per MN/m^2
$\alpha = 1.4 \times 10^6$ MN/m^2

(iv) ν = constant = 0.20.

In order to obtain the Laplace transformation of the creep solution, it is necessary to replace p by p/s and $1/E_c$ by $1/E_c + \rho/s + g_o\alpha/(s+\alpha)$. If this is done, and the numerical values (i) to (iv) above are substituted, we obtain,

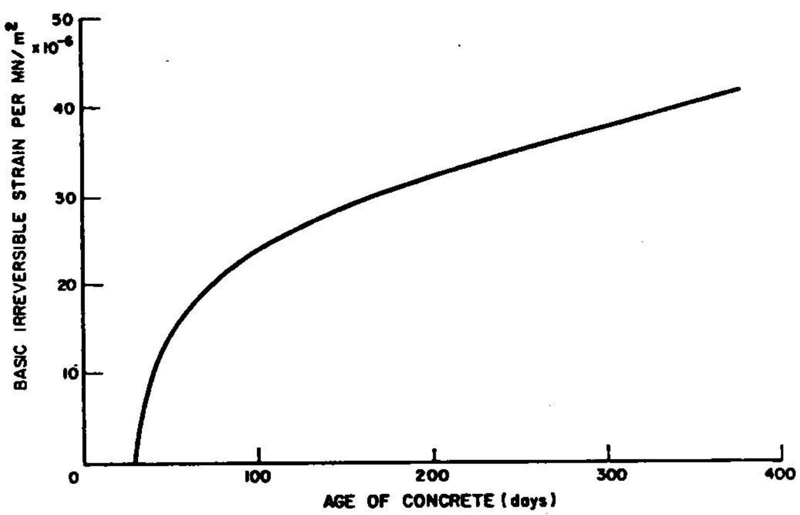


Fig. 7. Basic irreversible creep per unit stress from reference [15].

Fig. 7. Le fluage irréversible fondamental par unité de contrainte, d'après la référence [15].

Bild 7. Nicht umkehrbares Grundfließen pro Spannungseinheit (nach Lit. [15]).

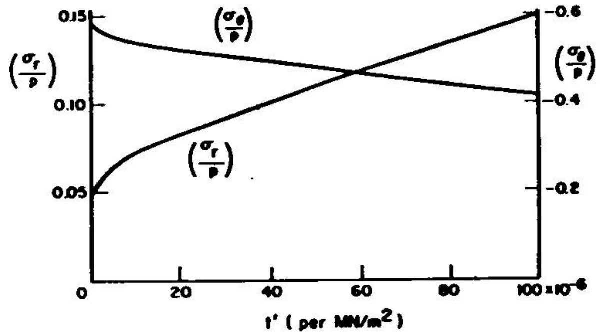
for $r_2 = 2r_1$ and $A_s/r_2 = .01$:

$$\bar{p}(s)\bar{A}(s) = p\left(\frac{.294}{s} - \frac{.0152}{s+1,440,000} - \frac{.546}{s+2450}\right)$$

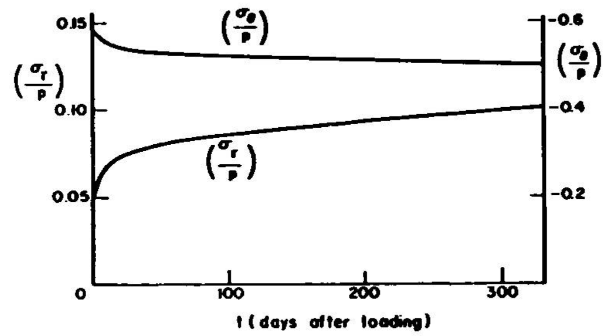
which, when inverted gives

$$A(t') = p\left(.294 - .0152e^{-1.44t' \times 10^6} - .546e^{-2.45t' \times 10^3}\right) \tag{26}$$

The values of σ_r and σ_θ at any value of t' are found from equation (24) upon substitution of the appropriate values of A from equation (26). As would be expected, the values of radial stress σ_r increase with time (except at $r = r_1$) while the values of σ_θ diminish. Fig. 8(a) shows the variation of the stresses at the interface between the steel and the concrete, i.e., at $r = r_2$. The variation is shown against values of pseudo-time t', while Fig. 8(b) shows the values plotted against true time t.



(a)



(b)

Fig. 8. σ_r and σ_t in thick cylinder at various values of t' and t; $r = r_2$.

Fig. 8. σ_r et σ_t dans un cylindre épais à des valeurs diverses de t' et de t; $r = r_2$.

Bild 8. Dis Spannungen σ_r und σ_t in einem dicken Zylinder für $r = r_2$ und verschiedene Werte von t und t'.

Laterally Restrained
Concrete Cylinder

We consider a cylindrical concrete core, of radius r, reinforced with steel around its circumference so as to restrain lateral movement. The cylinder is loaded axially with a stress σ_z and the stress imparted to the concrete by radial reinforcement is σ_r. For simplicity, no longitudinal steel is included. The elastic solution to the problem is:

$$\sigma_r = \nu\left(\frac{E_s}{E_c}\right)\sigma_z \Bigg/ \left\{\frac{r}{A_s} + \frac{E_s}{E_c}(1-\nu)\right\} \tag{27}$$

where ν, E_s, E_c have the same meaning as before. A_s is also, as previously, the cross sectional area of lateral reinforcement per unit length of the cylinder.

The same data for the concrete and steel as in the previous example will be used with the following exceptions:

(a) $A_s/r = .04$

(b) Fig. 7 gives t' in terms of t but we will take $\rho = 2$

(c) Two assumptions for Poisson's ratio will be used:
  (1) ν = constant = 0.2 (for all strains)
  (2) Poisson's ratio = .2 for elastic and reversible creep strains and equal to .1 for irreversible creep, i.e., $\rho = 2$ and $\nu = .2$ in equation (21).

The Laplace transformations of the creep solution corresponding to the two assumptions above are

$$\bar{\sigma}_r(s) = .0541\sigma_z\left(\frac{4.63}{s} - \frac{.198}{s+1.48\times10^6} - \frac{3.43}{s+.00981\times10^6}\right)$$

and
$$\bar{\sigma}_r(s) = .0541\sigma_z\left(\frac{2.05}{s} - \frac{.198}{s+1.48\times10^6} - \frac{.857}{s+.0110\times10^6}\right)$$

which when inverted give

$$\sigma_r(t') = .0541\sigma_z\left(4.63-.198e^{-1.48\times10^6t'}-3.43e^{-.00981\times10^6t'}\right) \tag{28}$$

and
$$\sigma_r(t') = .0541\sigma_z\left(2.05-.198e^{-1.48\times10^6t'}-.857e^{-.0110\times10^6t'}\right) \tag{29}$$

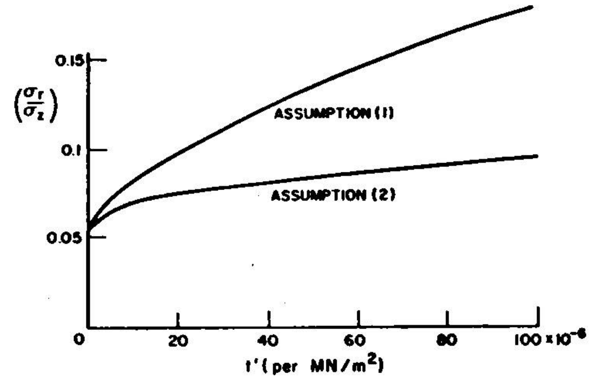
It has been assumed in the above that σ_z is applied at $t' = 0$ and remains constant thereafter.

Values of σ_r are plotted against t' and t in Fig. 9. It is seen that there is a significant difference in the two results corresponding to the two assumptions regarding Poisson's ratio. However, both values are a small proportion of σ_z and so is the difference between the two, e.g., at $t = 300$ days the difference is less than 5 per cent of σ_z. This supports the conclusion made in the following section that the additional accuracy that follows from equation (21) for drying concrete is generally small.

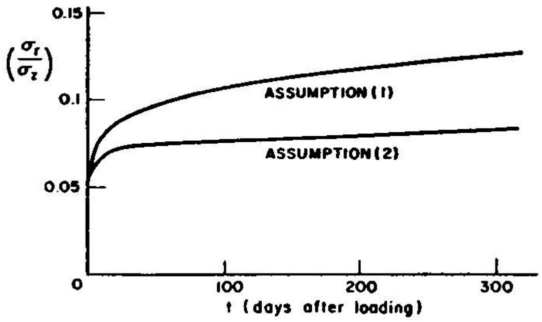
CONCLUDING REMARKS

The method which forms the basis of this paper allows the calculation of creep effects in many problems in which the elastic solution is known. The creep solution is obtained using the correspondence rule which links the elastic and viscoelastic solutions of problems and is restricted to linear material behaviour. The extension to problems in which elastic solutions are available in numerical form (e.g., in finite element solutions) is being studied at The University of Calgary. The emphasis is being placed on obtaining creep solutions directly from numerical values obtained from the elastic solutions.

In considering creep problems it is important to be able to detect those problems in which stress redistribution occurs; in the following discussion, only linear material behaviour is taken into account. For problems in which the uniaxial compliance only is important stresses remain constant while creep takes place provided that the supports to the structure remain fixed; in cases of settling supports in statically indeterminate structures, stress redistribution will occur. Other common cases in which stresses alter with time are those of non-homogeneity of the creeping material or those in which two or more materials are combined; in combinations of steel and concrete the stresses will tend to be transferred from the concrete to the steel. In



(a)



(b)

Fig. 9. σ_r for concrete cylindrical core at various values of t' and t, for two assumptions regarding Poisson's ratio.

Fig. 9. σ_r pour un cylindre en béton à de valeurs diverses de t' et de t, pour deux suppositions concernant la valeur de Poisson.

Bild 9. σ_r für den zylindrischen Betonkern für verschiedene Werte von t' und t und zwei verschiedene Annahmen bezüglich der Querdehnungszahl.

short, redistribution of stress will occur if, in the elastic solution, the stress depends on the elastic modulus, E_c, of the creeping material.

The general three-dimensional problem tends to be more complicated. The same reasoning as above applies in cases of settling supports and in problems involving non-homogeneity: the elastic stresses will depend on E_c and will therefore be time-dependent in the viscoelastic problem. Elastic solutions will also contain ν, the Poisson's ratio for the creeping material: in some cases ν alone (and not E_c) will appear, e.g., in one-material problems with rigid boundaries. It is accepted practice in such cases to assume that no stress redistribution occurs. This assumption is only valid if the Poisson's ratio for creep strains is constant and equal to the static elastic value [20]. For many materials, this assumption is not true: metals, for example, exhibit elastic dilatation and a time-dependent distortion.

In the case of concrete, if creep is basic (i.e., non-drying) the assumption of a constant Poisson's ratio equal to the static elastic value is consistent with the actual material response. For drying concrete (based on the limited experimental information that is available) small errors are introduced by this assumption (contrast curves (a) and (b) in Fig. 5). In the writer's opinion, the small additional accuracy introduced by the assumption embodied in equation (21) does not justify the extra complications that result, particularly if an average value is used to characterize the axial compliance of the material.

With regard to the axial compliance, a particular form has been used in this paper, which permits the correspondence principle of linear viscoelasticity to be used. The method provides adequate accuracy for basic creep, and for drying creep if an average value of axial compliance is used. If spatial variation of the creep parameters is to be incorporated in the analysis, the relationships shown in the paper are promising for situations where an average value of relative humidity over the time period to be considered is accurate enough for any particular distance from the surface. On the other hand, if the humidities are changing rapidly, then a more complicated function will have to be used to describe the creep behaviour. Experimental work should be directed towards determining these parameters.

Other forms of the uniaxial constitutive relationship, possibly requiring iterative calculations in time, may be used together with the three-dimensional relationships proposed in the paper. The stress level in all cases should be low enough so that linear material behaviour may be assumed.

References

1. Ross, A. D., England, G. L. and Suan, R. H. "Prestressed concrete beams under a sustained temperature crossfall." Magazine of Concrete Research, Vol. 17, 1965, pp. 117-126.

2. England, G. L. "The direct calculation of stresses in creeping concrete and concrete structures." Proceedings of the Southampton 1969 Civil Engineering Materials Conference, Wiley, 1971, pp. 1269-1276.

3. Goodall, I. W. and Irving, J. "On the approximate analysis of structures composed of non-homogeneous linear viscoelastic material." Nuclear Engineering and Design, Vol. 22, 1972, pp. 63-74.

4. Pickett, G. "The effect of change in moisture-content on the creep of concrete under a sustained load." American Concrete Institute Journal, Proceedings Vol. 13, No. 4, 1942, pp. 333-355.

5. Illston, J. M. "The creep of concrete in terms of its developing structure and moisture condition." Proceedings of the Southampton 1969 Civil Engineering Materials Conference, Wiley, 1971, pp. 1287-1297.

6. Gamble, B. R. and Jordaan, I. J. "A direct method of viscoelastic analysis for ageing concrete." Materials and Structures, RILEM (in press).

7. Illston, J. M. "The components of strain in concrete under sustained compressive stress." Magazine of Concrete Res. V.17, No. 50, 1965, pp. 21-28.

8. Volterra, V. "Theory of functionals and of integral and integro-differential equations." Dover Publications, New York, 1959.

9. Carlson, R. W. "Drying shrinkage of large concrete members." Journal of the American Concrete Institute, Proceedings Vol. 33, 1937, pp. 327-336.

10. Pihlajavaara, S. E. "On the main features and methods of investigation of drying and related phenomena in concrete." Doctor of Philosophy thesis, University of Helsinki, 1965.

11. Bazant, Z. P. and Najjar, L. J. "Drying of concrete as a nonlinear diffusion problem." Cement and Concrete Research, Vol. 1, 1971, pp. 461-473.

12. Illston, J. M., Jordaan, I. J. and Parrott, L. J. "The measurement and estimation of the strain in concrete under multiaxial stress." The Deformation and the Rupture of Solids Subjected to Multiaxial Stresses, Proceedings of RILEM International Symposium, Cannes, October, 1972, pp. 113-129.

13. Jordaan, I. J. and Illston, J. M. "The creep of sealed concrete under multiaxial compressive stresses." Magazine of Concrete Research, Vol. 21, No. 69, 1969, pp. 195-204.

14. Illston, J. M. and Jordaan, I. J. "Creep prediction for concrete under multiaxial stress." Journal of the American Concrete Institute, Proceedings Vol. 69, 1972, pp. 158-164.

15. Parrott, L. J. "Some observations on the components of creep in concrete." Magazine of Concrete Research, Vol. 22, No. 72, 1970, pp. 143-148.

16. Plowman, J. M. "Young's modulus and Poisson's ratio of concrete cured at various humidities." Magazine of Concrete Research, Vol. 15, No. 44, 1963, pp. 77-82.

17. Verbeck, G. J. and Helmuth, R. H. "Structure and physical properties of cement paste." Fifth Symposium on the Chemistry of Cement, Tokyo, 1968, Vol. III, pp. 1-32.

18. Meyer, H. G. "On the influence of water content and of drying conditions on lateral creep of plain concrete." Materials and Structures, Vol. 2, No. 8, 1969, pp. 125-131.

19. Hannant, D. J. "Strain behaviour of concrete up to 95°C under compressive stresses." Conference on Prestressed Concrete Pressure Vessels, 1967, London, Institution of Civil Engineers, 1968, pp. 177-191.

20. McHenry, D. "A new aspect of creep in concrete and its application to design." Proceedings of the American Society of Testing and Materials, Vol. 43, 1943, pp. 1069-1084.

SUMMARY

A method of direct analysis of linear concrete creep, which takes into account the effects of ageing, is described. The method relies on the substitution of a pseudo-time for the true time in the creep analysis. The analysis gives satisfactory results for sealed concrete, and for drying concrete if an average value of the creep compliance is used. Extension to three-dimensional analysis is given, treating the cases of basic creep and drying creep separately. Existing experimental data indicate that a constant Poisson's ratio equal to the static elastic value is satisfactory for analysis of basic creep, but for drying creep the values for static and creep strains are not equal. The difference in the assumptions for the two cases is small and it is further suggested that it is reasonable in practical calculations to assume a constant Poisson's ratio for all strains. Two examples of direct calculation of the time-dependent stresses in reinforced concrete structures are given.

RESUME

Une méthode est décrite pour le calcul direct du fluage linéaire du béton, en tenant compte des effets de son vieillissement. La méthode dépend de la substitution dans le calcul d'un temps fictif pour le temps réel. La méthode donne des résultats précis dans deux cas: (a) quand le mouvement de l'eau vers l'éxterieur du béton est empêché (b) dans le cas du béton en train de sécher, si une valeur moyenne du fluage est utilisée. La méthode est appliquée séparément dans chacun de ces deux cas pour le calcul de fluage en trois dimensions. Les données expérimentales actuelles indiquent que l'utilisation d'une valeur de Poisson égale à la valeur élastique statique est raisonable pour le Cas (a), mais pour le Cas (b) les deux valeurs ne sont pas égales. La différence dans les deux cas est minime. Il est donc proposé d'utiliser dans la pratique une valeur de Poisson constante pour le calcul de tous les allongements spécifiques. Exemples de calcul direct des contraintes qui dépendent du temps dans deux structures en béton armé sont donnés.

ZUSAMMENFASSUNG

Es wird ein Verfahren zur direkten Berechnung des linearen Betonkriechens beschrieben, das den Einfluß des Alterns einschließt. Die Methode für die Kriechberechnung verwendet eine Ersatzzeit anstatt der tatsächlichen Zeit. Die Berechnung ergibt zufriedenstellende Ergebnisse für versiegelten Beton; für austrocknenden Beton sollte ein Mittelwert für das Kriechen eingeführt werden. Eine Erweiterung auf dreidimensionales Kriechen ist ebenfalls angegeben, wobei der Fall des Grundfließens und des Trocknungsfließens getrennt behandelt werden. Aus bestehenden Versuchsdaten wird gefolgert, daß eine konstante Querdehnungszahl (gleich dem elastischen Wert unter Kurzzeitbelastung) für das Grundfließen zu zufriendenstellenden Resultaten führt, daß aber für das Trocknungskriechen die beiden Werte nicht gleich sind. Der Unterschied zwischen den für die beiden Fälle getroffenen Annahmen is gering, und es wird daher vorgeschlagen, für praktische Berechnungen die verschiedenen Querdehnungszahlen als Konstante anzunehmen. Zwei Beispiele direkter Berechnungen von zeitabhängigen Spannungen in Stahlbetontragwerken werden gegeben.